

Forecasting oil price realized volatility using information channels from other asset classes

Stavros Degiannakis^{1,2} and George Filis^{1,3,*}

¹Department of Economics and Regional Development, Panteion University of Social and Political Sciences, 136 Syggrou Avenue, 17671, Greece.

²Postgraduate Department of Business Administration, Hellenic Open University, Aristotelous 18, 26 335, Greece.

³Department of Accounting, Finance and Economics, Bournemouth University, BH8 8EB, United Kingdom.

*Corresponding author: email: gfilis@bournemouth.ac.uk

Abstract

Motivated from Ross (1989) who maintains that asset volatilities are synonymous to the information flow, we claim that cross-market volatility transmission effects are synonymous to cross-market information flows or “information channels” from one market to another. Based on this assertion we assess whether cross-market volatility flows contain important information that can improve the accuracy of oil price realized volatility forecasting. We concentrate on realized volatilities derived from the intra-day prices of the Brent crude oil and four different asset classes (*Stocks*, *Forex*, *Commodities* and *Macro*), which represent the different “information channels” by which oil price volatility is impacted from. We employ a HAR framework and estimate forecasts for 1-day to 66-days ahead. Our findings provide strong evidence that the use of the different “information channels” enhances the predictive accuracy of oil price realized volatility at all forecasting horizons. Numerous forecasting evaluation tests and alternative model specifications confirm the robustness of our results.

Keywords: Volatility forecasting, realized volatility, crude oil futures, risk management, HAR, asset classes.

JEL: C22, C53, G13, Q47.

1. Introduction

Crude oil price movements are of major importance for the global economy. Elder and Serletis (2010) opine that oil price uncertainty exercises significant impact on the economy. It is no coincidence that since the second half of 2015 the plunge of oil prices and its economic effects have monopolised media attention from the most widely circulated financial press. Even more, this fall in oil prices has resulted in increased oil price volatility, which is an essential input in many macroeconomic models, as well as, in option pricing and value at risk.

Furthermore, oil price volatility forecasts are particularly important nowadays due to the fact that the increased participation of hedge funds in the oil market over the last decade or so has resulted in the financialisation of the market (Le Pen and Sévi, 2013; Fattouh *et al.*, 2013; Buyuksahin and Robe, 2014). In addition, we observe that financial institutions now consider the oil market as a profitable alternative investment for their portfolios, which renders the oil price volatility forecasting important (see, for example, Silvennoinen and Thorp, 2013). Lastly, oil price volatility forecasting is also essential for oil risk management and the management of oil contingent claims as noted by Giot and Laurent (2003), Cabedo and Moya (2003) and Sévi (2014).

Thus, accurate forecasts of oil price volatility are both timely and essential for policy makers, oil traders, and researchers. However, it is interesting to note a paradox in the field of oil price volatility forecasting. Despite the fact that the importance of oil price fluctuations and volatility on the economy and financial markets have long been established¹ and researchers have forecasted asset market volatility since the 80's², the earliest study in the field of oil volatility forecasting dates from as recently as 2006 by Sadorsky³.

Even more, the majority of the existing papers use daily oil prices and forecast the conditional oil price volatility; yet, ultra-high frequency data, which are used for the construction of the realized volatility, can produce more accurate forecasts (see,

¹ See, for instance, Hamilton, 1983; Jones and Kaul (1996), Kilian and Park (2009), Filis (2010), Arouri *et al.* (2011), Filis *et al.* (2011), Degiannakis *et al.* (2013), Rahman and Serletis (2011), Baumeister and Peersman (2013), Filis and Chatziantoniou (2014).

² See, Bollerslev *et al.* (1992), Andersen *et al.* (2003, 2005), Degiannakis (2004), Hansen and Lunde (2005), Angelidis and Degiannakis (2008), Fuertes *et al.* (2009), Frijns *et al.* (2010) among others.

³ This is in contrast to the literature on oil price forecasting, which has attracted the attention of the research community since the 1990s. See, for example, Abramson and Finizza (1991), Huntington (1994), Alquist *et al.* (2013), Baumeister and Kilian (2010, 2014, 2015).

for instance, Hansen and Lunde, 2005; Engle and Sun, 2007; Tay *et al.*, 2009). To date, there are only five studies that concentrate their attention on forecasting oil price realized volatility (i.e., Haugom *et al.*, 2014; Sévi, 2014; Prokopczuk *et al.*, 2015; Phan *et al.*, 2015; Wen *et al.*, 2016).

Hence, we add to the scarce literature of oil price realized volatility forecasting using the current state-of-the-art Heterogeneous AutoRegressive model for Realized Volatility (HAR-RV), which we extend in a number of ways. (i) We consider 14 exogenous variables (using HAR-RV-X models), which are categorized into four different asset classes (*Stocks*, *Forex*, *Commodities* and *Macro*) and we investigate whether their realized volatilities improve the oil volatility forecasts. (ii) We provide a method that handles exogenous variables in a HAR model in order to proceed with the forecasts. (iii) We assess the forecasting accuracy of the HAR-RV-X models based on each individual asset and asset class, their combined forecasts, as well as the forecast-averaging. (iv) We assess the forecasting accuracy of our models during economic turbulent periods, such as the Global Financial Crisis (GFC) of 2007-08. (v) We use the newly developed Model Confidence Set, the Direction-of-Change (DoC) and trading games to evaluate the forecasting accuracy of the competing models. (vi) We use forecasting horizons ranging from 1-day to 66-days ahead, given that different stakeholders have different predictive needs.

In short, we report the following regularities. (i) The exogenous volatilities improve the forecasting accuracy of the simple HAR-RV at all forecasting horizons. (ii) The HAR-RV-X models that combine asset volatilities from all asset classes are the best performing models, since they capture the different “information channels” that impact on oil price volatility at different times. (iii) The DoC suggests that all HAR models are highly accurate in predicting the movements of oil price volatility. Thus, we maintain that HAR-RV-X models should be used from stakeholders who are interested in the accuracy of the forecasts, whereas those interested only in the movement of oil price volatility can be limited to HAR-RV. (iv) The trading games confirm that the HAR-RV-X models which combine multiple asset volatilities generate higher positive returns compared to both the Random Walk and the HAR-RV model. (v) The findings are robust even when we concentrate only on turbulent economic periods.

The rest of the paper is structured as follows. Section 2 reviews the existing literature. Section 3 provides a detailed description of the data. Section 4 explains the

construction of the realized volatility, whereas Section 5 describes the econometric approach employed in this paper. Section 6 explains the forecasting evaluation techniques. Section 7 analyses the findings of the study and Section 8 includes the robustness checks. Section 9 concludes the study.

2. Review of the literature

The earliest study on oil price volatility forecasting is this by Sadorsky (2006). Sadorsky forecasts the squared daily returns of oil futures prices (as a proxy of volatility) using GARCH, TGARCH and Exponential Smoothing, VAR and BEKK models. The VAR and BEKK models also include the squared returns of other petroleum futures (such as heating oil, gasoline and natural gas). He finds that the GARCH-family models are able to outperform the random walk model, which is used as the benchmark. Sadorsky and McKenzie (2008) second Sadorsky's (2006) findings, showing that the GARCH-type models produce more accurate forecasts than any other competing model, although only in the longer-horizons. They claim that in shorter-horizons, it is the power autoregressive model that produces the best forecasts of oil price volatility.

Following Sadorsky (2006) and Sadorsky and McKenzie (2008), an increasing number of authors have turned their attention to oil price volatility forecasting. For example, Kang *et al.* (2009) use daily oil spot prices in order to forecast the 1-day, 5-days and 20-days ahead conditional volatilities by means of CGARCH, FIGRACH and IGARCH models. Their findings suggest that the CGARCH and FIGARCH models are more useful in modelling and forecasting the volatility in the crude oil prices.

More recently, Nomikos and Pouliasis (2011) and Kang and Yoon (2013) consider oil futures prices to estimate and forecast oil price conditional volatility. Nomikos and Pouliasis (2011) use Mix-GARCH and MRS-GARCH models to forecast the 1-day-ahead oil price volatility and find that both models are able to outperform the forecasts of the simple GARCH model. Kang and Yoon (2013), on the other hand, combine ARFIMA models with GARCH models to produce 1-day, 5-days and 20-days ahead forecasts. They claim that while ARFIMA-FIGARCH models are better in modelling oil price conditional volatility, no model consistently outperforms all other competing ones.

Similarly, several other authors model the conditional volatility of oil prices and forecast these volatilities, using univariate models such as the FIAPARCH, HYGARCH, EGARCH, FIGARCH, APARCH, as well as multivariate models such as the BEKK, VAR and Risk Metrics (see, Agnolucci, 2009; Wei *et al.*, 2010; Aroui *et al.*, 2012; Hou and Suardi, 2012; Chkili *et al.*, 2014). For the multivariate models, they consider conditional volatilities of other energy commodities, similar to those of Sadorsky (2006). The general consensus is that the univariate GARCH-type models are able to produce more accurate forecasts than any other competing models. It is worth noting that the majority of these papers evaluate the forecasting accuracy of their models in 1-day, 5-days and 20-days ahead horizons.

A study that is quite distinct is this of Efimova and Serletis (2014), as it is the first paper to consider the inclusion of an additional asset class in their models in order to assess if this yields better forecasts for the oil price volatility. More specifically, all previous papers, which have estimated multivariate models, have considered prices only from other energy markets (e.g. heating oil, gasoline, etc.). By contrast, Efimova and Serletis (2014) include the S&P500 daily returns to their models, as well as, oil spot prices to model and forecast the 1-day ahead oil conditional volatility using univariate GARCH-type models and multivariate models such as BEKK, DCC and VARMA-GARCH. Their findings, though, corroborate these of previous literature, suggesting that the univariate models are able to produce more accurate forecasts and that the inclusion of the S&P500 daily returns did not produce better forecasts.

All aforementioned papers use daily oil prices and forecast the conditional oil price volatility. Nevertheless, empirical evidence (primarily from the finance literature) has long suggested that intraday (ultra-high frequency) data are more information-rich and thus they can produce more accurate estimates of the daily volatility (see, *inter alia*, Andersen *et al.*, 2001, 2003, 2010; McAleer and Medeiros, 2008). More specifically, Andersen and Bollerslev (1998) introduce an alternative measure of daily volatility which considers intraday data, the Realized Volatility (*RV*). Realized volatility is based on the idea of using the sum of squared intraday returns to generate more accurate daily volatility measures.

Numerous studies have shown that intraday data are able to produce better forecasts, compared to daily data (see, for instance, Hansen and Lunde, 2005; Engle and Sun, 2007; Tay *et al.*, 2009).

However, until very recently the use of ultra-high frequency data for volatility forecasting has concentrated only on stock market and exchange rate volatilities (see, among others, Bollerslev *et al.*, 1992; West and Cho, 1995; Andersen *et al.*, 2003, 2005; Hansen and Lunde, 2005; Angelidis and Degiannakis, 2008).

Studies on oil price volatility forecasting using ultra-high frequency data started in 2014. One of the early studies is this by Haugom *et al.* (2014), who construct the realized volatility for the WTI crude oil futures and assess whether the CBOE Crude Oil Volatility Index (OVX) and variables such as volume, open interest, daily returns, bid-ask spread and the slope of the futures curve can improve the forecasts of the WTI realized volatility. The authors use data from the WTI crude oil futures from May, 2007 to May, 2012, considering the front-month futures contracts only. They use the HAR model of Corsi (2009) to forecast the realized oil volatility, given its superiority in forecasting this volatility measure (see, *inter alia*, Andersen *et al.*, 2007; Corsi, 2009; Busch *et al.*, 2011; Fernandes *et al.*, 2014)⁴, and find that the exogenous variables improve the forecasting accuracy of WTI realized volatility.

Sévi (2014) also forecasts the realized volatility of oil futures prices for the front-month futures contracts. More specifically, the author considers 5min intraday oil price returns to construct the daily realized volatility. He then uses several extensions of the HAR model in order to consider the jump component, semi variances, leverage effects and asymmetries in these components. The data range from January, 1987 to December, 2010. Despite the fact that Sévi (2014) considers nine different HAR models in total, he concludes that none of these models is able to outperform the forecasting accuracy of the simple HAR model, which is based only on the oil realized volatility (HAR-RV), in any forecasting horizon (i.e. 1-day to 66-days ahead).

More recently, Prokopczuk *et al.* (2015) use intraday data to forecast the realized volatility of crude oil prices, gasoline, heating oil and natural gas for three forecasting horizons, namely 1-day, 5-days and 22-days ahead. Their data span from January 2007 to June 2012. In order to construct their realized volatilities for the three

⁴ The HAR model considers information of the previous day's, week's and month's volatility and thus, it is able to accommodate the heterogeneous beliefs of traders in the oil market. Bollerslev and Wright (2001) maintain that any volatility series exhibits long-memory behaviour and so a model which considers this stylized fact (such as the HAR model) is able to produce better forecasts. Andersen *et al.* (2007), Corsi (2009), Busch *et al.* (2011) and Fernandes *et al.* (2014) also argue that HAR models are more successful in forecasting asset price volatility as they are parsimonious and they capture the long-memory that is observed in asset price volatility.

time-series, the authors choose a sampling frequency of 15min. As in Haugom *et al.* (2014) and Sévi (2014), Prokopczuk *et al.* (2015) also use a HAR model for their forecasting exercise. Similarly with Sévi (2014), they also consider several extensions of the HAR-RV model in order to capture whether the jump detection produces better forecasts. Their findings corroborate those of Sévi (2014), showing that the modelling of jumps does not improve the forecast accuracy of the simple HAR-RV model.

Phan *et al.* (2015), on the other hand, examine whether the S&P500 volatility improves the oil price realized volatility forecasts. The authors consider 5min intraday data to construct the realized volatility measure; nevertheless, they use an EGARCH(1,1) model rather than a HAR-RV one. They report that the cross-market volatility interaction improves the forecasts for the oil price volatility. Finally, Chatrath *et al.* (2015) also forecast the oil price volatility, using a sampling frequency of 5min to construct their realized volatility measure. The authors employ similar regressions to those by Christensen and Prabhala (1998) and Jiang and Tian (2005) and find that the incorporation of the crude oil implied volatility improves the forecasting of realized volatility.

Our paper directly extends the previous contributions on oil price realized volatility forecasting, using ultra-high frequency data.

3. Data Description

In this study we use tick by tick transaction data of the front-month futures contracts for the following series: Brent Crude Oil (ICE Futures Europe), GBP/USD (CME Group), CAD/USD (CME Group), EUR/USD (CME Group), FTSE100 (ICE Futures Europe), S&P500 (CME Group), Hang Seng (Hong Kong Stock Exchange), Euro Stoxx 50 (Eurex), Gold (CME Group), Copper (CME Group), Natural Gas (CME Group), Palladium (CME Group), Silver (CME Group) and the US 10yr T-bills (CME Group). All data are obtained from Tick Data. We use an additional US macroeconomic volatility indicator which is available in daily frequency, namely the Economic Policy Uncertainty (EPU)⁵ Index by Baker *et al.* (2013). The period of our

⁵ As indicated by Baker *et al.* (2013), EPU index is constructed based on three types of underlying components. The first component quantifies newspaper coverage of policy-related economic uncertainty. The second component reflects the number of federal tax code provisions set to expire in future years. The third component uses disagreement among economic forecasters as a proxy for uncertainty. For more information the reader is directed to <http://www.policyuncertainty.com/>.

study spans from August 1, 2003 to August 5, 2015 and is dictated by the availability of intraday data for the Brent Crude oil futures contracts.

The choice of variables is justified by the fact that there is a growing literature that confirms the cross-market transmission effects (either of returns or volatilities) between oil and four main asset classes (i.e., *Stocks*, *Forex*, *Commodities* and *Macro*)⁶. Given these interactions, we posit that the volatilities of these four asset classes contain important information for the future movements of the oil price volatility. This is related to Ross (1989) who maintains that volatilities are synonymous to the information flow and thus the cross-market transmission effects are synonymous to cross-market information flows or “information channels”.

We consider some specific variables among the four asset classes, largely because they are some of the most tradable futures contracts globally⁷, but also due to the following reasons.

Specifically, for the stock market indices we choose the key US, EU and Asian indices as (i) their combined trading spans across a full day (24-hours) and (ii) they represent the stock market indices of the largest economies in the world. However, we also include the FTSE100 index futures, given that we forecast the Brent crude oil volatility.

As far as the foreign exchange variables are concerned, we maintain that the EUR/USD is the main currency that exercises an impact on oil fluctuations, while the use of the GBP/USD futures is incontestable, given that it is related to the Brent crude oil. Finally, the choice of the CAD/USD is motivated by Chen *et al.* (2010) who maintain that currencies of commodity exporters contain important information for the future movements of commodity prices.

Lastly, as macroeconomic indicators we use the US 10yr T-bill futures and the US EPU, as recent studies have shown that oil price volatility is responsive to change in the economic conditions (see, for instance, Antonakakis *et al.*, 2014). We treat both the US 10yr T-bill and the US EPU as variables that approximate global economic developments, given the importance of the US in the global economy.

⁶ See, *inter alia*, Aloui and Jammazi (2009), Sari *et al.* (2010), Arouri *et al.* (2011), Aloui *et al.* (2013), Souček and Todorova (2013, 2014), Mensi *et al.* (2014), Antonakakis *et al.* (2014), Sadorsky (2014), Phan *et al.* (2015), IEA (2015), Ferraro *et al.* (2015).

⁷ Although, this is not the case for the Hang Seng index, given that the most traded Chinese index futures is the CSI 300. Nevertheless, intraday data for the CSI 300 index futures are available after 2008 and thus we had to replace this index with Hang Seng, which is among the most traded index futures in the Asian region.

Important milestones for the construction of the intra-day time series are the following:

- (i) Trading day: In our paper we define as trading day the period between 21:01 GMT the night before until 21:00 GMT that evening. This particular definition of the trading day is motivated by Andersen *et al.* (2001, 2003, 2007).
- (ii) Holidays: We exclude several fixed and moving holidays from our series, such as Christmas, Martin Luther King day, Washington's Birthday, Good Friday, Easter Monday, Memorial day, July 4th, Labour day and Thanksgiving and the day after.
- (iii) Non-trading hours: We remove any trading that takes place between Friday 21:01 GMT until Sunday 21:00 GMT.
- (iv) Brent Crude Oil 2-hours Sunday trading session: We use two approaches for the additional 2-hour trading session that occurs in the Brent Crude Oil futures on Sundays. The first approach is to disregard these observations, whereas the second approach is to incorporate these observations to the Monday's trading day. The results of our forecasting exercise are not affected by the choice of the approach. Given the indifference in the results, we have decided to follow the second approach as it is more instructive to consider all available information in the construction of the realized volatility measure.
- (v) Calendar or business-time sampling: We choose the calendar sampling as it is most commonly used in the literature and thus, allows for comparability of the results. Furthermore, as Sévi (2014) explains, the use of business-time sampling is not recommended as its asymptotic properties are less well-known.
- (vi) Common sample: Finally, to arrive to a common sample across all series, we have considered the trading days when the Brent Crude Oil is traded⁸.

After the aforementioned considerations, our final sample consists of $T = 3028$ trading days.

4. Realized volatility

According to Andersen and Bollerslev (1998) the daily realized volatility is estimated to be the sum of squared intra-day returns, as shown in eq.1:

$$DRV_t^{(\tau)} = \sqrt{\sum_{j=1}^{\tau} \left(\log P_{t_j} - \log P_{t_{j-1}} \right)^2}, \quad (1)$$

⁸ If on any given day we have an oil observation when it is not a trading day for one of the other variables, we use the value that this variable had the day before.

where P_{t_j} are the observed prices of the asset at trading day t , and τ are the equidistant intraday time intervals.

The realized volatility converges to the integrated volatility as the sampling frequency (m) goes to zero and the number of time intervals (τ) approaches infinity. Nevertheless, more noise is added to the estimated volatility when the sampling frequency converges on zero, due to microstructure frictions. Thus, there is a trade-off between the bias that is inserted in the realized volatility measure and its accuracy. Andersen *et al.* (2006) suggested the construction of the volatility signature plot, which depicts the average realized volatility against the sampling frequency. Based on the volatility signature plot, the optimal sampling frequency is the one where the autocovariance bias is minimum. In order to identify the point where the realized volatility appears to stabilise, we decompose the inter-day variance $(\log P_t - \log P_{t-1})^2$ into the intra-day variance $(DRV_t^{(\tau)})^2$ and the intra-day autocovariance $(\sum_{j=1}^{\tau-1} \sum_{i=j+1}^{\tau} (\log P_{t_i} - \log P_{t_{i-1}}) (\log P_{t_{i-j}} - \log P_{t_{i-j-1}}))$, as in eq.2:

$$(\log P_t - \log P_{t-1})^2 = (DRV_t^{(\tau)})^2 + 2 \sum_{j=1}^{\tau-1} \sum_{i=j+1}^{\tau} (\log P_{t_i} - \log P_{t_{i-1}}) (\log P_{t_{i-j}} - \log P_{t_{i-j-1}}). \quad (2)$$

The intra-day autocovariance represents the bias that is inserted in the realized volatility measure, with $E((\log P_{t_i} - \log P_{t_{i-1}}) (\log P_{t_{i-j}} - \log P_{t_{i-j-1}})) = 0$, for $j \neq 0$. Thus, the optimal sampling frequency (m) is the highest frequency that minimises the autocovariance bias. Table 1 shows the optimal sampling frequencies for our series.

[TABLE 1 HERE]

Furthermore, it is well established that when markets are closed, i.e. during night-time periods, holidays, and weekends information still flows. The existing literature has proposed different approaches to dealing with this issue. For instance, authors such as Andersen *et al.* (2001), Thomakos and Wang (2003) or Wu (2011), opine that night-time periods and weekends could be ignored from the construction of the realized volatility. By contrast, Hansen and Lunde (2005) maintain that a good proxy of the true volatility should accommodate the fact that information flows when

markets are closed and thus, they propose to adjust the intra-day volatility with the close-to-open inter-day volatility, as shown in eq.3:

$$DRV_{(HL),t}^{(\tau)} = \sqrt{\omega_1 (\log P_{t_1} - \log P_{t_{-1\tau}})^2 + \omega_2 \sum_{j=2}^{\tau} (\log P_{t_j} - \log P_{t_{j-1}})^2}, \quad (3)$$

where the weights ω_1 and ω_2 are such that minimise the difference between the realized volatility and the integrated volatility, i.e. minimise the variance of the realized volatility, $\min V(DRV_{t,(HL)}^{(\tau)})$. In this paper we are in line with Hansen and Lunde (2005). Table 2 presents the descriptive statistics of our annualised realized volatility series $RV_{(HL),t}^{(\tau)}$:

$$RV_{(HL),t}^{(\tau)} = \sqrt{252} \times DRV_{(HL),t}^{(\tau)}, \quad (4)$$

for all variables and Figure 1 portrays their plots over the sample period.

[TABLE 2 HERE]

[FIGURE 1 HERE]

From Table 2 we notice that EPU has the highest average value and that it is very volatile, given its maximum, minimum and standard deviation values. From the realized volatilities, it is the natural gas (NG) that exhibits the highest average volatility, followed by palladium (PA), silver (SV) and oil (CO). On the contrary, the lowest average volatilities are observed in T-bills (TY) and the three exchange rate volatilities (BP, CD and EC). It is also evident that none of the series under consideration are normally distributed, as they exhibit excess kurtosis and positive skewness. Another interesting point is the average number of 1min observations that each series has, with the Eurostoxx 50 (XX), FTSE100 (FT) and Hang Seng (HI) showing the lowest figures, due to the shorter trading sessions that these markets have. The unit root test results support the hypothesis of stationary realized volatilities.

Furthermore, as it is apparent from Figure 1, volatility clustering of high values is observed for all series during the GFC of 2007-08, although additional clusters of high volatility are evident in different periods for each series. Focusing on the Brent Crude Oil volatility, a second cluster of high volatility appears in the late 2014 – early 2015 period, mainly due to the plunge of the oil prices. Finally, we should mention that all autocorrelations (not shown here for brevity) decrease monotonically, suggesting long-memory processes for our series.

5. Econometric specifications

5.1. Naïve models

We consider two naïve models, namely a simple Random Walk (RW) without a drift and an Autoregressive model of order 1, or AR(1), as shown in eqs. 5 and 6, respectively:

$$\log \left(RV_{(HL),OIL,t}^{(\tau)} \right) = \log \left(RV_{(HL),OIL,t-1}^{(\tau)} \right) + \varepsilon_t, \quad (5)$$

$$\log \left(RV_{(HL),OIL,t}^{(\tau)} \right) = \varphi_0^{(t)} \left(1 - \varphi_1^{(t)} \right) + \varphi_1^{(t)} \log \left(RV_{(HL),OIL,t-1}^{(\tau)} \right) + \varepsilon_t, \quad (6)$$

where $RV_{(HL),OIL,t}^{(\tau)}$ is the annualised realized volatility of the Brent crude oil at time t , $\varphi_0^{(t)}$, $\varphi_1^{(t)}$ are coefficients to be estimated and ε_t is a white noise.

5.2. HAR-RV model

We employ the HAR model by Corsi (2009), which is recently implemented in Haugom *et al.* (2014), Sévi (2014) and Prokopczuk *et al.* (2015)⁹. Eq. 7 presents the HAR-RV model:

$$\begin{aligned} \log \left(RV_{(HL),OIL,t}^{(\tau)} \right) = & \\ w_0^{(t)} + w_1^{(t)} \log \left(RV_{(HL),OIL,t-1}^{(\tau)} \right) + w_2^{(t)} \left(5^{-1} \sum_{k=1}^5 \log \left(RV_{(HL),OIL,t-k}^{(\tau)} \right) \right) + & (7) \\ w_3^{(t)} \left(22^{-1} \sum_{k=1}^{22} \log \left(RV_{(HL),OIL,t-k}^{(\tau)} \right) \right) + \varepsilon_t, & \end{aligned}$$

where $w_0^{(t)}$, $w_1^{(t)}$, $w_2^{(t)}$, $w_3^{(t)}$ are parameters to be estimated. The HAR-RV model relates the current trading day's realized volatility of the Brent crude oil to the daily, weekly and monthly realized volatilities of the same asset.

5.3. HAR-RV-X model

We augment the simple HAR-RV model to embody exogenous variables, as discussed in Section 2. The HAR-RV-X model is shown in the following equation:

$$\begin{aligned} \log \left(RV_{(HL),OIL,t}^{(\tau)} \right) = & \\ w_0^{(t)} + w_1^{(t)} \log \left(RV_{(HL),OIL,t-1}^{(\tau)} \right) + w_2^{(t)} \left(5^{-1} \sum_{k=1}^5 \log \left(RV_{(HL),OIL,t-k}^{(\tau)} \right) \right) + & (8) \\ w_3^{(t)} \left(22^{-1} \sum_{k=1}^{22} \log \left(RV_{(HL),OIL,t-k}^{(\tau)} \right) \right) + w_{(a),4}^{(t)} \log \left(RV_{(HL),X(a),t-1}^{(\tau)} \right) + & \end{aligned}$$

⁹ We do not consider the jump components in this study, given that the existing literature provides evidence that their inclusion does not produce better forecasts.

$$w_{(a),5}^{(t)} \left(5^{-1} \sum_{k=1}^5 \log \left(RV_{(HL),X_{(a)},t-k}^{(\tau)} \right) \right) + \\ w_{(a),6}^{(t)} \left(22^{-1} \sum_{k=1}^{22} \log \left(RV_{(HL),X_{(a)},t-k}^{(\tau)} \right) \right) + \varepsilon_t,$$

where $X_{(a)}$ denotes each of the fourteen (14) alternative exogenous realized volatilities that are used in this paper.

5.4. HAR-RV-Asset Class, HAR-RV-Combined and HAR-RV-Average models

In order to reveal the predictive information from all the asset volatilities within an asset class without imposing selection and look-ahead biases, we employ the Principal Component Analysis (PCA). In general the techniques for data dimensionality reduction have been successfully applied in now-casting macroeconomic variables (see, for instance, Giannone *et al.*, 2008; Stock and Watson, 2002, among others). PCA enables us to reduce the dimensionality of the existing dataset so that we can reveal the predictive information of asset volatilities, within a single asset class, without losing important information. Hence, most of the available information per asset class is exploited, without imposing selection and look-ahead biases.

More specifically, we construct the HAR-RV-STOCKS, HAR-RV-FOREX, HAR-RV-COMMODITIES and HAR-RV-MACRO models. For each asset class, we use the volatilities that belong to this asset class. E.g. for the HAR-RV-STOCKS we use the volatilities of the four stock market indices to estimate the principal components. For g denoting the number of asset volatilities within the class, the HAR-RV-Asset Class model is illustrated in the following framework:

$$\begin{bmatrix} RV_{(HL),X_{(1)},t}^{(\tau)} \\ \vdots \\ RV_{(HL),X_{(g)},t}^{(\tau)} \end{bmatrix} = \mathbf{A}^{(g)} \begin{bmatrix} RV_{(PCA),X_{(1)},t}^{(\tau)} \\ \vdots \\ RV_{(PCA),X_{(g)},t}^{(\tau)} \end{bmatrix} + \mathbf{e}_t^{(g)}, \quad (9)$$

$$\log \left(RV_{(HL),OIL,t}^{(\tau)} \right) = \\ w_0^{(t)} + w_1^{(t)} \log \left(RV_{(HL),OIL,t-1}^{(\tau)} \right) + w_2^{(t)} \left(5^{-1} \sum_{k=1}^5 \log \left(RV_{(HL),OIL,t-k}^{(\tau)} \right) \right) + \\ w_3^{(t)} \left(22^{-1} \sum_{k=1}^{22} \log \left(RV_{(HL),OIL,t-k}^{(\tau)} \right) \right) + w_{(a),4}^{(t)} \log \left(RV_{(PCA),X_{(a)},t-1}^{(\tau)} \right) + \\ w_{(a),5}^{(t)} \left(5^{-1} \sum_{k=1}^5 \log \left(RV_{(PCA),X_{(a)},t-k}^{(\tau)} \right) \right) + \quad (10)$$

$$w_{(a),6}^{(t)} \left(22^{-1} \sum_{k=1}^{22} \log \left(RV_{(PCA),X_{(a)},t-k}^{(\tau)} \right) \right) + \varepsilon_t,$$

where $\mathbf{\Lambda}^{(g)}$ is the matrix of factor loadings, $\begin{bmatrix} RV_{(PCA),X_{(1)},t}^{(\tau)} \\ \vdots \\ RV_{(PCA),X_{(g)},t}^{(\tau)} \end{bmatrix}$ is the vector with the

common factors, and $\mathbf{e}_t^{(g)}$ is the vector of the idiosyncratic component. The $X_{(a)}$ denotes the common factors that are incorporated in the HAR model for each asset class. The HAR model can be extended to accommodate up to g principal components.

Similarly, we construct the HAR-RV-COMBINED model, which includes the principal components from all 14 asset volatilities, so to capture simultaneously the various “information channels” that could enhance the oil price volatility predictions.

Finally, we also consider the HAR-RV-AVERAGE, which produces the average forecasts from the four HAR-RV-Asset Class models.

5.5. Forecasting realized volatility

Equations 7 - 10 are estimated in the natural logarithms of the realized volatilities. However, we are interested in forecasting the realized volatility, which is the variable of interest for traders, portfolio managers and policy makers. Thus, in our forecasts we concentrate on the estimator of the $RV_{(HL),OIL,t}^{(\tau)}$, which is the $\exp \left(\log \left(RV_{(HL),OIL,t}^{(\tau)} \right) + 1/2 \hat{\sigma}_\varepsilon^2 \right)$. The HAR-RV 1-day-ahead forecast is as follows:

$$\begin{aligned} RV_{(HL),OIL,t+1|t}^{(\tau)} = & \exp \left(\hat{w}_0^{(t)} + \hat{w}_1^{(t)} \log \left(RV_{(HL),OIL,t}^{(\tau)} \right) \right. \\ & + \hat{w}_2^{(t)} \left(5^{-1} \sum_{k=1}^5 \log \left(RV_{(HL),OIL,t-k+1}^{(\tau)} \right) \right) \\ & \left. + \hat{w}_3^{(t)} \left(22^{-1} \sum_{k=1}^{22} \log \left(RV_{(HL),OIL,t-k+1}^{(\tau)} \right) \right) + 1/2 \hat{\sigma}_\varepsilon^2 \right) \end{aligned} \quad (11)$$

The one-day-ahead forecast of the HAR-RV-X models is shown in eq.12:

$$\begin{aligned}
RV_{(HL),OIL,t+1|t}^{(\tau)} &= \exp \left(\hat{w}_0^{(t)} + \hat{w}_1^{(t)} \log \left(RV_{(HL),OIL,t}^{(\tau)} \right) \right. \\
&+ \hat{w}_2^{(t)} \left(5^{-1} \sum_{k=1}^5 \log \left(RV_{(HL),OIL,t-k+1}^{(\tau)} \right) \right) \\
&+ \hat{w}_3^{(t)} \left(22^{-1} \sum_{k=1}^{22} \log \left(RV_{(HL),OIL,t-k+1}^{(\tau)} \right) \right) \\
&+ \hat{w}_{(a),4}^{(t)} \log \left(RV_{(HL),X_{(a)},t}^{(\tau)} \right) \\
&+ \hat{w}_{(a),5}^{(t)} \left(5^{-1} \sum_{k=1}^5 \log \left(RV_{(HL),X_{(a)},t-k+1}^{(\tau)} \right) \right) \\
&\left. + \hat{w}_{(a),6}^{(t)} \left(22^{-1} \sum_{k=1}^{22} \log \left(RV_{(HL),X_{(a)},t-k+1}^{(\tau)} \right) \right) + 1/2 \hat{\sigma}_\varepsilon^2 \right)
\end{aligned} \tag{12}$$

The s -days-ahead forecasts ($s = 2, \dots, 66$ days) are estimated in a similar fashion. More specifically, the s -days-ahead forecast of the HAR-RV model, for horizon ($s \geq 2$):

$$\begin{aligned}
RV_{(HL),OIL,t+s|t}^{(\tau)} &= \\
&\exp \left(\hat{w}_0^{(t)} + \hat{w}_1^{(t)} \log \left(RV_{(HL),OIL,t+s-1|t}^{(\tau)} \right) + \right. \\
&\hat{w}_2^{(t)} \left(s^{-1} \sum_{k=1}^{s-1} \log \left(RV_{(HL),OIL,t-k+s|t}^{(\tau)} \right) + \right. \\
&(5-s)^{-1} \sum_{k=s}^5 \log \left(RV_{(HL),OIL,t-k+s}^{(\tau)} \right) \left. \right) + \\
&\hat{w}_3^{(t)} \left(s^{-1} \sum_{k=1}^{s-1} \log \left(RV_{(HL),OIL,t-k+s|t}^{(\tau)} \right) + \right. \\
&\left. (22-s)^{-1} \sum_{k=s}^{22} \log \left(RV_{(HL),OIL,t-k+s}^{(\tau)} \right) \right) + 1/2 \hat{\sigma}_\varepsilon^2
\end{aligned} \tag{13}$$

Finally, the s -days-ahead forecast of the HAR-RV-X models, for horizon ($s \geq 2$):

$$\begin{aligned}
RV_{(HL),OIL,t+s|t}^{(\tau)} &= \\
&\exp \left(\hat{w}_0^{(t)} + \hat{w}_1^{(t)} \log \left(RV_{(HL),OIL,t+s-1|t}^{(\tau)} \right) + \right. \\
&\hat{w}_2^{(t)} \left(s^{-1} \sum_{k=1}^{s-1} \log \left(RV_{(HL),OIL,t-k+s|t}^{(\tau)} \right) + \right. \\
&(5-s)^{-1} \sum_{k=s}^5 \log \left(RV_{(HL),OIL,t-k+s}^{(\tau)} \right) \left. \right) + \\
&\hat{w}_3^{(t)} \left(s^{-1} \sum_{k=1}^{s-1} \log \left(RV_{(HL),OIL,t-k+s|t}^{(\tau)} \right) + \right.
\end{aligned} \tag{14}$$

$$\begin{aligned}
& (22 - s)^{-1} \sum_{k=s}^{22} \log \left(RV_{(HL), OIL, t-k+s}^{(\tau)} \right) + \hat{w}_{(a),4}^{(t)} \log \left(RV_{(HL), X_{(a)}, t-k+s|t}^{(\tau)} \right) + \\
& \hat{w}_{(a),5}^{(t)} \left(s^{-1} \sum_{k=1}^{s-1} \log \left(RV_{(HL), X_{(a)}, t-k+s|t}^{(\tau)} \right) + \right. \\
& \left. (5 - s)^{-1} \sum_{k=s}^5 \log \left(RV_{(HL), X_{(a)}, t-k+s}^{(\tau)} \right) \right) + \\
& \hat{w}_{(a),6}^{(t)} \left(s^{-1} \sum_{k=1}^{s-1} \log \left(RV_{(HL), X_{(a)}, t-k+s|t}^{(\tau)} \right) + \right. \\
& \left. (22 - s)^{-1} \sum_{k=s}^{22} \log \left(RV_{(HL), X_{(a)}, t-k+s}^{(\tau)} \right) \right) + 1/2 \hat{\sigma}_{\varepsilon}^2
\end{aligned}$$

Equivalently, for the HAR-RV-ASSET CLASS and HAR-RV-COMBINED models, in eqs. (12) and (14) we replace $RV_{(HL), X_{(a)}}^{(\tau)}$ with $RV_{(PCA), X_{(a)}}^{(\tau)}$.

It is important, though, to explain here how we proceed with the out-of-sample forecasts of 1-day ahead to 66-days ahead, as far as the HAR-RV and HAR-RV-X models are concerned. For the 1-day ahead forecast of the Brent Crude oil, the models use data that belong to the information set at time t and thus, they are known to the forecaster at the time of the forecasting exercise. Nevertheless, from the 2-days ahead forecasts onwards (i.e. $t + 2, \dots, t + 66$), the forecasts of the HAR-RV-X models of eq. (8), as well as, the HAR-RV-ASSET CLASS and HAR-RV-COMBINED models of eq. (10), require the use of future data that do not belong to the information set at time t . For example, for the $t + 2$ forecast we need to know the $t + 1$ volatility values of all variables. As far as the Brent Crude oil volatility is concerned, there is no problem as the model uses the 1-day ahead forecast, i.e. at $t + 1$. Turning to the exogenous variables, there are three possible choices to overcome the issue of using future data that do not belong to the information set at time t .

The first choice is to assume a zero value from $t + 1$ onwards for the volatilities of the exogenous variable(s), since the information is not available at the time of the forecast.

The second choice is to assume that at time $t + 1$ onwards the volatility of the exogenous variable remains constant, i.e. $E \left(RV_{(HL), X_{(a)}, t+1}^{(\tau)} \right) = RV_{(HL), X_{(a)}, t}^{(\tau)}$. The concept that the best forecast of the next days' volatility value is today's value (plus a random component) is referred to as the random walk and it is based on the Efficient Market Hypothesis.

The third choice is to forecast volatilities of the exogenous variables. Thus, the exogenous volatilities data that are required for the estimation of the $t + 2, \dots, t + 66$

forecasts of the Brent crude oil volatility (which are not available at time t), are taken from the forecasted values of these exogenous volatilities.

Between the first two alternatives, the latter is clearly preferred on the grounds that it is closely related to the finance literature and it is easy to implement. To proceed with the second choice, though, we would need to confirm that the RW generates the most accurate forecasts for the exogenous variables and thus to confirm the Efficient Market Hypothesis. To do so, we forecast each of exogenous variables using both a RW model and the HAR-RV model of eq. (7). Our results (not shown here for brevity but they are available upon request) reveal that the HAR-RV model is able to outperform the RW for each of the exogenous variables. Thus, we reject the second choice and we proceed with the Brent Crude oil forecasts based on the third choice. The third choice is shown in eq. (14), where we denote the information of the previous week's and previous month's exogenous variables as $\left(s^{-1} \sum_{k=1}^{s-1} \log \left(RV_{(HL),X_{(a)},t-k+s|t}^{(\tau)}\right) + (5-s)^{-1} \sum_{k=s}^5 \log \left(RV_{(HL),X_{(a)},t-k+s}^{(\tau)}\right)\right)$ and $\left(s^{-1} \sum_{k=1}^{s-1} \log \left(RV_{(HL),X_{(a)},t-k+s|t}^{(\tau)}\right) + (22-s)^{-1} \sum_{k=s}^{22} \log \left(RV_{(HL),X_{(a)},t-k+s}^{(\tau)}\right)\right)$, respectively. The first term represents the information from the forecasted exogenous variables, while the second term indicates the information from the constructed exogenous variables¹⁰.

This is an important innovation in our approach. The existing literature either ignores this particular procedure (hence, the stated forecasting accuracies can be put into question) or fails to explain it.

6. Forecast evaluation

The initial sample period is $\tilde{T} = 1000$ days and we use the remaining $\tilde{T} = 2028$ for our out-of-sample forecasting period. The $\tilde{T} = 1000$ is justified by the fact that (i) we require a large sample size for the estimation of the models and (ii) we need our initial sample to stop before the GFC of 2007-08. This allows us to include the recession period in our out-of-sample period. For the first out-of-sample forecasts for 1-day to 66-days ahead, we use the initial sample period $\tilde{T} = 1000$. For each

¹⁰ We do not explicitly consider a multivariate HAR model. The idea of a multivariate HAR model is to capture bidirectional effects between variables. We capture these effects in our model as we consider the predicted values of the exogenous variables, where needed, which allows us the estimation of a quasi-multivariate HAR model. Thus, we remove part of the complexity of the model, without losing any significant information.

subsequent forecast, we use a rolling window approach with fixed length of 1000 days. Engle *et al.* (1993) and Angelidis *et al.* (2004) maintain that the use of restricted samples is capable of capturing changes in the market activity more accurately.

6.1 Evaluation functions

The forecasting accuracy of the models illustrated in Section 5 is initially evaluated using two well established evaluation functions, namely the Mean Squared Predicted Error (MSE) and the Mean Absolute Predicted Error (MAE):

$$MSE^{(s)} = T^{-1} \sum_{t=1}^T \left(RV_{(HL),OIL,t+s|t}^{(\tau)} - RV_{(HL),OIL,t+s}^{(\tau)} \right)^2, \quad (15)$$

and

$$MAE^{(s)} = T^{-1} \sum_{t=1}^T \left| RV_{(HL),OIL,t+s|t}^{(\tau)} - RV_{(HL),OIL,t+s}^{(\tau)} \right|, \quad (16)$$

where $RV_{(HL),OIL,t+s|t}^{(\tau)}$ is the s -days-ahead oil realized volatility forecast, whereas $RV_{(HL),OIL,t+s}^{(\tau)}$ is the Brent Crude oil realized volatility at time $t+s$.

6.2 Model Confidence Set

We further employ the newly established Model Confidence Set (MCS) procedure by Hansen *et al.* (2011), which identifies the set of the best models, as these are defined in terms of a specific loss function, without an *a priori* choice of a benchmark model¹¹. In our case, the two loss functions are the MSE and MAE¹².

The MCS explores the predictive ability of an initial set of M^0 models and investigates, at a predefined level of significance, which group of models survive an elimination algorithm. Let us define as $\Psi_{n,t}$ the evaluation function of model n at day t , and $d_{n,n^*,t} = \Psi_{n,t} - \Psi_{n^*,t}$ as the evaluation differential for $n, n^* \in M^0$. For example, the evaluation function may be the Mean Squared Error, so $\Psi_{n,t} \equiv \left(RV_{(HL),OIL,t+s|t}^{(\tau)} - RV_{(HL),OIL,t+s}^{(\tau)} \right)^2$, where $RV_{(HL),OIL,t+s|t}^{(\tau)}$ is the s -days-ahead oil realized volatility forecast. The hypotheses that are being tested are:

¹¹ Most papers compare the forecasts from a variety of models against a benchmark model, utilising the Diebold-Mariano test (Diebold and Mariano, 1995), for a pairwise comparison, the Equal Predictive Accuracy test (Clark and West, 2007) for nested models, or the Reality Check for Data Snooping (White, 2000) and the Superior Predictive Ability test (Hansen, 2005) for multiple comparisons. We depart from this standard setup as in our case we have a number of forecasting models that need to be evaluated simultaneously and not against a benchmark model.

¹² For brevity, we report only the results from the MAE. The results from the MSE loss function are qualitatively similar and available upon request.

$$H_{0,M}: E(d_{n,n^*,t}) = 0, \quad (17)$$

for $\forall n, n^* \in M$, $M \subset M^0$ against the alternative hypothesis $H_{1,M}: E(d_{n,n^*,t}) \neq 0$, for some $n, n^* \in M$. The elimination algorithm, based on an equivalence test and an elimination rule, employs the equivalence test for investigating the $H_{0,M}$ for $\forall M \subset M^0$ and the elimination rule to identify the model n to be removed from M in the case that $H_{0,M}$ is rejected.

6.3 Direction-of-Change

We also consider the Direction-of-Change (DoC) as an additional forecasting evaluation technique. The DoC is particularly important for market timing, which is essential for asset allocation and trading strategies. The DoC reports the proportion of forecasts that have correctly predicted the direction (up or down) of the volatility movement. Let us denote as $P_{n,i}^{(s)}$ a dummy variable that takes the value of 1 for each trading day i that model n correctly predicts the direction of volatility movement s trading days ahead, and zero otherwise, i.e.:

$$P_{n,i}^{(s)} = \begin{cases} 1 & \text{if } RV_{(HL),OIL,t+s}^{(\tau)} > RV_{(HL),OIL,t}^{(\tau)} \text{ and } RV_{(HL),OIL,t+s|t}^{(\tau)} > RV_{(HL),OIL,t}^{(\tau)} \\ 1 & \text{if } RV_{(HL),OIL,t+s}^{(\tau)} < RV_{(HL),OIL,t}^{(\tau)} \text{ and } RV_{(HL),OIL,t+s|t}^{(\tau)} < RV_{(HL),OIL,t}^{(\tau)} \\ 0 & \text{otherwise.} \end{cases} \quad (18)$$

Then, the proportion of forecasted values that have correctly predicted the direction of the volatility movement ($DoC_{prop}^{(s)}$) is shown in eq. 19:

$$DoC_{prop}^{(s)} = \frac{\sum_{i=1}^{\tilde{T}} P_{n,i}^{(s)}}{\tilde{T}} \times 100, \quad (19)$$

where \tilde{T} is the number of out-of-sample forecasted values. The statistical significance of the directional accuracy is gauged by the Pesaran and Timmermann (2009) test, under the null hypothesis of no directional accuracy.

7. Empirical results

7.1. MAE and MSE

We evaluate the forecasting accuracy of our models for 1-day to 66-days ahead, although we report six different horizons, namely 1-day, 5-days, 10-days, 22-day, 44-days and 66-days ahead. The results for the MAE and MSE are shown in

Tables 3a, 3b and 4. In the first column we report the value of the loss functions for the RW model, whereas in the remaining columns we report the loss functions' ratios, relative to the RW model. A score below 1 denotes that the forecasts of the HAR-RV models outperform these of the RW.

[TABLE 3a HERE]

[TABLE 3b HERE]

[TABLE 4 HERE]

Primarily, it is evident in Tables 3a, 3b and 4 that all HAR-RV models are able to significantly outperform the RW forecasts. In more detail, focusing on Tables 3a and 3b, we show that almost all HAR-RV-X models with a single exogenous volatility are able to generate superior forecasts even when compared to the HAR-RV. This is particularly evident for the HAR-RV-X models with the foreign exchange volatilities and the US T-bill volatility and to a lesser extend for the commodities and stock markets. The only exceptions are the EPU and the Natural Gas volatility, which yield comparable results with the simple HAR-RV model.

More interesting findings, though, are reported in Table 4, which shows the forecasting performance of the HAR-RV-Asset Class models, as well as, the HAR-RV-COMBINED and HAR-RV-AVERAGE models. In particular, we find that the HAR-RV-COMBINED model is able to generate significantly improved forecasts not only relatively to the RW, but also to the simple HAR-RV model. This holds true for all forecasting horizons. Even more, the predictive gains from the HAR-RV-COMBINED model relatively to the RW and the simple HAR-RV are higher as the forecasting horizon increases. In greater detail, the HAR-RV-COMBINED is able to reduce the forecasting error, in terms of MAE, by more than 50% (compared to the RW) in short-run horizons and more than 23% in long-run horizons. Equivalently, the HAR-RV-COMBINED model reduces the forecasting error by more than 10% in the long-run horizons, compared to the single HAR-RV. A plausible explanation as to why the HAR-RV-COMBINED is the best performing model lies to the fact that oil price volatility is not influenced by a single asset class throughout the sample period, but rather it is impacted by different asset classes and, thus, different “information channels”. Interestingly enough, the HAR-RV-AVERAGE model does not manage to improve further the forecasting accuracy of the HAR-RV-COMBINED.

This finding is of particular importance as the existing literature on the forecast of the oil realized volatility suggests that the HAR-RV generates the most

accurate forecasts. In this paper we provide evidence that the incorporation of different asset classes' volatilities is capable of generating superior forecasts compared to the HAR-RV model, since they accommodate for the fact that oil volatility is impacted by different “information channels”.

7.2. Model Confidence Set (MCS) procedure

Next, we discuss the results from the MCS procedure, reported in Table 5. Even though the results from Tables 3a, 3b and 4 suggest that the inclusion of the exogenous volatilities significantly improves the forecasting performance of the HAR-RV model, it is vital to assess whether the simple HAR-RV can be included among the best performing models.

[TABLE 5 HERE]

From Table 5 we can make the following observations. First and foremost, the two naïve models and the HAR-RV are never among the best performing models and the same holds for the HAR-RV-X models with the single asset volatility¹³. We also note that the highest probability is assigned to the HAR-RV-COMBINED model, which is the HAR-RV models that is augmented with the use of multiple exogenous volatility “information channels”. This holds across all horizons. Another interesting finding from Table 5 is the fact that only in the 1-day ahead forecasting horizon, the HAR-RV-STOCKS, HAR-RV-FOREX and HAR-RV-AVERAGE are also included in the set of the best performing models. Overall, the findings presented in Table 5 strengthen the conclusion that the predictive accuracy of the oil price realized volatility increases when we include the exogenous volatilities of multiple asset classes.

7.3. Direction-of-Change results

The DoC results are shown in Table 6, which reports the proportion of forecasted values that have correctly predicted the direction of volatility movement, as well as the DoC performance of each HAR-RV model against the RW.

[TABLE 6 HERE]

Table 6 shows that all HAR-RV models exhibit statistically significant directional accuracy of the oil volatility movements. Interestingly enough, even

¹³ The only exception is the HAR-RV-SP at the 1-day ahead forecast.

though the simple HAR-RV model is not included among the best models (as suggested by the MCS test) its ability to predict the direction of change is comparable to that of all HAR-RV-X models. From Table 6 we further notice that all HAR-RV models are able to predict the direction of change at a much higher rate compared to the RW model in all forecasting horizons.

7.4. Summary of findings

Overall, evidence suggests that the use of the exogenous volatilities of different asset classes results in substantial improvement in the forecasting accuracy of Brent Crude oil volatility. More importantly though, we highlight that as we move towards longer-run forecasting horizons, where accurate forecasts are more difficult to make, the predictive gains from the HAR-RV-COMBINED are becoming more prevalent. On the other hand, focusing on the DoC, we maintain that all models are highly accurate in predicting the direction of oil volatility movements. Thus, the combination of the MCS and the DoC results reveals a very important finding which has not been previously discussed in this strand of the literature.

These findings reveal that the simple HAR-RV model would be adequate for the stakeholders interested in the future movement of oil price volatility. Nevertheless, stakeholders who put more emphasis on the accuracy of the forecasts should use the HAR-RV-X models and more specifically, the HAR-RV-COMBINED model. Finally, the fact that the HAR-RV-COMBINED outperforms all other models provides support to our claim that different asset classes provide different information to oil price volatility and thus, their combination improves the forecasting accuracy.

8. Robustness and further tests

8.1 Distribution of forecast errors

The first robustness check is related to the distribution of the forecast errors. More specifically, the squared difference and the absolute difference between the oil realized volatility forecast $\left(RV_{(HL),OIL,t+s}^{(\tau)}\right)$ and the realized volatility $\left(RV_{(HL),OIL,t+s}^{(\tau)}\right)$ is highly asymmetric. This suggests that the use of the median deviation may report a more accurate picture of the forecasting errors, not in terms of which the best model is, but regarding the magnitude of the forecasting errors. Thus, for example, even though the HAR-RV-COMBINED model undoubtedly exhibits

higher forecasting accuracy, the actual deviation between the model's predicted volatilities and the actual values may be lower than the reported ones from MSE and MAE. To illustrate this, we first present the distribution of the absolute and squared deviations between the forecasted values from HAR-RV-COMBINED and the actual oil realized volatility (see Figure 2).

[FIGURE 2 HERE]

As evident from Figure 2, the distribution of the deviations is highly skewed, which provides support to our claim that it is instructive to use the median deviations (i.e. the Median Absolute Error – MeAE or the Median Squared Error - MeSE), as they may assess the magnitude of the prediction error more accurately.

From Table 7 and Figure 3, we observe that as the forecasting horizon increases, the magnitude of the prediction errors differs greatly between the mean and the median deviation. For example, the MAE (MSE) for the 1-day ahead forecast is reported to be 5.3457 (69.3662), whereas the MeAE (MeSE) is estimated at 3.6661 (13.4401). Equivalently, for the 66-days ahead, even though the MAE (MSE) reports values of the magnitude of 9.2670 (221.7807), the MeAE (MeSE) are only 5.7089 (32.5920).

[TABLE 7 HERE]

[FIGURE 3 HERE]

8.2 Predictive accuracy during crisis periods

As a further robustness check we assess the validity of our findings in extreme economic conditions, such as the GFC of 2007-08. We follow the same forecasting evaluation procedure and we evaluate our forecasts only for the period between August, 2007 and June, 2009. For brevity, we only present the results from the MCS procedure and we concentrate only on the HAR-RV-ASSET CLASS, HAR-RV-COMBINED and HAR-RV-AVERAGE models (see, Table 8).

[TABLE 8 HERE]

Table 8 suggests that the inclusion of exogenous asset classes' volatilities improves the forecasting ability of the simple HAR-RV model, even during turbulent times. More specifically, the HAR-RV is not included in the set of the best performing models at any forecasting horizon. Furthermore, it is evident that in crisis period, the HAR-RV-COMBINED is not the only best model but the HAR-RV-FOREX is also always included in the set of the best performing models.

Furthermore, we observe that in the short-run horizons, the HAR-RV-MACRO and HAR-RV-AVERAGE models are also among the models with the best predictive accuracy. Overall, the MCS results shown in Table 8 corroborate the findings from Table 5, and it also highlights the fact that during turbulent times, the *Forex* can also provide important predictive information at all horizons. Overall, we maintain that the evidence provided by the robustness check validates the proposed forecasting strategy plan, as it is effective even under extreme economic conditions.

8.3 Incremental value of the HAR-RV-X models

We shall remind the reader that the MCS test, shown in Section 7.2, provides convincing evidence that the HAR-RV-COMBINED model is always included in the set of the best performing models, whereas the simple HAR-RV model is never included in this set. It is, thus, important to show the incremental value of the incorporation of exogenous volatilities in the HAR-RV model, over time. To do so, we compare the HAR-RV-COMBINED with the HAR-RV model, given that the former is our best performing model, whereas the latter is the best model based on the existing literature.

The cumulative incremental value is estimated by deducting the MAE score of the HAR-RV model from the MAE score of the HAR-RV-COMBINED for the whole out-of-sample forecasting period. Thus, an upward movement of the line suggests that the HAR-RV-COMBINED is having a positive incremental value compared to the HAR-RV. The reverse holds true when the line is moving downwards. For brevity, Figure 4 shows the cumulative incremental value for the 1-day ahead forecasts, based on the MAE loss function.

[FIGURE 4 HERE]

Overall, we find that the incremental value of the use of exogenous asset volatilities is substantial over the forecasting period. More importantly though, we observe that during turbulent periods (such as the GFC of 2007-08 and the oil price collapse in 2014-15) the incremental value of the HAR-RV-COMBINED model is sizeable, as indicated by the steep upward movement of the line.

8.4 Forecast evaluation based on a trading strategy

Furthermore, we compare the forecasting performance of the HAR-RV-COMBINED model against the RW and the HAR-RV models based on a naïve day-

trading strategy. Indicatively, for the 1-day ahead forecasts, we assume that a trader assumes a long position in an asset that resembles the performance of the oil realized volatility when the $t + 1$ forecasted oil price volatility of model n is higher compared to the actual volatility at time t . By contrast, if the $t + 1$ forecasted volatility of model n is lower compared to the actual volatility at time t , then the trader assumes a short position. The trading game is constructed in a similar fashion for the remaining forecasting horizons. Volatility portfolio returns are then computed as the average daily returns over the investment horizon, which equals our out-of-sample forecasting period of $\tilde{T}=2028$ days.

The results of the trading strategy are reported in Table 9. It is evident that the HAR-RV-COMBINED provides greater positive returns compared to both the RW and the HAR-RV models. As expected, the cumulative difference in returns is greater between the HAR-RV-COMBINED and RW as opposed to the HAR-RV-COMBINED and HAR-RV. Nevertheless, the difference in returns is sizeable in both cases. This holds true even for the crisis periods, such as the GFC of 2007-08 and the oil collapse in 2014-15. Overall, these findings confirm superiority of the HAR-RV-COMBINED model.

[TABLE 9 HERE]

8.5 Forecast evaluation based on option straddles trading profitability metrics

Based on Andrada-Felix *et al.* (2016), Angelidis and Degiannakis (2008), Engle *et al.* (1993) and Xekalaki and Degiannakis (2005), we additionally employ an options straddle trading strategy as an additional economic criterion to evaluate the volatility forecasts of different models. In particular, we allow investors to go long (short) in a straddle when the forecasted volatility at time $t+s$ is higher (lower) than the oil realized volatility at the present time t .

The volatility forecasts are evaluated according to the Black and Scholes (1973) formula that is widely accepted and straightforwardly computed. The straddle trading, which is the purchase (or sale) of both a call and a put option with the same maturity day, is used as it is a delta neutral option trading strategy. Thus, the straddle holder's rate of return is indifferent to any change in the underlying asset price and is affected only from changes in volatility.

The expected price of a straddle on a \$1 share of the Brent crude oil at the next trading day with s days to maturity and \$1 exercise price is computed as:

$$S_{t+1|t} = 2N\left(\frac{rf_t\sqrt{s}}{\overline{RV}_{t+s|t}} + \frac{\overline{RV}_{t+s|t}\sqrt{s}}{2}\right) - 2e^{-rf_t s}N\left(\frac{rf_t\sqrt{s}}{\overline{RV}_{t+s|t}} - \frac{\overline{RV}_{t+s|t}\sqrt{s}}{2}\right) + e^{-rf_t s} - 1, \quad (20)$$

where $N(\cdot)$ denotes the cumulative normal distribution function,

$\overline{RV}_{t+s|t} = \frac{1}{s-1} \sum_{i=1}^s \left(\frac{RV_{(HL),OIL,t+i|t}^{(\tau)}}{\sqrt{252}} \right)$ is the average volatility forecast during the life of the option, and rf_t is the risk-free interest rate. The daily profit from holding the straddle is: $\pi_{t+1} = \max(e^{y_{t+1}} - e^{rf_{t+1}}, e^{rf_{t+1}} - e^{y_{t+1}})$, for y_t denoting the Brent crude oil daily log-returns.

We assume the existence of three investors who trade their volatility forecasts. Each investor i prices the straddles, $S_{t+1|t}^{(i)}$, every trading day according to one of the three volatility forecasting models; i.e. RW, HAR-RV and HAR-RV-COMBINED. A trade between two investors, i and j , is executed at the average of their forecasting prices, yielding to investor i a profit of:

$$\pi_t^{(i,j)} = \begin{cases} \pi_{t+1} - (S_{t+1|t}^{(i)} + S_{t+1|t}^{(j)}) & \text{if } S_{t+1|t}^{(i)} > S_{t+1|t}^{(j)} \\ (S_{t+1|t}^{(i)} + S_{t+1|t}^{(j)}) - \pi_{t+1} & \text{if } S_{t+1|t}^{(i)} < S_{t+1|t}^{(j)} \end{cases}. \quad (21)$$

As an economic evaluation criterion, we define the cumulative returns computed as $\pi = \frac{1}{2} \sum_{t=1}^{\check{T}} \sum_{j=1}^2 \pi_t^{(i,j)}$. Table 10 presents the cumulative returns, for \check{T} trading days, of investors who pricing their straddles according the RW, HAR-RV and HAR-RV-COMBINED volatility forecasts. The results show that the HAR-RV-COMBINED is once again superior compared to the other models¹⁴.

[TABLE 10 HERE]

8.6 HAR-RV-X models using time-varying correlations

Finally, we repeat the same forecasting strategy replacing the exogenous asset volatilities in the HAR-RV-X models with the time-varying correlations between the volatilities of the Brent crude oil and the 14 remaining assets. We do this in order to

¹⁴ For robustness purposes, the straddles trading game has been repeated for i) the exercise price being equal to the exponent of the risk free rate of return and ii) various levels of exercise prices; i.e. e^{-3rf_t} . However, the results remain qualitatively similar.

assess whether we can enhance the predictive accuracy of oil price volatility using a different measure to extract information from the four aforementioned asset classes.

For the estimation of the time-varying correlations we use the multivariate DCC model of Engle (2002). We denote the vector that comprises the log-realized

volatilities of the oil price and the other 14 assets as $\mathbf{Y}_t \equiv \begin{bmatrix} \log \left(RV_{(HL),OIL,t}^{(\tau)} \right) \\ \log \left(RV_{(HL),X_{(1)},t}^{(\tau)} \right) \\ \vdots \\ \log \left(RV_{(HL),X_{(14)},t}^{(\tau)} \right) \end{bmatrix}$. The

DCC model is estimated in the form:

$$\begin{aligned} \mathbf{Y}_t &= \mathbf{B}_0 + \boldsymbol{\varepsilon}_t, \\ \boldsymbol{\varepsilon}_t &= \mathbf{H}_t^{1/2} \mathbf{z}_t, \end{aligned} \quad (22)$$

where \mathbf{B}_0 is a vector of constants and $\mathbf{z}_t \sim N(\mathbf{0}, \mathbf{I})$. The variance-covariance matrix is decomposed as:

$$\mathbf{H}_t = \boldsymbol{\Sigma}_t^{1/2} \mathbf{C}_t \boldsymbol{\Sigma}_t^{1/2}. \quad (23)$$

The $\boldsymbol{\Sigma}_t^{1/2}$ is a diagonal matrix with the conditional standard deviations along the diagonal defined as GARCH(1,1) processes, whereas the

$\mathbf{C}_t = \begin{bmatrix} 1 & cor_{OIL,X_{(1)},t} & \cdots & cor_{OIL,X_{(14)},t} \\ cor_{OIL,X_{(1)},t} & 1 & \cdots & cor_{X_{(1)},X_{(14)},t} \\ \vdots & \vdots & \ddots & \vdots \\ cor_{OIL,X_{(14)},t} & cor_{X_{(1)},X_{(14)},t} & \cdots & 1 \end{bmatrix}$ is the conditional correlations' matrix¹⁵.

Then, we employ eqs. 8, 10, 12 and 14 to forecast the oil price realized volatility using the alternative HAR-RV-X models. More specifically, the HAR-RV-X model is modified in the following form:

$$\begin{aligned} \log \left(RV_{(HL),OIL,t}^{(\tau)} \right) &= \\ w_0^{(t)} &+ w_1^{(t)} \log \left(RV_{(HL),OIL,t-1}^{(\tau)} \right) + w_2^{(t)} \left(5^{-1} \sum_{k=1}^5 \log \left(RV_{(HL),OIL,t-k}^{(\tau)} \right) \right) + \\ &w_3^{(t)} \left(22^{-1} \sum_{k=1}^{22} \log \left(RV_{(HL),OIL,t-k}^{(\tau)} \right) \right) + w_4^{(t)} cor_{OIL,X_{(a)},t-1} + \\ &w_5^{(t)} \left(5^{-1} \sum_{k=1}^5 cor_{OIL,X_{(a)},t-k} \right) + w_6^{(t)} \left(22^{-1} \sum_{k=1}^{22} cor_{OIL,X_{(a)},t-k} \right) + \varepsilon_t, \end{aligned} \quad (24)$$

where $X_{(a)}$ denotes each of the alternative exogenous variables.

¹⁵ The matrix of conditional correlations is estimated as $\mathbf{C}_t = \mathbf{Q}_t^{*-1/2} \mathbf{Q}_t \mathbf{Q}_t^{*-1/2}$, where $\mathbf{Q}_t = (1 - a - b)\bar{\mathbf{Q}} + a(\mathbf{z}_{t-1}\mathbf{z}_{t-1}') + b\mathbf{Q}_{t-1}$, for $\bar{\mathbf{Q}}$ being the unconditional covariance of the standardized residuals and $\mathbf{Q}_t^{*-1/2}$ denoting the diagonal matrix composed of the square roots of the inverse of the diagonal elements of \mathbf{Q}_t .

We further repeat the same forecasting strategy using the log-returns' time-varying correlations; i.e. the DCC model is estimated for $\mathbf{Y}_t \equiv \begin{bmatrix} y_{OIL,t} \\ y_{X_{(1)},t} \\ \vdots \\ y_{X_{(14)},t} \end{bmatrix}$, where $y_{X_{(a)},t}$

denotes the log-returns of the $X_{(a)}$ asset. Hence, the conditional correlations' matrix \mathbf{C}_t contains the conditional correlations between oil log-returns and exogenous variables' log-returns. For brevity, we only present the ratio of the MAE and MSE between the HAR-RV-X models based on the asset volatilities and the time-varying correlations. The results are shown in Table 11.

[TABLE 11 HERE]

All ratios are below one, which suggests that the HAR-RV-X models based on the asset volatilities are able to provide superior predictive accuracy compared to the alternative HAR-RV-X models, which are based on the time-varying correlations of either volatilities or returns. Indicatively, the 0.8395 MAE ratio, in the 66-days ahead horizon under the HAR-RV-COMBINED column, suggests that the time-varying return correlations provide about 16% lower predictive accuracy compared to the asset volatilities.

9. Conclusion

The aim of this paper is to contribute to the limited but growing literature on oil price realized volatility forecasting. To do so we use tick by tick data of the front-month futures contracts for 14 asset prices. The period of our study spans from August 1, 2003 to August 5, 2015, which provides us with a total of 3028 trading days. Our forecasting horizons range from 1-day to 66-days ahead, given that different stakeholders have different predictive needs.

The current consensus provides evidence that the HAR-RV model outperforms all other competing forecasting models (see, Haugom *et al.*, 2014; Sévi, 2014; Prokopczuk *et al.*, 2015). Our paper builds upon these previous contributions and extends them in multiple ways.

In short, our out-of-sample results suggest that the HAR-RV models with exogenous volatilities from different asset classes (i.e. HAR-RV-X) outperform the forecasting accuracy of the HAR-RV at all forecasting horizons. In particular, we show that the HAR-RV-X model that combines multiple asset classes' volatilities

(HAR-RV-COMBINED) is the best performing models. Interestingly enough, the Direction of Change suggests that all HAR models are highly accurate in predicting the movements of oil price volatility. Thus, we maintain that HAR-RV-X models should be used by stakeholders who are interested in the accuracy of the forecasts, whereas those interested only in the movement of oil price volatility could be limited to HAR-RV. It is important to note that our findings are robust even when we concentrate only on turbulent economic periods, such as the Global Financial Crisis of 2007-08. Finally, the trading games confirm that the HAR-RV model which is augmented with the use of exogenous volatilities from multiple asset classes offers higher returns compared to both the RW and the simple HAR-RV.

More importantly, the fact that HAR-RV-COMBINED model is the best performing model provides strong support to our argument that there are different “information channels” through which different asset classes could impact oil price volatility and thus, their combination enhances the predictive accuracy of the simple HAR-RV model.

An interesting direction for further research would be the use of our forecasting strategy for the prediction of other assets’ volatilities. Finally, it would also be research-worthy to investigate whether the predictive added-value of the “information channels” featured in this paper would remain qualitatively comparable if we considered alternative measures of volatility, such as the bi-power variation, the median realized variance and the realized semi-variance.

Acknowledgements

The authors would like to thank Prof. Kees Koedijk (Editor) and the anonymous reviewer for their helpful comments on a previous version of this paper. The authors acknowledge the support of the European Union's Horizon 2020 research and innovation programme, which has funded them under the Marie Skłodowska-Curie grant agreement No 658494. We also thank Lutz Kilian, Simos Meintanis, Stephen Millard, Nikos Nomikos, Perry Sadorsky, Apostolos Serletis, Lorenzo Trapani and Benoît Sévi for their valuable comments on an earlier version of the paper. Finally, we would also like to thank the participants of the ISEFI 2016 conference, as well as, the participants of the research seminars at the Bank of Greece, Athens University of Economics and Business, Panteion University of Social and Political Sciences and University of Athens.

References

- Abramson, B., & Finizza, A. (1991).** Using belief networks to forecast oil prices. *International Journal of Forecasting*, 7(3), 299-315.
- Agnolucci, P. (2009).** Volatility in crude oil futures: a comparison of the predictive ability of GARCH and implied volatility models. *Energy Economics*, 31(2), 316-321.
- Aiolfi, M., & Favero, C. A. (2005).** Model uncertainty, thick modelling and the predictability of stock returns. *Journal of Forecasting*, 24(4), 233-254.
- Aloui, C., & Jammazi, R. (2009).** The effects of crude oil shocks on stock market shifts behaviour: a regime switching approach. *Energy Economics*, 31(5), 789-799.
- Aloui, R., Aïssa, M. S. B., & Nguyen, D. K. (2013).** Conditional dependence structure between oil prices and exchange rates: a copula-GARCH approach. *Journal of International Money and Finance*, 32, 719-738.
- Alquist, R., Kilian, L., & Vigfusson, R. J. (2013).** Forecasting the price of oil. *Handbook of Economic Forecasting*, 2, 427-507.
- Andersen, T. G., & Bollerslev, T. (1998).** Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *International Economic Review*, 39, 885-905.
- Andersen, T. G., Bollerslev, T., Diebold, F. X., & Labys, P. (2001).** The distribution of realized exchange rate volatility. *Journal of the American Statistical Association*, 96(453), 42-55.
- Andersen, T. G., Bollerslev, T., Diebold, F. X., & Labys, P. (2003).** Modeling and forecasting realized volatility. *Econometrica*, 71, 579-625.
- Andersen, T. G., Bollerslev, T., & Meddahi, N. (2005).** Correcting the errors: Volatility forecast evaluation using high-frequency data and realized volatilities. *Econometrica*, 73(1), 279-296.
- Andersen, T. G., Bollerslev, T., Christoffersen, P. F., & Diebold, F. X. (2006).** Volatility and correlation forecasting. *Handbook of Economic Forecasting*, 1, 777-878.
- Andersen, T. G., Bollerslev, T., & Diebold, F. X. (2007).** Roughing it up: Including jump components in the measurement, modeling, and forecasting of return volatility. *The Review of Economics and Statistics*, 89(4), 701-720.

- Andersen, T. G., Bollerslev, T., Frederiksen, P., & Ørregaard Nielsen, M. (2010).** Continuous-time models, realized volatilities, and testable distributional implications for daily stock returns. *Journal of Applied Econometrics*, 25(2), 233-261.
- Andrada-Felix, J., Fernandez-Rodriguez, F. and Fuertes, A.M. (2016).** Combining nearest neighbor predictions and model-based predictions of realized variance: Does it pay? *International Journal of Forecasting*, 32, 695–715.
- Angelidis, T., Benos, A., & Degiannakis, S. (2004).** The use of GARCH models in VaR estimation. *Statistical Methodology*, 1(1), 105-128.
- Angelidis, T., & Degiannakis, S. (2008).** Volatility forecasting: Intra-day versus inter-day models. *Journal of International Financial Markets, Institutions and Money*, 18(5), 449-465.
- Antonakakis, N., Chatziantoniou, I., & Filis, G. (2014).** Dynamic spillovers of oil price shocks and economic policy uncertainty. *Energy Economics*, 44, 433-447.
- Arouri, M. E. H., Jouini, J., & Nguyen, D. K. (2011).** Volatility spillovers between oil prices and stock sector returns: implications for portfolio management. *Journal of International Money and Finance*, 30(7), 1387-1405.
- Arouri, M. E. H., Lahiani, A., Lévy, A., & Nguyen, D. K. (2012).** Forecasting the conditional volatility of oil spot and futures prices with structural breaks and long memory models. *Energy Economics*, 34(1), 283-293.
- Baker, S. R., Bloom, N., & Davis, S. J. (2013).** Measuring economic policy uncertainty. *Chicago Booth research paper*, (13-02).
- Baumeister, C., & Kilian, L. (2012).** Real-time forecasts of the real price of oil. *Journal of Business & Economic Statistics*, 30(2), 326-336.
- Baumeister, C., & Kilian, L. (2014).** What central bankers need to know about forecasting oil prices. *International Economic Review*, 55(3), 869-889.
- Baumeister, C., & Kilian, L. (2015).** Forecasting the Real Price of Oil in a Changing World: A Forecast Combination Approach. *Journal of Business & Economic Statistics*, 33(3), 338-351.
- Baumeister, C., & Peersman, G. (2013).** Time-Varying Effects of Oil Supply Shocks on the US Economy. *American Economic Journal: Macroeconomics*, 5(4), 1-28.

- Black, F. and Scholes, M. (1973).** The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3), 637-654.
- Bollerslev, T., Chou, R. Y., & Kroner, K. F. (1992).** ARCH modeling in finance: A review of the theory and empirical evidence. *Journal of Econometrics*, 52(1), 5-59.
- Bollerslev, T., & Wright, J. H. (2001).** High-frequency data, frequency domain inference, and volatility forecasting. *Review of Economics and Statistics*, 83(4), 596-602.
- Busch, T., Christensen, B. J., and Nielsen, M. Ø. (2011).** The role of implied volatility in forecasting future realized volatility and jumps in foreign exchange, stock, and bond markets. *Journal of Econometrics*, 160, 48–57.
- Buyuksahin, B., & Robe, M. A. (2014).** Speculators, commodities and cross-market linkages. *Journal of International Money and Finance*, 42, 38-70.
- Cabedo, J. D., & Moya, I. (2003).** Estimating oil price ‘value at risk’ using the historical simulation approach. *Energy Economics*, 25, 239–253.
- Chatrath, A., Miao, H., Ramchander, S., & Wang, T. (2015).** The Forecasting Efficacy of Risk-Neutral Moments for Crude Oil Volatility. *Journal of Forecasting*, 34(3), 177-190.
- Chen, Y-C., Rogoff, S.K., & Rossi, B. (2010).** Can exchange rates forecast commodity prices?. *The Quarterly Journal of Economics*, 125(3), 1145-1194.
- Chkili, W., Hammoudeh, S., & Nguyen, D. K. (2014).** Volatility forecasting and risk management for commodity markets in the presence of asymmetry and long memory. *Energy Economics*, 41, 1-18.
- Christensen, B. J., & Prabhala, N. R. (1998).** The relation between implied and realized volatility. *Journal of Financial Economics*, 50(2), 125-150.
- Clark, T.E. and West, K.D. (2007).** Approximately normal tests for equal predictive accuracy in nested models, *Journal of Econometrics*, 138, 291–311.
- Corsi, F. (2009).** A simple approximate long-memory model of realized volatility. *Journal of Financial Econometrics*, 7, 174–196.
- Degiannakis, S. (2004).** Volatility forecasting: evidence from a fractional integrated asymmetric power ARCH skewed-t model. *Applied Financial Economics*, 14(18), 1333-1342.

- Degiannakis, S., Filis, G., & Floros, C. (2013).** Oil and stock returns: Evidence from European industrial sector indices in a time-varying environment. *Journal of International Financial Markets, Institutions and Money*, 26, 175-191.
- Diebold, F. X., & Mariano, R. S. (1995).** Comparing predictive accuracy. *Journal of Business & Economic Statistics*, 13, 253-263.
- Efimova, O., & Serletis, A. (2014).** Energy markets volatility modelling using GARCH. *Energy Economics*, 43, 264-273.
- Elder, J., & Serletis, A. (2010).** Oil price uncertainty. *Journal of Money, Credit and Banking*, 42(6), 1137-1159.
- Engle, R.F., Hong, C.H., Kane, A. and Noh, J. (1993).** Arbitrage Valuation of Variance Forecasts with Simulated Options, *Advances in Futures and Options Research*, 6, 393-415.
- Engle, F. R. (2002).** Dynamic conditional correlation: a simple class of multivariate GARCH models. *Journal of Business and Economic Statistics*, 20, 339–350.
- Engle, E., & Sun, Z. (2007).** When is noise not noise—a microstructure estimate of realized volatility.
- Fattouh, B., Kilian, L., & Mahadeva, L. (2013).** The Role of Speculation in Oil Markets: What Have We Learned So Far?. *The Energy Journal*, 34(3).
- Fernandes, M., Medeiros, M. C., & Scharth, M. (2014).** Modeling and predicting the CBOE market volatility index. *Journal of Banking & Finance*, 40, 1-10.
- Ferraro, D., Rogoff, K., & Rossi, B. (2015).** Can oil prices forecast exchange rates? An empirical analysis of the relationship between commodity prices and exchange rates. *Journal of International Money and Finance*, 54, 116-141.
- Filis, G. (2010).** Macro economy, stock market and oil prices: Do meaningful relationships exist among their cyclical fluctuations?. *Energy Economics*, 32(4), 877-886.
- Filis, G., Degiannakis, S., & Floros, C. (2011).** Dynamic correlation between stock market and oil prices: The case of oil-importing and oil-exporting countries. *International Review of Financial Analysis*, 20(3), 152-164.
- Filis, G., & Chatziantoniou, I. (2014).** Financial and monetary policy responses to oil price shocks: evidence from oil-importing and oil-exporting countries. *Review of Quantitative Finance and Accounting*, 42(4), 709-729.

- Frijns, B., Tallau, C., & Tourani-Rad, A. (2010).** The information content of implied volatility: evidence from Australia. *Journal of Futures Markets*, 30(2), 134-155.
- Fuertes, A. M., Izzeldin, M., & Kalotychou, E. (2009).** On forecasting daily stock volatility: The role of intraday information and market conditions. *International Journal of Forecasting*, 25(2), 259-281.
- Giannone, D., L. Reichlin and D. Small (2008).** Nowcasting: The real- time informational content of macroeconomic data. *Journal of Monetary Economics*, 55(4), 665–676.
- Giot, P., & Laurent, S. (2003).** Market risk in commodity markets: A VaR approach. *Energy Economics*, 25, 435–457.
- Hamilton, J. D. (1983).** Oil and the macroeconomy since World War II. *The Journal of Political Economy*, 228-248.
- Hansen, P.R. (2005).** A Test for Superior Predictive Ability. *Journal of Business and Economic Statistics*, 23, 365-380.
- Hansen, P. R., & Lunde, A. (2005).** A forecast comparison of volatility models: does anything beat a GARCH (1, 1)?. *Journal of Applied Econometrics*, 20(7), 873-889.
- Hansen, P. R., Lunde, A., & Nason, J. M. (2011).** The model confidence set. *Econometrica*, 79(2), 453-497.
- Haugom, E., Langeland, H., Molnár, P., & Westgaard, S. (2014).** Forecasting volatility of the US oil market. *Journal of Banking & Finance*, 47, 1-14.
- Hou, A., & Suardi, S. (2012).** A nonparametric GARCH model of crude oil price return volatility. *Energy Economics*, 34(2), 618-626.
- Huntington, H. G. (1994).** Oil price forecasting in the 1980s: what went wrong?. *The Energy Journal*, 1-22.
- IEA (2015).** What drives crude oil prices? US International Energy Administration, July 07. https://www.eia.gov/finance/markets/spot_prices.cfm
- Jiang, G. J., & Tian, Y. S. (2005).** The model-free implied volatility and its information content. *Review of Financial Studies*, 18(4), 1305-1342.
- Jones, C. M., & Kaul, G. (1996).** Oil and the stock markets. *The Journal of Finance*, 51(2), 463-491.
- Kang, S. H., Kang, S. M., & Yoon, S. M. (2009).** Forecasting volatility of crude oil markets. *Energy Economics*, 31(1), 119-125.

- Kang, S.-H., & Yoon, S.-M. (2013).** Modelling and forecasting the volatility of petroleum futures prices. *Energy Economics*, 36, 354–362.
- Kilian, L., & Park, C. (2009).** The impact of oil price shocks on the US stock market. *International Economic Review*, 50(4), 1267-1287.
- Le Pen, Y., & Sévi, B. (2013).** Futures trading and the excess comovement of commodity prices. Available at SSRN 2191659.
- McAleer, M., & Medeiros, M. C. (2008).** Realized volatility: A review. *Econometric Reviews*, 27(1-3), 10-45.
- Mensi, W., Hammoudeh, S., Nguyen, D. K., & Yoon, S. M. (2014).** Dynamic spillovers among major energy and cereal commodity prices. *Energy Economics*, 43, 225-243.
- Nomikos, N. K., & Pouliasis, P. K. (2011).** Forecasting petroleum futures markets volatility: The role of regimes and market conditions. *Energy Economics*, 33(2), 321-337.
- Pesaran, M. H., & Timmermann, A. (2009).** Testing dependence among serially correlated multicategory variables. *Journal of the American Statistical Association*, 104(485), 325-337.
- Phan, D. H. B., Sharma, S. S., & Narayan, P. K. (2015).** Intraday volatility interaction between the crude oil and equity markets. *Journal of International Financial Markets, Institutions and Money*, forthcoming.
- Prokopczuk, M., Symeonidis, L., & Wese Simen, C. (2015).** Do Jumps Matter for Volatility Forecasting? Evidence from Energy Markets. *Journal of Futures Markets*, 1-35.
- Rahman, S., & Serletis, A. (2011).** The asymmetric effects of oil price shocks. *Macroeconomic Dynamics*, 15(S3), 437-471.
- Ross, S. A. (1989).** Information and volatility: The no-arbitrage martingale approach to timing and resolution irrelevancy. *The Journal of Finance*, 44(1), 1-17.
- Sadorsky, P. (2006).** Modeling and forecasting petroleum futures volatility. *Energy Economics*, 28, 467–488.
- Sadorsky, P. (2014).** Modelling volatility and correlations between emerging market stock prices and the prices of copper, oil and wheat. *Energy Economics*, 43, 72-8.
- Sadorsky, P., & McKenzie, M. D. (2008).** Power transformation models and volatility forecasting. *Journal of Forecasting*, 27, 587–606.

- Samuels, J. D., & Sekkel, R. M. (2013).** *Forecasting with many models: Model confidence sets and forecast combination* (No. 2013-11). Bank of Canada Working Paper.
- Sari, R., Hammoudeh, S., & Soytas, U. (2010).** Dynamics of oil price, precious metal prices, and exchange rate. *Energy Economics*, 32(2), 351-362.
- Sévi, B. (2014).** Forecasting the volatility of crude oil futures using intraday data. *European Journal of Operational Research*, 235(3), 643-659.
- Silvennoinen, A., & Thorp, S. (2013).** Financialization, crisis and commodity correlation dynamics. *Journal of International Financial Markets, Institutions and Money*, 24, 42-65.
- Souček, M., & Todorova, N. (2013).** Realized volatility transmission between crude oil and equity futures markets: A multivariate HAR approach. *Energy Economics*, 40, 586-597.
- Souček, M., & Todorova, N. (2014).** Realized volatility transmission: The role of jumps and leverage effects. *Economics Letters*, 122(2), 111-115.
- Stock, J. H. and M. W. Watson (2002).** Forecasting using principal components from a large number of predictors. *Journal of the American Statistical Association*, 97(460), 1167–1179.
- Tay, A., Ting, C., Tse, Y. K., & Warachka, M. (2009).** Using high-frequency transaction data to estimate the probability of informed trading. *Journal of Financial Econometrics*, 7(3), 288-311.
- Thomakos, D. D., & Wang, T. (2003).** Realized volatility in the futures markets. *Journal of Empirical Finance*, 10(3), 321-353.
- Timmermann, A. (2006).** Forecast combinations. *Handbook of economic forecasting*, 1, 135-196.
- Wei, Y., Wang, Y., & Huang, D. (2010).** Forecasting crude oil market volatility: Further evidence using GARCH-class models. *Energy Economics*, 32, 1477–1484.
- Wen, F., Gong, X., & Cai, S. (2016).** Forecasting the volatility of crude oil futures using HAR-type models with structural breaks. *Energy Economics*, 59, 400-413.
- West, K. D., & Cho, D. (1995).** The predictive ability of several models of exchange rate volatility. *Journal of Econometrics*, 69(2), 367-391.
- White, H. (2000).** A Reality Check for Data Snooping. *Econometrica*, 68, 1097–1126.

- Wu, L. (2011).** Variance dynamics: Joint evidence from options and high-frequency returns. *Journal of Econometrics*, 160(1), 280-287.
- Xekalaki, E. and Degiannakis, S. (2005).** Evaluating volatility forecasts in option pricing in the context of a simulated options market. *Computational Statistics and Data Analysis*, 49, 611–629.

Table 1: Optimal Sampling frequencies for the realized volatility construction.

Variable	Futures ticker	Market	m
Brent Crude Oil	CO	ICE Futures Europe	23
GBP/USD	BP	CME Group	30
CAD/USD	CD	CME Group	25
EUR/USD	EC	CME Group	16
FTSE100	FT	ICE Futures Europe	1
S&P500	SP	CME Group	6
Hang Seng	HI	Hong Kong Stock Exchange	60
Euro Stoxx 50	XX	Eurex	3
Gold	GC	CME Group	15
Copper	HG	CME Group	20
Natural Gas	NG	CME Group	10
Palladium	PA	CME Group	90
Silver	SV	CME Group	28
US T-bill 10yr	TY	CME Group	15

Note: m denotes the optimal sampling frequency.

Table 2: Descriptive statistics of the series under investigation. The sample runs from 1st August 2003 to 5th August 2015.

	<i>OIL</i>	<i>FOREX</i>			<i>STOCK MARKETS</i>			
	CO	BP	CD	EC	SP	XX	FT	HI
Mean	28.670	8.841	9.558	9.402	15.826	20.849	16.595	18.807
Maximum	164.219	49.515	47.253	35.554	147.168	150.027	143.174	158.970
Minimum	5.075	2.187	2.396	2.258	4.435	6.324	4.302	2.295
Std. Dev.	14.911	4.352	4.313	3.914	11.383	11.585	11.192	13.899
Skewness	2.447	2.509	1.982	1.675	3.850	3.231	3.485	3.202
Kurtosis	13.119	13.146	10.641	8.200	26.030	20.522	22.222	19.995
Average 1min obs/day	1308	1362	1362	1364	1332	752	663	453
Daily obs	3028	3028	3028	3028	3028	3028	3028	3028
	<i>COMMODITIES</i>					<i>MACRO</i>		
	GC	HG	NG	PA	SV	TY	EPU	
Mean	17.782	27.337	46.720	32.653	31.118	5.937	100.953	
Maximum	98.135	144.822	424.579	141.978	200.658	48.187	626.028	
Minimum	4.247	7.133	14.861	4.470	8.418	1.874	7.396	
Std. Dev.	8.606	14.178	21.813	18.956	15.995	2.957	68.328	
Skewness	2.616	2.355	4.301	1.902	2.946	3.075	1.780	
Kurtosis	15.390	11.903	53.480	8.278	19.073	25.931	8.343	
Average 1min obs/day	1211	1210	1229	1205	1207	1219		
Daily obs	3028	3028	3028	3028	3028	3028	3028	

Note: The values here are based on the annualised realized volatilities that have been scaled according to Hansen and Lunde's (2005) approach. CO=Brent Crude oil, BP=GBP/USD, CD=CAD/USD, EC=EUR/USD, SP=S&P500, XX=Euro Stoxx 50, FT=FTSE100, HI=Hang Seng, GC=Gold, HG=Copper, NG=Natural gas, PA=Palladium, SV=Silver, TY=US 10yr T-Bills, EPU=Economic Policy Uncertainty.

All series are stationary according to the ADF and PP unit root tests. The Jarque-Bera test suggests that none of the series are normally distributed.

Table 3a: MAE Ratios of AR(1), HAR-RV and HAR-RV-X single asset models to the RW model.

Days ahead	RW	AR(1)	HAR-RV	HAR-RV-FT	HAR-RV-SP	HAR-RV-XX	HAR-RV-HI	HAR-RV-BP	HAR-RV-CD
1	11.3344	0.5988	0.4797	0.4780	0.4741	0.4782	0.4789	0.4778	0.4789
5	11.3832	0.8722	0.5408	0.5363	0.5327	0.5379	0.5371	0.5362	0.5383
10	11.4482	0.9635	0.5821	0.5747	0.5677	0.5771	0.5741	0.5758	0.5772
22	11.5643	0.9969	0.6588	0.6479	0.6337	0.6505	0.6428	0.6462	0.6446
44	11.7839	0.9987	0.7752	0.7546	0.7423	0.7628	0.7438	0.7506	0.7385
66	12.0567	0.9987	0.8552	0.8321	0.8227	0.8409	0.8164	0.8245	0.8012
Days ahead		HAR-RV-EC	HAR-RV-GC	HAR-RV-HG	HAR-RV-NG	HAR-RV-PA	HAR-RV-SV	HAR-RV-TY	HAR-RV-EPU
1		0.4790	0.4779	0.4777	0.4800	0.4801	0.4778	0.4771	0.4798
5		0.5388	0.5365	0.5370	0.5406	0.5412	0.5377	0.5363	0.5407
10		0.5784	0.5736	0.5730	0.5809	0.5814	0.5732	0.5695	0.5815
22		0.6471	0.6438	0.6431	0.6566	0.6550	0.6397	0.6340	0.6643
44		0.7426	0.7430	0.7442	0.7777	0.7689	0.7393	0.7219	0.8081
66		0.8030	0.8129	0.8113	0.8628	0.8458	0.8147	0.7810	0.9068

Note: BP=GBP/USD, CD=CAD/USD, EC=EUR/USD, SP=S&P500, XX=Euro Stoxx 50, FT=FTSE100, HI=Hang Seng, GC=Gold, HG=Copper, NG=Natural gas, PA=Palladium, SV=Silver, TY=US 10yr T-Bills, EPU=Economic Policy Uncertainty.

Values represent ratios of HAR-RV-X models to RW model. A ratio below 1 suggests that the MAE of the HAR-RV-X models outperform those of the RW model. The actual MAE values are reported for the RW model.

Table 3b: MSE Ratios of AR(1), HAR-RV and HAR-RV-X single asset models to the RW model.

Days ahead	RW	AR(1)	HAR-RV	HAR-RV-FT	HAR-RV-SP	HAR-RV-XX	HAR-RV-HI	HAR-RV-BP	HAR-RV-CD
1	289.4508	0.4148	0.2442	0.2417	0.2401	0.2406	0.2440	0.2407	0.2429
5	290.9137	0.8444	0.3100	0.3037	0.2984	0.3042	0.3070	0.3004	0.3052
10	292.9822	0.9598	0.3574	0.3451	0.3358	0.3460	0.3480	0.3396	0.3473
22	296.8560	0.9963	0.4716	0.4438	0.4212	0.4437	0.4453	0.4230	0.4404
44	303.9693	0.9984	0.6668	0.6266	0.5954	0.6291	0.6163	0.5788	0.5992
66	312.2399	0.9986	0.8478	0.8071	0.7691	0.8119	0.7874	0.7476	0.7554
Days ahead		HAR-RV-EC	HAR-RV-GC	HAR-RV-HG	HAR-RV-NG	HAR-RV-PA	HAR-RV-SV	HAR-RV-TY	HAR-RV-EPU
1		0.2429	0.2431	0.2424	0.2436	0.2438	0.2411	0.2410	0.2425
5		0.3044	0.3062	0.3084	0.3091	0.3082	0.3026	0.3010	0.3051
10		0.3452	0.3482	0.3537	0.3543	0.3536	0.3419	0.3385	0.3487
22		0.4299	0.4484	0.4644	0.4665	0.4627	0.4341	0.4237	0.4584
44		0.5693	0.6254	0.6604	0.6659	0.6544	0.5939	0.5653	0.6643
66		0.7063	0.7986	0.8383	0.8638	0.8348	0.7577	0.6988	0.8696

Note: BP=GBP/USD, CD=CAD/USD, EC=EUR/USD, SP=S&P500, XX=Euro Stoxx 50, FT=FTSE100, HI=Hang Seng, GC=Gold, HG=Copper, NG=Natural gas, PA=Palladium, SV=Silver, TY=US 10yr T-Bills, EPU=Economic Policy Uncertainty.

Values represent ratios of HAR-RV-X models to RW model. A ratio below 1 suggests that the MSE of the HAR-RV-X models outperform those of the RW model. The actual MSE values are reported for the RW model.

Table 4: MAE and MSE ratios of HAR-RV and HAR-RV-X asset class models, HAR-RV-COMBINED and HAR-RV-AVERAGE to the RW model.

<i>MAE results</i>								
Days ahead	RW	HAR-RV	HAR-RV-STOCKS	HAR-RV-FOREX	HAR-RV-COMMODITIES	HAR-RV-MACRO	HAR-RV-COMBINED	HAR-RV-AVERAGE
1	11.3344	0.4797	0.4745	0.4743	0.4765	0.4769	0.4716	0.4732
5	11.3832	0.5408	0.5323	0.5297	0.5335	0.5365	0.5257	0.5299
10	11.4482	0.5821	0.5683	0.5652	0.5677	0.5733	0.5565	0.5647
22	11.5643	0.6588	0.6354	0.6284	0.6310	0.6460	0.6141	0.6280
44	11.7839	0.7752	0.7398	0.7177	0.7229	0.7687	0.7012	0.7265
66	12.0567	0.8552	0.8169	0.7833	0.7904	0.8484	0.7686	0.7974
<i>MSE results</i>								
Days ahead	RW	HAR-RV	HAR-RV-STOCKS	HAR-RV-FOREX	HAR-RV-COMMODITIES	HAR-RV-MACRO	HAR-RV-COMBINED	HAR-RV-AVERAGE
1	289.4508	0.2442	0.2413	0.2384	0.2439	0.2397	0.2396	0.2391
5	290.9137	0.3100	0.3028	0.2952	0.3048	0.2992	0.2954	0.2982
10	292.9822	0.3574	0.3418	0.3315	0.3440	0.3381	0.3280	0.3352
22	296.8560	0.4716	0.4337	0.4076	0.4399	0.4322	0.4043	0.4217
44	303.9693	0.6668	0.6086	0.5408	0.6064	0.6025	0.5488	0.5778
66	312.2399	0.8478	0.7845	0.6930	0.7759	0.7717	0.7103	0.7404

Note: HAR-RV-STOCKS includes the principal component of the four stock market volatilities (SP=S&P500, XX=Euro Stoxx 50, FT=FTSE100, HI=Hang Seng). HAR-RV-FOREX includes the principal component of the three foreign exchange volatilities (BP=GBP/USD, CD=CAD/USD, EC=EUR/USD). HAR-RV-COMMODITIES includes the principal component of the five commodity volatilities (GC=Gold, HG=Copper, NG=Natural gas, PA=Palladium, SV=Silver). HAR-RV-MACRO includes the principal component of the two macroeconomic volatilities (TY=US 10yr T-Bills, EPU=Economic Policy Uncertainty). HAR-RV-COMBINED includes the principal components of the 14 volatility series. HAR-RV-AVERAGE is the average of the forecasts of HAR-RV-STOCKS HAR-RV-FOREX, HAR-RV-COMMODITIES and HAR-RV-MACRO models.

Values represent ratios of HAR-RV-X models to RW model. A ratio below 1 suggests that the MSE of the HAR-RV-X models outperform those of the RW model. The actual MSE values are reported for the RW model.

Table 5: MCS p-values for 1-day to 66-days ahead.

Loss function: MAE	Forecasting Horizon					
	1-day	5-days	10-days	22-days	44-days	66-days
	p-values are reported					
RW	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(1)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RV	0.0023	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RV-FT	0.0407	0.0002	0.0000	0.0000	0.0000	0.0000
HAR-RV-SP	0.4677	0.0023	0.0001	0.0000	0.0000	0.0000
HAR-RV-XX	0.0318	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RV-HI	0.0023	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RV-BP	0.0466	0.0002	0.0000	0.0000	0.0000	0.0000
HAR-RV-CD	0.0019	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RV-EC	0.0043	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RV-GC	0.0334	0.0002	0.0000	0.0000	0.0000	0.0000
HAR-RV-HG	0.0888	0.0002	0.0001	0.0000	0.0000	0.0000
HAR-RV-NG	0.0060	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RV-PA	0.0015	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RV-SV	0.0781	0.0002	0.0001	0.0000	0.0000	0.0000
HAR-RV-TY	0.0888	0.0002	0.0001	0.0000	0.0000	0.0356
HAR-RV-EPU	0.0012	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RV-STOCKS	0.2745	0.0016	0.0001	0.0000	0.0000	0.0000
HAR-RV-FOREX	0.4677	0.0767	0.0025	0.0006	0.0018	0.0356
HAR-RV-COMMODITIES	0.0781	0.0016	0.0001	0.0000	0.0000	0.0000
HAR-RV-MACRO	0.0407	0.0001	0.0000	0.0000	0.0000	0.0000
HAR-RV-COMBINED	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-RV-AVERAGE	0.4677	0.0016	0.0001	0.0000	0.0000	0.0000

Note: Figures in bold denote that the model belongs to the confidence set of the best performing models. The interpretation of the MCS p-value is analogous to that of a classical p-value; a $(1 - \alpha)$ confidence interval that contains the 'true' parameter with a probability of no less than $(1 - \alpha)$.

BP=GBP/USD, CD=CAD/USD, EC=EUR/USD, SP=S&P500, XX=Euro Stoxx 50, FT=FTSE100, HI=Hang Seng, GC=Gold, HG=Copper, NG=Natural gas, PA=Palladium, SV=Silver, TY=US 10yr T-Bills, EPU=Economic Policy Uncertainty. HAR-RV-STOCKS includes the principal component of the four stock market volatilities (SP=S&P500, XX=Euro Stoxx 50, FT=FTSE100, HI=Hang Seng). HAR-RV-FOREX includes the principal component of the three foreign exchange volatilities (BP=GBP/USD, CD=CAD/USD, EC=EUR/USD). HAR-RV-COMMODITIES includes the principal component of the five commodity volatilities (GC=Gold, HG=Copper, NG=Natural gas, PA=Palladium, SV=Silver). HAR-RV-MACRO includes the principal component of the two macroeconomic volatilities (TY=US 10yr T-Bills, EPU=Economic Policy Uncertainty). HAR-RV-COMBINED includes the principal components of the 14 volatility series. HAR-RV-AVERAGE is the average of the forecasts of HAR-RV-STOCKS HAR-RV-FOREX, HAR-RV-COMMODITIES and HAR-RV-MACRO models.

Table 6: Direction-of-Change results.

Days ahead	HAR-RV	HAR-RV-STOCKS	HAR-RV-FOREX	HAR-RV-COMMODITIES	HAR-RV-MACRO	HAR-RV-COMBINED	HAR-RV-AVERAGE
1	0.6838*	0.6874***	0.6869**	0.6930***	0.6889***	0.6889***	0.6884***
5	0.6700**	0.6741**	0.6792**	0.6711***	0.6716**	0.6823**	0.6762**
10	0.6655*	0.6685*	0.6655*	0.6690**	0.6645**	0.6731**	0.6721**
22	0.6496*	0.6451*	0.6486*	0.6451*	0.6425**	0.6466*	0.6486*
44	0.6160**	0.6191***	0.6201***	0.6308***	0.6150***	0.6293***	0.6267***
66	0.6012	0.5987*	0.6073**	0.6186*	0.5936**	0.6145**	0.6037**
Days ahead	$\frac{HAR - RV}{RW}$	$\frac{HAR - RV - STOCKS}{RW}$	$\frac{HAR - RV - FOREX}{RW}$	$\frac{HAR - RV - COMMODITIES}{RW}$	$\frac{HAR - RV - MACRO}{RW}$	$\frac{HAR - RV - COMBINED}{RW}$	$\frac{HAR - RV - AVERAGE}{RW}$
1	1.1600	1.1661	1.1652	1.1756	1.1687	1.1687	1.1678
5	1.1375	1.1446	1.1532	1.1394	1.1403	1.1584	1.1481
10	1.1299	1.1351	1.1299	1.1359	1.1281	1.1429	1.1437
22	1.1040	1.0962	1.1023	1.0962	1.0919	1.0988	1.1014
44	1.0504	1.0547	1.0565	1.0747	1.0478	1.0721	1.0643
66	1.0297	1.0253	1.0402	1.0594	1.0166	1.0524	1.0306

Note: HAR-RV-STOCKS includes the principal component of the four stock market volatilities (SP=S&P500, XX=Euro Stoxx 50, FT=FTSE100, HI=Hang Seng). HAR-RV-FOREX includes the principal component of the three foreign exchange volatilities (BP=GBP/USD, CD=CAD/USD, EC=EUR/USD). HAR-RV-COMMODITIES includes the principal component of the five commodity volatilities (GC=Gold, HG=Copper, NG=Natural gas, PA=Palladium, SV=Silver). HAR-RV-MACRO includes the principal component of the two macroeconomic volatilities (TY=US 10yr T-Bills, EPU=Economic Policy Uncertainty). HAR-RV-COMBINED includes the principal components of the 14 volatility series. HAR-RV-AVERAGE is the average of the forecasts of HAR-RV-STOCKS HAR-RV-FOREX, HAR-RV-COMMODITIES and HAR-RV-MACRO models.

A ratio above 1 suggests that the DoC values of the HAR-RV-X models outperform those of the RW model. The statistical significance of the directional accuracy is tested based on the Pesaran and Timmermann (2009) under the null hypothesis of no directional accuracy. ***, **, * denotes significance at 10%, 5% and 1% level, respectively.

Table 7: Comparison between MAE (MSE) and MeAE (MeSE).

	1-day	5-days	10-days	22-days	44-days	66-days
MAE	5.3457	5.9838	6.3710	7.1022	8.2624	9.2670
MeAE	3.6661	4.0515	4.2615	4.6663	5.4378	5.7089
MSE	69.3662	85.9406	96.1096	120.0241	166.8111	221.7807
MeSE	13.4401	16.4149	18.1607	21.7741	29.5694	32.5920
<i>Note:</i> These values correspond to the HAR-RV-COMBINED model. The HAR-RV-COMBINED includes the principal components of the 14 volatility series.						

Table 8: MCS p-values for 1-day to 66-days ahead: During crisis period.

<i>Loss function: MAE</i>	Forecasting Horizon					
	1-day	5-days	10-days	22-days	44-days	66-days
	p-values are reported					
RW	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(1)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0012
HAR-RV	0.0076	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-RV-STOCKS	0.0663	0.0011	0.0000	0.0000	0.0000	0.0000
HAR-RV-FOREX	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-RV-COMMODITIES	0.0264	0.0031	0.0008	0.0000	0.0000	0.0000
HAR-RV-MACRO	0.7125	0.1808	0.2278	0.0103	0.0002	0.6177
HAR-RV-COMBINED	0.9278	0.4731	0.4781	0.5657	0.2129	0.6177
HAR-RV-AVERAGE	0.7125	0.1808	0.0202	0.0001	0.0000	0.0000

Note: Figures in bold denote that the model belongs to the confidence set of the best performing models. The interpretation of the MCS p-value is analogous to that of a classical p-value; a $(1 - \alpha)$ confidence interval that contains the 'true' parameter with a probability of no less than $(1 - \alpha)$.

HAR-RV-STOCKS includes the principal component of the four stock market volatilities (SP=S&P500, XX=Euro Stoxx 50, FT=FTSE100, HI=Hang Seng). HAR-RV-FOREX includes the principal component of the three foreign exchange volatilities (BP=GBP/USD, CD=CAD/USD, EC=EUR/USD). HAR-RV-COMMODITIES includes the principal component of the five commodity volatilities (GC=Gold, HG=Copper, NG=Natural gas, PA=Palladium, SV=Silver). HAR-RV-MACRO includes the principal component of the two macroeconomic volatilities (TY=US 10yr T-Bills, EPU=Economic Policy Uncertainty). HAR-RV-COMBINED includes the principal components of the 14 volatility series. HAR-RV-AVERAGE is the average of the forecasts of HAR-RV-STOCKS HAR-RV-FOREX, HAR-RV-COMMODITIES and HAR-RV-MACRO models.

We exclude the HAR-RV models based on single exogenous asset volatility given that their forecasting performances are inferior to the HAR-RV-ASSET CLASSES. Results are available upon request.

Table 9: Cumulative difference in trading returns between HAR-RV-COMBINED, HAR-RV and RW.

<i>Full Sample Period</i>		
Days ahead	HAR-RV-COMBINED vs RW	HAR-RV-COMBINED vs HAR-RV
1	109.7344	3.0103
5	125.5096	0.6620
10	140.9243	19.4403
22	95.8202	26.9962
44	115.0214	54.0649
66	101.1175	92.7130
<i>Global Financial Crisis 2007-08</i>		
Days ahead	HAR-RV-COMBINED vs RW	HAR-RV-COMBINED vs HAR-RV
1	34.9137	4.5681
5	37.7158	-0.9249
10	45.0679	8.7431
22	19.0589	3.3751
44	20.1203	17.7008
66	6.9904	22.5461
<i>2014-15 Oil Price Collapse</i>		
Days ahead	HAR-RV-COMBINED vs RW	HAR-RV-COMBINED vs HAR-RV
1	22.1712	-1.4895
5	31.7253	-0.2619
10	38.1548	4.9162
22	25.6630	2.9230
44	7.8582	1.6065
66	1.7541	24.4823

Note: HAR-RV-COMBINED includes the principal components of the 14 volatility series. The figures refer to percentages. Positive values suggest that the HAR-RV-COMBINED model generates higher trading returns.

Table 10: Cumulative difference in trading straddle options between HAR-RV-COMBINED, HAR-RV and RW

Days ahead	<i>Full Sample Period</i>		
	RW	HAR-RV	HAR-RV-COMBINED
1	-9.7230	0.3611	9.3619
5	-24.0335	-0.3757	24.4092
10	-34.9487	4.7603	30.1885
22	-53.0814	10.0182	43.0632
44	-80.0632	14.8515	65.2117
66	-102.7904	17.4487	85.3417

Note: HAR-RV-COMBINED includes the principal components of the 14 volatility series. The numbers refer to cumulative returns. Positive values suggest that the HAR-RV-COMBINED model generates higher trading returns.

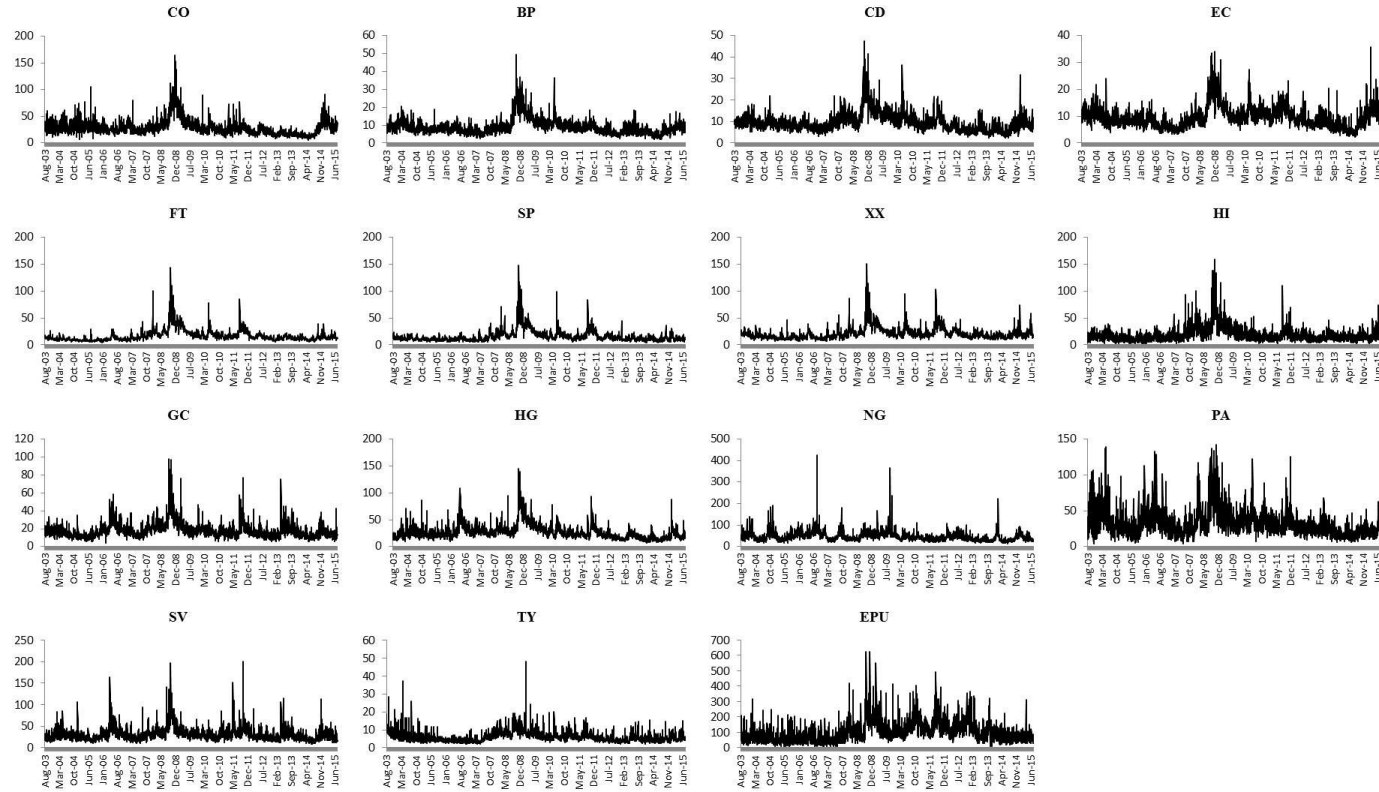
Table 11: MAE and MSE ratios of the HAR-RV-X models based on the asset volatilities and the time-varying correlations.

Days ahead	HAR-RV-STOCKS	HAR-RV-FOREX	HAR-RV-COMMODITIES	HAR-RV-MACRO	HAR-RV-COMBINED	HAR-RV-AVERAGE
<i>MAE ratio: Volatilities vs. Return correlations</i>						
1	0.9765	0.9790	0.9918	0.9941	0.9684	0.9897
5	0.9580	0.9729	0.9811	0.9927	0.9552	0.9809
10	0.9388	0.9543	0.9696	0.9774	0.9338	0.9663
22	0.9136	0.9324	0.9429	0.9648	0.9071	0.9431
44	0.9154	0.8840	0.9026	0.9645	0.8637	0.9216
66	0.9147	0.8489	0.8930	0.9578	0.8395	0.9139
<i>MAE ratio: Volatilities vs. Volatility correlations</i>						
1	0.9819	0.9887	0.9936	0.9965	0.9840	0.9881
5	0.9698	0.9774	0.9860	0.9960	0.9708	0.9804
10	0.9588	0.9682	0.9813	0.9964	0.9612	0.9747
22	0.9374	0.9455	0.9775	0.9958	0.9442	0.9621
44	0.9082	0.8819	0.9642	1.0000	0.8996	0.9315
66	0.8946	0.8549	0.9630	0.9915	0.8867	0.9193
<i>MSE ratio: Volatilities vs. Return correlations</i>						
1	0.9676	0.9702	0.9949	0.9912	0.9743	0.9924
5	0.9285	0.9644	0.9880	0.9869	0.9557	0.9793
10	0.8725	0.9423	0.9671	0.9463	0.9185	0.9445
22	0.7860	0.8744	0.9332	0.8924	0.8518	0.8832
44	0.7595	0.7898	0.9030	0.8389	0.7819	0.8382
66	0.8000	0.7561	0.9177	0.8306	0.7677	0.8439
<i>MSE ratio: Volatilities vs. Volatility correlations</i>						
1	0.9874	0.9865	0.9929	0.9962	0.9901	0.9886
5	0.9735	0.9658	0.9777	0.9956	0.9666	0.9748
10	0.9576	0.9465	0.9687	0.9982	0.9467	0.9626
22	0.9269	0.8933	0.9629	0.9943	0.9127	0.9385
44	0.9044	0.8111	0.9598	0.9998	0.8742	0.9085
66	0.8946	0.7973	0.9719	0.9957	0.8774	0.9047

Note: “Return correlations” denotes the time-varying correlations between the log-returns of the 14 assets and Brent crude oil. “Volatility correlations” denotes the time-varying correlations between the realized volatilities of the 14 assets and Brent crude oil. The nominator for all ratios is the HAR-RV-X model based on the asset volatilities, in all cases.

A ratio below 1 suggests that the HAR-RV-X models based on volatilities outperform those of the HAR-RV-X models based on correlations.

Figure 1: Realized volatility plots.



Note: From the top left to the bottom right the variables are as follows: CO=Brent Crude oil, BP=GBP/USD, CD=CAD/USD, EC=EUR/USD, FT=FTSE100, SP=S&P500, XX=Euro Stoxx 50, HI=Hang Seng, GC=Gold, HG=Copper, NG=Natural gas, PA=Palladium, SV=Silver, TY=US 10yr T-Bills, EPU=Economic Policy Uncertainty. All value in the y-axis refers to percentages.

Figure 2: Distribution of the absolute and squared deviations between the actual oil realized volatility and the predicted values.

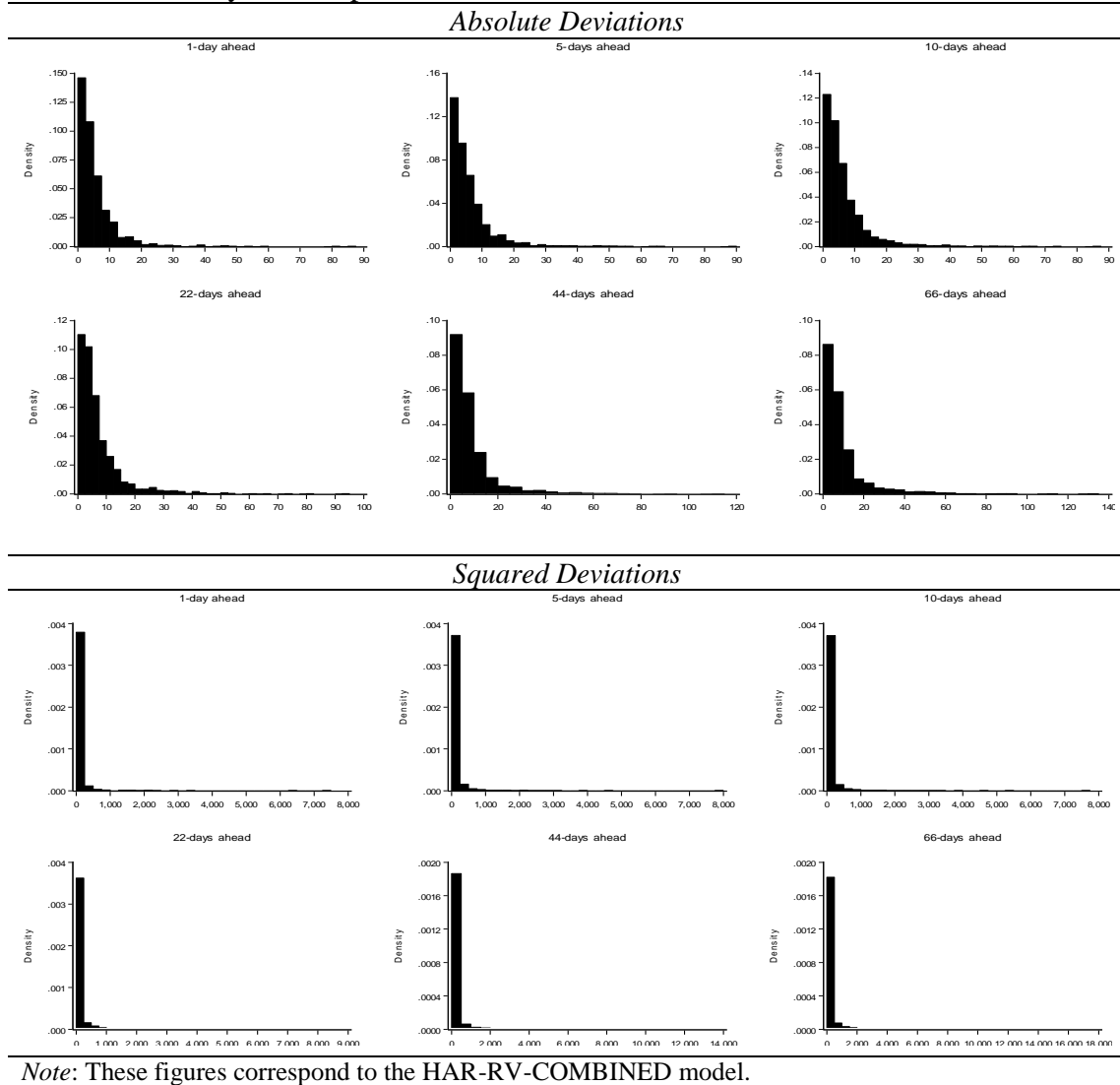
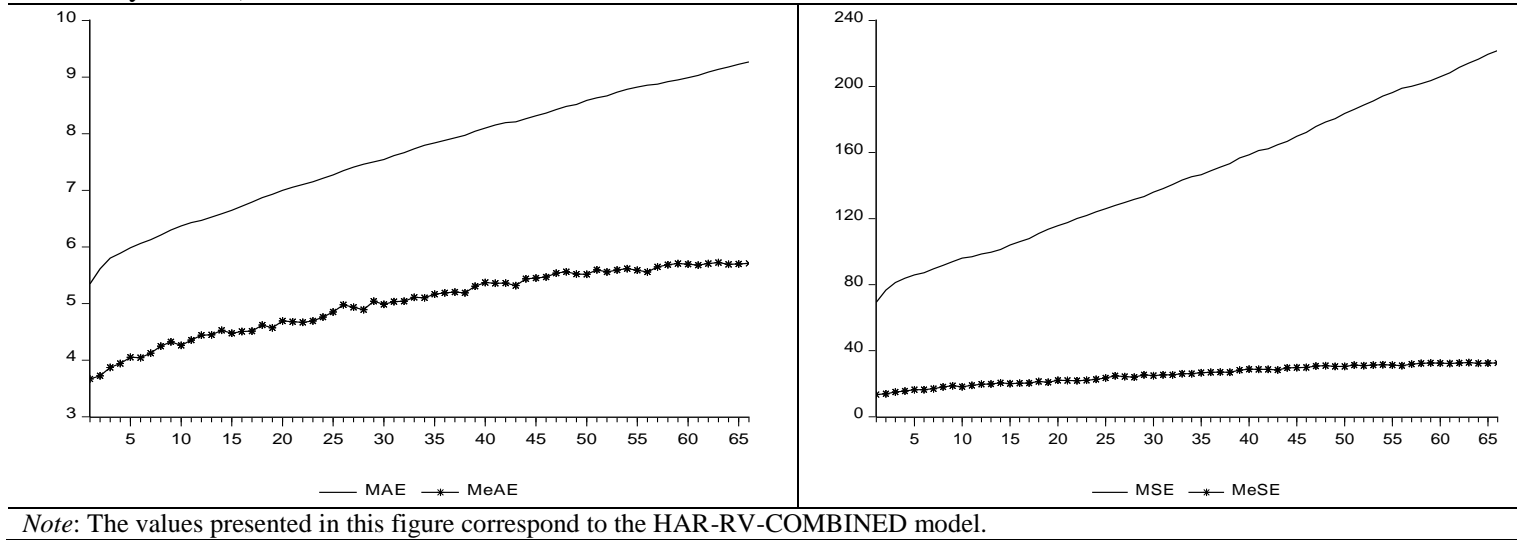
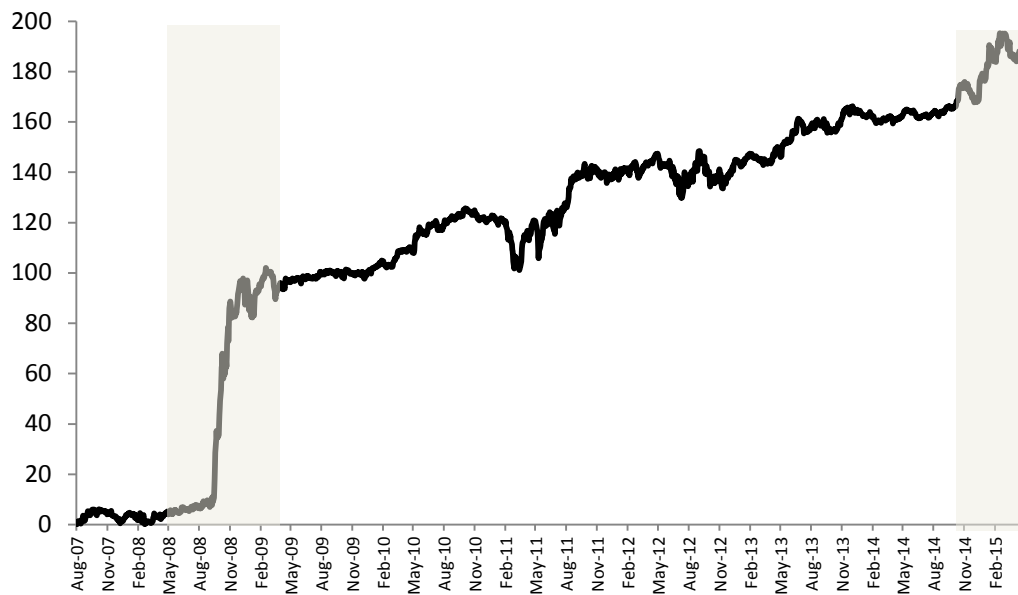


Figure 3: Mean and median values of the squared difference and the absolute difference between the oil realized volatility forecast $(RV_{(HL),OIL,t+s|t}^{(\tau)})$ and the realized volatility $(RV_{(HL),OIL,t+s}^{(\tau)})$ across the forecasting horizons (1-day to 66-days ahead).



Note: The values presented in this figure correspond to the HAR-RV-COMBINED model.

Figure 4: Cumulative incremental value of the HAR-RV-COMBINED model vs. the HAR-RV for the 1-day ahead, based on the MAE.



Note: Upward (downward) movements denote that HAR-RV-COMBINED (HAR-RV) is closer to the actual RV. The first shaded-area denotes the period of the Global Financial Crisis 2007-08, whereas the second shaded-area denotes the 2014-15 oil price collapse.