Motion Capture Data Completion via Truncated Nuclear Norm Regularization

Wenyu Hu, Zhao Wang, Shuang Liu, Xiaosong Yang, Gaohang Yu and Jian J. Zhang

***Abstract*—The objective of mocap data completion is to recover missing measurement of the body markers from motion capture. It becomes increasingly challenging as the missing ratio and duration of mocap data grow. Traditional approaches usually recast this problem as a low-rank matrix approximation problem based on the nuclear norm. However, the nuclear norm defined as the sum of all the singular values of a matrix, is not a good approximation to the rank of mocap data. This paper proposes a novel approach to solve mocap data completion problem by adopting a new matrix norm, called truncated nuclear norm (TrNN). An efficient iterative algorithm is designed to solve this problem based on the augmented Lagrange multiplier. The convergence of the proposed method is proved mathematically under mild conditions. To demonstrate the effectiveness of the proposed method, various comparative experiments are performed on synthetic data and mocap data. Compared to other methods, the proposed method is more efficient and accurate.**

***Index Terms*—Motion capture, Low rank, Truncated nuclear norm, Augmented Lagrange multiplier.**

I. INTRODUCTION

Motion capture (mocap) is widely used for acquiring and analyzing human articulations in computer animation, movie production, virtual reality, and medical rehabilitation [1], [2], [3]. However, the motion data captured is not complete even from professional systems, for example, some markers cannot be recorded due to occlusion, ambiguities or other factors. This problem is more severe for the mocap data acquired by Microsoft Kinect [4], [5]. Therefore, it is essential to complete the missing entries before further applications. Many efforts have been devoted to deal with this issue [6], [7], [8], [9].

In recent years, a new class of matrix-based methods was applied to mocap data completion. The main idea of these methods is to exploit the low-rank property of motion matrix to remove noises and estimate the missing markers [2], [5], [10], [11], [12], [13]. Theoretically, Candès and Recht [14] proved that a low-rank matrix can be accurately recovered from the observations of a small fraction of its entries by solving a nuclear norm minimization problem. From this, Lai et al. [10] applied the low-rank matrix completion model to recover the missing mocap data:

$$\min_{X} \|X\|_* \quad \text{s.t. } P_\Omega(X) = P_\Omega(D), \tag{1}$$

where $\|\cdot\|_*$ denotes the nuclear norm of a matrix, $D \in \mathbb{R}^{m\times n}$ is the observed incomplete motion data, each column of D represents the 3D coordinates of body markers in each frame ($m = 3*$marker number, $n =$frame number), X is the corresponding complete and clean motion data and P_Ω denotes the orthogonal projection of a matrix onto the subspace of matrices which has non-zero entries corresponding to the observed entries in Ω and 0 otherwise. Based on [10], Feng *et al.* [12] additionally took the temporal stability and noise effect of mocap data into account, obtaining a model as follow:

$$\min_{X,S} \|X\|_* + \lambda\|S\|_1 + \frac{\mu}{2}\Theta(X), \ \text{s.t. } P_\Omega(X+S) = P_\Omega(D), \tag{2}$$

where S represents the sparse noises and outliers in the observed part and $\Theta(\cdot)$ is a temporal smoothing penalty term. However, due to the application of nuclear norm, this method has two major limitations. On one hand, the employed iterative solution method involves the expensive computational task of singular value decomposition (SVD) at each iteration, which becomes increasingly costly as the frame numbers of motion sequences grow [15], [16], [17], [18]. On the other hand, nuclear norm minimization makes all of the singular values simultaneously minimized, and thus the rank may not be well approximated in practice [19].

As shown in Fig. 1, the information of motion sequences is commonly dominated by the top r (≤ 30) singular values. Motivated by this observation, this paper proposes a novel mocap data completion method by replacing the nuclear norm with a new matrix norm, called truncated nuclear norm (TrNN) which is defined as the sum of the smallest $\min(m,n) - r$ singular values.

Till now, TrNN has been successfully applied in many fields, such as image inpainting [20], [21], [22], background subtraction [19], multi-class classification [23], photometric stereo [24], [25] and high dynamic range imaging [26], [27]. To the best of our knowledge, this is the first time to use TrNN tackling the mocap data completion problem.

The main contributions of this paper can be summarized as follows: (1) A new but effective mocap data completion model is presented based on the truncated nuclear norm. (2) An efficient iterative algorithm is developed to solve the model, where each subproblem has a closed-form solution. Moreover, a convergence analysis of the proposed algorithm is given.

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W. Hu and G. Yu are with the College of Mathematics and Computer Science, Gannan Normal University, Ganzhou, China (email: wenyu.huu@gmail.com).

Z. Wang, S. Liu, X. Yang and J.J. Zhang are with National Centre for Computer Animation, Bournemouth University, Poole, UK (email: xyang@bournemouth.ac.uk).

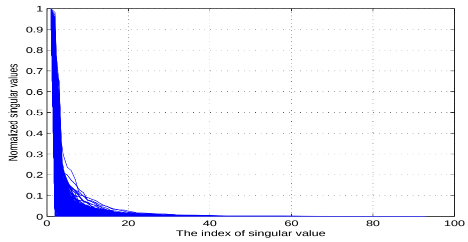


Fig. 1. Normalized singular values of motions collected from CMU mocap database which totally includes 2605 motion sequences. All the singular values are normalized to $[0, 1]$. It's shown that the main information roughly lies in the first 30 largest singular values.

II. TrNN REGULARIZED MOCAP DATA COMPLETION

A. Problem formulation

From the above discussion in Section I, the mocap data completion model considered in this paper is

$$\min_{X,S} \|X\|_r + \lambda\|S\|_1 + \frac{\mu}{2}\Theta(X), \text{ s.t. } P_\Omega(X+S) = P_\Omega(D), \quad (3)$$

where $\|X\|_r = \sum_{i=r+1}^{\min(m,n)} \sigma_i(X)$ denotes the truncated nuclear norm (TrNN) of X ($r \le \min(m,n)$), and $\sigma_i(X)$ is the ith largest singular value of X. Since TrNN minimization is not a convex problem, we cannot solve (3) directly as [12]. The following lemma gives a good surrogate of TrNN, which was proved in [28].

Lemma 1 **([28]).** *The truncated nuclear norm of X can expressed as*

$$\begin{aligned} \|X\|_r = \min_{L,R,U,V} & \frac{1}{2}(\|L\|_F^2 + \|R\|_F^2) - tr(ULR^TV^T), \\ & s.t.\ X = LR^T, UU^T = VV^T = I_r, \end{aligned} \quad (4)$$

where $L \in \mathbb{R}^{m\times d}$, $R \in \mathbb{R}^{n\times d}$, $U \in \mathbb{R}^{r\times m}$ and $V \in \mathbb{R}^{r\times n}$ for any $d \ge rank(X)$, and I_r denotes the $r \times r$ identity matrix.

To make (3) more tractable, we use the following lemma.

Lemma 2. *The model* (3) *is equivalent to the following problem:*

$$\begin{aligned} \min_{X,S} & \|X\|_r + \lambda\|P_\Omega(S)\|_1 + \frac{\mu}{2}\Theta(X), \\ & s.t.\ P_\Omega(X+S) = P_\Omega(D), P_{\Omega^c}(S) = \mathbf{0}. \end{aligned} \quad (5)$$

The proof of this lemma (similar to Lemma 2 in [29]) is given in the Supplementary Material.

By Lemma 1 and Lemma 2, we obtain the following equivalent form to (3):

$$\begin{aligned} \min_{L,R,S,U,V} & \frac{1}{2}(\|L\|_F^2 + \|R\|_F^2) - tr(ULR^TV^T) \\ & + \lambda\|P_\Omega(S)\|_1 + \frac{\mu}{2}\Theta(LR^T), \\ \text{s.t. } & LR^T + S = P_\Omega(D), UU^T = VV^T = I_r, \end{aligned} \quad (6)$$

where the complete and clean motion matrix is $X = LR^T$ and the entries of S in Ω^c are assumed to take the opposite numbers of the corresponding missing values, rather than 0.

To enforce the smoothness of X in the temporal direction, we incorporate the similarity penalty between neighboring frames into the term $\Theta(\cdot)$ as follows:

$$\Theta(X) = \|XO\|_F^2, \quad (7)$$

where $O \in \mathbb{R}^{n\times n}$ is a symmetrical matrix defined by

$$O = \begin{pmatrix} -1 & 1 & & & & \\ 1 & -2 & 1 & & & \\ & \ddots & \ddots & \ddots & & \\ & & & 1 & -2 & 1 \\ & & & & 1 & -1 \end{pmatrix}. \quad (8)$$

This term was interpreted in [12], [30], [31].

B. Solving scheme

To solve the problem (6), we develop an iterative scheme based on the augmented Lagrange multiplier (ALM) method.

Firstly, we introduce a slack variable M and rewrite the problem (6) together with (7) as follow:

$$\begin{aligned} \min_{L,R,S,M,U,V} & \frac{1}{2}(\|L\|_F^2 + \|R\|_F^2) - tr(ULR^TV^T) \\ & + \lambda\|P_\Omega(S)\|_1 + \frac{\mu}{2}\|MO\|_F^2, \\ \text{s.t. } & LR^T + S = P_\Omega(D),\ M = LR^T, \\ & UU^T = VV^T = I_r. \end{aligned} \quad (9)$$

The introduction of M ensures that the solution to each of the following subproblems is simple and has a closed-form.

Next, we can define the partial augmented Lagrangian function $\pounds := \pounds(L, R, S, M, U, V, Y_1, Y_2, \eta)$ of (9) as

$$\begin{aligned} \pounds = & \frac{1}{2}(\|L\|_F^2 + \|R\|_F^2) - tr(ULR^TV^T) + \lambda\|P_\Omega(S)\|_1 \\ & + \frac{\mu}{2}\|MO\|_F^2 + \langle Y_1, M - LR^T\rangle + \langle Y_2, P_\Omega(D) - LR^T - S\rangle \\ & + \frac{\eta}{2}(\|M - LR^T\|_F^2 + \|P_\Omega(D) - LR^T - S\|_F^2), \end{aligned}$$

where Y_1, Y_2 are the Lagrange multiplier matrices and $\eta > 0$ is a penalty parameter. Then, we can employ alternating direction scheme to update each variable while fixing the other variables, and summarize as Algorithm 1.

Specially, by simply computing, we update L, R and S as:

$$\begin{cases} L^{k+1} = P_k R^k \left(I_d + 2\eta^k (R^k)^T R^k\right)^{-1}, \\ R^{k+1} = P_k^T L^{k+1} \left(I_d + 2\eta^k (L^{k+1})^T L^{k+1}\right)^{-1}, \\ S^{k+1} = P_{\Omega^c}(Q_k) + P_\Omega\left(\mathcal{S}_{\lambda/\eta^k}(Q_k)\right), \end{cases} \quad (10)$$

where we denote

$$P_k := (U^k)^T V^k + (Y_1^k + Y_2^k) + \eta^k(M^k + P_\Omega(D) - S^k),$$
$$Q_k := P_\Omega(D) - L^{k+1}(R^{k+1})^T + Y_2^k/\eta^k,$$

and $\mathcal{S}_\tau$ is an element-wise soft-thresholding operator defined as $\mathcal{S}_\tau(x) = \text{sgn}(x)\max(|x| - \tau, 0)$ for any number x [32].

To update M, we fix the other variables and obtain

$$M^{k+1} = \left(\eta^k L^{k+1}(R^{k+1})^T - Y_1^k\right)\left(\eta^k I_n + \mu O^2\right)^{-1}.$$

Since calculating matrix inversion is much costly when n is very large, we analyze the eigen-decomposition $O = U\Lambda U^T$, where the eigenvector matrices U^T and U actually the n-by-n type-2 discrete cosine transform (DCT) and inverse DCT (IDCT) matrices, respectively [33]. As a consequence, we can get a much more economical updating rule for M:

$$\begin{aligned} M^{k+1} & = [(M^{k+1})^T]^T \\ & = \left\{\text{IDCT}\left[\Gamma\text{DCT}\left(\eta^k R^{k+1}(L^{k+1})^T - (Y_1^k)^T\right)\right]\right\}^T, \end{aligned} \quad (11)$$

Algorithm 1 Mocap data completion via TrNN

Input: $D, \Omega, O, d, r, \lambda, \mu, \rho, \eta^0, \max_\eta, \varepsilon$.

Initialize: set $(R^0, S^0, M^0, U^0, V^0, Y_1^0, Y_2^0)$.

while not converge **do**

  Update $L^k, R^k, S^k, M^k, U^k, V^k$ using (10), (11), (13);

  Update $Y_1^k = Y_1^{k-1} + \eta^{k-1}(M^k - L^k(R^k)^T)$,
  $Y_2^k = Y_2^{k-1} + \eta^{k-1}(P_\Omega(D) - L^k(R^k)^T - S^k)$;

  $\eta^k = \min\{\rho\eta^{k-1}, \max_\eta\}$;

  Check the convergence conditions:
  $\frac{\|P_\Omega(D) - L^k(R^k)^T - E^k\|_F}{\|P_\Omega(D)\|_F} \le \varepsilon$,
  $\frac{\|L^k(R^k)^T - L^{k-1}(R^{k-1})^T\|_F}{\|L^k(R^k)^T\|_F} \le \varepsilon$;

end while

Output: $\tilde{X} = L^k(R^k)^T$, the perfect motion matrix.

where Γ is a diagonal matrix with the diagonal elements

$$\Gamma_{ii} = \left(\eta^k + 16\mu \sin^4 \frac{(i-1)\pi}{2n}\right)^{-1}, \ i = 1, 2, \cdots, n.$$

Finally, we update U and V based on the following lemma.

Lemma 3. *Assume $G \in \mathbb{R}^{n\times r} (r < n)$. Then one of the optimal solutions of the following problem*

$$\max_{U \in \mathbb{R}^{r\times n}} tr(UG) \quad s.t. \quad UU^T = I_r \tag{12}$$

is $U^* = QP^T$, where P and Q are given by the reduced SVD of G: $G = P\Sigma Q^T$, $P \in \mathbb{R}^{n\times r}$, $Q \in \mathbb{R}^{r\times r}$, $\Sigma \in \mathbb{R}^{r\times r}$, $P^T P = I_r$ and $Q^T Q = QQ^T = I_r$.

Therefore, we obtain

$$\begin{aligned} U^{k+1} &= \operatorname*{argmax}_{U \in \mathbb{R}^{r\times m}} tr(UL^{k+1}(R^{k+1})^T(V^k)^T) \text{ s.t. } UU^T = I_r \\ &= Q_u^{k+1}(P_u^{k+1})^T, \\ V^{k+1} &= \operatorname*{argmax}_{V \in \mathbb{R}^{r\times n}} tr(VR^{k+1}(L^{k+1})^T(U^{k+1})^T) \text{ s.t. } VV^T = I_r \\ &= Q_v^{k+1}(P_v^{k+1})^T, \end{aligned} \tag{13}$$

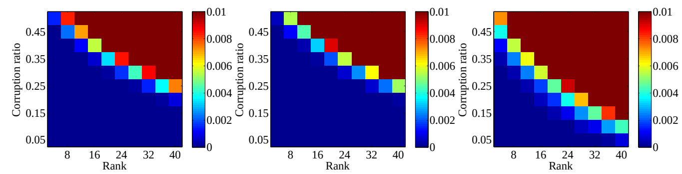
where P_u^{k+1}, Q_u^{k+1} and P_v^{k+1}, Q_v^{k+1} are given by the reduced SVDs of $L^{k+1}(R^{k+1})^T(V^k)^T \in \mathbb{R}^{m\times r}$ and $R^{k+1}(L^{k+1})^T(U^{k+1})^T \in \mathbb{R}^{n\times r}$, respectively. Since $r \ll \min(m, n)$, it's very efficient to implement the reduced SVDs.

C. Convergence analysis

As we know, for the nonconvex problems or convex problems with multiple blocks, there is no theoretical guarantee for its global convergence of ALM. However, we can give a convergence result for Algorithm 1 under mild conditions, following the analysis in [34], [35].

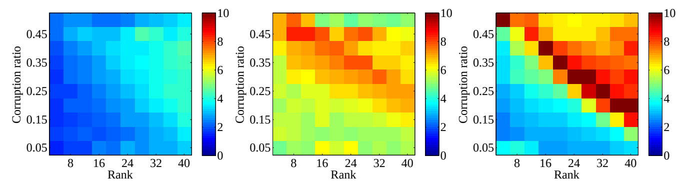
Theorem 1. *Let $W \triangleq (L, R, S, M, U, V, Y_1, Y_2)$. Assume that the sequence $\{W^k\}$ generated by Algorithm 1 is bounded and satisfies $\lim_{k\to\infty} (W^{k+1} - W^k) = 0$. Then, any accumulation point of $\{W^k\}$ satisfies the KKT conditions of* (9).

More implementation details and the complexity analysis of Algorithm 1, the proofs of Lemma 3 and Theorem 1 are given in the Supplementary Material due to the space limit.



(a) TrNN (b) PSVT [24] (c) IALM [15]

Fig. 2. Comparison of NRE by varying rank r_0 and corruption ratio cr. (a) the proposed approach (TrNN), (b) PSVT [24], and (c) IALM [15]. The color magnitude represents NRE.



(a) TrNN (b) PSVT [24] (c) IALM [15]

Fig. 3. Comparison of execution time by varying rank r_0 and corruption ratio cr. (a) the proposed approach (TrNN), (b) PSVT [24], and (c) IALM [15]. The color magnitude represents execution time in seconds.

III. Experimental Results

In this section, the performance of the proposed method is measured on synthetic and mocap data. All the experiments were implemented and timed on a PC with an Intel Core i5 CPU at 2.4GHz and with 4GB of memory.

A. Synthetic data

By setting $\mu = 0$ and Ω as the whole index set, the problem (3) is reduced to the problem of robust principal component analysis (RPCA). Thus, we first demonstrate the effectiveness of the proposed approach on synthetic data.

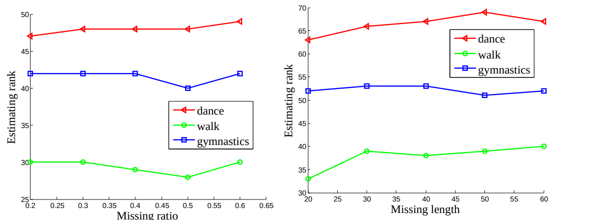
Following [15], we use r_0 and cr to represent the rank and the corruption ratio of the given matrix. Let the raw data matrix be $D = X^* + S^*$. The low-rank matrix X^* is written as the product of a $m \times r_0$ matrix and a $r_0 \times n$ matrix, whose entries are generated independently from standard Normal distribution. The sparse error matrix S^* is uniformly chosen at random, whose nonzero entries are independent and uniformly distributed in $U[-10, 10]$.

We fix $m = 1000$ and $n = 200$, and set $\lambda = \frac{1}{\sqrt{\max(m,n)}}$, $d = r = r_0$, $\rho = 1.5$, $\eta^0 = 1.25/\|D\|_2$ and the stopping tolerance $\varepsilon = 10^{-7}$. We compare our approach (TrNN) with PSVT [24][1] and IALM [15][2] by evaluating the normalized reconstruction error (NRE) $\frac{\|\tilde{X} - X^*\|_F}{\|X^*\|_F}$ and execution time over various settings of matrix rank and corruption ratio. The experiments are repeated by 30 times.

Fig. 2 reports the results of NRE and shows that TrNN and PSVT can produce comparable results, but outperform IALM as the matrix rank r_0 and corruption ratio cr increase. This is because the methods TrNN and PSVT are based on the truncated nuclear norm, while IALM is based on the traditional nuclear norm. Moreover, Fig. 3 reports the execution time, which shows the proposed method TrNN performs much more efficiently than PSVT and IALM.

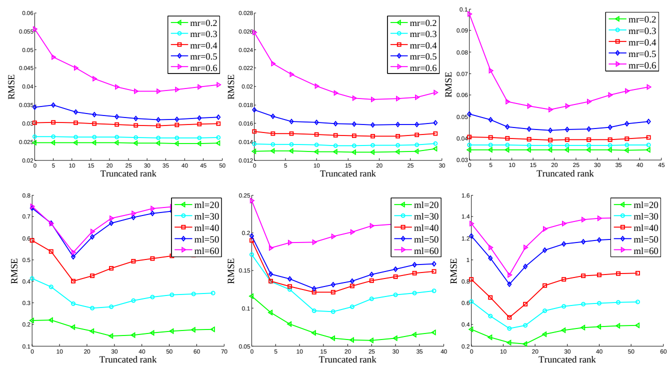
[1]MATLAB code: http://rcv.kaist.ac.kr/v2/bbs/board.php?bo_table=rs_publications&wr_id=483

[2]MATLAB code: http://www.cis.pku.edu.cn/faculty/vision/zlin/zlin.htm



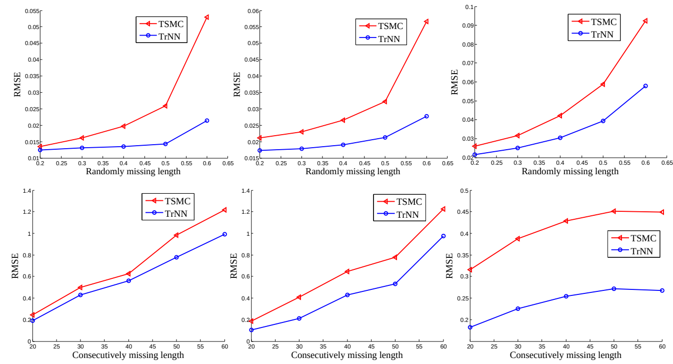
(a) Randomly missing (b) Continuously missing

Fig. 4. Robustness of rank estimation with respect to the randomly missing ratio mr and continuously missing length ml. (a) Estimated ranks for $mr = [0.2 : 0.1 : 0.6]$, (b) Estimated ranks for $ml = [20 : 10 : 60]$.



(a) Dance (b) Walk (c) Gymnastics

Fig. 5. Illustration of the effect of truncated rank to the completing results via RMSE. The **upper** row shows the RMSE for the randomly missing case with $mr = [0.2 : 0.1 : 0.6]$, while the **lower** row shows the RMSE for the continuously missing case with $ml = [20 : 10 : 60]$.



(a) Jump (b) Score (c) Boxing

Fig. 6. Comparison of completion results between the proposed method (TrNN) and TSMC [12]. The **upper** row shows the RMSE for the randomly missing case with $mr = [0.2 : 0.1 : 0.6]$, while the **lower** row shows the RMSE for the continuously missing case with $ml = [20 : 10 : 60]$.

B. Mocap data

Like [12], denoting mr and ml as the randomly missing ratio and continuously missing length, we generate two kinds of incomplete mocap data: randomly missing data and continuously missing data. The first one is obtained by randomly removing mr markers from each frame, while the second one is continuously removing ml frames for 10 randomly selected missing markers in each frame. In this part, we present the completion results of six motion sequences (i.e. dance, walk, gymnastics, jump, score and boxing[3]) from CMU mocap database[4] in the following experiments. We perform the simulations by fixing $\lambda = \frac{100}{\sqrt{\max(m,n)}}, \mu = 100, \rho = 1.4, \eta^0 = 10^{-5}$ and $\varepsilon = 10^{-4}$, and repeat 20 times.

Due to the definition of the truncated nuclear norm in (4), it is necessary to estimate the rank rank(X) of the incomplete motion data at first. To this end, we first give an initial guess for missing data by adopting the linear interpolation scheme along the temporal direction, and then detect the largest jump between adjacent singular values from SVD as in [36], [37]. Specifically, the estimated rank is set as the largest index where the jump is beyond a specified threshold, namely, $\text{rank}(X) \approx \tilde{r} := \max\{i : |\sigma_i - \sigma_{i+1}| \geq \theta\}$ (here $\theta = 0.1$). For both of the data-missing modes, we verify the robustness of rank estimation with respect to the missing ratio mr and missing length ml in Fig. 4. We observe that the estimated ranks keep much stable as mr and ml increase. It is reasonable because human motion is highly articulated so that most of the missing information can be revived through interpolation from neighboring markers.

Next, we illustrate the effect of truncated rank r to the motion completion results, where we adopt the metric of Root Mean Squared Error (RMSE) defined by RMSE $= \frac{\|(X-\tilde{X})|_{\Omega^c}\|_F}{\sqrt{|\Omega^c|}}$. As shown in Fig. 5, using the truncated nuclear norm improves the completion results a lot except for the randomly missing cases with mr below 0.4. This is due to the results of these cases are already good enough (because RMSE ≤ 0.03), and besides a small amount of randomly missing markers don't make the original motion structures badly damaged, so they can be recovered well from their (dense) neighboring markers. Most significantly, we observe that regardless of what values mr and ml take, the RMSE of all the cases approximately attain its minima when the truncated rank is $r = 15$. Thus, we always set $r = \min(15, \tilde{r})$ in the following experiments.

Finally, we evaluate the performance of proposed approach (also called TrNN as in Section III-A) by comparing to Feng et al.'s approach (TSMC) [12]. Since Feng et al. have demonstrated TSMC's superior performance over many other methods such as linear/spline interpolation, Dynammo [8] and SVT [10], we here only compare TrNN with TSMC. In Fig. 6, we report the completion results of three motions ('jump', 'score' and 'boxing') whose frame numbers are respectively 439, 801 and 4840. It is shown that TrNN outperforms TSMC and its advantage becomes even more obvious as mr and ml grow. Accordingly, the average execution time of TrNN for completing these motions is 2.8976 s, 6.3003 s and 63.2687 s respectively, while TSMC took 3.7151 s, 16.1250 s and 690.2639 s. Therefore, TrNN is much more efficient than TSMC especially for long motion sequences (e.g. more than ten times faster for the motion 'boxing').

IV. Conclusion

In this paper, we analyze the problem of mocap data completion based on the truncated nuclear norm which takes into account the prior information of motion matrix rank. An efficient optimization algorithm based on the augmented Lagrange multiplier is proposed. Extensive experiments show

[3]The indices of the selected motions are 05_13, 12_02, 49_02, 13_13, 10_01 and 13_17, which consist of multiple types of action.

[4]http://mocap.cs.cmu.edu/.

our approach outperforms the state-of-the-art methods on both low-rank matrix recovery and mocap data completion.

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