Emergence of Option Prices in Markets Populated by Portfolio-Holders

Sarvar Abdullaev[1] and **Peter McBurney**[2] and **Katarzyna Musial**[3]

Abstract. Options constitute integral part of modern financial trades, and are priced according to the risk associated with buying or selling certain asset in future. Financial literature mostly concentrates on risk-neutral methods of pricing options such as Black-Scholes model. However, using trading agents with utility function to determine the option's potential payoff is an emerging field in option pricing theory. In this paper, we use one of such methodologies developed by Othman and Sandholm to design portfolio-holding agents that are endowed with popular option portfolios such as bullish spread, bearish spread, butterfly spread, straddle, etc to price options. Agents use their portfolios to evaluate how buying or selling certain option would change their current payoff structure. We also develop a multi-unit direct double auction which preserves the atomicity of orders at the expense of budget balance. Agents are simulated in this mechanism and the emerging prices are compared to risk-neutral prices under different market conditions. Through an appropriate allocation of option portfolios to trading agents, we can simulate market conditions where the population of agents are bearish, bullish, neutral or non-neutral in their beliefs.

1 Introduction

The classic finance literature on derivatives is mostly based on Black-Scholes framework [1] which prices options from the perspective of no arbitrage assumption. According to this assumption, if there is a strategy with other financial instruments in the market which could simulate the payoff structure of the created financial contract, the value of such contract must be equal to the total cost of running this strategy. European option is one example of such financial contracts that gives the right to its holder to buy or sell certain asset at an agreed price in future. Black and Scholes showed that the payoff from holding an option (i.e. European option onwards) can be replicated by taking positions in two different markets: one is risk-free investments market, and the other is risky assets market. There is a mathematical solution which requires the parameters of these underlying markets to be set in order to compute the *risk-neutral* price of given option. In its initial formulation, Black-Scholes framework models the risky underlying market as Geometric Brownian Motion (GBM) which also implies the efficiency of that market. Moreover, the risk-free market was assumed to be static, so that the risk-free rate the investor has chosen to price the option remained constant throughout the lifespan of the option. Since then, similar models have been developed under different assumptions about the underlying market, and some of the important ones are Black-Scholes-Merton model which assumes that the asset prices are discontinuous [11] and Heston model which assumes that the volatility of the asset prices also changes according to certain stochastic model [7].

However it is known that contemporary financial markets are populated with heterogeneous traders using different methodologies that model the behaviour of markets. These are the crucial factors for each trader to make buying or selling decision. There have been many researches which propose agent-oriented approaches to price options, in contrast to the previous models which were directed at modeling the behaviour of the markets as a whole. In agent-oriented approaches, the behaviour of an individual trader is designed, so they can be simulated to obtain aggregate prices. This approach is also referred as *indifference pricing*, so the agents are indifferent to the exposed risk of buying or selling a contract based on their individual utility function. Another important aspect of indifference pricing is that there is no unique price as it happens to be in monolithic frictionless markets described in classic finance, but different bidding and asking quotes for each agent using different utility function. Gerber and Pafum described risk-averse traders based on an exponential utility function which could produce a bid-ask spread around risk-neutral option prices [4]. The width of the bid-ask spread could be specified by the risk-averseness factor of the exponential utility function. The other application of utility functions in pricing derivatives can be the use of intrinsic aspects of the agent's implementation such as portfolio, budget constraints, transaction costs and the other market related frictions. Carmona [2] and Henderson etal [6] provide extensive overview of indifference pricing methodologies used in practice.

Beyond designing the trader's behaviour, there has been a considerable advancement in designing market mechanisms too. Such mechanisms could determine the equilibrium prices and efficient allocations of goods from the corresponding behaviour of participating agents. Typical implementations of auctions have realised these objectives along with other important properties such as incentive compatibility, individual rationality and budget-balance. Strategyproof auctions can always guarantee truthfulness of participating agents through revelation principle and some even allow traders to be more expressive in revealing their combinatorial preferences. Parkes etal [13] and Parsons etal [14] provided up-to-date survey of different auction protocols, their designs and implementations.

The key question that we are posing in this paper is what option prices emerge if the traders come to the market already owning some option portfolios, and they also make their pricing decisions based on this fact. How would the prices be different from traditional risk-neutral prices? In this paper, we develop an agent-based system which uses direct double auction mechanism designed for trading multi-unit orders which also preserve the atomicity of orders. We

[1] King's College London, UK, email: sarvar.abdullaev@kcl.ac.uk
[2] King's College London, UK, email: peter.mcburney@kcl.ac.uk
[3] Bournemouth University, UK, email: kmusialgabrys@bournemouth.ac.uk

use inventory-based Logarithmic Market Scoring Rule (LMSR) option trader developed by Othman and Sandholm [12] to enable the option pricing based on the trader's current portfolio. Such agents price options based on the payoff structure of their current portfolio. For example, if the agent already owns short and long market positions on certain number of Out-of-The-Money (OTM), At-The-Money (ATM) and In-The-Money (ITM) options, his payoff can be different based on the underlying asset price on maturity date. And by pricing any given option with respect to his portfolio, agent computes the logarithmic score difference between his current payoff structure and the new payoff structure after buying or selling the option. We endow the LMSR traders with commonly used option portfolios such as *bullish spread*, *bearish spread*, *butterfly spread* etc. and run them in our proposed mechanism. This allows us to set up different scenarios in the market and observe the changes in option prices to evaluate their sensitivity to certain factors such as changes in the asset price or time-to-maturity of the option. The resulted prices are also compared with the theoretical Black-Scholes prices as well as their theoretical sensitivity to different factors. Besides that, we show the range of accepted bids and asks on each trading day, and analyse the relative efficiency and the distribution of differences in mechanisms budget. The key contribution of this paper is that it shows how option prices may differ from the theoretical prices when the option traders utility is based on their corresponding portfolios. Moreover, it proposes a new methodology in option pricing via double auctions which would enable the analysts to mix different indifference pricing methodologies together to obtain competitive option prices.

2 Options

Option is the type of financial derivative that enables its holder (i.e. owner) to buy or sell specified assets at certain future price to writer (i.e. issuer) of the option. Holder of the option buys for an additional cost (i.e option premium) determined by the market or the writer of the option. On the other hand, the writer of the option sells by taking future obligation to trade assets if holder chooses to exercise his right to buy or sell. Option contract must specify the underlying asset to be traded, its *volume*, *strike price* and expiration date. European options can be exercised only on their maturity date, while American options on any date until expiration. We will use only European options in the scope of this paper.

Options are defined as *put* or *call* options depending on rights and obligations that they bear. Put option gives its holder the right to sell underlying assets at agreed strike price, where the writer has the liability to buy them when holder exercises his right. Call option gives its holder the right to buy at agreed strike price, while the writer has the liability to sell. Option's value usually depends on several parameters of the underlying market such as spot asset price S_0, risk-free rate r and asset price volatility σ, and the conditions written in the option contract such as strike price K and time to maturity T. The other parameter of the asset (if it is a company stock) is the dividend it yields annually. This is normally subtracted from the overall return the asset is likely to make, but in this paper, we assume that the asset does not yield any dividend. There is an established relationship between put and call options with the same strike price and maturity date. This relationship results from the possibility of buying the one and selling the other. Using the put-call parity relationship, we can easily convert call prices to put prices, and vice versa. Therefore in our simulation, we only price call options, because the put price can be directly obtained from call price. Interested reader can look up the Black-Schole's formula[1] for risk-neutral pricing of options. Option

	OTM	ATM	ITM
CALL	$K > S_t$	$K = S_t$	$K < S_t$
PUT	$K < S_t$	$K = S_t$	$K > S_t$

Table 1: Options by Moneyness

belongs to different *moneyness* range at any given time t depending on whether its strike is greater or less than the current asset price. Table 1 summarises the options by moneyness.

2.1 Greeks

Greeks analysis provides set of measurements for evaluating the sensitivity of option price on different factors in the market. They have important role in hedging portfolios and evaluating the volatility of the asset prices. We consider two of them for purpose of our analysis of option prices obtained from the simulation.

1. *Delta* $\Delta = \frac{\partial c}{\partial S}$: It measures the rate of change of the option price with respect to the change in price of the underlying asset. For example, if the delta of the call option is 0.4, then it means that if there is small change in the underlying asset's price, there will be a change in call options price in 0.4 of that amount. Delta is defined as the partial derivative of option's price function with respect to underlying asset price.
2. *Theta* $\Theta = -\frac{\partial c}{\partial \tau}$: It can be defined as the sensitivity of the option price to the passage of time, or 'time decay'. Its value is always negative, as option price becomes less sensitive to time as it approaches its maturity date. In other words, the payoff the option yields is more certain near its maturity.

2.2 Option Portfolios

We review different types of option portfolios used in practice. Traders can take different positions with options of different moneyness and create option portfolios which can align with their forecast and at same time limit their loss in case if their forecast is not true. Cohen counts more than 40 option portfolios and classifies them based on their market direction (i.e. bullishness or bearishness), volatility level, riskiness and gain [3]. We will not use all of them, but consider only the ones used in the scope of this work.

Let us consider, butterfly spread. This type of spread involves taking positions in options with three different strike prices. In butterfly call spread, trader has an estimate that the price is not going to change sharply, so he wants to stay neutral. He buys 2 call options: one ITM with low K_1 and one OTM with high K_3. At the same time, he sells 2 ATM calls with K_2, where K_2 is halfway between the range of K_1 and K_3. This spread leads to a profit if the asset price will not go far from its current spot price. It will incur in fixed loss if the asset price changes sharply in either directions. Butterfly spread can be created using put options as well. Figure 1 illustrates payoff structure of a butterfly call spread.

We can summarize the option portfolios used in the scope of this paper in Table 2 where c_A stands for ATM call, p_A ATM put, and so forth.

3 Portfolio-holding Trading Agent

In prediction markets [15], the informants are allowed to change the aggregator's payoff structure for a corresponding payment. For example, if aggregator is accepting bets for teams A and B on a football

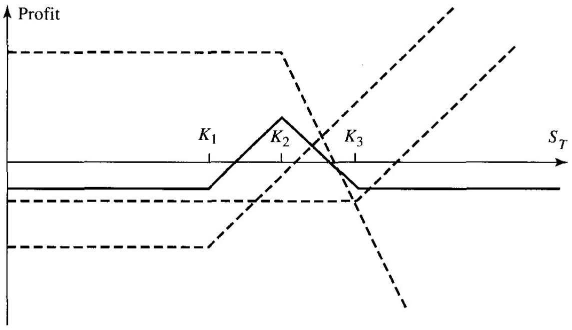

Figure 1: Butterfly Spread with Call Options.

Name	c_A	p_A	c_O	p_O	c_I	p_I
Bull Call Spread	0	0	-1	0	1	0
Bear Call Spread	0	0	1	0	-1	0
Butterfly Call Spread	-2	0	1	0	1	0
Long Call Ladder	-1	0	-1	0	1	1
Short Call Ladder	1	0	1	0	-1	0
Iron Butterfly	-1	-1	1	1	0	0
Long Straddle	1	1	0	0	0	0
Long Strangle	0	0	1	1	0	0
Short Strangle	0	0	-1	-1	0	0
Strip	1	2	0	0	0	0

Table 2: Some of the Popular Option Portfolios

match, and his current payoff structure is (300,200) meaning that the aggregator has to pay \$300 in total if team A wins, and \$200 if team B wins. However one would like to bet on team A, and he expects to receive \$50 if his bet is achieved. The aggregator changes his payoff structure to (350,200) by accepting the bet, and he also needs to decide how he can charge the client for accepting his bet. The most common method for evaluating the cost of accepting the bet in prediction markets, LMSR [5] and it is defined as a cost function for the vector of payoffs $\mathbf{x} = \{x_1, x_2, \ldots, x_n\}$ on the probability space of events $\Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$:

$$C(\mathbf{x}) = b \log \left(\sum_i \exp(x_i/b) \right) \tag{1}$$

where $b > 0$ is a liquidity parameter. The larger values of b produce tighter bid/ask spreads, but may also incur larger worst-case losses capped by $b \log(n)$ [12].

The agent who wishes to change the payoff from $\mathbf{x}$ to $\mathbf{y}$ has to pay the difference between the costs $C(\mathbf{y}) - C(\mathbf{x})$. In our above example, given that $b = 100$, the aggregator accepting bets must charge the client $C((350, 200)) - C((300, 200)) \approx$ \$39 for the bet.

The same principle can be used for the option trader who holds a certain portfolio of options that generate certain payoffs for different asset price outcomes, and prices other options from the point of his own payoff structure. The agent can virtually simulate buying or selling particular type of option and compute the changes it makes to his current payoff structure. For example, let agent take butterfly call spread by buying ITM call at strike $K_1 = 80$ and OTM call at $K_3 = 120$, and selling 2 ATM calls at $K_2 = 100$. We can compute his discounted payoffs for the range of possible prices where the asset price can end up at time T. Let this payoff structure be $\mathbf{x}$, and it can be depicted as in Table 3. The trader feels bullish and wants to buy one more call option at strike $K_4 = 130$. This should change his payoff structure to $\mathbf{y}$ as shown in Table 4.

Asset Prices	**Payoffs x**
< 70	0.00
75	0.00
80	0.00
85	4.75
90	9.51
95	14.26
100	19.02
105	14.27
110	9.51
115	4.75
120	0.00
> 125	0.00

Table 3: Potential payoffs of the trader holding butterfly call spread BEFORE buying call at $K_4 = 130$

Asset Prices	**Payoffs y**
< 70	0.00
75	0.00
80	0.00
85	4.75
90	9.51
95	14.26
100	19.02
105	14.27
110	9.51
115	4.75
120	0.00
125	0.00
130	0.00
135	4.75
140	9.51
> 145	$e^{-rT}(S_t - 130)$

Table 4: Potential payoffs of the trader holding butterfly call spread AFTER buying call at $K_4 = 130$

As it can be seen from tables 3 and 4, buying a certain type of option can significantly change the payoff structure of the trader. The agent's bid for buying an OTM option at $K_4 = 130$ can be computed from the difference of his payoff structures $C(\mathbf{y}) - C(\mathbf{x})$, and in our particular case is \$1.42 given that the liquidity parameter is $b = 2500$. We can also compute the Black-Scholes price of such option given parameters $T = 1$, $r = 0.05$, $S_0 = 100$, $\sigma = 0.02$ and it is \$2.52. This would mean that the trader places a bid for given OTM option less than its risk-neutral value.

We have simulated bids and asks for the call option with different strikes setting the liquidity parameter $b = 100$ and compared it with Black-Scholes prices. Figure 2 illustrates the bids and asks of LMSR trader holding butterfly call spread. This shows the breadth of bid-ask spread for the call option under different strikes.

To sum up, the indifference pricing methods like LMSR takes into account the agent's current portfolio and make option pricing decisions based on this information. We can use this principle in populating the option market by traders holding various portfolios, and observe the resulted option prices.

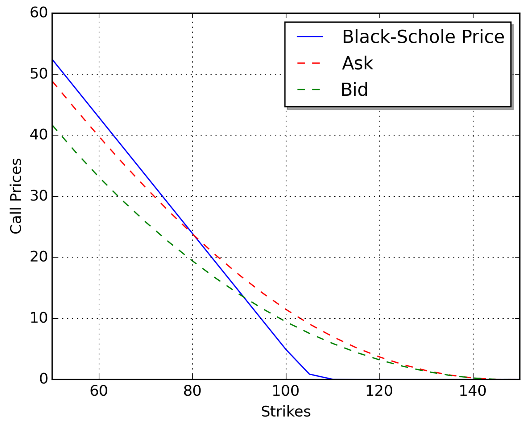

Figure 2: Bids and asks of LMSR trader holding butterfly call spread in comparison with Black-Scholes prices.

4 Direct Double Auction

We extend McAfee's double auction [10] to a multi-unit double auction, but in the process we have to give up its weakly budget-balanced property and introduce an agent who has to subsidise the exposed multi-unit bid or ask in order to preserve strategyproofness of the mechanism and the atomicity of orders. There have been a number of multi-unit double auction designs proposed previously [8, 9] which support weak budget-balance property. However these mechanisms partially satisfy the orders to spread the burden of overdemand or oversupply. In this design, we propose a multi-unit double auction that preserves the atomicity of orders at the expense of budget-balance. The reason for such a requirement is that it is crucial for the option trader who uses option portfolios. Because option portfolio determines exactly at which quantity each type of option needs to be sold or bought, trader cannot take quantity less than requested. The violation of atomicity of orders would result in the distortion of option portfolio as a whole.

Consider multi-unit bid as a tuple $\mathbf{b}_i = (b_i, q_i)$ where b_i is per unit bid, and q_i is the amount demanded. The same is defined for multi-unit ask. We can split this tuple into set of equally-valued single-unit bids $\mathbf{b}_i = \bigcup_{t=1}^{q_i} b_{i,t}$. This can be done to asks as well. Then we have complete set of bids $\mathbf{b} = \bigcup_{i=1}^{n} \mathbf{b}_i$ and asks $\mathbf{a} = \bigcup_{i=1}^{n} \mathbf{a}_i$. We can use single-unit McAfee's mechanism to find the allocation and payment. However, we can observe below that not all bids/asks can be fully satisfied.

Lemma 4.1. *In multi-unit McAfee's mechanism, there exists at most one multi-unit bid/ask which is partially satisfied, and the remaining winning bids/asks are fully satisfied.*

Proof. Let us assume that we use McAfee's matching rule for expanded set of single-unit bids $\mathbf{b}$ and asks $\mathbf{a}$ ordered subsequently by its host multi-unit bid or ask. Then we should have some k such that $b_{(k)} \geq a_{(k)}$ and $b_{(k+1)} < a_{(k+1)}$ for their constituent single-unit bids and asks. We can also claim, without loss of generality, that there exists such a multi-unit bid $\mathbf{b}_i$ such that two of its bids $b_{(k)}, b_{(k+1)} \in \mathbf{b}_i$. This would imply that $b_{(k)} = b_{(k+1)}$. However, there cannot be some multi-unit ask $\mathbf{a}_j$ having asks such that $a_{(k)} = a_{(k+1)}$, because it contradicts with $b_{(k)} \geq a_{(k)}$ and $b_{(k+1)} < a_{(k+1)}$. Hence, $a_{(k)}$ and $a_{(k+1)}$ must belong to different multi-unit asks. It must also be the case that the multi-unit ask which owns $a_{(k)}$ is fully satisfied, and so do other preceding winning multi-unit bids and asks. □

We can formulate an LP problem for for multi-unit bids and asks where $\lambda_i \in [0, 1]$ now. So it is not binary any more, and takes any value between 0 and 1. When it takes 1, the multi-unit bid/ask is fully satisfied, zero means it is rejected. But when $\lambda_i \in (0, 1)$, the agent i is partially satisfied. For given vectors of valuations and quantities (v, q), allocation rule for multi-unit double auction is:

$$\max_{\lambda} \sum_i q_i \lambda_i v_i \tag{2}$$

$$s.t. \quad \lambda_i \in [0, 1] \quad \forall i \tag{3}$$

$$\sum_i q_i \lambda_i = 0 \tag{4}$$

where $q_i \in \mathbb{Z}$ represents quantities, v_i is the agent's valuation, λ_i is an allocation decision variable.

The solution of above allocation problem can be used to find the volume demanded and supplied. Below are the formulas for computing the volumes of matched multi-unit bids V_b and asks V_a.

$$V_b = \sum_i q_i \quad \text{s.t. } q_i > 0, \lambda_i > 0 \tag{5}$$

$$V_a = \sum_i |q_i| \quad \text{s.t. } q_i < 0, \lambda_i > 0 \tag{6}$$

Let us denote the number of multi-unit bids matched (both fully and partially) as K, and for multi-unit asks L. Also Kth multi-unit bid would mean the lowest bid matched, and Lth multi-unit ask would mean highest ask matched. I denote their quoted valuations as b_K and a_L, and quantities as bq_K and aq_L respectively. From Lemma 4.1, we know that there is at most one $\lambda_i \in (0, 1)$ exists, so let us denote this as λ^*. It can also be noted that if such λ^* exists, it either belongs to Kth multi-unit bid, or Lth multi-unit ask. Now depending on whether λ^* exists, and if it exists, to whom it is assigned to, we apply appropriate payment rule. There are 3 cases that can emerge in this mechanism:

1. No λ^*: This would mean that supply and demand is matched exactly, hence $V_a = V_b$. In this case, buyers pay at b_{K+1}, sellers receive at good a_{L+1}. Because $b_{K+1} < a_{L+1}$, mechanism subsidises the deficit of $V_a(a_{L+1} - b_{K+1})$.
2. λ^* is assigned to buyer: This means that there is an over-demand, hence $V_b > V_a$. In this case, mechanism rejects Kth multi-unit bid. If there is a tie, it is randomly resolved. The remaining $K - 1$ buyers pay b_K per unit, L sellers receive a_{L+1} per unit. As the implication of Kth buyer rejection, a number of sellers at the bottom of the list can be exposed to $V_a - V_b + bq_K$ number of goods unmatched. So mechanism pays out $a_{L+1}(V_a - V_b + bq_K)$ to them. Because $b_K < a_{L+1}$ and number of full matches is $V_b - bq_K$, mechanism subsidises in total the deficit of $(V_b - bq_K)(a_{L+1} - b_K) + a_{L+1}(V_a - V_b + bq_K)$.
3. λ^* is assigned to seller: This means that there is an over-supply, hence $V_b < V_a$. In this case, mechanism rejects Lth multi-unit ask. If there is a tie, it is randomly resolved. The remaining $L - 1$ sellers receive a_L per unit, K buyers pay b_{K+1} per unit. As the implication of Lth seller rejection, a number of buyers at the bottom of the list can be exposed to $V_b - V_a + aq_L$ number of goods unmatched. So mechanism sells out in total $b_{K+1}(V_b - V_a + aq_L)$ worth of goods, and generates income. Because $b_{K+1} < a_L$ and number of full matches is $V_a - aq_L$, mechanism subsidises in total the deficit of $(V_a - aq_L)(a_{L+1} - b_K) - b_{K+1}(V_b - V_a + aq_L)$.

In above payment rules, mechanism is not only taking loss from clearing bids and asks at their offsetting prices, but also covering the exposed bids and asks resulting from the rejection of least efficient traders. Although the first part of the mechanism's loss can be insignificant in competitive markets due to narrow difference between inefficient bid and ask, the second part contributes the large portion of it, as the mechanism takes the responsibility to cover the exposed bids or asks. Given that the difference between $a_{L+1} - b_K$ is insignificant, the worst case budget-deficit for the mechanism is given below:

$$\bar{q}(K-1)(a_{L+1} - b_K) + a_{L+1}(\bar{q} - 1) \quad (7)$$

In worst case budget-deficit scenario, all buyers submit cap quantities $\bar{q}$, and Kth multi-unit bid is covered for $\bar{q} - 1$ of its bid. The mechanism rejects Kth bid, and leaves $\bar{q} - 1$ quantities for matched asks exposed. Mechanism spends extra $a_{L+1}(\bar{q} - 1)$ to cover these exposed asks. Hence it is the incentive of the mechanism to keep $\bar{q}$ as low possible to minimise its loss.

Theorem 4.1. *Proposed multi-unit double auction is Dominant Strategy Incentive Compatible (DSIC) and individual rational.*

Proof. Proof is done using Vickrey's argument. Without loss of generality, let us assume buyer i submits multi-unit bid (b_i, q_i) and $b_i > v_i$.

1. No λ^*: Then the clearing price is b_{K+1}, K buyers trade and there is no partially satisfied bid. If buyer gets fully satisfied, then $b_i \geq b_{K+1}$. So buyers utility is $v_i - b_{K+1}$, and in case if it is $v_i < b_{K+1}$ buyer gets negative utility, while if he posted v_i he would not trade and his utility would be zero. If $v_i \geq b_{K+1}$, the utility is indifferent to truthful bidding. If his bid is rejected, buyer is also indifferent.
2. λ^* assigned to buyers: Then the clearing price is b_K, $K-1$ buyers trade and there is one partially satisfied bid. If buyer gets fully satisfied, the above Vickrey's argument applies for critical bid b_K. If buyer gets rejected, he is indifferent to truthful bidding. However if buyer is partially satisfied, then $b_K = b_i > v_i$, he is rejected and he would be rejected for submitting v_i. So he is indifferent.
3. λ^* assigned to sellers: Then the clearing price is b_{K+1}, K buyers trade and there is no partially satisfied bid. The same argument for no λ^* case applies here.

In case if bidder submits $b_i < v_i$.

1. No λ^*: Then the clearing price is b_{K+1}, K buyers trade and there is no partially satisfied bid. If buyer gets fully satisfied, then $v_i > b_i \geq b_{K+1}$ and buyer has the same positive utility. If buyer gets rejected, and $v_i > b_{K+1}$, buyer misses the positive utility, otherwise he is indifferent.
2. λ^* assigned to buyers: Then the clearing price is b_K, $K-1$ buyers trade and there is one partially satisfied bid. If buyer gets fully satisfied, he is indifferent. If buyer gets rejected, the above Vickrey's argument applies for critical bid b_K. However if buyer is partially satisfied, then $b_K = b_i < v_i$, he is rejected and misses a positive utility.
3. λ^* assigned to sellers: Then the clearing price is b_{K+1}, K buyers trade and there is no partially satisfied bid. The same argument for no λ^* case applies here.

So there is a dominant strategy for buyer i, and it is $b_i = v_i$.

If buyer plays his dominant strategy, his utility is always non-negative. Hence, buyer is ex-post individual rational. Same argument applies to sellers. □

There two ways of looking at the efficiency of the mechanism we proposed. First way is computing the efficient trades happened within the mechanism. Because mechanism takes the place of Kth (Lth) rejected partially satisfied buyer (seller), the efficient trades are not lost. Hence mechanism can be considered efficient. However there is a partially satisfied bid (ask) rejected from the trade. In second way of looking at mechanism's efficiency, we can consider this rejected partially satisfied bid (ask) as the lost efficiency, because the traders are not benefiting from it. In this case, at most $\bar{q} - 1$ units of goods supposed for trade can be lost.

Also it is worthwhile to mention that the proposed mechanism is tractable, because it uses LP for determining the allocation which is polynomially solvable, and the payment rule is $O(1)$.

5 Experimental Results

We have simulated the asset prices using the Geometric Brownian Motion with a calibrated parameters according to the historic data of NASDAQ-100 index in 2014. The daily mean drift is computed as $\mu = 0.0007$, and and the volatility is $\sigma = 0.0089$. The figure 3 shows the instance of simulated asset prices that I use for all experiments. It can be seen that the initial asset price is the same as NASDAQ-100

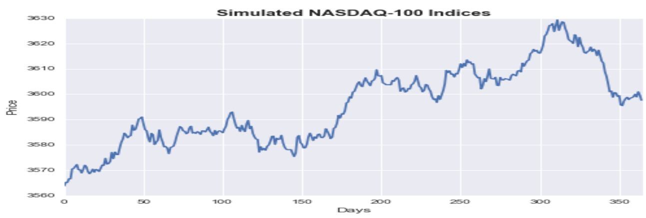

Figure 3: Simulated NASDAQ-100 Indices

on 2 January 2014, $S_0 = \$3563.57$, and the it is $S_T = \$3597.59$ at the end of the year. This particular instance of asset price is interesting because it includes dramatic fluctuation near the end of the year. This should enable us to stress test the option pricing methods that are proposed. We also analyse option with strike \$3563.00 which is ATM. Also we use only call options for the simulation, because put prices can be directly computed from the call price using put-call parity relationship.

LMSR traders create a positive bid-ask spread, which forbids them from trading if the market is uniformly populated with LMSR traders holding the same portfolio. Therefore LMSR trader should be also simulated in mixed groups each holding different set of portfolios, and thus produce different prices. It is also important to note that LMSR trader is deterministic in their pricing, because the only factor which affects their pricing decision is their portfolio and fixed range of events horizon that they use to compute their final payoff. Therefore two LMSR traders holding the same portfolio produce same bids or same asks. To make market more heterogeneous, I use most of the option portfolios given in Table 2 to simulate traders from neutral, non-neutral, bullish and bearish perspectives. The full list of LMSR traders with the portfolios they hold is given in Table 5.

After running several experiments with LMSR traders, we found out that liquidity $b = 100$ provides reasonable range of bids and asks which are likely to produce trades in the market. LMSR trader picks random quantities between -2000 and 2000 while submitting orders,

Trader Name	Belief	Portfolio
LMSR-NEUT1	Neutral	Butterfly Call Spread
LMSR-NEUT2	Neutral	Iron Butterfly
LMSR-NEUT3	Neutral	Long Call Ladder
LMSR-NEUT4	Neutral	Short Strangle
LMSR-NON-NEUT1	Non-Neutral	Short Call Ladder
LMSR-NON-NEUT2	Non-Neutral	Long Straddle
LMSR-NON-NEUT3	Non-Neutral	Long Strangle
LMSR-NON-NEUT4	Non-Neutral	Strip
LMSR-BULL	Bullish	Bullish Call Spread
LMSR-BEAR	Bearish	Bearish Call Spread

Table 5: LMSR Traders and their portfolios

so agent's decision to buy or sell is uniformly distributed. The negative quantities stand for asks, and the positive ones are bids. Table 6 lists some experimental scenarios using different LMSR traders together.

Groups	Traders	Population
NEUT,NON-NEUT	LMSR-NEUT1	25
	LMSR-NEUT2	25
	LMSR-NON-NEUT1	25
	LMSR-NON-NEUT2	25
ALL	LMSR-NEUT1	25
	LMSR-BULL	25
	LMSR-BEAR	25
	LMSR-NON-NEUT1	25
MORE BULL	LMSR-NEUT3	10
	LMSR-BULL	70
	LMSR-BEAR	10
	LMSR-NON-NEUT3	10
MORE BEAR	LMSR-NEUT4	10
	LMSR-BULL	10
	LMSR-BEAR	70
	LMSR-NON-NEUT4	10

Table 6: Experiments with LMSR Traders

Mechanism simulates 365 trading days going up to the point the option expires. It feeds the option market with new asset price information and collects corresponding bids and asks from LMSR traders. Because mechanism is direct, it clears order in one round and switches to the next trading day. Every trading day, the traders are re-instantiated with the same distribution of portfolios so they do not remember their previous choices. Greeks are simulated by linearly changing the control factors (asset price or time to maturity) and fixing the other parameters constant, and inputting the given setting to a mechanism populated with the same traders.

Figures from 4 to 7 represent the simulation of different groups of traders given in Table 6. The yellow shaded area indicates the range of accepted orders for a given day, the blue line is the Black-Scholes price which is accepted as a benchmark model, and the red line indicates the average of clearing bid and ask prices for given day. In Figure 4 we can see the trade between neutral and non-neutral portfolio holders. It can be seen that the prices are volatile around Black-Scholes prices. This is explained using the deterministic nature of LMSR traders. The whole market consists of 2 neutral LMSR traders and 2 non-neutral LMSR traders who output all together 8 different pricing quotes, 4 for bids and 4 for asks. So naturally, one of 4 bids and one of 4 asks are used as the clearing prices for the matched orders. Because mechanism has very few choices to determine the clearing price among mostly homogeneous quotes, the option price for each trading day differs significantly. In Figure 5, we can see that the prices start with the same volatility, but upon maturity they get close to the option's theoretical price and the volatility around it subsides. Figure 6 illustrates the market mostly populated with bullish traders. In this example, there is no much volatility, and the prices are generally close to risk-neutral price. This is because the upward trend of the asset prices correspond with the traders' expectations. However in the market of bearish traders as shown in Figure 7, the call options are initially underpriced, because the direction of asset prices is opposite to traders' belief and therefore considered less profitable for them. However the prices cross the risk-neutral price only after option lives the half of its lifespan. This is because the payoff from the option becomes more certain, as the asset prices continue to grow defying the bearish belief of traders.   We simulate the Greeks using

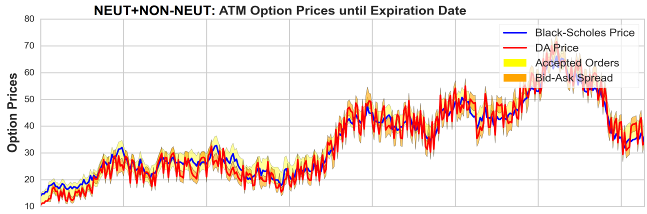

Figure 4: Market Simulation of Traders in NEUT and NON-NEUT Group

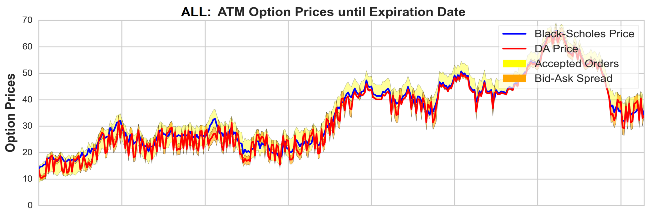

Figure 5: Market Simulation of Traders in ALL Group

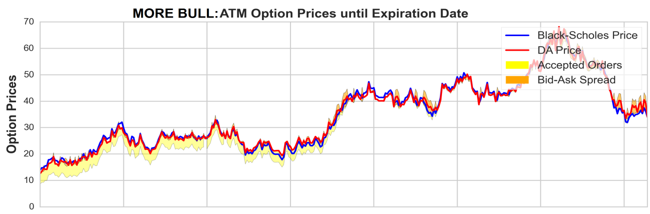

Figure 6: Market Simulation of Traders in MORE BULL Group

the mixed population (i.e. group ALL) of LMSR traders for OTM, ATM and ITM calls and compare them with Black-Scholes analytical solutions. Simulation of Greeks involves fixing all parameters of the market, except the one which is tested for sensitivity. For example, the delta is measured by linearly changing the asset prices in the mechanism while fixing the passage of time and other parameters such as the population of traders, risk-free interest rates, etc.

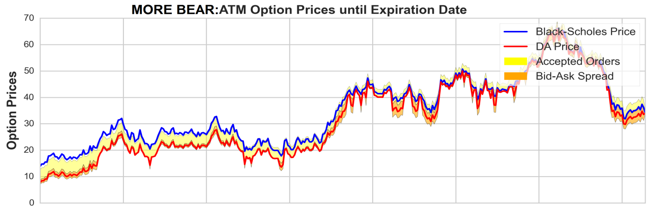

Figure 7: Market Simulation of Traders in MORE BEAR Group

Figure 8 shows the deltas obtained from the simulation, and it can be seen that they steeper compared to Black-Scholes' analytical delta. This means that in a market populated with LMSR traders the option prices are highly sensitive to the changes in the asset price. Similar to risk-neutral pricing, the option price is highly volatile when the asset price is around its corresponding strike. The steepness of delta can be explained using characteristics of the mechanism and the LMSR traders involved. Because LMSR traders produce limited number of different bids and asks for the same option, and the mechanism has to clear the orders using the critical bids and asks, the sharp jumps in option prices are plausible. We have also simulated the option theta

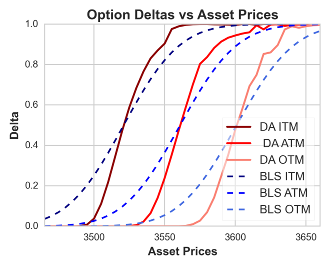

Figure 8: Option's delta

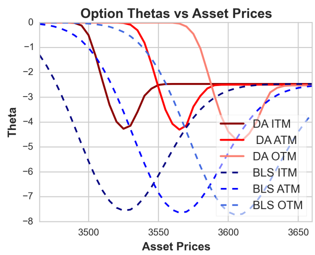

Figure 9: Option's theta

for the same configuration of the market. In Figure 9, we linearly changed the asset prices and computed the option's sensitivity to the change in time. We can see how the option price loses less than risk-neutral price when the asset price is around its strike as it approaches its maturity date. This is because LMSR traders are more inclined to their private beliefs related to the payoff from the portfolios they hold. This results in less change in option price compared to risk-neutral traders when the option is nearing its maturity date. We also observed that theta is less than the analytical solution if simulated per change in time-to-maturity.

From above simulations, we also found out that about 10% of efficient trades have been rejected by the mechanism to preserve the atomicity of other orders. It also means that mechanism had to cover up 10% of overall trades due to this rejection. Also surplus or deficiencies resulted from clearing the trades has been distributed around zero.

6 Concluding Remarks

In this paper, we have simulated LMSR-based option trading agents holding popular option portfolios in a multi-unit direct double auction to analyse the resulted option prices. Our simulation results have shown that pricing option via double auctions is a valid technique as the obtained prices were close to risk-neutral solution if the market was populated with traders having different beliefs. Besides that we were able to observe option prices under different population of traders with bearish, bullish, neutral and non-neutral beliefs modelled through their corresponding portfolios. For example, we have seen that the neutral and non-neutral traders create volatility as their portfolios consist of opposite payoff structure. Also in more bearish population of traders, the calls were initially underpriced until the option reached half-way to its maturity. When all traders are simulated together, the volatility of prices subsided as the option approached its maturity date. From Greeks analysis, we found out that the option prices are highly sensitive to the changes in the asset price, while they are less sensitive to the change in maturity date. This can be seen from the comparatively fixed width of accepted orders too in the simulation of the marketplace. Moreover we determined that in our current setting, we could generate enough volume if we set the liquidity parameter to $b = 100$ (although it is specific to particular price range of options, and not generic quality.). We also analysed the mechanism's relative efficiency and the budget balance.

This approach can be further improved using other incentive compatible mechanisms such as original McAfee's double auction with single-unit orders. Also the traders can be more sophisticated in making buy or sell decisions rather than randomly choosing the either action. Also other indifference pricing methods such as traders with exponential utility, zero-intelligence traders, etc can be simulated with our presented agents to observe their impact to the results obtained, and to analyse how they are different from the standard risk-neutral valuation techniques.

REFERENCES

[1] Fischer Black and Myron Scholes, 'The pricing of options and corporate liabilities', *The journal of political economy*, 637–654, (1973).

[2] René Carmona, *Indifference pricing: theory and applications*, Princeton University Press, 2009.

[3] G. Cohen, *The Bible of Options Strategies: The Definitive Guide for Practical Trading Strategies*, Prentice Hall, 2005.

[4] Hans U Gerber and Gérard Pafum, 'Utility functions: from risk theory to finance', *North American Actuarial Journal*, **2**(3), 74–91, (1998).

[5] R. Hanson, 'Logarithmic market scoring rules for modular combinatorial information aggregation', *Journal of Prediction Markets*, **1**(1), 1–15, (2007).

[6] Vicky Henderson and David Hobson, 'Utility indifference pricing-an overview', *Volume on Indifference Pricing*, (2004).

[7] Steven L Heston, 'A closed-form solution for options with stochastic volatility with applications to bond and currency options', *Review of financial studies*, **6**(2), 327–343, (1993).

[8] Pu Huang, Alan Scheller-Wolf, and Katia Sycara, 'Design of a multi–unit double auction e–market', *Computational Intelligence*, **18**(4), 596–617, (2002).

[9] Simon Loertscher and Claudio Mezzetti, 'A dominant strategy double auction with multi-unit traders', Technical report, mimeo, (2013).

[10] R Preston McAfee, 'A dominant strategy double auction', *Journal of economic Theory*, **56**(2), 434–450, (1992).

[11] Robert C Merton, 'Option pricing when underlying stock returns are discontinuous', *Journal of financial economics*, **3**(1), 125–144, (1976).

[12] A. Othman and T. Sandholm, 'Inventory-Based Versus Prior-Based Options Trading Agents', *Algorithmic Finance*, **1**(2), 95–121, (2012).

[13] D. Parkes and J. Kalagnanam, 'Auctions, bidding and exchange design', Technical report, IBM Research Division, (2004).

[14] Simon Parsons, Juan A Rodriguez-Aguilar, and Mark Klein, 'Auctions and bidding: A guide for computer scientists', *ACM Computing Surveys (CSUR)*, **43**(2), 10, (2011).

[15] David M Pennock and Rahul Sami, 'Computational aspects of prediction markets', *Algorithmic game theory*, 651–674, (2007).