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Key Words: Labour market integration, remittances, tax
JEL Classifications: F15, F22, F24, O24

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Labour Market Integration, Remittances and Optimal Tax Policy

Abstract: This paper, using a model comprising two labour sending countries and one labour receiving country, analyses the optimal tax/subsidy policies of countries to control international migration when labour markets are integrated. The analysis shows that the countries of emigration should tax the migrants to maximise national income. This result suggests that the developing and transitional countries need to re-evaluate their policies of supporting migration. The optimal policy of the receiving country is to use discriminatory tax rates where the sending country with higher labour endowment bears a higher tax burden.

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1. Introduction

Economic integration reduces the barriers of movement of goods and factors. Though many countries still maintain high barriers for the movement, especially for labour, it is fair to say that the world is gradually moving towards reduction of the barriers. Economic literature has to some extent already addressed integration that leads to reduction of barriers for the movement of goods and capital. The integration of the labour market, especially the link between labour market
integration and remittances is however, a relatively understudied field\textsuperscript{1}. This paper addresses this gap in the literature.

Integration of labour market should initiate the movement of labour from the low wage to the high wage countries. This phenomenon was observed following the enlargement of the European Union in 2004 which integrated Eastern and Western Europe. Many Eastern European and Turkish migrants regularly send remittances to home countries. In addition, we observe that the global labour market, especially of skilled workers, to some extent has become integrated through the advent of the internet. Many people are now searching for jobs in foreign countries, by themselves, using the internet which was unthinkable just ten years ago. Many of them send remittances to home countries. It can be observed that developing and transitional counties directly or indirectly subsidise/support this migration to increase the inflow of remittances. In doing so, the countries are indirectly competing with each other in the international labour market.

In this paper we question this policy of supporting migration for remittances when labour markets are integrated. We show that instead of subsidising, the home countries are better off by taxing migrants. To illustrate this, we propose a model where migration of labour takes place

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\textsuperscript{1}Chen (2009) and Haufler and Wooton (2010) studied product market integration and foreign direct investment; Haaland and Kind (2008), Braun (2008) and Morita (2012) studied the relationship between trade barriers, innovations and R&D subsidies. We found that Bretchger (2001) and Lundborg and Seregstrom (2002) have studied the linkage between labour market integration and innovation using a quality ladder growth model. Iranzo and Peri (2009) studied the link between migration flow and trade flow with reference to Eastern-Western European Integration. Haaland and Norman (1995) studied the effects of European Integration on production and factor prices. Lavenex and Uçarer (2003) addressed the issues relevant to immigration and asylum following European Integration.
from two home/sending countries to one host/receiving country. The existence of two sending
countries captures strategic interactions of labour sending countries in international migration.
The receiving countries, in reality, are often the rich developed countries who act as leaders in
international migration by setting their own policies first. In this paper also, the receiving country
moves first by setting the migration tax\(^2\). Given the policies of the receiving country, the sending
countries design tax/subsidy policies to maximise national output plus remittances. In doing so,
they engage in a Bertrand type competition with each other.

Our paper is aligned with the strategic trade policy literature, especially the model
proposed in Brander and Spencer (1985) where two countries engage in a tax/subsidy
competition to export goods to a third country. In a non-cooperative situation both countries
provide a positive subsidy to maximise welfare if the goods exported are substitutable. Gatsios
(1990) extended the model by looking at the optimal policies of the importing country. He found
that the importing country practises discriminatory tax policy to maximise welfare where the
most efficient producer receives a higher tax burden. Contrary to Brander and Spencer (1985) we
find that sending countries should tax the migrants and this tax policy improves the income of
the other sending country. In the case of the receiving country we see that the receiving country
uses discriminatory tax policy and the tax burden is higher for the country with the higher labour
endowment. This result is similar to Gatsios (1990) where the most efficient producer bears the
highest tax burden. The findings of this paper shed doubt on the policies of many developing
countries that promote migration. It is however to be noted that apart from remittances, migrants

\(^2\) Migration tax is not possible in an integrated market like the European Union. However it is already in use for
migration from developing to developed countries. It can be observed in the form of high visa application fees,
mandatory medical check-ups, limited entitlement of social security system etc.
are also regarded as the source of other types of benefits such as investment, human capital formation and business networks\(^3\). Nevertheless remittances are the most tangible and immediate benefit of migration and deserve adequate attention.

The competition of countries in international migration so far received little attention in the mainstream economic literature and this is really surprising given the importance of strategic trade to the economists and policy makers. Competition among countries for remittances is certainly having effects on the benefits of migration. However we can only find vague reference to competition in the policy literature, without any clarifying discussion. In the publication of International Organization of Migration (IOM) titled ‘Labour Migration in Asia’, for example, references of competition have sometimes been provided without any detailed discussion on the nature of such competition (IOM 2003). We believe that the paper will be able to fill this gap in the existing literature and will be useful in designing the migration policies of developing and transitional economies. Though the model we use heavily borrows from international trade literature, especially strategic trade policy, we greatly believe that similar analysis needs to be conducted to address ‘international migration’. Clemens (2011) interestingly mentioned that term ‘international trade’ is 13 times more frequent than ‘international migration’ in all the published article abstracts contained in the Research Papers in Economics(RePEc). Before proceeding with the main analysis we describe the structure of the paper. The second section describes the complete model. It also identifies the conditions for the equilibrium of migration. The third and fourth sections deal respectively with the optimum emigration and immigration taxes. The main results are given in proposition format in those sections. The fifth section is the concluding section and provides policy recommendations based on the analysis of the paper.

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3 Docquier and Rapoport (2012) provides a recent survey on migration, brain drain and economic development.
2. The Model

2.1. Basic Assumptions and Notations

We assume that there are three open economies \( C \), \( I \) and \( U \) producing two internationally traded goods \( X_1 \) and \( X_2 \). The prices of the goods are fixed. We select the units in a way that the relative price of two goods is just 1. Endowments of labour are fixed, denoted by \( L_i \) for country \( i \in \{C,I\} \) and \( L_u \) for country \( U \). The technologies and labour endowments of the countries are such that country \( C \) and \( I \) specialise in the production of good 1 and country \( U \) specialises in production of good 2. Hence the wage rates in \( C \) and \( I \) are given by the value of marginal products of labour in producing good 1 in these two countries. Similarly wage rate in \( U \) is given by the value of marginal product of labour in producing good 2 in \( U \). The assumption of specialisation helps us to simplify the analysis.

We assume that the wage rate in country \( U \) is greater than the wage rates in \( C \) and \( I \) which initiates migration following integration of labour markets. Migrants repatriate all the income back to the home country. The national income of the home is given by the production of \( X_1 \) plus the remittances received. We assume preferences are such that the countries always use a part of income to consume both goods. The countries export the goods in which they specialise and import the others. We assume that migration does not alter this pattern of comparative advantage. As migration affects the production and consumption of goods in the countries under consideration, it may also affect the world price levels and may reverse the pattern of the specialisation and wage differentials. As the aim of the analysis is not to evaluate the welfare
from trade and/or modification of comparative advantage, we assume fixed prices throughout to keep the analysis simple. The assumption of fixed price, additionally, allows us to work with national income equation instead of a welfare function, as when price is fixed, higher income implies higher welfare. It should also be noted that the world may consist of many goods, factors and countries whereas we are working with a partial equilibrium model with only three countries, two goods and one factor. In this sense our analysis is somewhat restrictive. The incorporation of more countries, sectors and factors may change our results substantially.

To facilitate further analysis we specify the forms of the production functions. For \( i \in \{C,I\} \), let us assume that the production of \( X_1 \) is depicted by \( X_{i1} = a_i L_{i1} - \frac{b_i}{2} L_{i1}^2 \), where \( X_{i1} \) denotes the amount of \( X_1 \) produced in \( i \in \{C,I\} \), \( L_{i1} \) is total amount of labour used in production of \( X_1 \), \( a_i \) and \( b_i \) are coefficients. Differentiating with respect to \( L_{i1} \), the value of the marginal product of labour in \( X_1 \) is obtained as \( a_i - b_i L_{i1} \). Therefore the value of the marginal product of labour varies between \( a_i \) and \( a_i - b_i L_i \).

The production of \( X_2 \) in \( i \in \{C,I\} \) is given by \( X_{i2} = a_2 L_{i2} \), where \( a_2 \) is the amount of labour required to produce one unit of \( X_2 \) and \( L_{i2} \) is the amount of labour used for production of \( X_2 \). Differentiating with respect to \( L_{i2} \), the value of marginal product of labour is obtained as \( a_2 \). Hence, country \( i \in \{C,I\} \) switches labour from \( X_2 \) to \( X_1 \) if the value of marginal product of labour in \( X_1 \) is greater than the value of marginal product of labour in \( X_2 \). That is, if \( a_i - b_i L_{i1} > a_2 \). Let us assume that \( a_i - b_i L_i > a_2 \). Thus with trade and without integration \( i \) necessarily specialises in production of \( X_1 \). Without integration the workers of \( i \) receive wage \( w_i = a_i - b_i L_i \).
On the other hand, assume that country $U$ has a linear production technology for $X_1$, which is written as $X_{u1} = a_{u1} L_{u1}$ where $L_{u1}$ is the amount of labour used and $a_{u1}$ is the coefficient. The value of the marginal product of labour is $a_{u1}$. The production of $X_2$ is given by $X_{u2} = a_{u2} L_{u2} - \frac{b_{u2}}{2} L_{u2}^2$, where $L_{u2}$ is the amount of labour used in production of $X_2$ in $U$. The value of the marginal product of labour in $X_2$ is $a_{u2} - b_{u2} L_{u2}$. Thus the marginal product varies between $a_{u2}$ and $a_{u2} - b_{u2} L_{u2}$. Country $U$ thus specialises in production of $X_2$ if $a_{u2} - b_{u2} L_{u2} > a_{u1}$. Assume that this assumption holds, hence $U$ specialises in production of $X_2$.

Without integration, the workers of country $U$ receive wage $\bar{w}_u = a_{u2} - b_{u2} L_{u2}$. With integration, the wage rate in $U$ becomes $\bar{w}_u = a_{u2} - b_{u2} \left( L_u + L_{C2} + L_{I2} \right)$. We assume that $\bar{w}_u > a_{u1}$ throughout as such the country still specialises in production of good $X_2$ after integration. Additionally $a_{u2} - b_{u2} L_{u} > a_i - b_i L_i$ such that, without integration, the wage received by labour in country $U$ is higher than that of $C$ and $I$. This wage gap initiates migration.

The game we consider is as follows. First the receiving country, that is, country $U$ sets its tax policies for migration. Given the tax rates, $C$ and $I$ set their emigration tax policies. Given the tax policies of $C$, $I$ and $U$, the labour from $C$ and $I$ migrate to country $U$. As labour can migrate freely, it is possible all the labour of $C$ and $I$ to migrate to $U$ if wage gaps are large enough. To avoid such extreme situation we assume that the equilibrium of labour market is reached well in advance of that.
2.2. Equilibrium Migration between Country $i$ and $U$

In the previous sub-sections we defined the model and showed the conditions under which $i \in \{C, I\}$ specialises in production of good $X_i$ and $U$ specialises in production of good $X_2$. In this section we calculate the equilibrium of labour migration. We assume that integration of labour markets equates the wage rate of $i$ and $U$ through migration. The equality of wage rates also implies full information and no travel costs, which are obviously two simplifying assumptions.

Instead of using direct quantitative restrictions, both sending and receiving countries use taxes to control migration. Let $D_i$ be the per capita tax imposed by country $i$. Let us call it the ‘domestic’ tax for $i$. Similarly let $T_i$ be the per capita tax imposed by country $U$ on the migrants of country $i$ and call it the ‘foreign’ tax for $i$. In the calculation we assume $T_i \neq T_j$. However the difference of tax rate depends on the receiving country, as it may well wish to set $T_i = T_j$ implying no discrimination policy. The following two equations must be satisfied in the equilibrium of migration assuming full information, perfect mobility and no travel costs:

\[
a_{u2} - b_{u2} \left( \bar{L}_u + L_{c2} + L_{f2} \right) - T_c - D_c = a_1 - b_1 \left( \bar{L}_1 - L_{c2} \right) \tag{1}
\]

\[
a_{u2} - b_{u2} \left( \bar{L}_u + L_{c2} + L_{f2} \right) - T_i - D_f = a_1 - b_1 \left( \bar{L}_1 - L_{f2} \right) \tag{2}
\]

Hence the wage of migrants minus domestic tax and foreign tax must be equal to the wage received by domestic workers. As shown in the equation, the migrants face two barriers.
One comes from the domestic tax and another comes from the foreign tax. The domestic tax can be assumed to be a proxy for some kinds of barriers, similar to the ‘Bhagwati Tax’ (Bhagwati and Dellalfar 1973) aiming to discourage migration and increase national income. The foreign tax, however, has a different purpose. The foreign tax transfers a part of the income of the migrants to the national income of the receiving country. The foreign tax is a proxy of monetary entry barriers which is above the administrative cost of managing the inflow of labour.

Solving the equations the following solutions of equilibrium migrations are obtained,

\[ L^*_{c2} = V^{-1}(G_c(b_i + b_{u2}) - G_ib_{u2}) \]  
\[ L^*_{i2} = V^{-1}(G_i(b_i + b_{u2}) - G_c b_{u2}) \]

where

\[ V = b_i(b_i + 2b_{u2}) \]  
\[ G_i = -(a_i - b_i L_i) + (a_{u2} - b_{u2} T_i) - T_i - D_i \]

\( V^{-1} \) is inverse of \( V \). We assume that \( G_i > 0 \), because to initiate migration the wage rate in \( U \) must be higher than the wage rates in \( i \) plus taxes\(^4\). Equations (1) and (2) have been plotted in Fig. 1. \( L_{c2} \) has been plotted in the vertical and \( L_{i2} \) has been plotted in the horizontal axis. The line 1 corresponds to equation (2) of country \( I \) and line 2 corresponds to equation (1) of country \( C \). Two lines have been assumed to intersect at point \( A \). We must assume the values

\(^4\) It necessary implies that tax rates are not prohibitive and the solutions for labour migration are positive.
of $L_i$, $L_u$, $a_i$, $b_1$, $b_{u2}$ in a way that the conditions of intersection of the two lines in the positive quadrant are satisfied. To ensure stability it is also needed that the intercept of line 1 of country $I$ in vertical axis is higher than that of country $C$.

It is observed in Fig. 1 that a country can affect the other country’s labour migration by altering its own domestic tax rate. By reducing the domestic tax rate, country $C$ shifts the line 2 to line 3. The stability requirement implies that the new equilibrium of labour migration would be on the vertical axis where migration from country $I$ is nil. Nevertheless, it is not the case that a reduced tax rate is always associated with a net welfare gain for the labour sending country. The changes in tax rates can have beneficial or detrimental effects, which we analyse in the next sections.

3. **Imposition of a Domestic Tax**

In the previous section we have defined our model and found the equilibrium amount of labour migration given the domestic and foreign tax rates. In this section we analyse how the sending countries determine their domestic tax. It can be considered as the second stage of the game. We assume that the sending countries determine their domestic tax rates given the tax
rates already selected by the receiving country in the first stage of the game. To analyse the
process of determination of tax rates by the sending counties, we first look at the effects of the
domestic taxes on the equilibrium labour migration. Differentiating equations (3) and (4) with
respect to $D_i$ we obtain that the marginal changes of labour migration of the countries are,

$$\frac{\partial L_{i2}}{\partial D_i} = -V^{-1}(b_1 + b_{u2})$$

and,

$$\frac{\partial L_{j2}}{\partial D_j} = V^{-1}(b_{u2})$$

Adding the two we obtain

$$\frac{\partial L_{i2}}{\partial D_i} + \frac{\partial L_{j2}}{\partial D_j} = -V^{-1}(b_i).$$

Thus when a country increases its
domestic tax rate, labour migration of that country decreases and that of the other country
increases but in total the overall labour migration decreases. This result is obtained as the labour
migration of a country responses more in absolute value to the changes of a country’s own tax
rate, compared to the response it has to the changes of other country’s tax rate. As the country $i$
increases its domestic tax rate, the migrants of $i$ find that the income at home is higher than the
net income they receive by working in $U$. Hence some migrants withdraw from $U$. As wage
rate in $U$ increases, migration from $j$ increases, however in total, the stock of migrants
decreases.

Below we state one of the most important results of the paper:

**Proposition 1:** The optimal domestic tax rate is positive.
Proof:

In order to prove the proposition we first need to define the national income equations of the sending countries. The national income equation of country \( i \), after cancelling out the domestic tax, is written as

\[
Y_i = \left( a_i \left( L_i - L_{i2} \right) - \frac{b_i}{2} \left( L_i - L_{i2} \right)^2 \right) + \left( a_{u2} - b_{u2} \left( L_u + L_{j2} \right) - T_i \right) L_{i2}
\]

(5)

Hence national income is the summation of the output produced at home and the wage remitted by migrants working in \( U \). The domestic tax cancels out in the above equation as migrants cannot send the same amount as remittances that they have already paid to their countries as domestic tax. By differentiating the income equation of \( i \) in equation (5) we obtain the marginal change in the income,

\[
\frac{\partial Y_i}{\partial D_i} = -\left( a_i - b_i \left( L_i - L_{i2} \right) \right) \frac{\partial L_{i2}}{\partial D_i} + \left( a_{u2} - b_{u2} \left( L_u + L_{j2} \right) - T_i \right) \frac{\partial L_{i2}}{\partial D_i} - b_{u2} L_{i2} \frac{\partial L_{i2}}{\partial D_i} - b_{u2} L_{j2} \frac{\partial L_{j2}}{\partial D_i}
\]

Using the migration equilibrium conditions of equation (1) or (2):

\[5\] We assume that migrants remit all the income back to home. In reality migrants do keep some income in the country of residence, consume and make investment. Some migrants, especially permanent migrants, may not remit anything. We, however, assume full repatriation of remittances to simplify analysis. In the appendix 2, further calculations have been done assuming that migrants do not remit all the income home.
\[
\frac{\partial Y_i}{\partial D_i} = -V^{-1}(D_i(b_1 + b_{u2}) - b_{u2}L_{i2}b_1)
\]

Setting \(\frac{\partial Y_i}{\partial D_i} = 0\) we get \(D_i > 0\), or for \(D_i = 0\) we have \(\frac{\partial Y_i}{\partial D_i} = V^{-1}b_{u2}L_{i2}b_1 > 0\). Therefore the optimal domestic tax rate is positive. □

The proposition has substantial implications for the migration policies of developing and transitional countries. It shows that labour sending countries cannot depend only on remittances to maximise national income. They also need to control migration by imposing positive tax rates. The result can be compared with that of Brander and Spencer (1985) where the subsidy acts as a transfer but a positive subsidy is required for profit shifting effects. Here, however, we see that instead of a subsidy the country should tax the migrants. The reason why this result is obtained explained further as follows. When the domestic tax is zero, given any foreign tax, migration is at the maximum. The imposition of a positive domestic tax discourages some people from migration. Hence the marginal product of labour at home falls and the marginal product at \(U\) increases. The migration from the other sending country increases but that does not reduce the wage rate to the previous level. The difference between the remittances and the domestic wage rate of a country is given by the domestic tax rate of that country which is actually equal to the loss of per capita remittances. When the domestic tax is zero, the loss of per capita remittances is zero, hence domestic tax increases national income as the output of the country is now higher.

We are also interested to know if the positive domestic tax has anything to do with the existence of another country and a positive foreign tax. Let us first assume that country \(j\) is
inactive such that out of equation (1) and (2) we have only one equation\(^6\). The remaining
equation therefore has no \(L_{j2}\) term. Therefore differentiating the income equation,

\[
\frac{\partial Y_i}{\partial D_i} = -(a_1 - b_1(L_i - L_{i2})) \frac{\partial L_{i2}}{\partial D_i} + (a_{u2} - b_{u2}(L_u + L_{i2})) \frac{\partial L_{i2}}{\partial D_i} - b_{u2}L_{i2} \frac{\partial L_{i2}}{\partial D_i} - T_i \frac{\partial L_{i2}}{\partial D_i}
\]

Using the migration equilibrium condition stated in Footnote 5,

\[
\frac{\partial Y_i}{\partial D_i} = -\frac{D_i}{(b_1 + b_{u2})} + \frac{b_{u2}L_{i2}}{(b_1 + b_{u2})}
\]

Setting \(\frac{\partial Y_i}{\partial D_i} = 0\) we get \(D_i > 0\). Therefore the optimal domestic tax is again positive.

Thus, with or without the presence of competition from the other sender, a country should pursue
the policy of a positive domestic tax rate. Interestingly a positive domestic tax rate has nothing to
do with the foreign tax being positive. Irrespective of the foreign tax rate, the country must
pursue a positive domestic tax policy. The following proposition however shows that, in a
strategic situation, domestic taxes imposed by the countries do have effects on each other.

**Proposition 2:** Marginal increase in domestic tax rate by country \(i\) implies increase of income of
country \(j\).

\(^6\) Equilibrium condition is \(a_{u2} - b_{u2}(L_u + L_{i2}) - T_i - D_i = a_1 - b_1(L_i - L_{i2})\), the solution is,

\[
L_{i2} = \frac{-(a_1 - b_1L_i) + (a_{u2} - b_{u2}L_u) - T_i - D_i}{(b_1 + b_{u2})}
\]
Proof:

The income equation of \( j \) is

\[
Y_j = \left( a_1 (\overline{L}_j - L_{j2}) - \frac{b_1}{2} (\overline{L}_j - L_{j2})^2 \right) + \left( a_{u2} - b_{u2} (\overline{L}_u + L_{u2} + L_{j2}) \right) L_{j2} - T_j L_{j2} \quad (6)
\]

Differentiating the income equation of \( j \) in equation (6) with respect to \( D_i \) and after necessary calculation we get the marginal change of the income of \( j \) as

\[
\frac{\partial Y_j}{\partial D_i} = V^{-1} \left( D_j b_{u2} + b_{u2} L_{u2} b_i \right) > 0
\]

Hence the country \( j \) experiences an increase in income. \( \square \)

This proposition is quite obvious, but interesting. If country \( i \) increases its domestic tax, it reduces migration from \( i \) and increases wage rate in \( U \). Hence, with same migration, country \( j \) is better off. The migration from \( j \) increases, but not by that much to bring the wage rate down exactly to the previous level. Thus with higher wage rate and higher migration, \( j \) experiences higher income.

Therefore, the competition among the countries is not really harming each other. The countries have a unilateral incentive to use positive tax but it does not reduce the income of the other country, instead increases it. On the contrary, if a country decides to follow the policy of
subsidising migration or to reduce the domestic tax rate, it is harmful for the other country. As obtained in Proposition (1), a migration subsidy cannot be the optimal policy response of a sending country.

3.1. The Optimum Domestic Tax

As already mentioned, the selection of optimum domestic tax can be considered as the second stage of the game. In the first stage the leader \( U \) sets the foreign tax. In the second stage country \( C \) and \( I \) set domestic tax rates to maximise national incomes. Thus the game is to be solved using backward induction which involves finding the solutions for optimum domestic tax rates assuming the tax rates of \( U \) given. The country \( i \) when selecting the domestic tax rate \( D_i \) considers that the other country \( j \) keeps domestic tax rate \( D_j \) fixed. Hence the countries engage in a Bertrand type competition for setting domestic tax rates. We have already seen from Proposition (1) that \( i \) will have a positive domestic tax in order to maximise national income irrespective of the domestic tax rate of \( j \). Therefore positive solutions of domestic tax rates exist that satisfy the first order conditions of maximisation. The solutions (calculation in Appendix 1) for the domestic tax rates are

\[
D_C = H^{-1} ((w_{DC} - T_C)A - (w_{DI} - T_I)B) 
\]

\[
D_I = H^{-1} ((w_{DI} - T_I)A - (w_{DC} - T_C)B) 
\]
where $w_{Di}$, $H$, $A$ and $B$ are as defined in the Appendix. $H^{-1}$ is the inverse of $H$. Proposition (1) shows that optimum domestic tax rates are positive. But it is not directly observable from equations (7) and (8) as tax rates depend on the values of the other parameters of the model namely $a_i$, $a_{u2}$, $b_1$, $b_{u2}$, $\bar{L}_i$ and $\bar{L}_u$. Later we assume $b_1 = b_{u2} = 1$ to look at the solutions a bit more clearly. From equations (7) and (8) it can nevertheless be seen how changes in foreign tax rates have effects on domestic tax rates. This is stated in the following proposition:

**Proposition 3:** If the foreign tax rate for $i$ increases then the optimal domestic tax rate of $i$ decreases and that of $j$ increases.

Proof:

Differentiating the solutions of $D_i$ from (7) or (8) with respect to $T_i$ we get

$$\frac{\partial D_i}{\partial T_i} = -H^{-1}A$$

and

$$\frac{\partial D_j}{\partial T_i} = H^{-3}B$$

Therefore the optimal domestic tax rate for $i$ decreases and that of $j$ increases. □
This result is an important proposition from the policy makers’ perspective. If the tax rate for \(i\) is increased, then migration from that country decreases. This implies that the domestic tax rate for \(i\) is no longer in equilibrium and is higher than the amount required for controlling migration. Thus the equilibrium domestic tax rate of \(i\) decreases. On the other hand, more people from country \(j\) migrate to \(U\) as the wage rate is now higher. Thus the domestic tax rate for \(j\) becomes lower than the equilibrium value. Consequently, the equilibrium domestic tax rate of \(j\) increases.

It may be of some interest to look at the effects in national output of \(U\) when the sending countries increase the domestic tax rates. To see it note that the national output equation of \(U\) is

\[
Y_U = a_{u2}(T_u + L_{i2} + L_{j2}) - \frac{b_{u2}}{2}(T_u + L_{i2} + L_{j2})^2
\]

Therefore,

\[
\frac{\partial Y_U}{\partial D_i} = (a_{u2} - b_{u2}(T_u + L_{i2} + L_{j2})) \left( \frac{\partial L_{i2}}{\partial D_i} + \frac{\partial L_{j2}}{\partial D_i} \right).
\]

As \(\frac{\partial L_{i2}}{\partial D_i} + \frac{\partial L_{j2}}{\partial D_i} < 0\), national output decreases when the importing countries increase their domestic tax rates. It implies that \(U\) will prefer to have no domestic tax imposed by the sending countries.

4. **Imposition of Foreign Tax by \(U\)**

The first stage of the game involves setting of the foreign tax by country \(U\). We assume in this section that \(U\) sets \(T_i \neq T_j\) which implies possible discrimination. It is not possible in an integrated zone like European Union. However it is possible when migrants move from a
developing to a developed country by job search using the internet. For example, it is possible that the developed country may use different entry barriers for the entrants of different countries. This assumption of different tax rates brings us close to works like Gatsios (1990) and Hwang and Mai (1991) where the tax policies of the receiving country were analysed. The reason to adopt a similar model is the presence of market power of the rich developed countries. As the rich countries are powerful, it is often possible for them to move first by setting policies to fulfil their own objectives. The poor countries merely follow the path paved by such policies. Thus in our model the importing country \( U \) acts as a Stackelberg leader by setting its policies first. The country sets the tax rates contemplating that country \( C \) and \( I \) will set their domestic taxes to control migration after fixation of foreign tax rates. By foreseeing this \( U \) chooses the tax rates to maximise its objective function. Below, we state some results in propositions:

**Proposition 4:** If \( U \) increases the tax rate for \( i \) then the labour migration of \( i \) decreases and that of \( j \) increases. The total labour migration decreases.

Proof:

To prove the proposition we need to recall the solutions of labour migration in equations (3) and (4). \( L_{i2} \), that is, labour migration is function of \( D_i \) and \( T_i \). But \( D_i \) is also a function of \( T_i \). In the Appendix 2 it has been shown by further calculation that \( \frac{\partial L_{i2}}{\partial T_i} < 0, \frac{\partial L_{i2}}{\partial T_j} > 0 \) and

\[
\frac{\partial L_{i2}}{\partial T_i} + \frac{\partial L_{i2}}{\partial T_j} < 0. \square
\]
The proposition shows that the labour migration of country $i$ decreases and that of country $j$ increases. We have already illustrated that a country decreases the domestic tax rate if foreign tax rate of own migrants is increased, thereby partially offsets the rate of decrease of migration. However, if the other country’s foreign tax is increased then the country increases its own domestic tax rate, thereby partially offsets the rate of increase of migration. As the overall labour migration decreases, the imposition of positive tax rates by $U$ decreases the national output of $U$. Hence maximisation of national output cannot be the reason for a positive migration tax. Below we explore the cases where the country may use positive tax rates.

**Proposition 5:** $U$ may impose a positive foreign tax in order to maximise the income of the permanent residents.

**Proof:**

The objective of the receiving country may be to maximise the income of the permanent residents of the country. Thus it maximises the national output plus tax minus the wage of migrant workers. The objective function is therefore,

$$Y_{UN} = Y_U - \left( a_{u2} - b_{u2} \left( \bar{L}_u + L_{i2} + L_{j2} \right) \right) \frac{L_{i2} + L_{j2}}{L_{i2} + L_{j2}} + T_i L_{i2} + T_j L_{j2}$$

Differentiating with respect to $T_j$ and assuming tax rates are initially zero we obtain,
\[
\frac{\partial Y_{UN}}{\partial T_i} = b_{u2} L_{i2} + L_{j2} \left( \frac{\partial L_{i2}}{\partial T_i} + \frac{\partial L_{j2}}{\partial T_i} \right) + L_{i2}
\]

Thus the first part of the expression is negative whereas the second part of the expression is positive. Therefore, if a tax is imposed it can increase or decrease the income of residents. Therefore the country may also subsidise migration to increase national income.\(^7\) □

One question that may arise here is whether this tax rate has anything to do with competition of the sending countries. It actually does not alter the result significantly. If we have just one country the change in the income of permanent residents is
\[
\frac{\partial Y_{UN}}{\partial T_i} = b_{u2} L_{i2} \frac{\partial L_{i2}}{\partial T_i} + L_{i2}.
\]
As again the first term is negative and the second term is positive, the sign depends on the relative magnitude of the two terms.

Proposition (5) shows that the power of the receiving country depends on how sending countries respond to the changed tax rates. If \( \frac{\partial L_{i2}}{\partial T_i} \) and \( \frac{\partial L_{j2}}{\partial T_i} \) are almost equal in absolute value, then the first term of the above expression is almost zero and the national income of \( U \) may increase. That is, if the migrants of country \( i \) are to a large extent replaced by the migrants of country \( j \), then country \( U \) does not experience production and tax revenue losses and accordingly the income of residents increases. This is apparently happening in most developed countries. These countries are highly dependent on foreign labour force, but still maintain high

---

\(^7\) Further calculation here will not change the qualitative result. To see it let us assume that \( b_1 = b_{u2} = 1 \). Hence we obtain
\[
\frac{\partial Y_{U}}{\partial T_i} = -\frac{2}{7} \left( L_{i2} + L_{j2} \right) + L_{i2}.
\]
Therefore if \( L_{i2} = L_{j2} \), the income increases but if \( L_{i2} \neq L_{j2} \), income may increase or decrease.
entry barriers. The reason is probably the existence of large pools of skilled and unskilled workers in the global market who are willing to migrate and work in developed countries. These barriers are effectively increasing the entry cost for foreign labour force. Proposition (5) also suggesting that the countries may practise the policy of subsidising migrants.

Another issue we wish to analyse is the relationship between the labour endowment and the tax rate faced by a country. It is an important issue to analyse as it is well known that a monopolist discriminates on the basis of the willingness of a consumer to pay in purchasing a good. In our analysis we may regard the receiving country a monopolist who is allowing the migrants to work in its territory. Additional workers increase output, but the country acting as a monopolist may also discriminate in designing the tax policy with an aim to maximise the national income. Here is it very easy to discriminate as arbitrage is nearly impossible in international labour migration. We however do not completely ignore the possibility that migrants from a sending country can first move to the other sending country before moving to the receiving country.

To understand what may form the basis of discrimination, we work with a case where the country $U$ wants to maximise the income of its permanent native workers. Not surprisingly, if the country wants to maximise the income of permanent workers, that is, wage plus tax revenue, then it imposes a positive tax rate. The income of the permanent workers is,

$$Y_{LU} = (a_2 - b_2 \left( L_u + L_{i2} + L_{j2} \right))L_u + T_i L_{i2} + T_j L_{j2}$$

(10)

The calculation here is complicated. Hence we do the calculation using the assumption that $b_1 = b_{u2} = 1$. Though it eases the calculations a lot, we also need to remember that this
assumption does not automatically satisfy all the conditions stated in equations (1) to (4). Therefore we must assume that there exists $a_1$, $a_{u_2}$, $L_i$ and $L_u$ such that with $b_1 = b_{u_2} = 1$, they satisfy the conditions as stated in equations (1) to (4).

The calculations are given in Appendix 3. According to the simplification mentioned previously we obtain

$$T_i = 0.50L_i + 0.50w_{Di} \quad (11)$$

and,

$$T_j = 0.50L_u + 0.50w_{Dj} \quad (12)$$

where $w_{Di}$ and $w_{Dj}$ are as defined before. We are now in a position to see which country bears higher per capita tax burden and sends more worker. This is given in the following proposition:

**Proposition 6:** The foreign tax rate and labour migration is higher for the country with a higher labour endowment compared to the country with a lower labour endowment.

**Proof:**

In order to find which country receives higher tax burden, subtract equation (12) from (11):

$$T_i - T_j = 0.50(w_{Di} - w_{Dj}) = 0.50(L_i - L_j)$$
Therefore the foreign tax rate and labour migration are higher if labour endowment is higher. Further the calculations in Appendix 4 show that $L_{i2} > L_{j2}$ if $\bar{L}_i > \bar{L}_j$. □

The wage rate of the country with a higher labour endowment is low. Hence the difference between the foreign wage and the domestic wage is large. The receiving country can exploit this gap by imposing a higher per capita tax rate on the highly labour endowed country. The result is to some extent similar to that of Gatsios (1990) and Hwang and Mai (1991). In general it has been found that if the importing country practises a discriminatory tariff policy, the tariff rate is higher for the most cost-efficient exporter. In our case, the country with a higher labour endowment has a lower opportunity cost of migration, and that country faces higher tax burden. It is also interesting to note that (shown in Appendix 4) we obtain $D_i > D_j$ if $\bar{L}_i > \bar{L}_j$. Hence the sending country should also impose a higher tax rate if endowed with a large pool of willing migrants.

Proposition (6) is another important result of the paper. It shows that how the importing country may resort to discrimination. If a country is more endowed with labour, that is, where workers are relatively poor, that country receives a higher per capita tax burden. One may ask about the empirical validity of this finding. As already discussed, migrants do pay taxes in many forms, but such taxes shall be the same for all type of migrants. To find out discriminatory tax rates one needs to look at monetary and non monetary conditions attached to entry and residence. (Winters 2005) mentioned that the developed countries are currently entering into bilateral agreement with different countries to target specific skills from specific countries. How foreign
workers face facing discrimination in wage and non-wage terms remains as an interesting empirical issue to analyse further.

5. Conclusions

In this paper we worked with a model to analyse the tax/subsidy policies of two labour sending countries and one labour receiving country following the integration of labour markets. We assumed that migration was initiated by the wage differentials that existed between the sending and the receiving countries. Labour migration reached equilibrium when the net income from migration was equal to home wage rate. As migration could not be directly prohibited, the labour sending countries used taxes to control migration. The analysis showed that to maximise national income, sending countries should use a positive emigration tax. The result was contrary to the fact that many countries are now actually subsidising/encouraging migration. The result to some extent was in line with the idea of the ‘Bhagwati Tax’ (Bhagwati and Delleffar 1973) though, in this paper, tax was favoured not because of the externalities that arose from migration. Without tax too many people migrated, which made wage earned abroad low. An emigration tax restricted migration and wages earned by the migrants rose.

In analysing how a receiving country imposes a tax we have found that to maximise the income of permanent residents, the country might impose a positive or negative (subsidy) immigration tax. However, when the objective of the country was to maximise the income of permanent workers then it resorted to a positive tax. We have found that it imposed different tax rates for different countries and the country with higher labour endowment had higher tax capita
tax burden. As the wage rate in the country with more labour endowment was low, the earnings they received from migration were initially relatively high. The receiving country exploited this gap by imposing a higher tax rate.

What lessons can we learn from the analysis? The global labour market is moving toward integration because of the internet despite the barriers used by many countries. Mostly, skilled workers migrate in this way but sooner or later migration of unskilled workers will follow this path. The paper is suggesting that subsidising migration for remittances is not a correct policy when labour markets are integrated.

The policy recommendation for the developed countries is that they should pursue a discriminatory tax policy. To what extent it is happening is an empirical question but we need to note that tax rate in the paper acted as a proxy of monetary barriers of entry and residence. Migrants from developing countries are in reality facing relatively higher barriers but as the income gap is very high, the barriers are not overpowering the willingness of people to emigrate. The barriers can come in the form of visa processing time, high application cost, mandatory medical check-ups or language proficiency requirement. In addition, the immigrants remain ineligible to receive social benefits for a substantial period of time (e.g. 5 years), though they may remain liable to pay all types of taxes for the whole period of time. Because of a high income gap, these barriers have little effect on emigration.

In summary, this paper is suggesting that the developing and transitional countries should not take the benefits of migration for granted. Policies need to be devised to control migration and secure the benefits. If the outflow of people is not beneficial then the country should increase exit barriers. In this regard the governments can consider taxing the organisations providing emigration services. This will be an immense task given the social-political influence of the
groups to be effected by such policy changes. Another solution is to ensure the benefit of all parties by mutually beneficial negotiation such that the whole world acts cooperatively on migration matters. Given the segmentations of the world, such negotiation is unlikely to take place in the near future.
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References


Appendix

1. Calculation of domestic tax

The national income equation of country $C$ is given as

$$Y_C = \left( a_1 (\overline{L}_C - L_{C2}) - \frac{b_1}{2} (\overline{L}_C - L_{C2})^2 \right) + \left( a_{u2} - b_{u2} (\overline{L}_u + L_{C2} + L_{I2}) \right) L_{C2} - T_C L_{C2}$$

Differentiating with respect to $D_C$ and setting the derivative zero we obtain,

$$\frac{\partial Y_C}{\partial D_C} = -\left( a_1 - b_1 (\overline{L}_C - L_{C2}) \right) \frac{\partial L_{C2}}{\partial D_C} + \left( a_{u2} - b_{u2} (\overline{L}_u + L_{C2} + L_{I2}) \right) \frac{\partial L_{C2}}{\partial D_C} - b_{u2} L_{C2} \frac{\partial L_{C2}}{\partial D_C}$$

$$- b_{u2} L_{C2} \frac{\partial L_{I2}}{\partial D_C} - T_C \frac{\partial L_{C2}}{\partial D_C} = 0$$

Using migration equilibrium condition of equation (1):

$$\frac{\partial Y_C}{\partial D_C} = D_C \frac{\partial L_{C2}}{\partial D_C} - b_{u2} L_{C2} \frac{\partial L_{C2}}{\partial D_C} - b_{u2} L_{C2} \frac{\partial L_{I2}}{\partial D_C} = 0$$

Now using the equation for equilibrium labour migration of (3) and (4) and as $w_{Di} = -(a_i - b_i \overline{L}_i) + (a_{u2} - b_{u2} \overline{L}_u)$ we obtain by further calculation the reaction function of country $C$:
\[- D_c (b_1 + b_{u_2} (b_1 + 3b_{u_2}) + D_k b_{u_2} b_{u_2} = -b_{u_2} (b_1 + b_{u_2}) (w_{DC} - T_c) + b_{u_2} b_{u_2} (w_{DI} - T_l) \]

Similarly the reaction function of country $I$ is obtained as

\[ D_c b_{u_2} b_{u_2} - D_i (b_1 + b_{u_2} (b_1 + 3b_{u_2}) = -b_{u_2} (b_1 + b_{u_2}) (w_{DI} - T_l) + b_{u_2} b_{u_2} (w_{DC} - T_c) \]

The second derivatives have signs as required for stability as:

\[
\frac{\partial^2 Y}{\partial D_i^2} = -b_{u_2}^2 \left( \frac{b_1 + b_{u_2}}{b_1 (b_1 + 2b_{u_2})^2} \right) < 0
\]

\[
\frac{\partial^2 Y}{\partial D_i \partial D_j} = \frac{b_{u_2}^2}{b_1 (b_1 + 2b_{u_2})^2} > 0
\]

From the two equations the following two solutions for the domestic tax rates are obtained:

\[ D_c = H^{-1}((w_{DC} - T_c) A - (w_{DI} - T_l) B) \]

\[ D_l = H^{-1}((w_{DI} - T_l) A - (w_{DC} - T_c) B) \]
where,

\[ H = (b_1 + b_{u2})^2 (b_1 + 3b_{u2})^2 - (b_{u2})^4 = b_1^4 + 8b_1^3 b_{u2} + 22b_1^2 b_{u2}^2 + 24b_1 b_{u2}^3 + 8b_{u2}^4 > 0 \]

\[ A = b_{u2} ((b_1 + b_{u2})(b_1 + 3b_{u2}) - b_{u2}^3) = b_1^3 b_{u2} + 5b_1^2 b_{u2}^2 + 7b_1 b_{u2}^3 + 2b_{u2}^4 > 0 \]

\[ B = b_{u2}^2 (b_1 + 2b_{u2}) = b_1^2 b_{u2}^2 + 3b_1 b_{u2}^3 + 2b_{u2}^4 > 0 \]

2. Calculation of proposition 1 assuming migrates do not remit all the income to home

The national income equation of country \( C \) is given as

\[ Y_C = \left( a_1 (L_C - L_{C2}) - \frac{b_1}{2} (L_C - L_{C2})^2 \right) + \delta \left( a_{u2} - b_{u2} (L_{u} + L_{C2} + L_{I2} - D_C - T_C) \right) L_{C2} + D_C L_{C2} \]

Where \( 0 \leq \delta \leq 1 \). If \( \delta = 1 \), migrants send all the income back home. Differentiating with respect to \( D_C \) we obtain,

\[ \frac{\partial Y_C}{\partial D_C} = -\left( a_1 - b_1 (L_C - L_{C2}) \right) \frac{\partial L_{C2}}{\partial D_C} + \left( a_{u2} - b_{u2} (L_{u} + L_{C2} + L_{I2}) \right) \frac{\partial L_{C2}}{\partial D_C} - b_{u2} L_{C2} \frac{\partial L_{C2}}{\partial D_C} + L_{C2} \frac{\partial L_{C2}}{\partial D_C} + (1 - \delta)D_C \frac{\partial L_{C2}}{\partial D_C} + (1 - \delta)L_{C2} \]

Using the labour market equilibrium condition of (1),

\[ \frac{\partial Y_C}{\partial D_C} = -D_C \frac{\partial L_{C2}}{\partial D_C} - b_{u2} L_{C2} \frac{\partial L_{C2}}{\partial D_C} - b_{u2} L_{C2} \frac{\partial L_{I2}}{\partial D_C} + (1 - \delta)D_C \frac{\partial L_{C2}}{\partial D_C} + (1 - \delta)L_{C2} \]
Assuming \(D_c > 0\) we obtain that \(\frac{\partial Y_c}{\partial D_c} > 0\). It implies that when tax rate is zero imposition of a positive tax increases the income. The result is hence at par with proposition 1. However note that we cannot get an unambiguous result using first order condition that is assuming, \(\frac{\partial Y_c}{\partial D_c} = 0\).

3. **Proposition 4**

By differentiating equation (3) and using solutions from equation (7), (8) and Proposition (3) we obtain,

\[
\frac{\partial L_{i2}}{\partial T_i} = V^{-1}\left( - (b_1 + b_{u2}) - \frac{\partial D_i}{\partial T_i}(b_1 + b_{u2}) + \frac{\partial D_j}{\partial T_j}b_{u2} \right) = V^{-1}\left( (b_1 + b_{u2}) + H^{-1}A(b_1 + b_{u2}) + H^{-1}Bb_{u2} \right)
\]

Similarly from equation (4) we obtain,

\[
\frac{\partial L_{i2}}{\partial T_i} = V^{-1}\left( - \frac{\partial D_i}{\partial T_i}(b_1 + b_{u2}) + b_{u2} + \frac{\partial D_j}{\partial T_j}b_{u2} \right) = V^{-1}\left( b_{u2} - H^{-1}B(b_1 + b_{u2}) - H^{-1}Ab_{u2} \right)
\]

As

\[
H^{-1}A = \frac{b_1 b_{u2}^3 + 5b_1^2 b_{u2}^2 + 7b_1 b_{u2}^3 + 2b_{u2}^4}{b_1^4 + 8b_1^3 b_{u2} + 22b_1^2 b_{u2}^2 + 24b_1 b_{u2}^3 + 8b_{u2}^4} < 1,
\]

35
\[ H^{-1}B = \frac{b_1^2 b_{u_2}^2 + 3b_1 b_{u_2}^3 + 2b_{u_2}^4}{b_1^4 + 8b_1^3 b_{u_2} + 22b_1^2 b_{u_2}^2 + 24b_1 b_{u_2}^3 + 8b_{u_2}^4} < 1 \] and
\[ H^{-1}A + H^{-1}B = \frac{b_1^3 b_{u_2} + 6b_1^2 b_{u_2}^2 + 10b_1 b_{u_2}^3 + 4b_{u_2}^4}{b_1^4 + 8b_1^3 b_{u_2} + 22b_1^2 b_{u_2}^2 + 24b_1 b_{u_2}^3 + 8b_{u_2}^4} < 1 \]

We obtain,
\[
\frac{\partial L_{i_2}}{\partial T_i} = V^{-1} \left[ -b_1 \left( 1 - H^{-1}A \right) - b_{u_2} \left( 1 - H^{-1}A - H^{-1}B \right) \right] < 0
\]

and
\[
\frac{\partial L_{i_2}}{\partial T_i} = V^{-1} \left( b_{u_2} - H^{-1}B b_{u_2} - H^{-1}A b_{u_2} - H^{-1}B b_{i_2} \right)
\]
\[= V^{-1} \left( \frac{b_1^4 b_{u_2} + 6b_1^3 b_{u_2}^2 + 13b_1^2 b_{u_2}^3 + 12b_1 b_{u_2}^4 + 6b_{u_2}^5}{b_1^4 + 8b_1^3 b_{u_2} + 22b_1^2 b_{u_2}^2 + 24b_1 b_{u_2}^3 + 8b_{u_2}^4} \right) > 0\]

In addition,
\[ A - B = b_{u_2} \left( (b_1 + b_{u_2})(b_1 + b_{u_2})(b_1 + 3b_{u_2}) - b_{u_2} \right) - b_{u_2} \left( b_1 + b_{u_2} \right) (b_1 + 2b_{u_2})
\]
\[= b_{u_2} \left[ \left( b_1^2 + 2b_1 b_{u_2} + b_{u_2}^2 \right) (b_1 + 3b_{u_2}) - b_{u_2} \left( b_1^2 + 3b_1 b_{u_2} + 2b_{u_2}^2 \right) \right]
\]
\[= b_{u_2} \left[ b_1^3 + 2b_1^2 b_{u_2} + b_1 b_{u_2}^2 + 3b_1^2 b_{u_2} + 6b_1 b_{u_2}^2 + 3b_{u_2}^3 - b_{u_2}^3 \right] - \left( b_1^2 b_{u_2} + 3b_1 b_{u_2}^2 + 2b_{u_2}^3 \right]
\]
\[= b_{u_2} \left[ b_1^3 + 5b_1^2 b_{u_2} + 7b_1 b_{u_2}^2 + 2b_{u_2}^3 - b_1^2 b_{u_2}^2 - 3b_1 b_{u_2}^2 - b_{u_2}^3 \right]
\]
\[= b_{u_2} \left[ b_1^3 + 2b_1^2 b_{u_2} + 4b_1 b_{u_2}^2 \right] = b_{u_2} b_1^3 + 4b_1 b_{u_2}^2 + 4b_1 b_{u_2}^3
\]

Therefore
\[
\frac{\partial L_{ij}}{\partial T_i} + \frac{\partial L_{ij}}{\partial T_j} = V^{-1}b_1\left( -1 + H^{-1}A - H^{-1}B \right) = \\
V^{-1}b_1\left( -1 + \frac{b_u b_i^3 + 4b_u^2 b_{i2} b_i b_i}{b_i^4 + 8b_i^3 b_{i2} + 22b_i^2 b_{i2}^2 + 24b_i b_{i2}^3 + 8b_{i2}^4} \right) < 0
\]

4.

The income of permanent workers is given as

\[
Y_{LL} = (a_{u2} - b_{u2}(T_u + L_{i2} + L_{j2}))L_u + T_i L_{i2} + T_j L_{j2}
\]

By differentiating

\[
\frac{\partial Y_{LL}}{\partial T_i} = -b_{u2} T_u\left( \frac{\partial L_{i2}}{\partial T_i} + \frac{\partial L_{i2}}{\partial T_j} \right) + L_{i2} + T_i \frac{\partial L_{i2}}{\partial T_i} + T_j \frac{\partial L_{i2}}{\partial T_j}
\]

When tax rates are initially zero, the value of the derivatives are positive as

\[
\frac{\partial Y_{LL}}{\partial T_i} = -b_{u2} T_u\left( \frac{\partial L_{i2}}{\partial T_i} + \frac{\partial L_{i2}}{\partial T_j} \right) + L_{i2} > 0
\]
From Proposition (4) we have \( \left( \frac{\partial L_{i2}}{\partial T_i} + \frac{\partial L_{j2}}{\partial T_j} \right) < 0 \). Therefore, the country uses positive foreign tax on the migrants. By differentiating again with respect to \( T_i \) and \( T_j \) we see that the second derivatives have desired signs as required for stability:

\[
\frac{\partial^2 Y_{LU}}{\partial T_i^2} = 2 \frac{\partial L_{i2}}{\partial T_i} < 0 \quad \text{and} \quad \frac{\partial^2 Y_{LU}}{\partial T_i \partial T_j} = 2 \frac{\partial L_{i2}}{\partial T_j} > 0
\]

In order to obtain the solutions of tax rates we set the derivatives equal to zero. We substitute \( L_{i2} \) and \( L_{j2} \) from (3) and (4) and then substitute \( D_i \) and \( D_j \) from (7) and (8). Only carrying the calculation for tax rate for country \( C \) we get

\[
T_C \left( -2(b_1 + b_{u_2}) + 2H^{-1}A(b_1 + b_{u_2}) + 2H^{-1}Bb_{u_2} \right) + T_1 \left( 2b_{u_2} - 2H^{-1}Ab_{u_2} - 2H^{-1}B(b_1 + b_{u_2}) \right)
\]

\[
= b_{u_2} T_u b_1 \left( -1 + H^{-1}(A - B) \right) - w_{DC}(b_1 + b_{u_2}) + H^{-1}\left( (w_{DC})A - (w_{DI})B \right)(b_1 + b_{u_2})
\]

\[
+ w_{DI} b_{u_2} - H^{-1}\left( (w_{DI})A - (w_{DC})B \right) b_{u_2}
\]

where \( w_{DI} \), \( H \), \( A \) and \( B \) are as defined before. We also need to carry out the calculations for country \( I \) too and then can find the solutions using Cramer’s rule. Assuming \( b_1 = b_{u_2} = 1 \) we obtain following solutions,

\[
T_i = 0.50T_u + 0.50w_{DI} \quad (11)
\]

and,

\[
T_j = 0.50T_u + 0.50w_{DJ} \quad (12)
\]
Note that we have also utilised the fact that the solution of domestic tax rates in equations (7) and (8) are given now as \( D_i = \frac{1}{21}(5(w_{D_i} - T_i) - 2(w_{D_j} - T_j)) \).

5.

\[
L^*_c - L^*_{12} = V^{-1}(G_c(b_1 + b_{u_2}) - G_t b_{u_2}) - V^{-1}(G_t(b_1 + b_{u_2}) - G_c b_{u_2}) \\
= V^{-1}(b_1 + 2b_{u_2})G_c - G_t = (w_{D_i} - T_i - D_i - w_{D_j} + T_j + D_j) \\
= \frac{1}{3}(- (a_i - \overline{L}_i) + (a_{u_2} - \overline{L}_u) + (a_i - \overline{L}_j) - (a_{u_2} - \overline{L}_u)) \\
= \frac{1}{3}(- (a_i - \overline{L}_i) + (a_i - \overline{L}_j)) = \frac{1}{3}(\overline{L}_i - \overline{L}_j)
\]

Also

\[
D_i - D_j = \frac{1}{21}(5(w_{D_i} - T_i) - 2(w_{D_j} - T_j)) - \frac{1}{21}(5(w_{D_j} - T_j) - 2(w_{D_i} - T_i)) \\
= \frac{1}{3}(0.50w_{D_i} - 0.50w_{D_j}) = \frac{1}{6}(- (a_i - \overline{L}_i) + (a_{u_2} - \overline{L}_u) + (a_i - \overline{L}_j) - (a_{u_2} - \overline{L}_u)) \\
= \frac{1}{6}(- (a_i - \overline{L}_i) + (a_i - \overline{L}_j)) = \frac{1}{6}(\overline{L}_i - \overline{L}_j)
\]
Fig 1. Labour Migration Equilibrium