Space-Time Transfinite Interpolation of Volumetric Material Properties

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Abstract—The paper presents a novel technique based on extension of a general mathematical method of transfinite interpolation to solve an actual problem in the context of a heterogeneous volume modelling area. It deals with time-dependent changes to the volumetric material properties (material density, colour, and others) as a transformation of the volumetric material distributions in space-time accompanying geometric shape transformations such as metamorphosis. The main idea is to represent the geometry of both objects by scalar fields with distance properties, to establish in a higher-dimensional space a time gap during which the geometric transformation takes place, and to use these scalar fields to apply the new space-time transfinite interpolation to volumetric material attributes within this time gap. The proposed solution is analytical in its nature, does not require heavy numerical computations and can be used in real-time applications. Applications of this technique also include texturing and displacement mapping of time-variant surfaces, and parametric design of volumetric microstructures.

Index Terms—Scalar field, material distribution, texturing, transfinite interpolation, volumetric metamorphosis, heterogeneous volume modelling

1 Introduction

Heterogeneous volume modelling deals with objects with complex internal structures composed of diverse materials and possessing diverse densities and other physical properties reflecting their heterogeneous volumetric nature. The range of its application areas spans from biological and medical research, bio-engineering, multi-material design and fabrication, geology and physical simulations to digital entertainment applications (computer animation, computer games, and visual effects).

Volumetric material properties include such characteristics as the density distribution of mixed materials, material colour, opacity, parameters of microstructures and their spatial variations within the object volume. Representing volumetric material properties with continuous fields rather than discrete samples (as voxel grids or point clouds) increases the model accuracy required in critical applications such as medicine or bio-engineering and can increase the level of realism or verisimilitude in digital entertainment.

Changes of volumetric material properties in time occur in various situations such as object deformation or more general transformation, cracking and breaking, ageing and chemical evolution. Geometric metamorphosis (or morphing) in 2D and 3D space is an example of time-variant shape transformation that can also cause changes in volumetric material properties. The metamorphosis has been relatively well studied including the topological changes in genus and number of components. While morphing between two boundary representation objects is possible, it is often complicated, in particular if these objects do not have the same topology or shape complexity. Scalar field based methods are simpler, more robust and handle topological changes automatically [1], [2], [3], [4].

Transformations of material properties in metamorphosis are not well understood especially in the cases of radical topology changes, although there are several approaches aiming at handling these transformations to some extent using scalar fields [5], [6], [7]. These approaches are based on solving differential equations or tracking particle systems. We, on the other hand, are looking for a compact closed-form solution, with an acceptable quality of results, that is essential for near real-time rendering in interactive volume modelling and digital entertainment applications.

There exists a similar problem in engineering relating to the design of heterogeneous volume objects with functionally graded materials. Some regions with known geometry and materials are defined within such an object. These regions are usually called material features or sources [8], [9]. The spatial gaps between the material features of an object are filled with intermediate materials using some interpolation scheme.

A general approach to the construction of a single function taking predefined values on some collection of point sets is called a transfinite interpolation [10]. This type of interpolation was applied in [9] to the modelling of gradual material distribution in static 3D objects. The proposed solution is independent of the material features geometry, topology and even of the number of material features. The only requirement is that each material feature be represented by a scalar field with a distance property.

We propose to extend this approach to spatio-temporal variations in heterogeneous volume objects and to formulate a space-time transfinite interpolation. We treat time-variant changes in volumetric material properties as transformations of their volumetric distributions in space-time and then we apply a transfinite interpolation. The

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Manuscript received 25 Oct. 2013; revised 2 Aug. 2014; accepted 16 Aug. 2014. Date of publication 7 Sept. 2014; date of current version 5 Jan. 2015.
Recommended for acceptance by A. Sheffer.
For information on obtaining reprints of this article, please send e-mail to: reprints@ieee.org, and reference the Digital Object Identifier below.
Digital Object Identifier no. 10.1109/TVCG.2014.2356196

Fig. 1. Metamorphosis with time-variant volumetric materials starting from a cartoon character to a cut watermelon, and then to a banana using the proposed space-time transfinite interpolation.

main idea is to create a property scalar field in space-time independent of the object geometry and to assign corresponding property attribute values to the points of the variable object shape floating in this field. For the metamorphosis shape transformation, our approach is to represent the geometry of both given objects by scalar fields with distance properties, to establish a time gap, a time-interval in space-time during which the geometry transition occurs, and to use these scalar fields in order to apply the transfinite interpolation to the material attributes during this time gap. The main constraint is that the given attributes of the initial and target objects are matched exactly by the interpolation.

In our approach, two given objects with their volumetric material properties are considered as material features for the transfinite interpolation. A scalar field for each object provides a weight for the interpolation of a volumetric material property at any location in time and space. The object changing geometry and topology in time passes through these generated fields and takes the corresponding attributes at each point for the given instance of time, regardless of the method used for the shape transformation (as shown in Fig. 1).

There are several different scenarios where our approach is applicable. The main situation is when two static objects are given and metamorphosis is applied for changing the shape from one shape to another. All the time-variant volumetric material attributes can be obtained using the space-time transfinite interpolation. Our approach can also be applied to a single static object with two completely different material distributions. The geometry will not change, but the material will change from one to another through time. The second object may be used in this situation only to carry the target material properties. This emulates, for instance, a chameleon or an octopus blending in with its environment.

1.1 Contributions

The contributions of this work are as follows:

- We have formulated the new problem of finding a compact closed-form solution for time-variant transformations of volumetric material properties;
- The general mathematical method of the transfinite interpolation is extended to space-time to solve the stated problem;
- We provide a formulation for volumetric metamorphosis where both geometry and volumetric properties change in time as two interrelated processes;
- Our approach applies to any material definition, be it a procedural definition using scalar fields, space partitioning or surface textures;
- We propose applications of our approach to such well-known surface decoration techniques as colour texturing, bump mapping, and displacement mapping for time-variant surfaces. In the case of surface textures, a method to create a volumetric texture from the surface is required. We present a few simple solutions to achieve this;
- The proposed approach can be applied to a parametric design with "time" used as a design parameter. Design of volumetric microstructures is given as an example.

2 Related Works

The general area of this work is heterogeneous volumetric objects modelling [11]. There are two main groups of related works: texturing time-evolving implicit surfaces and modelling functionally graded materials in engineering.

Texturing of implicit surfaces is still an open research problem. 2D surface texturing is often used to add detail and to emulate material complexity on the surface. The most common solution to this problem is to use solid texturing [12] for implicit surfaces. However, this approach is not user friendly and it is often difficult for artists to control. It also introduces the artefact of sliding texture for time-variant surfaces in metamorphosis.

Several attempts have been made to produce a 2D parameterisation for surfaces defined by scalar fields. Gradient vector fields have been used to project points to a source shape which is trivial to parameterise [13]. The texture coordinates of any point in space can be found be letting that point flow, as a particle, through the gradient vector field until it hits the support surface. When the support surface is hit, the world coordinates of the point are converted into texture coordinates. However, this solution requires the use of support shapes with similar complexity or the particle may be trapped in a local minimum and fail to intersect the support shape.

In the context of animation and metamorphosis, textures tend to glide or jump through time, causing unwanted artefacts. Drastic changes in topology make this effect more difficult to control. The work of [5] improved on the above mentioned particle system to get good results for animated implicit surfaces. A set of points on the surface at the initial shape (virtual skin), parameterised in 2D, are transformed using different vector fields defining the particle velocities. Several effects such as clay style texturing can be achieved and splitting or merging objects are handled correctly. The vector fields are however difficult to create and often exhibit case specific problems.

The work of [14] extended texturing through the use of particles to deal with gradient discontinuities that may arise with constructive models and distance functions. The force applied to the particle is a weighted sum of forces. The gradient vector field is used to repel the particle, but the support shape also acts as an attractor. Several operations are also extended to blend textures and surface materials together.

In [6] a method was introduced to track surfaces with corresponding attributes in time by solving PDEs. This solution gives consistent results with any topological changes including splits and holes. In the work of [7], the authors used a similar method to track properties (such as colours and displacement information) during the shape transformation process in morphing or in fluid simulation with heterogeneous fluids. This method is able to handle drastic topology changes. A similar approach is suggested in [15] for attributes interpolation by using mass transport. However, solving the PDEs is time consuming and makes this solution impractical for real time applications, in long animation sequences or when applied to large meshes.

Modelling material distribution in engineering faced similar issues where only few elements with known materials are properly defined within a part and material in gaps between them has to be calculated. The work of [8] introduced the idea of source-based heterogeneous modelling. It redefines the usual operators (of union, intersection, and difference) regarding material properties, and allows for gaps in the material definition. These material gaps are filled with the weighted-sum of the neighbouring source materials. The transfinite interpolation with distance fields of [10] provides a general solution to the interpolation of scalar values defined at the given point sets. In [9], the transfinite interpolation is applied to define material properties throughout the object volume. A set of non-overlapping sub-sets have material properties defined by scalar values. In the rest of the object volume, the material properties are interpolated as a composition of the given materials. Transfinite interpolation was applied in a similar fashion in [16] in order to interpolate between parameters of two given volumetric microstructures. We propose to extend this approach to spatio-temporal variations in volumetric material properties, to formulate and to apply a space-time transfinite interpolation for this purpose.

3 Background

In this work, geometric objects and geometric operations are defined using scalar fields (continuous real functions of point coordinates). In this section we describe function-based set operations and shape metamorphosis operations used in this paper.

3.1 Set Operations

For two given objects defined by the functions f_1 and f_2, the set-theoretic operations (of union, intersection and subtraction) can be defined with R-functions [17] as follows:

- $f_3 = f_1 \vee_a f_2$ defines a union
- $f_3 = f_1 \wedge_a f_2$ defines an intersection
- $f_3 = f_1 \setminus_a f_2$ defines a subtraction

The most popular solution is to use minimum and maximum functions. These operations have C^1 discontinuities whenever $f_1 = f_2$. We prefer to use another class of R-functions which have C^1 discontinuities only at the intersection of the two surfaces.

- $f_1 \vee_0 f_2 = f_1 + f_2 + \sqrt{f_1^2 + f_2^2}$
- $f_1 \wedge_0 f_2 = f_1 + f_2 - \sqrt{f_1^2 + f_2^2}$
- $f_1 \setminus_0 f_2 = f_1 - f_2 - \sqrt{f_1^2 + f_2^2}$

Other functions for set-theoretic operations can be used, for example presented in [18] or [19]. However they are computationally inefficient and are generally used in the case when the properties of the resulting field after set-theoretic operations are more important than the efficiency.

3.2 Shape Metamorphosis Operations

A number of different metamorphosis operations can be performed on shapes defined by scalar fields. The easiest solution is to use linear interpolation between the value of the first function and the value of the second function at any point in the space:

$$m_{linear}(\mathbf{x}, t) = f_1(\mathbf{x}) \cdot (1 - t) + f_2(\mathbf{x}) \cdot t.$$

However it lacks any control and quite often results in disconnected components in the intermediate shapes. More complex methods for metamorphosis include space-time blending [20] and feature-based metamorphosis. One of the most recent papers [21] includes a survey of existing methods and presents another solution for metamorphosis between two shapes with improved artistic control.

4 Space-Time Transfinite Interpolation

In this section, we introduce the concept of space-time transfinite interpolation and provide its mathematical formulation. First, we discuss time-variant transformations between two given volumetric objects with constant material attributes assigned to each of the single-partition objects, then extend our discussion to objects with multiple partitions and with a constant attribute per partition and finally we discuss the case of attributes continuously varying in space throughout the volumes of both objects. The case of established correspondences between partitions of the given objects is also briefly discussed. Note that the proposed space-time transfinite interpolation can be applied to interpolate either the material attribute field values directly or parameters influencing the fields.

4.1 Single-Partition Objects

Let $f_1(x, y, z)$ and $f_2(x, y, z)$ be defining functions of the initial object G_1 and target object G_2, and c_1 and c_2 be respective colours or other constant material attributes assigned to each of the objects respectively. We assume that the

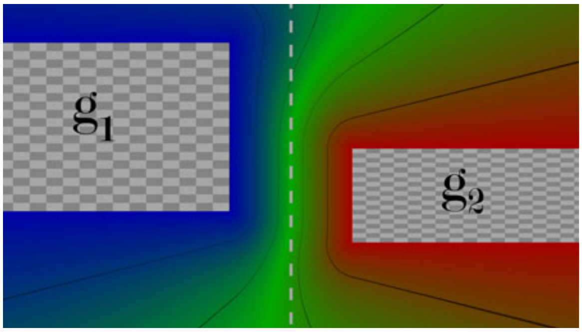


Fig. 2. The half-cylinders g_1 and g_2 are the known material features in space-time. The time-interval is filled with intermediate materials according to an interpolation scheme called transfinite interpolation.

functions f_1 and f_2 have a distance property, i.e., they assume a zero value on the surface of the respective object, they are positive inside and negative outside the object, and their absolute values grow with the distance from the respective surface. We are interested in a shape transformations $G(t)$ between objects G_1 and G_2 and an attribute transformation $\mathbf{c}(t)$ between attributes $\mathbf{c}_1$ and $\mathbf{c}_2$ on the interval of time $t = [0, 1]$, such that $G(0) = G_1$ and $G(1) = G_2$, $\mathbf{c}(0) = \mathbf{c}_1$ for all the points with $f_1 \geq 0$ (on the surface and the interior of G_1) and $\mathbf{c}(1) = \mathbf{c}_2$ for all the points with $f_2 \geq 0$ (on the surface and the interior of G_2). The pair $(G(t), \mathbf{c}(t))$ makes a function-based model of a volumetric metamorphosis where both geometry and volumetric properties change in time as two interrelated processes. While we can apply a variety of time-variant shape transformations (such as linear metamorphosis [2], space-time blending [20] or a 4D variational formulation [3]) to get intermediate shapes between G_1 and G_2, our arsenal of attribute transformations is not as rich.

The basic formulation for the attribute interpolation is as follows:

$$\mathbf{c}(t) = w_1(t) \cdot \mathbf{c}_1 + w_2(t) \cdot \mathbf{c}_2, \tag{1}$$

where $0 \leq t \leq 1$, and the constraints on the weighting functions provide the partition of unity:

- $w_1(0) = 1$ and $w_2(0) = 0$ for $f_1 \geq 0$ (on the surface and the interior of G_1)
- $w_1(1) = 0$ and $w_2(1) = 1$ for $f_2 \geq 0$ (on the surface and the interior of G_2)
- $w_1 + w_2 = 1$.

The continuity of the interpolation is a fundamental requirement, the differentiability is a desirable property, which provides absence of sharp creases in the material property distribution. The simplest linear interpolation is arrived at by assigning $w_1(t) = 1 - t$ and $w_2(t) = t$, but this is a naive solution that does not take any particular features of the geometry into consideration and results in unintuitive attributes in the transitional stages (see Fig. 8).

A similar problem appears in material distribution design for heterogeneous objects with functionally graded materials, where some areas called material features with known geometry and materials are given within an object. The gaps between material features within the object are filled with intermediate materials according to some interpolation scheme. In [9], transfinite interpolation was applied to define material properties in 3D space throughout the object volume.

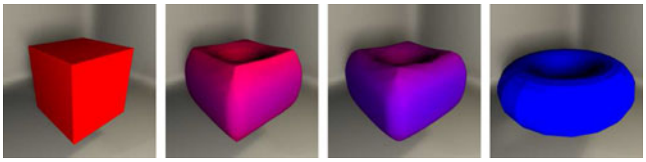


Fig. 3. Space-time transfinite interpolation of colours during the metamorphosis.

We propose to extend the transfinite interpolation to the 4D case of space-time and to apply it in order to define time-variant volumetric material properties. To formulate the new concept of space-time transfinite interpolation, we introduce two space-time material features with the time gap on the interval $0 \leq t \leq 1$:

$$g_1(x, y, z, t) = f_1(x, y, z) \wedge_0 (-t), \tag{2}$$

$$g_2(x, y, z, t) = f_2(x, y, z) \wedge_0 (t - 1), \tag{3}$$

where $\wedge_0$ is a R-function for the set intersection. Here the function $g1$ defines a space-time half-cylinder (semi-infinite cylinder) with the boundary $t = 0$. In the other words, the initial object G_1 exists at any time $t \leq 0$ and then disappears. The function $g2$ defines a space-time half-cylinder with the boundary $t = 1$, which means the target object G_2, appears at the time $t = 1$ and exists for any $t \geq 1$ (see Fig. 2). By analogy with the 3D weighting expressions of [9], we apply transfinite interpolation to these 4D space-time material features and thus define the weighting functions $w_1(X) = w_1(x, y, z, t)$ and $w_2(X) = w_2(x, y, z, t)$ using a normalisation of each of the inverse defining functions:

$$w_1(X) = \frac{\frac{1}{g_1(X)}}{\frac{1}{g_1(X)} + \frac{1}{g_2(X)}} = \frac{g_2(X)}{g_1(X) + g_2(X)}, \tag{4}$$

$$w_2(X) = \frac{\frac{1}{g_2(X)}}{\frac{1}{g_1(X)} + \frac{1}{g_2(X)}} = \frac{g_1(X)}{g_1(X) + g_2(X)}. \tag{5}$$

Note that $g_1(X) = 0$ for $t = 0$ on the surface and at internal points of G_1 and $g_2(X) = 0$ for $t = 1$ on the surface and at internal points of G_2, thus the weighting functions w_1 and w_2 satisfy the above constraints of the partition of unity for the interval $0 \leq t \leq 1$.

Fig. 3 illustrates the application of the above technique to the colour interpolation in the process of metamorphosis between two simple shapes. The colours are constant for the initial objects and represent their homogeneous material properties. From this figure it is apparent that the intermediate colour behaviour of the metamorphosing object is non-linear and geometry dependent in nature. The colour of the intermediate surface converges faster to the target colour (depicted in blue) when this surface is closer to the target geometry. Similarly, source features far from any of the target surface points remain closer to the original source properties (depicted in red).

Note also that the presented formulation for the space-time transfinite interpolation can be easily extended to any material properties function of space. $\mathbf{c}_1$ and $\mathbf{c}_2$ can be

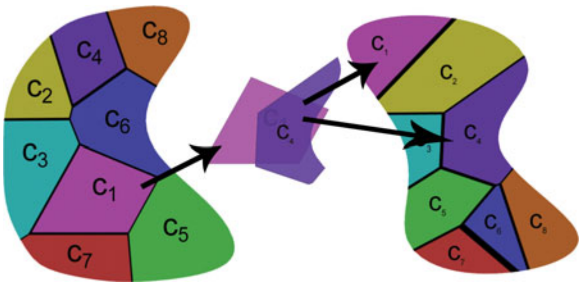


Fig. 4. Two partitioned objects with multiple corresponding material features (partitions) indicated by corresponding colours and the case of the overlapping partitions at an intermediate stage.

rewritten as $\mathbf{c}_1(x,y,z)$ and $\mathbf{c}_2(x,y,z)$. More precisely, this means that solid texturing can be used, as well as any function providing values of material property attributes at any point in space.

4.2 Partitioned Objects

Let one of the objects G_i have multiple semi-disjoint partitions with a constant attribute $\tilde{\mathbf{c}}_j$ assigned to jth partition with the defining function $\tilde{f}_j(\mathbf{x})$. Examples of partitioned objects are shown in Fig. 4. Also a method to define heterogeneous objects with multiple partitions with different volumetric attributes is shown in [22]. The overall contribution of the object to the attribute value at a given external point X can be represented by:

$$\mathbf{c}_i(\mathbf{x}) = \frac{\sum_{j=1}^{N} \tilde{w}_j(\mathbf{x})\tilde{\mathbf{c}}_j}{\sum_{j=1}^{N} \tilde{w}_j(\mathbf{x})}, \tag{6}$$

where N is a number of partitions, and the weight of each partition $\tilde{w}_j(\mathbf{x}) = \frac{1}{\tilde{f}_j(\mathbf{x})}$.

For example, if G_i is represented by a voxel array with an attribute such as colour assigned to each voxel, then this expression defines a summation by all the voxels and $\tilde{f}_j$ represents the distance from the given point to the voxel center.

If the number of partitions becomes infinite, i.e., an individual attribute value $\tilde{\mathbf{c}}_j(\tilde{\mathbf{x}})$ is assigned at each point $\tilde{\mathbf{x}}$ of the object point set Ω_j, the above finite summation is transformed into integration as follows:

$$\mathbf{c}_i(\mathbf{x}) = \frac{\int_{\Omega_i} \tilde{w}_i(\tilde{\mathbf{x}})\tilde{\mathbf{c}}_i(\tilde{\mathbf{x}})d\Omega}{W(\mathbf{x})}, \tag{7}$$

where

- $W(\mathbf{x}) = \int_{\Omega_i} \tilde{w}_i(\tilde{\mathbf{x}})d\Omega$
- $\tilde{w}_i(\tilde{\mathbf{x}}) = \frac{1}{d(\mathbf{x},\tilde{\mathbf{x}})}$

$d(\mathbf{x},\tilde{\mathbf{x}})$ being the distance between the points $\mathbf{x}$ and $\tilde{\mathbf{x}}$. The attribute field $\mathbf{c}_i(X)$ can be considered a volume potential with the given density $\tilde{\mathbf{c}}_i(\tilde{\mathbf{x}})$. In fact, the voxel array example mentioned above represents a numerical implementation of this integral formulation.

4.2.1 Partitioned Objects with No Established Correspondence

If both given objects have multiple partitions and there is no established correspondence between the partitions of the two objects, after defining $\mathbf{c}_1(\mathbf{x})$ and $c_2(\mathbf{x})$ using Equation (6), one can apply Equations (1)-(5) to interpolate this attribute over time. The volumetric metamorphosis from a cartoon character to a cut watermelon in Fig. 1 is an example for objects with multiple partitions and no established correspondence between them.

4.2.2 Partitioned Objects with Established Correspondence

Let us consider the case when correspondences are established between partitions of two objects as shown in Fig. 4, such that the shape transformation starts with a partition of the initial object and finishes with the corresponding partition of the target object. The object can be represented as a union of all the partitions and therefore the defining function for both source and target objects can be written as:

$$f_1(\mathbf{x}) = p_1(\mathbf{x}) \vee p_2(\mathbf{x}) \vee \ldots \vee p_n(\mathbf{x})$$

$$f_2(\mathbf{x}) = r_1(\mathbf{x}) \vee r_2(\mathbf{x}) \vee \ldots \vee r_n(\mathbf{x})$$

where n is the number of partitions, p_i is the defining function for ith partition of the source object and r_i is the defining function for ith partition of the target object.

The geometry transformation is done separately for each pair of corresponding partitions:

$$f_{mi} = m_i(p_i(\mathbf{x}), r_i(\mathbf{x}), t).$$

Here i is the index of the given partition, m_i denotes the selected operation for geometry metamorphosis (see Section 3.2) for the given pair of source and target geometry within the given partition. Note that different operations can be used for different partitions.

For partitioned objects, instead of weights $\tilde{w}$ in Equation (6) defined with the field value, we calculate the weights based on the geometry of the partitions at the given point $\mathbf{x}$. It should be noted that during the intermediate stages the point can belong to two and more partitions because the geometry can overlap (see the middle of Fig. 4). In our experiments we distinguish two most useful cases:

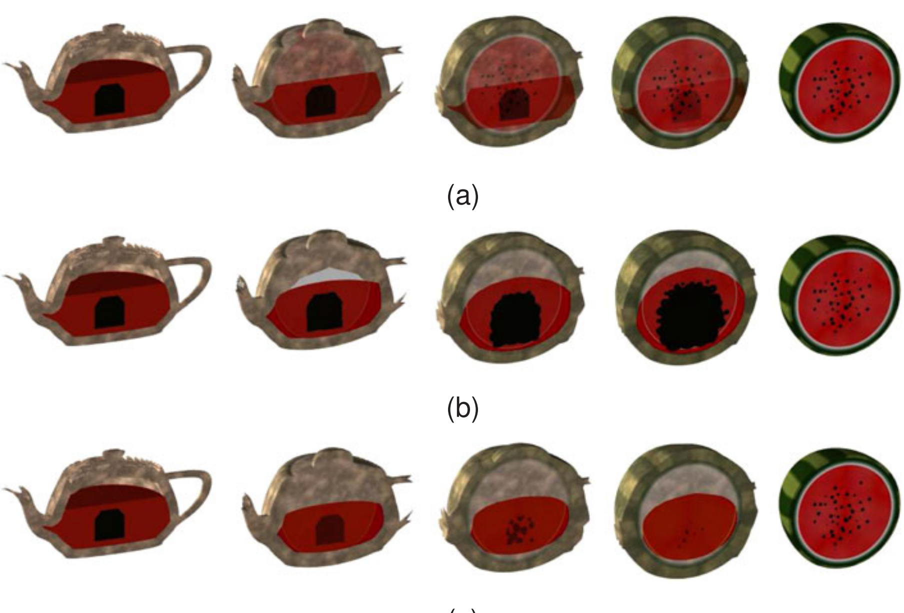
1) Priorities are set up for the partitions and the colour of the partition with a highest priority is chosen
2) The colours of all partitions enclosing the given point at the intermediate stage are mixed.

In the case of priorities we set up the priority P for each partition, such that $P_i \neq P_j \forall i,j$. The weight for the given partition at the given point depends on the other partitions that enclose this point on the given frame:

$$\tilde{w}_j = \begin{cases} 1, & \max_i(P_i) = P_j \forall i : f_{mi} \geq 0, \\ 0, & otherwise, \end{cases} \tag{8}$$

Note that some points can belong to zero objects in the intermediate stages. In this case we either need to define "null" partition by taking the complement point set to the union of the geometry of the partitions on each frame or to define the "master" partition whose weight is set to 1 if no partitions influence to the given point.

In the case of mixing the colours of all partitions the weights are set as following:



(a)

(b)

(c)

Fig. 5. Interpolation of attributes for partitioned objects: a) No correspondence established, b) Correspondence established, priorities are set for the partitions, c) Correspondence established, the colour of all the contributing partitions are mixed.

$$\tilde{w}_j = \begin{cases} 1, & f_{mj} \geq 0, \\ 0, & otherwise. \end{cases} \tag{9}$$

As the colour can be defined procedurally for each partition (in this case we actually consider the material associated with the partition while the colour serves to represent it), the Equation (6) takes the following form:

$$\mathbf{c}_i(\mathbf{x}) = \frac{\sum_{j=1}^{N} \tilde{w}_j(\mathbf{x}) \tilde{\mathbf{c}}_j(\mathbf{x})}{\sum_{j=1}^{N} \tilde{w}_j(\mathbf{x})}. \tag{10}$$

In Fig. 5 we show how the attributes of partitioned objects can be mixed based on the different scenarios. When no correspondence is established (Fig. 5a), it can be seen that on the intermediate stages all partitions from both source and target object influence the colour of the given point. When the correspondence is established, the geometry of the intermediate stage of two corresponding partitions influences the colour of the given point. It can be either one-to-one correspondence in case when the priorities are used (see Fig. 5b) or colours can be mixed from intermediate stages of various partitions. Note that different methods can be used for obtaining the intermediate stages of the geometry of the partitions, as shown in Figs. 5b and 5c.

5 User Control

For a tool to be useful, it needs to be easy to set up, and have intuitive parameters. In this section we introduce two parameters, time-interval and balance, to better control the effect.

5.1 Shape Driven Against Time Driven

In order to control the influence of geometry or time for the attribute distribution, we can stretch or shrink the time-interval. g_1 and g_2 can be rewritten as follows:

$$g_1(x,y,z,t) = f_1(x,y,z) \wedge_0 (-\alpha t)$$
$$g_2(x,y,z,t) = f_2(x,y,z) \wedge_0 (\alpha t - \alpha).$$

When α is small, the geometry influences the material distribution more than time. On the other hand, when α is large, then we achieve results which are close to linear interpolation. In Fig. 3 the value of α is set to one.

Fig. 6 shows the material distribution in space at several instances of time during the interpolation process while using different α values. The top row uses $\alpha = 0.1$ which translate to clear geometric features. The interpolation is strongly shape driven. The middle row uses $\alpha = 1$ where features are still visible, but the blending is faster. Finally, the last row uses $\alpha = 20$, making the material distribution almost equivalent to a linear interpolation.

5.2 Object Influence

To improve the user control, a balance parameter can be used to increase the influence of one object over the other. The parameter β is given a real value in the range $[0, 1]$ which is used to scale the values of the object functions. The function g_1 and g_2 are rewritten once more as:

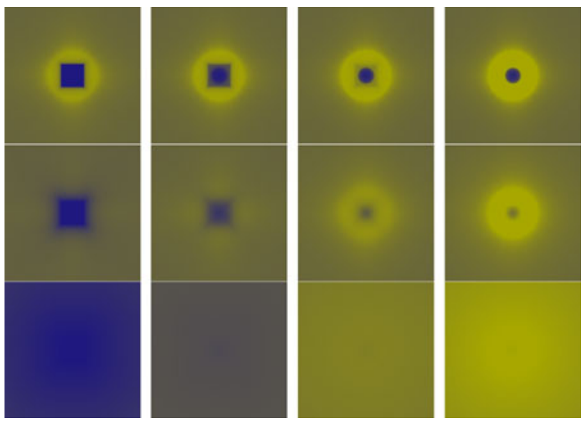


Fig. 6. Material distribution using a time gap of 0.1 (top row), 1 (middle row), 20 (bottom row), at time $t = 0$ (first column), $t = \frac{1}{3}$ (second column), $t = \frac{2}{3}$ (third column) and $t = 1$ (fourth column).

Fig. 7. The influence of the balance parameter on the metamorphosis between two objects.

$$g_1(x,y,z,t) = (f_1(x,y,z)\cdot 2(1-\beta)) \wedge_0 (-\alpha t)$$
$$g_2(x,y,z,t) = (f_2(x,y,z)\cdot 2\beta) \wedge_0 (\alpha t - \alpha).$$

Fig. 7 shows how the balance parameter can affect the attributes. The armadillo in brown is morphed with a gargoyle in grey (top row). The bottom row shows the same intermediate shape, with different balance parameters. This process can be viewed as a scheduling task. The first frame and last frame of the animation will match the source and target object materials. However, the β parameter modifies the weight curves to favour one object in order to give more artistic control to the animator in an intuitive way.

6 Implementation

As we have stated above, the geometry transformation is independent of our method, thus making it useful for a wide range of the applications. In the context of metamorphosis, the choice of method for the geometry transformation depends on the specific requirements of applications. In interactive applications the metamorphosis process should be as fast as possible, while in off-line geometry processing applications the result should be as close as possible to what the user requires. In our tests we have used space-time blending [20], as it provides better results than linear metamorphosis, and in addition is computationally inexpensive.

Note that the presented colour transformation technique allows the objects to be separated in space as well as to occupy the same space. Fig. 8 shows that the proposed method provides superior results than the linear colour

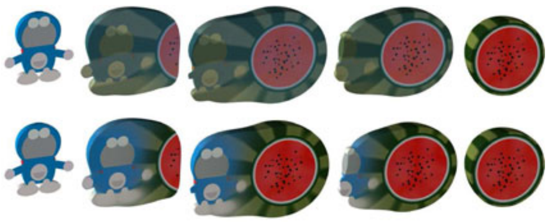


Fig. 8. Top image: linear interpolation in LAB space for volumetric material distribution; bottom image: volume material distribution with the presented method.

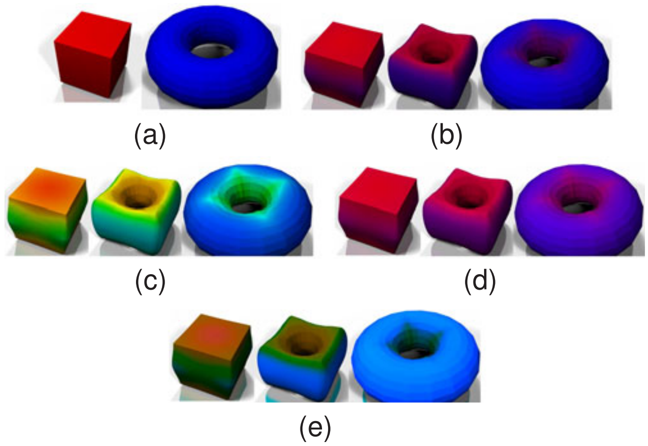


Fig. 9. Influence of different colour spaces for colour mixing in intermediate shapes: a) The source and the target models, b) RGB, c) HSV, d) CIELab, e) CIELCh.

interpolation technique. In particular, one can observe the better transfer of colours for the points in space located far from the initial object. It is worth noting that the proposed solution does not require heavy numerical computations essential to particle-based methods and can be implemented in real-time applications.

While the choice of colour space is which the mixing of colours is performed is done by the user, for practical purposes we can suggest either RGB or CIELab colour spaces as they provided best results for our purposes. Fig. 9 presents several popular colour spaces to show the motivation of this decision.

We now present several applications implemented in Maya or in GLSL with real-time rendering frame rates. All these applications can generate smooth animation without any jumps or discontinuities often associated with particle based methods.

6.1 Volumetric Material Properties

Constant colours are not very useful in the definition of realistic looking objects. However, in order to interpolate material and other attributes distribution, we need a volumetric representation of such attributes. To do so, we used three different approaches (procedural, space enumeration and surface texture extrapolation).

- Solid texturing can be used to define the material properties or any tri-variate function. They have been successfully used to describe marble, wood, and many other natural heterogeneous materials (see the "watermelon" model in Fig. 1). While those methods are the most accurate and explicit methods, they are unpopular because they often require knowledge of scripting or other technical skills.
- Space enumeration is defined by a constant colour for a given space partition (see the "cartoon cat" model in Fig. 1). Note that space enumeration can be combined with solid texturing. Some particular care has to be taken for points which belong to several partitions or to none at all. In the case of several partitions, a common approach is to give a higher priority to a particular partition over the others. If the

Fig. 10. The animation from a Buddha to a teapot, with a box cut out to see the material inside the intermediate shapes.

point does not belong to any partition, then we can use a default colour, such as white or grey.

- Surface textures are popular in creating heterogeneous looking surface colours and properties. But they only define properties at the surface. A volumetric representation of those surface textures can be achieved using the closest point of the mesh approach. This is neither an accurate nor a continuous representation, but gives good results especially during an animation sequence. The "banana" model in Fig. 1 uses this technique. However, to achieve better results, alternative techniques can be used such as the mean value coordinates technique [23]. Unfortunately, mean value coordinates are time consuming and often impractical for real meshes. This is not suitable for interactive applications but gives smoother and more intuitive results.

Fig. 10 shows a few frames from an animation generated in Maya. A segment of the shape is deliberately removed on every frame so that we can see the internal distribution of colours. The material properties inside the intermediate shapes show that locations where the head of the Buddha used to be still retain the Buddha's material properties mostly, while the inside of the teapot is a more even blend of both materials.

6.2 Displacement and Other Attributes

Space time transfinite interpolation can also be used for other properties such as displacement. Displacement is an important visual clue to help us understand the material properties of an object, just like pigmentation (colours). The displacement is applied on the intermediate shapes following the basic shape formation. The displacement information is available at any point in space, but renderers will only evaluate it at the surface. It is purely cosmetic in nature and the interior of the shape remains completely solid.

The example in Fig. 11 defined two displacement values through solid texturing. We then blend the two attributes using Equations (4) and (5). The displacement information is a single value in the range $[0, 1]$. Our method interpolates this value during the animation and provides at render time, a single displacement per surface sample which can be used to offset the surface along its normal. Our method is equally applicable to any attribute which can be linearly interpolated, such as roughness, opacity and reflectivity.

6.3 The Octopus Effect

An interesting property of our method is that it can mimic propagation of attributes through space. Animals such as the octopus and the chameleon have the ability to blend in with their environment. The look of the material they are standing on seems to propagate across their skin.

Fig. 12 shows how our method can be used to simulate octopus blending into its environment. The octopus can take the colours of its environment in order to hide. The colour of its skin changes by propagation. The basic formulation stated that $0 \leq t \leq 1$ because it assumes we apply the colours to a metamorphosis but if $t \geq 1$, then everything around the destination object will slowly tend towards the target material. In Fig. 12, the octopus is the source object, and the rock is the target. No transformation is applied to the objects, and the octopus slowly takes on the material properties of its environment (rock) as t increases until it is fully hidden. The octopus skin colour changes faster in the areas where it is closer to the target surface.

6.4 Real-Time Application

Our technique was successfully implemented in real-time in a metamorphosis application exploited within a computer game for people with limited abilities. The original objects could be described using function-based geometry, or using signed distance fields for meshes. In the case of meshes, we also store the UV coordinates of the closest point along the signed distance. The metamorphosis is performed on the fly using the two 3D textures and ray marching. Once the surface is found, we can retrieve the colour of each object at

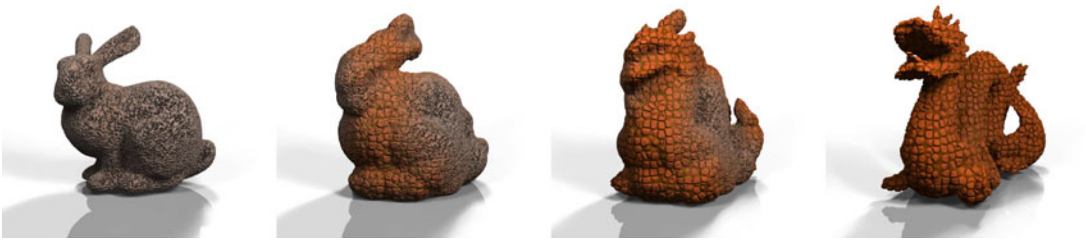


Fig. 11. The Stanford bunny is transformed into the Stanford dragon, intermediate colours and displacement are produced by our method.

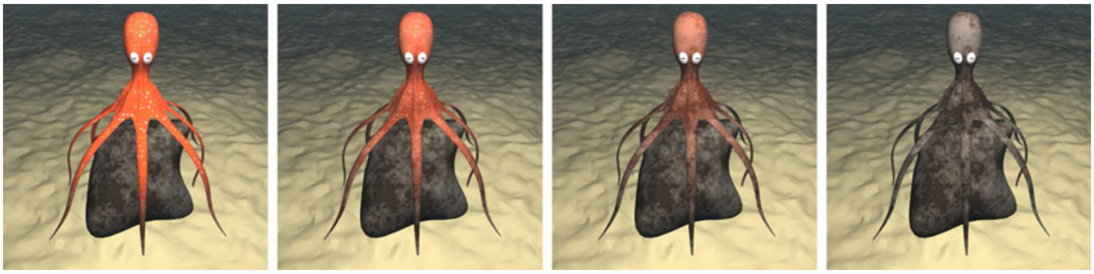


Fig. 12. Four frames from the blending animation of the octopus changing its colours from its natural appearance to its rock-like appearance.

that point in space. If it is a mesh, we use the texture coordinates from the closest point and sample the surface textures. We can then apply an interpolation between the two colours. The performance of our technique is constant regardless of the mesh complexity. The space-time transfinite interpolation computation times are negligible compared to the surface intersection test costs.

There are several limitations to this method:

- The discretization of the field onto a grid limits the accuracy of the two objects, both in shape and texture information.
- The UV coordinates needs to be continuous. That is, the UV coordinate of a vertex is the same regardless of the face it belongs to. While this may occur, it is often hidden in creases of the mesh (such as a character's armpits). This can become a source of noise especially in regions far from the surface, since the UV coordinates retrieved from the closest point are ambiguous. When a region is far from the mesh, it is likely that the closest feature is a vertex or an edge.

Our application runs at real-time frame rates (approx 30 frames per second) on an Intel Xeon W3680 CPU with NVIDIA GeForce GTX 560 Ti graphics card using volume textures with 256^3 voxels per object.

6.5 Microstructures

Microstructures are internal spatial geometric structures with size of detail orders of magnitude smaller than the overall size of the object. Microstructures can be described by a function with several parameters such as orientation and thickness. As any point attribute, they can be interpolated using our technique.

In Fig. 13, two types of microstructures are defined. Microstructure A is a regular lattice represented by the function m_a, and microstructure B, which is made up of interlaced threads, is represented by the function m_b. The parameter interpolated by our method is the mix parameter ω, set to 0 for the teapot, and 1 for the bunny. The final microstructure function is defined as follows:

$$m_f = m_a \cdot (1 - \omega) + m_b \cdot \omega.$$

A blended union is performed on the objects, and then filled with the microstructure represented by m_f. The time parameter is used as a designer tool to help determine how much each of the source objects should influence the resulting internal microstructure.

6.6 Corresponding Partitions

As discussed at the end of Section 4, the user may want to establish correspondences between particular partitions of the initial and final objects and thus to associate particular materials to each other. Our technique could be easily adapted to enforce this constraint given a set of priorities and appropriate metamorphosis operations for the corresponding partitions.

In Fig. 14, each object has three corresponding partitions: the outer skin, the flesh, and the inner element (core). The Equations (1)-(5) are separately applied to each pair of partitions and we have three transformation processes going on for the three pairs of partitions. To identify the material attribute value at each point at the given time, the innermost partitions are given the highest priority. This means that to define the material properties at a point where two partitions overlap, we use the partition with the highest priority. This is the most naive but an intuitive solution in this case. Other solutions could include blending based on the distances between the partitions or on averaging the attribute values of these partitions. In this example, space-time blending is used to transform the corresponding partitions to each other. The proposed space-time transfinite interpolation is then applied independently to each intermediate partition. The function representation makes it easy to detect which particular partition the given point belongs to. The results shown in Fig. 14 were produced using our Maya plug-in by creating two sets of meshes, each of which had its own material definition.

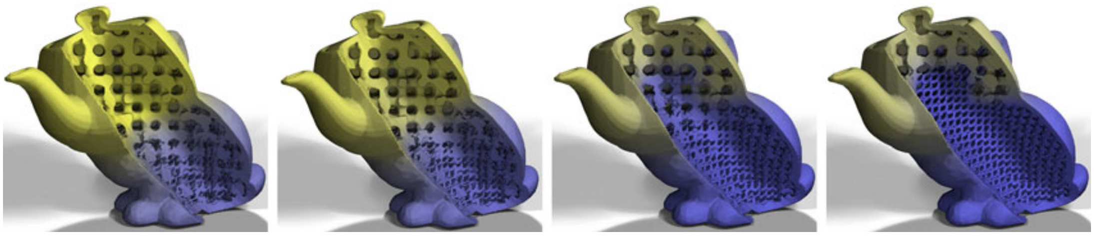


Fig. 13. The microstructure parameters are interpolated across the volume. Here the parameter is the mix amount between the lattice microstructure and the interlaced thread microstructure.

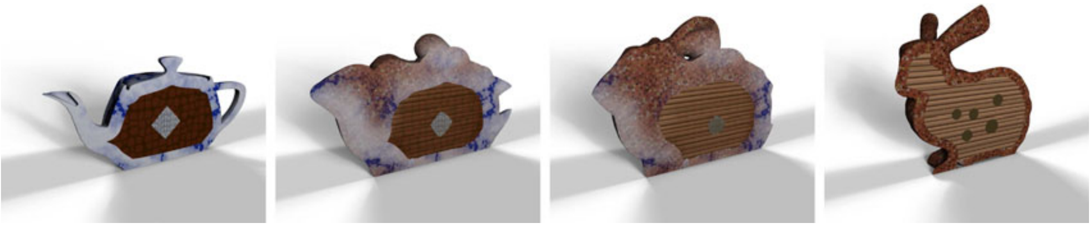


Fig. 14. Partitions can be mapped to each other correspondingly, creating more complex effects. Here the outer skin, the flesh, and the core consist of different materials in each of the two objects.

7 Conclusion

In this paper we have tackled a practical problem in heterogeneous volume modelling concerning the changing volumetric material properties (represented with continuous fields) over time. We presented a novel closed form solution for the time-variant transformation of volumetric material properties independent of the method of the object geometry and topology transformation. This solution is based on an extension of a general mathematical method of the transfinite interpolation to a specific higher-dimensional space-time technique. The continuity and differentiability of the proposed interpolation are provided by the continuity of the object defining functions, using of proper R-functions in Equations (2), (3) and the formulation of the space-time transfinite interpolation in Equations (4), (5).

The proposed transformation of properties can accompany any shape transformation and is not limited to metamorphosis. As soon as the applied shape transformation is symmetric, the proposed attributes interpolation is symmetric as well and we will get the same interpolation path, if we switch the source and target shape. Our approach does not involve the time-consuming solution of partial differential equations and particle tracking, and thus it is suitable for real-time applications. Several tools for intuitive user control were proposed, such as variations of the time gap to control the balance between the shape and time influence, and special parameters to control the influence of the source and target objects.

The proposed solution is applicable to any volumetric distributions of object properties including parameters of volumetric microstructures. A volumetric representation of properties by scalar fields is required, but textured meshes can be also handled with volumetric propagation of surface properties. In this way, the proposed solution is also useful for modelling time-evolving surface rendering attributes such as smooth transitions of reflectivity, shininess, pigment, and displacement maps. An interesting application was the modelling of the octopus or chameleon behaviour with time-variant material property for static geometry under the influence of the environment.

The requirement of the volumetric properties representation can be considered as a limitation of the proposed approach. While some simple solutions work to extend surface properties within a volume, none of them are very convincing. Our future work will deal with this issue involving smooth volume interpolation schemes for surface properties such as mean-value coordinates.

Another open problem is that of maintaining the integrity of the objects with corresponding multiple partitions such that the object does not fall apart in the transformation process, although this is mainly a geometric transformation issue.

Acknowledgments

This work was partially sponsored by the EU Interreg IVA project 5-023-FR_SHIVA. The Stanford bunny, Happy Buddha, Stanford Dragon, and Armadillo models are courtesy of the Stanford University Computer Graphics Laboratory.

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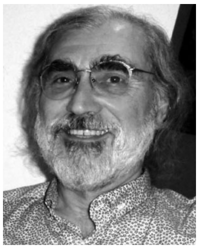

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