Received: 2 July 2018 | Revised: 9 October 2019 | Accepted: 18 June 2020

DOI: 10.1002/ijfe.1911

RESEARCH ARTICLE

WILEY

# Oil and currency volatilities: Co-movements and hedging opportunities

**Aleksander Olstad**[1] | **George Filis**[1] | **Stavros Degiannakis**[2]

[1]Department of Accounting, Finance and Economics, Bournemouth University, Bournemouth, UK

[2]Department of Economics and Regional Development, Panteion University of Social and Political Sciences, Athens, Greece

**Correspondence**
George Filis, Department of Accounting, Finance and Economics, Bournemouth Business School, The Executive Business Centre, 89 Holdenhurst Road, Bournemouth University, Bournemouth BH8 8EB, UK.
Email: gfilis@bournemouth.ac.uk

**Funding information**
H2020 Marie Skłodowska-Curie Actions, Grant/Award Number: 658494

**Abstract**

The current literature concentrates its attention to the interactions between oil and exchange rates, focusing only on the first moments. Extending this line of research, we investigate the time-varying correlation between the volatilities of two oil benchmarks (Brent and WTI) and six currencies of the major oil-importers and oil-exporters, for the period from February 1, 1999 to May 30, 2016, using a Diag-BEKK model. The optimal portfolio weights and hedge ratios for portfolios comprised of the aforementioned volatilities, are also examined. The analysis reveals that oil and currency volatilities exhibit positive correlations during major global or region-specific economic events (such as the Global Financial Crisis of 2007–2009 and the EU debt crisis period). By contrast, country-specific events yield heterogeneous time-varying correlations between oil and the different currencies in our sample. Both the optimal portfolio weights and optimal hedge ratios estimations demonstrate a time-varying behaviour, suggesting that continuous portfolio rebalancing is necessary for diversification purposes. The findings also show that risk reduction based on the optimal portfolio weight strategy is primarily beneficial for oil volatility investors, whereas currency volatility investors achieve better hedging using the optimal hedge ratio strategy.

**KEYWORDS**

dynamic correlation, exchange rate volatility, hedging strategy, oil price volatility, optimal portfolio

## 1 | INTRODUCTION

In this paper, the time-varying correlation between oil and foreign exchange volatility, for currencies of the major oil-importing and oil-exporting economies, is examined. Furthermore, we assess two risk management strategies, namely optimal portfolio weights and optimal hedge ratios for portfolios that are comprised of the aforementioned volatilities.

The relationship between oil and foreign exchange markets has been at the centre of the international finance literature for a long time, with studies primarily focusing on the returns' relationship using static frameworks (see, inter alia, Amano & Van Norden, 1998a, 1998b; Aloui, Aïssa, & Nguyen, 2013; Beckmann & Czudaj, 2013a, Beckmann & Czudaj, 2013b; Bodart, Candelon, & Carpantier, 2015; Chen, Choudhry, & Wu, 2013; Golub, 1983; Krugman, 1980, 1983;

Reboredo, Rivera-Castro, & Zebende, 2014; Yousefi & Wirjanto, 2004). The novelty of this paper is that we consider (a) their volatility relationship (rather than returns), which is an area where the literature has remained relatively silent and (b) a time-varying framework, given that there is a strand in the literature which emphasizes on the dynamic relationship between oil and other asset classes (see, for instance, Broadstock & Filis, 2014; Ciner, Gurdgiev, & Lucey, 2013; Filis, Degiannakis, & Floros, 2011; Sadorsky, 2014).

Several studies provide convincing evidence that a dollar depreciation may well lead to increased oil prices, whereas decreases in oil prices tend to lead to dollar depreciation (see, for instance Beckmann & Czudaj, 2013b; Blomberg & Harris, 1995; Krugman, 1983; Yousefi & Wirjanto, 2004). Furthermore, it is long established that asset volatility tends to be higher when markets are bearish, while the reverse holds true during bullish periods. Thus, we maintain that a depreciation of the dollar currency could increase its volatility, which can then spillover to the oil price volatility, and vice versa. Hence, it is important to study the relationship between the volatilities of these two asset classes.

This line of enquiry is both important and current given the fact that oil prices have experienced an increased volatility over the last few years, for example, during the Global Financial Crisis of 2007–2009 oil prices fluctuated from about \$60 to a record high of \$145 and subsequently dropped sharply to about \$30 or during the period 2014–2015, where oil lost about 75% of its price. At the same time the foreign exchange market also experienced similar price swings, where, for instance, the dollar exchange rate against numerous other currencies experienced increased volatility during the Global Financial Crisis.

Even more, crude oil is subject to significant trade on a global scale; its demand is expected to increase gradually by a total of 18.4 million barrels per day (mb/d) by 2040 (OPEC, 2015), with derivatives positions in this market exceeding such numbers, proportionally in turnover, which is the results of the financial investors' presence in trading energy commodities, and in particular the presence of hedge funds. Similarly, the foreign exchange market has seen a significant increase in its turnover, where USD, in particular, experienced a 56% increase in its daily average turnover of over-the-counter (OTC) derivatives for the period 2007–2016 (Bank of International Settlements, 2016a). On the other hand, oil-traded currencies (such as CAD, GBP and JPY) also experienced high growth rates in their average daily turnovers during the same period of about 83%, 32% and 91%, respectively (Bank of International Settlements, 2016b). Overall, the significant increase in hedge funds participating in the foreign exchange markets (Galati & Heath, 2007; Lowes & Nenova, 2013), as well as, the oil market, which led to the financialization of the latter (see, Fattouh, Kilian, & Mahadeva, 2013; Juvenal & Petrella, 2015; Tang & Xiong, 2012, among others) renders the examination of their volatility dynamics of major importance.

Against this backdrop, the aim of this paper is to assess the time-varying relationship between oil and foreign exchange volatility of the major oil-importers and oil-exporters and to address the issue of risk exposure to oil and foreign exchange volatility traders by evaluating different risk management strategies. Our study provides recommendations on how alternative risk management strategies can be employed to mitigate exposure in volatility portfolios.

Our findings can be described succinctly, as follows. First, major geopolitical and economic events impact upon the dynamic conditional correlation of the volatilities of crude oil and currencies in a similar fashion. We further show that correlations reach unprecedented levels during the Global Financial Crisis of 2007–2009, which highlights the impact of the crisis in the volatilities of both assets. Second, the optimal portfolio weights and optimal hedge ratios show that investors should engage in a dynamic risk management strategy, given the time-varying relationship between the volatilities of the two assets. Finally, we show that the risk reduction effectiveness of the optimal portfolio weight strategy, compared to the effectiveness of the optimal hedge ratios, is found to be significantly higher in the case of an oil volatility investor, whereas a currency volatility investor should prefer to engage in an optimal hedge ratio strategy, as opposed to a optimal portfolio weights strategy.

The rest of the paper is structured as follows. Section 2 provides a literature review on the relationship between crude oil and foreign exchange rates, as well as, on research regarding optimal hedging strategies. Section 3 provides a description of the data and Section 4 details the methodology applied in the paper. Section 5 discusses the findings of the study, before Section 6 draws a conclusion of the research.

## 2 | LITERATURE REVIEW

### 2.1 | Crude oil and exchange rates

Oil prices and their importance upon macroeconomic factors and in particular their influence upon exchange rates has been subject to vast research over time, in which the focus has been turned to the potential channels by which crude oil finds itself to be a determinant of exchange rates. There are two main channels which have been identified by the literature, namely, the terms of trade and wealth effect channels.

The terms of trade channel suggests that increases in real oil prices leads to a depreciation of the real exchange rates. This relationship may be explained by the increase in the price of tradable goods[1] relative to non-tradable goods in both the US and a foreign economy, and as such a depreciation of the foreign currency, relative to the US dollar, providing that the foreign economy is more dependent on oil-imports compared to the US. Authors who subscribe to this belief includes, McGuirk (1983), Amano and Van Norden (1998b), Amano and Van Norden (1998a), Chen and Chen (2007), among others.

On the other hand, several authors suggest that the effects of oil on exchange rates stems from the wealth effect channel (see, e.g., Krugman, 1983; Golub, 1983; Habib, Bützer, & Stracca, 2016). This channel maintains that rising oil prices are followed by increases in the wealth of the oil-exporting countries, which, under the assumption that the increased revenues are invested into US dollar denominated assets, this is expected to lead to a short-run appreciation of the US dollar relative to the foreign currency.

Despite the fact that ample evidence exists on the effects of oil on exchange rates, there is also a strand in the literature that provides evidence of the reverse causality, that is, where the US dollar exchange rate fluctuations influence crude oil prices (see, e.g., Blomberg & Harris, 1995; Brahmasrene, Huang, & Sissoko, 2014; Cheung, Chinn, & Marsh, 2004; Ding & Vo, 2012; Razgallah & Smimou, 2011; Sadorsky, 2000; Yousefi & Wirjanto, 2004).

For instance, Yousefi and Wirjanto (2004) suggests that US dollar exchange rate fluctuations against major currencies initiate an oil price adjustment mechanism so as to enable oil-exporting countries to sustain their purchasing power of revenues generated from oil exports and the market demand, that further allows them to maintain market shares. Sadorsky (2000) also reports that US dollar exchange rates and crude oil price are cointegrated in the long-run, whereas in the short-run the causality runs from the exchange rates to the oil price changes. Furthermore, Beckmann and Czudaj (2013a), also present a study based on oil-exporting and oil-importing countries and find a bidirectional relationship between oil and exchange rates, although the effect primarily flows from the exchange rates to the oil prices, whereas US dollar depreciation causes oil prices to increase. Similarly, Wu, Chung, and Chang (2012) also report that there is a dependence structure between exchange rates and crude oil returns, which primarily runs from the former to the latter (a depreciation of the US dollar leads to oil price increases). More importantly, they show that this dependence gradually increases from the global financial crisis of 2007–2009 onwards.

Lizardo and Mollick (2010), though, report that oil prices cause fluctuations in the US dollar exchange rates; although, the actual effects depend on whether the US dollar exchange rate is against an oil-importing or oil-exporting economy. In particular, they find that oil price increases tend to lead to a depreciation of the US dollar against the oil-exporting currencies. By contrast, an increase in oil prices would result in an appreciation of the US dollar relative to oil-importing currencies.

Turning to volatilities, the literature is extremely scarce. In particular, Ding and Vo (2012) examines volatility interactions between oil and foreign exchange markets for both oil-importing and oil-exporting countries. They approximate daily volatility for both the oil prices and exchange rates, as the absolute daily log-returns. Using data for the Canadian Dollar, Norwegian Krone and Mexican Peso (representing exporters), and for the European Euro, Indian Rupee, Japanese Yen and Brazilian Real (representing importers), relative to the US dollar, they report that the exchange rates and oil price volatilities do not interact during tranquil times. By contrast, during turbulent times the authors find a bidirectional volatility spillover effects between the two markets.

Our paper builds on Ding and Vo (2012) paper, focusing on the effectiveness of two well-established hedging strategies, namely, optimal portfolio weights and optimal hedge ratios, when investing in portfolios comprised oil and currency volatilities.

## 2.2 | Hedging strategies

The literature has suggested both unconditional and conditional optimal hedge ratios. Authors opine that the former are preferred due to their parsimony with respect to the model specification along with their relatively simple management requirement. Conditional optimal hedge ratios, on the other hand, require frequent rebalancing and as such the resulting transaction costs may deteriorate or even eliminate gains from implementing a conditional hedge (see, inter alia, Baillie & Myers, 1991; Chakraborty & Barkoulas, 1999; Cotter & Hanly, 2012; Fan, Li, & Park, 2015; Haigh, 2005; Kavussanos & Visvikis, 2008; Lee, Yoder, Mittelhammer, & McCluskey, 2006; Lee & Yoder, 2007; Xu & Yang, 2009).

Recently, several studies estimate time-varying optimal hedging strategies. For instance, Alizadeh, Nomikos, and Pouliasis (2008) focus on optimal hedging strategy and optimal hedge ratio estimations under the assumption of these being a function of market conditions by looking at data for WTI crude oil, heating oil and unleaded gasoline. They estimate both a conventional OLS model, as well as, dynamic conditional GARCH, MRS-GARCH and MRS-BEKK models and find that conditional hedge ratios are superior to the conventional ratios in terms of variance reduction, utility maximization and Value-at-Risk (VaR). Under a similar vein Chang, McAleer, and Tansuchat (2011), Toyoshima, Nakajima, and Hamori (2013), Pan, Wang, and Yang (2014) and Basher and Sadorsky (2016) also report that conditional optimal hedge ratios, based on time-varying volatility models (such as variations of the BEKK or the DCC models) yield better results.

Thus, in this study we will utilize a dynamic approach to optimal hedge ratios, using a time-varying correlation model. We will complement our study by examining the time-varying optimal portfolio weights in a two-asset portfolio, comprised of the volatilities of the oil and exchange rates.

## 3 | DATA DESCRIPTION

We consider daily closing spot prices for the Brent and the WTI crude oil benchmarks, as well as, for the currencies of oil-exporters and oil-importers, which are used to estimate the volatility measures of the two assets[2] . We implement two specific criteria as a means of selecting appropriate currencies for the analysis.

First, currencies need to correspond to the major oil-importers and -exporters. Second, the chosen currencies should have a significant volume turnover in the financial markets, measured by the monthly trade volume of foreign exchange futures on two of the most prominent global derivatives exchanges, namely the Chicago Mercantile Exchange (CME) and the Intercontinental Exchange Futures US market (ICE). The volume of foreign exchange futures traded in 1 month provide a reliable indicator for the extent to which the currencies chosen for the sample are among the most traded currencies globally. An overview of the first criterion is given in Table 1.

Furthermore, Table 2 provides an overview of monthly volumes of foreign exchange futures traded on CME and ICE, respectively, in which the currencies selected for the sample is found to be traded the most.

Based on Tables 1 and 2, we collect daily spot prices for six most tradable currencies of the major oil-importing and -exporting countries, with respect to crude oil. In particular our sample currencies are the Canadian Dollar (USD/CAD), the British Pound Sterling (USD/GBP) and the Norwegian Kroner (USD/NOK), which represent the currencies for oil-exporting countries (hereafter export-currencies), and the European Union Euro

**TABLE 1** Crude oil trade activity of major oil-importer and oil-exporters, by volume

| Exports | | | Imports | | |
|---|---|---|---|---|---|
| Rank | Country | Volume | Rank | Country | Volume |
| 1 | Saudi Arabia | 6,250 | 1 | United States | 9,812 |
| 2 | Russia | 4,871 | 2 | China | 4,082 |
| 3* | Canada | 2,470 | 3* | Japan | 3,724 |
| 4 | Iran | 2,297 | 4* | India | 3,185 |
| 5 | UAE | 2,181 | 5 | South Korea | 2,574 |
| 6 | Nigeria | 2,115 | 6* | Germany | 1,888 |
| 7 | Angola | 1,909 | 7 | Italy | 1,531 |
| 8 | Iraq | 1,903 | 8 | Netherlands | 1,274 |
| 9 | Venezuela | 1,594 | 9 | Spain | 1,233 |
| 10 | Kuwait | 1,495 | 10 | United Kingdom | 1,222 |
| 11 | Libya | 1,405 | 11 | France | 1,159 |
| 12 | Kazakhstan | 1,366 | 12 | Singapore | 1,078 |
| 13* | Norway | 1,324 | 13 | Taiwan | 946 |
| 14 | Mexico | 1,280 | 14 | Thailand | 803 |
| 15 | Algeria | 1,175 | 15 | Canada | 736 |
| 16 | Qatar | 1,041 | 16 | Belgium | 680 |
| 17 | Azerbaijan | 871 | 17 | Australia | 512 |

*Notes:* The respective countries in which its currency is selected for the sample is marked with *. The total number of oil-trading countries amounts to 80 oil-exporting countries and 85 oil-importing countries. All data is collected from the U.S. Energy Information Administration.

**TABLE 2** Foreign exchange futures traded on the Chicago Mercantile Exchange (CME) and Intercontinental Exchange (ICE), monthly volume

| CME[a] | | | ICE Futures U.S.[b] | | |
|---|---|---|---|---|---|
| **Abbr.** | **FX Futures** | **Monthly volume[c]** | **Abbr.** | **FX Futures** | **Monthly volume[d]** |
| EUR[e] | European Euro | 2,734,239 | EUR[e] | European Euro | 24,403 |
| JPY[e] | Japanese Yen | 2,632,892 | GBP[e] | British Pound | 3,700 |
| AUD | Australian Dollar | 2,337,080 | NOK[e] | Norwegian Krone | 2,400 |
| GBP[e] | British Pound | 1,743,259 | AUD | Australian Dollar | 1,133 |
| CAD[e] | Canadian Dollar | 1,240,452 | CAD[e] | Canadian Dollar | 407 |
| MXN | Mexican Peso | 842,397 | CHF | Swiss Franc | 241 |
| NZD | New Zealand Dollar | 405,823 | SEK | Swedish Krona | 162 |
| CHF | Swiss Franc | 328,087 | JPY[e] | Japanese Yen | 108 |
| BRL | Brazil Real | 63,316 | INR[e] | Indian Rupee | 27 |
| RUB | Russian Ruble | 24,535 | NZD | New Zealand Dollar | 18 |
| ZAR | S. African Rand | 11,611 | RUB | Russian Ruble | 0 |
| INR[e] | Indian Rupee | 9,985 | | | |
| CNY | Chinese Renminbi | 659 | | | |

[a]All data for CME is collected from Chicago Mercantile Exchange.
[b]All data for ICE Futures U.S. is collected from the Intercontinental Exchange (ICE).
[c]Monthly volume as of July 2016 (latest available data).
[d]Monthly volume as of April 2014 (latest available data).
[e]Currency selected for the sample.

(USD/EUR), the Indian Rupee (USD/INR and the Japanese Yen (USD/JPY), which represent the exchange rates for oil-importing countries (hereafter import-currencies).

The only currency that does not fulfil both criteria is the India Rupee. However, given India's status as one of the largest oil-importers and its status as one of the most important developing countries (Ding & Vo, 2012), we have decided to include this currency in our sample.[3] All exchange rate data, as well as, data on the Brent and WTI crude oil prices are collected from Datastream. The period of the study is February 1, 1999–May 30, 2016, which is purely dictated by the data availability of the chosen series, which allows for a common sample period.

## 3.1 | Volatility construction

There are numerous measures of realized volatility in the literature. In the spirit of Forsberg and Ghysels (2007), Ding and Vo (2012), Zhang and Wang (2014), Antonakakis and Kizys (2015) and Antonakakis, Cunado, Filis, Gabauer, and De Gracia (2018), we define daily volatility as the absolute value of daily log-returns, as follows:

$$v_{i,t} = |\log(p_{i,t}) - \log(p_{i,t-1})| \times 100 \quad (1)$$

where $p_{i,t}$ denotes the price of variable $i$ (i.e., oil prices and currencies) at time $t$.

According to the aforementioned authors, this volatility estimator yields better sampling error behaviour, along with demonstrating immunity to jumps. Alternative volatility approximations could be the squared log-returns, the standard deviation of log-returns and the implied volatility. However, the squared log-returns is an unbiased but extremely noisy estimator, whereas the recursive standard deviation of the most recent log-returns fails to capture the sudden changes in volatility as any moving average estimator. On the other hand, the implied volatility indices measure a weighted average of investors' sentiment for future volatility and not for the current volatility. Finally, the measure of realized volatility, which employs data at higher sampling frequency for computing volatility at a lower frequency, is an alternative measure with proper statistical properties, but it requires the availability of data at ultra-high sampling frequency (i.e., tick-by-tick).[4]

By plotting the volatilities along with geopolitical and economic-driven events (see Figures 1–8), it can clearly be observed how major country-specific, as well as, global events influence the series under investigation.

We notice that significant price innovations in Canadian commodity markets cause an increased volatility in

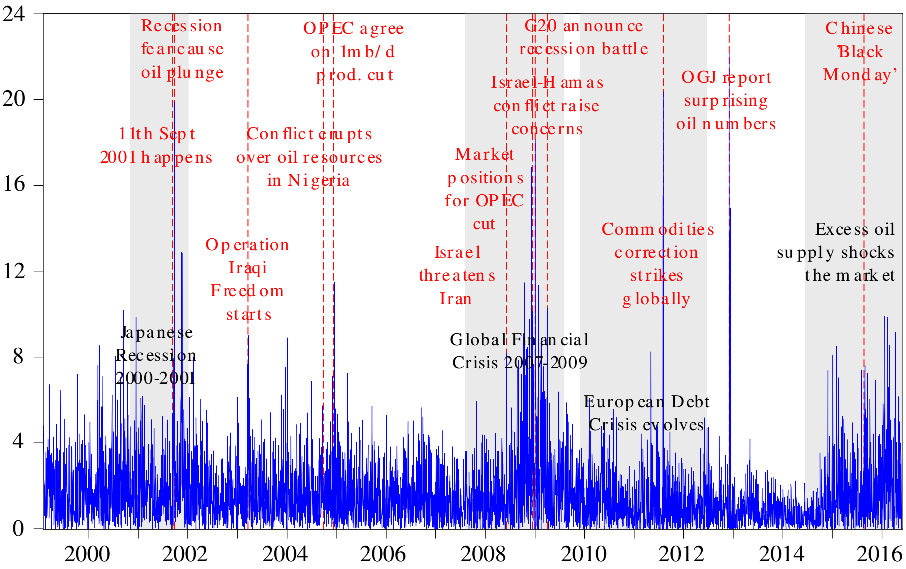



**FIGURE 1** Contemporary events and innovations to Brent oil price returns. *Note*: Grey-shaded areas indicate chronological time periods described in black text, and chronological red dashed lines indicate events described in chronological red text respectively [Colour figure can be viewed at wileyonlinelibrary.com]

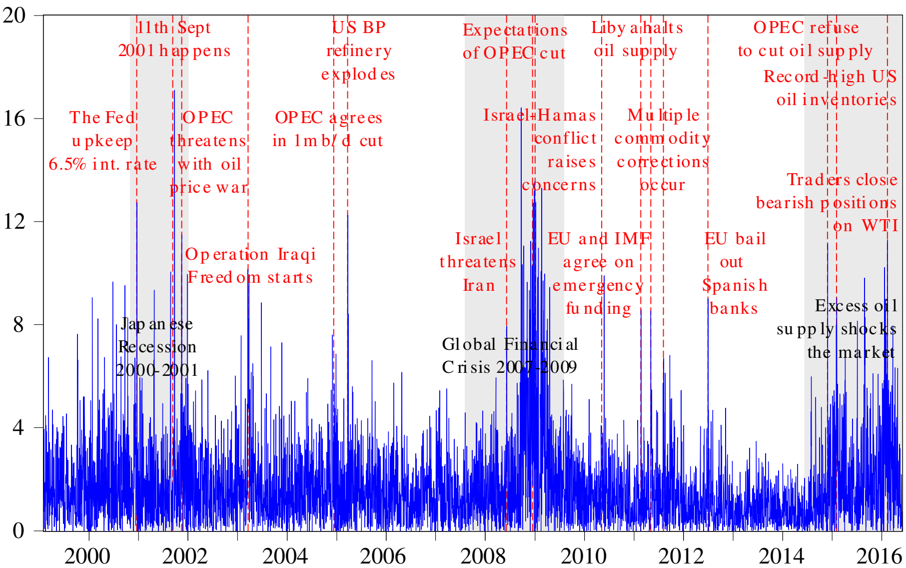



**FIGURE 2** Contemporary events and innovations to WTI oil price returns. *Note*: Grey-shaded areas indicate chronological time periods described in black text, and chronological red dashed lines indicate events described in chronological red text respectively [Colour figure can be viewed at wileyonlinelibrary.com]

the Canadian dollar during 2001–2006 (Powell, 2005). Even more, the joint central bank revaluation of the Euro on the September 22, 2000 caused the USD/EUR to experience increased volatility (ECB, 2000; The Financial Times, 2010). Other country-specific events that triggered increased volatility include the announcement of a substantial infrastructure project to provide fiscal stimulus to the Indian economy in August 2013 (BBC, 2013a; BBC, 2013b) and the volatile market reaction to a surprising monetary policy decision by the Norwegian Central Bank on 19th March 2015 to maintain its interest rate in a difficult time for the Brent crude oil price (The Wall Street Journal, 2015). Apart from these country-specific events, we can establish that major global events have materially influenced the crude oil volatility, as well as, currency volatility (i.e., early-2000 recession, Global Financial Crisis 2007–2009 and the excess oil supply during 2014–2015). It is also important to highlight the homogenous developments in the volatilities of the two different crude oil benchmarks, as expected, which is in contrast to the very heterogeneous volatility patterns of the six currencies, under investigation.

Based on the fact that both asset volatilities seem to be impacted by both country-specific, as well as, global events, we anticipate that significant spillover effects should exist across the two markets, which have implications on asset value and risk management policies for volatility portfolios comprised on these two assets.

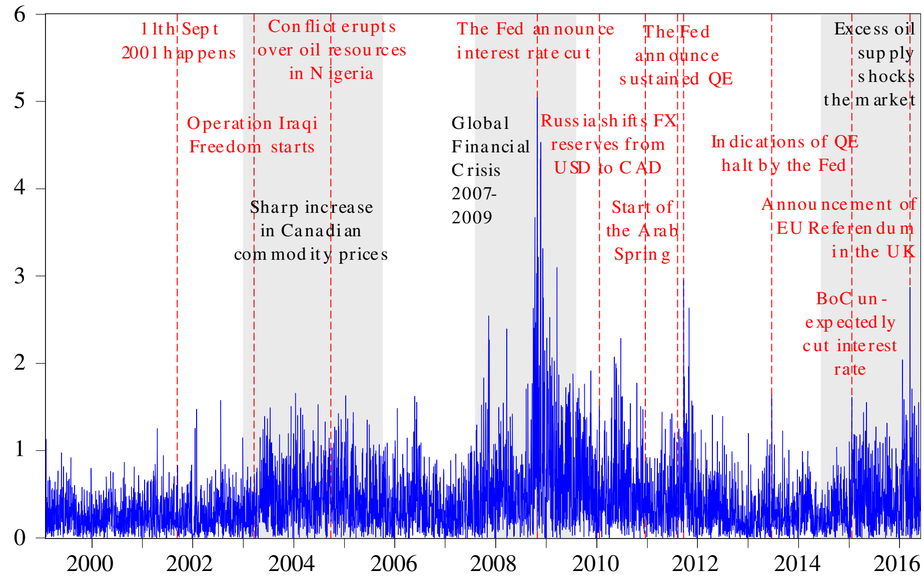



**FIGURE 3** Contemporary events and innovations to USD/CAD exchange rates. *Note*: Grey-shaded areas indicate chronological time periods described in black text, and chronological red dashed lines indicate events described in chronological red text respectively [Colour figure can be viewed at wileyonlinelibrary.com]

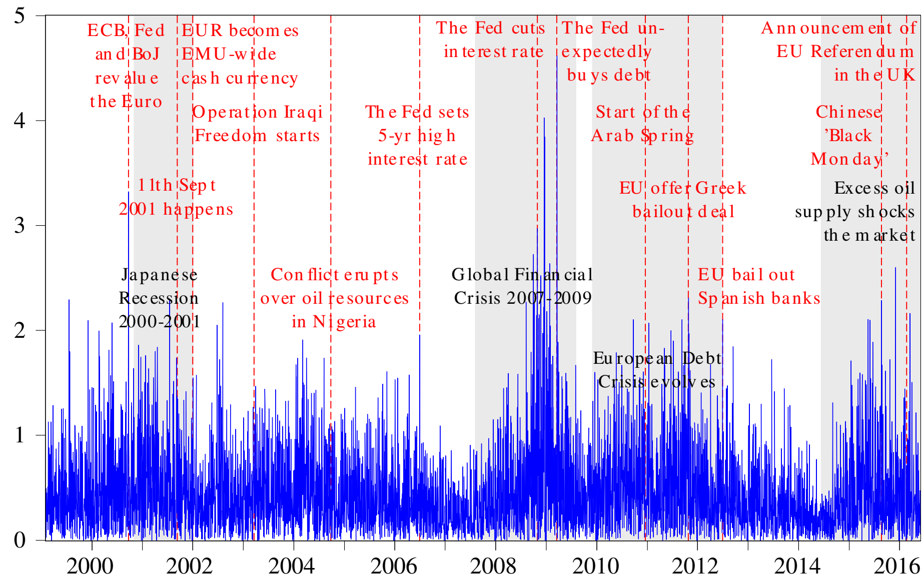



**FIGURE 4** Contemporary events and innovations to USD/EUR exchange rates. *Note*: Grey-shaded areas indicate chronological time periods described in black text, and chronological red dashed lines indicate events described in chronological red text respectively [Colour figure can be viewed at wileyonlinelibrary.com]

Table 3 provides the descriptive statistics for the volatility series. The standard deviation of the crude oil volatilities are significantly higher than those of the exchange rate volatilities, which is in line with existing literature that explains the crude oil prices to have much higher volatilities than currencies (Wu, Chung, Chang, 2012). However, for USD/NOK, the standard deviation of the volatility is relatively higher than the other currencies, whilst the standard deviation of the USD/INR volatility is the lowest. The lower volatility of the latter exchange rate may be explained by the fact that it is the least transacted currency in our sample.

Similarly to the findings of (Wu, Chung, Chang, 2012), the variables are found to have positive skewness and excess kurtosis, which indicates the presence of fat tails. Given the non-normality of the series, the Student-*t* distribution is more appropriate for our estimations.

## 4 | METHODS

### 4.1 | Diagonal BEKK

We start our analysis with the estimation of the time-varying correlations between oil and currency volatilities.

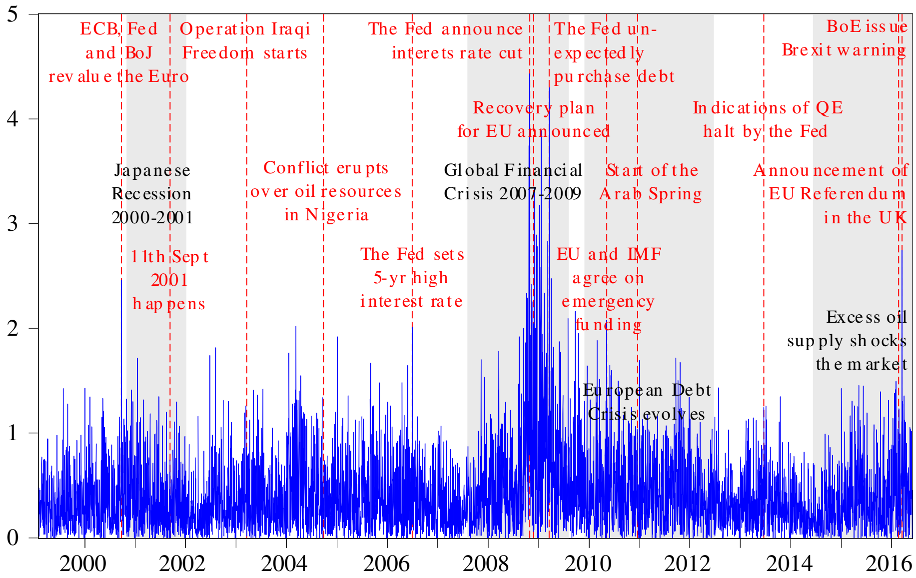



**FIGURE 5** Contemporary events and innovations to USD/GBP exchange rates. *Note*: Grey-shaded areas indicate chronological time periods described in black text, and chronological red dashed lines indicate events described in chronological red text respectively [Colour figure can be viewed at wileyonlinelibrary.com]

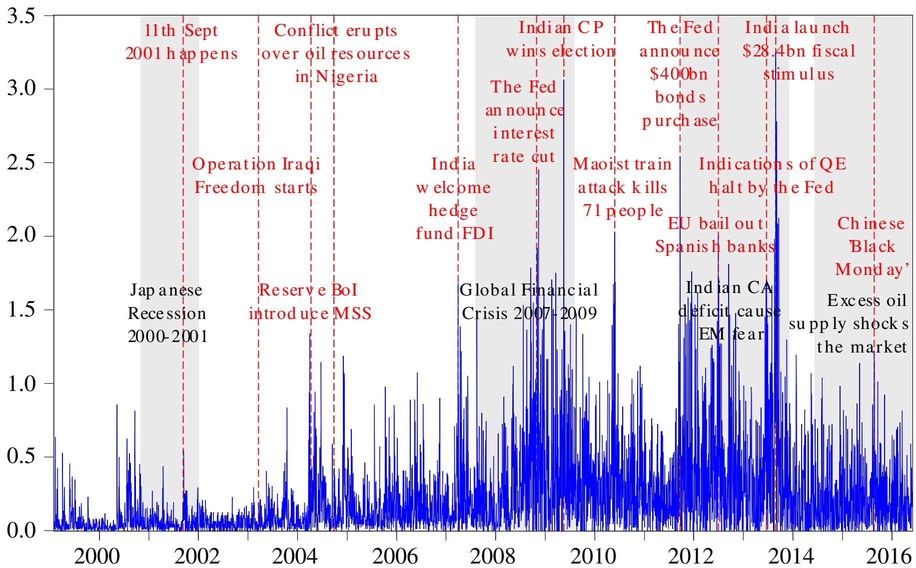



**FIGURE 6** Contemporary events and innovations to USD/INR exchange rates. *Note*: Grey-shaded areas indicate chronological time periods described in black text, and chronological red dashed lines indicate events described in chronological red text respectively [Colour figure can be viewed at wileyonlinelibrary.com]

Similarly to Boldanov, Degiannakis, and Filis (2016), we estimate a Diagonal BEKK model (Diag-BEKK; Engle & Kroner, 1995), where the restrictions allow for a reduction of estimated parameters to $(n \times [l+k]+3)$, where $n$ denotes the number of dependent variables, whereas and $l$ and $k$ are the lag orders. The Diag-BEKK guarantees that variance–covariance ($H_t$) to be positive-definite (Brooks, Henry, & Persand, 2002; Xu & Yang, 2009). The Diag-BEKK model for oil and currency volatilities is defined as:

$$V_t = \mu_t + \varepsilon_t, \quad \text{where} \quad \varepsilon_t \mid \Omega_{t-1} \sim t(0, H_t) \tag{2}$$

$$\varepsilon_t = H_t^{1/2} u_t, \quad \text{where} \quad u_t \sim t(0, I) \tag{3}$$

where $V_t = (v_{1,t}, v_{2,t})'$ is a $2 \times 1$ vector of volatilities (specifically, the *OIL* and *CURRENCY*). For *CURRENCY* we use the volatilities of the six chosen exchange rates, that is, USD/CAD, USD/EUR, USD/GBP, USD/INR, USD/JPY and USD/NOK, whereas for *OIL* we use both the WTI and Brent crude oil price volatility.

The Student $t$ probability density function for $\varepsilon_t$ is:

$$f(\varepsilon_t) = \frac{\Gamma\left(\frac{\lambda+1}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{\lambda}{2}\right)\sqrt{\frac{(\lambda-2)\varepsilon'\varepsilon}{\lambda}}} \times \left(\lambda + \frac{\lambda t^2}{(\lambda-2)\varepsilon'\varepsilon}\right)^{-(\lambda+1)/2} \tag{4}$$

The variance–covariance matrix is expressed as:

$$H_t = \Omega\Omega' + A\varepsilon_{t-1}\varepsilon'_{t\text{-}1}A' + \mathrm{B}H_{t-1}B' \tag{5}$$

where Ω, $A$ and $B$ are diagonal coefficient matrices. The conditional variance matrix of the model is a function of its own lagged values (indicating persistence in the volatility of crude oil and exchange rate return volatilities)

**FIGURE 7** Contemporary events and innovations to USD/JPY exchange rates. *Note*: Grey-shaded areas indicate chronological time periods described in black text, and chronological red dashed lines indicate events described in chronological red text respectively [Colour figure can be viewed at wileyonlinelibrary.com]

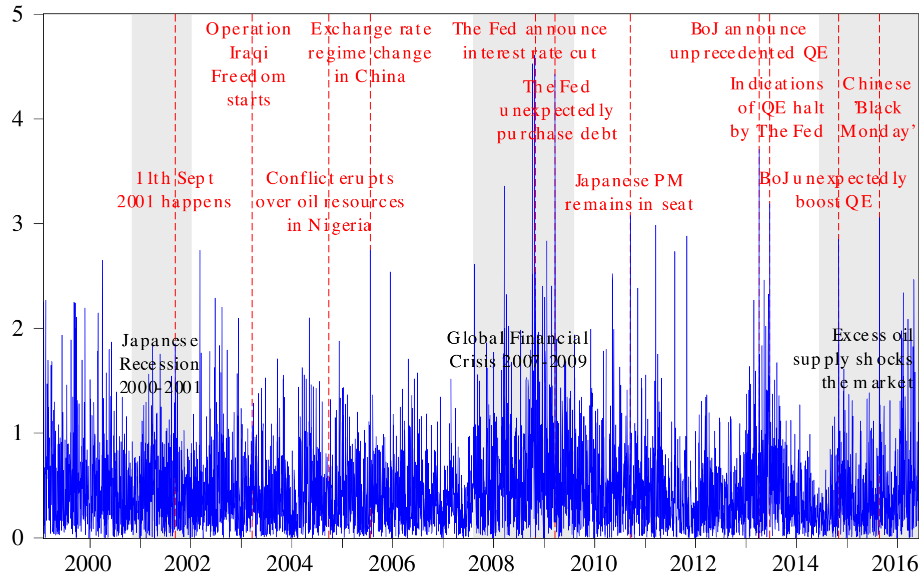
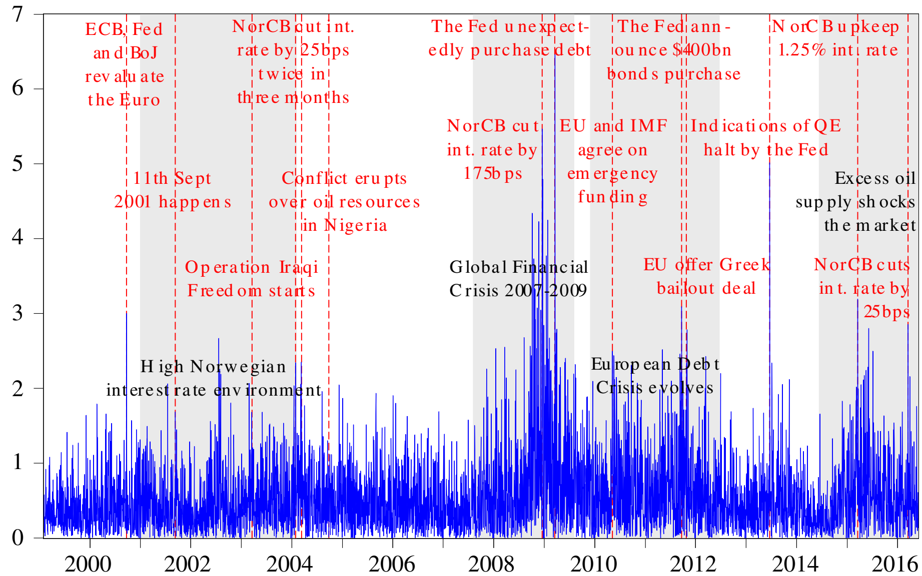



**FIGURE 8** Contemporary events and innovations to USD/NOK exchange rates. *Note*: Grey-shaded areas indicate chronological time periods described in black text, and chronological red dashed lines indicate events described in chronological red text respectively [Colour figure can be viewed at wileyonlinelibrary.com]




and the lagged squared error terms $\varepsilon_{t-1}$, which indicates lagged innovations in the volatility of the volatility of crude oil and exchange rate returns.

The conditional variance $\sigma^2_{o,t}$, $\sigma^2_{c,t}$ and covariance $\sigma_{oc,t}$ of the bivariate oil-currency volatility models are[5]:

$$H_t = \begin{pmatrix} \sigma^2_{o,t} & \sigma_{oc,t} \\ \sigma_{oc,t} & \sigma^2_{c,t} \end{pmatrix} = \begin{pmatrix} \omega_{o,t} & 0 \\ 0 & \omega_{c,t} \end{pmatrix} \begin{pmatrix} \omega_{o,t} & 0 \\ 0 & \omega_{c,t} \end{pmatrix}' + \begin{pmatrix} a_{o,t} & 0 \\ 0 & a_{c,t} \end{pmatrix} \begin{pmatrix} \varepsilon_{o,t-1} & \varepsilon_{o,t-1}\varepsilon_{c,t-1} \\ \varepsilon_{o,t-1}\varepsilon_{c,t-1} & \varepsilon_{c,t-1} \end{pmatrix} \times \begin{pmatrix} a_{o,t} & 0 \\ 0 & a_{c,t} \end{pmatrix}' + \begin{pmatrix} b_{o,t} & 0 \\ 0 & b_{c,t} \end{pmatrix} \begin{pmatrix} \sigma^2_{o,t-1} & \sigma_{oc,t-1} \\ \sigma_{oc,t-1} & \sigma^2_{c,t-1} \end{pmatrix} \begin{pmatrix} b_{o,t} & 0 \\ 0 & b_{c,t} \end{pmatrix}' \tag{6}$$

The estimated conditional variance-covariances are used to estimate the time-varying conditional correlations of oil-currency volatility combination, as:

$$\rho_{oc,t} = \frac{\sigma_{oc,t}}{\sqrt{\sigma^2_{o,t}\sigma^2_{c,t}}} \tag{7}$$

## 4.2 | Optimal portfolio weights

In order for volatility traders to protect their positions in crude oil (currency) volatility, the optimal portfolio weights are estimated. The optimal portfolio weights for a portfolio comprised by currency volatility and oil price volatility is estimated using the methodology adopted from Hammoudeh, Yuan, McAleer, and

**TABLE 3** Summary statistics for all volatilities

| | Mean | *SD* | Skewness | Kurtosis |
|---|---|---|---|---|
| *Commodities* | | | | |
| Brent | 1.623 | 1.694 | 3.143 | 23.527 |
| WTI | 1.722 | 1.727 | 2.371 | 12.326 |
| *Export currencies* | | | | |
| CAD | 0.412 | 0.402 | 2.648 | 17.291 |
| GBP | 0.411 | 0.394 | 2.303 | 14.081 |
| NOK | 0.550 | 0.519 | 3.413 | 14.991 |
| *Import currencies* | | | | |
| EUR | 0.462 | 0.422 | 1.850 | 9.578 |
| INR | 0.230 | 0.302 | 2.802 | 15.204 |
| JPY | 0.470 | 0.443 | 2.237 | 12.335 |

*Note:* The statistics are derived from the volatilities of the price returns as per Equation (1).

Thompson (2010), Arouri, Jouini, and Nguyen (2012), Lin, Wesseh, and Appiah (2014), Syriopoulos, Makram, and Boubaker (2015), Chkili (2016) and Antonakakis et al. (2018). More specifically, according to the aforementioned authors, in a two-asset \$1 USD portfolio of oil and currency volatility, the optimal weight of currency to be held is $w_{c,t}$, given as:

$$w_{c,t}=\begin{cases} 0 & \text{if} \quad \dfrac{\sigma_{o,t}^2-\sigma_{oc,t}}{\sigma_{c,t}^2+\sigma_{o,t}^2-2\sigma_{oc,t}}<0 \\ \dfrac{\sigma_{o,t}^2-\sigma_{oc,t}}{\sigma_{c,t}^2+\sigma_{o,t}^2-2\sigma_{oc,t}} & \text{if} \quad 0<\dfrac{\sigma_{o,t}^2-\sigma_{oc,t}}{\sigma_{c,t}^2+\sigma_{o,t}^2-2\sigma_{oc,t}}<1 \\ 1 & \text{if} \quad \dfrac{\sigma_{o,t}^2-\sigma_{oc,t}}{\sigma_{c,t}^2+\sigma_{o,t}^2-2\sigma_{oc,t}}>1 \end{cases} \tag{8}$$

which implicitly suggests that $w_{o,t}=1-w_{c,t}$, where $w_{o,t}$ denotes the optimal weight in oil to be held in a two-asset \$1 USD portfolio.

To enable a comparison between the performance of the optimal portfolio weights with the optimal hedging strategy, the estimation of the conditional variance of an optimally weighted volatility portfolio is computed as:

$$\sigma_{owp,t}^2=w_{o,t}^2\sigma_{o,t}^2+w_{c,t}^2\sigma_{c,t}^2+2w_{o,t}w_{c,t}\sigma_{oc,t} \tag{9}$$

where $\sigma_{owp,t}^2$ is the variance of the optimally weighted portfolio (in effect a two-asset portfolio).

## 4.3 | Optimal hedging ratios

An alternative risk management approach to optimal hedging can be also employed, where a portfolio of crude oil (currency) volatility is hedged with currency (oil) volatility. The portfolio return for a portfolio of oil volatility that is hedged with currency volatility is given:

$$R_{H,t}=R_{o,t}-\gamma_{c,t}R_{c,t} \tag{10}$$

where $R_{H,t}$ is the portfolio return for holding a long position in crude oil volatility hedged with currency volatility to the proportion of $\gamma_{c,t}$. $R_{o,t}$ and $R_{c,t}$ are the returns of holding positions in crude oil and currency volatility. $\gamma_{c,t}$ is the optimal hedge ratio at time $t$ which minimizes the conditional variance of the portfolio. As such, the variance of the hedged portfolio $R_{H,t}$ is given by:

$$\begin{aligned}\operatorname{var}(R_{H,t}|\Omega_{t-1})=\operatorname{var}(R_{o,t}|\Omega_{t-1})-2\gamma_{c,t}\operatorname{cov}(R_{o,t},R_{c,t}|\Omega_{t-1})\\+\gamma_{c,t}^2\operatorname{var}(R_{c,t}|\Omega_{t-1})\end{aligned} \tag{11}$$

where $\gamma_{c,t}$ is estimated as:

$$\gamma_{c,t}\mid\Omega_{t-1}=\frac{\sigma_{oc,t}}{\sigma_{c,t}^2} \tag{12}$$

The value $\gamma_{c,t}$ indicates the hedge ratio to which a \$1 USD long position in crude oil volatility can be hedged with a short position of $\gamma_{c,t}$ units of currency volatility used as a hedging asset. We also perform the same calculations for \$1 long position in every exchange rate volatility, which is hedged with a short position of $\gamma_{o,t}$ units of oil volatility.

## 4.4 | Hedging effectiveness

Finally, the hedging effectiveness ratio indicates the success of the hedging strategy to minimize the risk of the hedged portfolio, in which the higher ratio implies a higher risk reduction. As such, hedging effectiveness is estimated for both the optimal portfolio weights and optimal hedge ratio strategies so to allow comparison and provide recommendations for risk management purposes. The estimation of the hedging effectiveness ratio is achieved through estimating:

$$\mathrm{HE}=\left[\frac{\mathrm{V}_{\mathrm{unhedged}}-\mathrm{V}_{\mathrm{hedged}}}{\mathrm{V}_{\mathrm{unhedged}}}\right] \tag{13}$$

where $V_{unhedged}$ is the variance of a single-asset volatility portfolio, and $V_{hedged}$ is the variance of an optimally weighted ($\sigma_{owp,t}^2$) or optimally hedged ($\gamma_{c,t}$) oil-currency volatility portfolio.

Having detailed the methodology and model specifications, which are applied in the paper, we proceed with the analysis of the findings.

# 5 | EMPIRICAL FINDINGS

## 5.1 | Analysis of dynamic conditional correlations estimations

Figures 9 and 10 show a significant time-varying fluctuation in the volatility correlation of crude oil and exchange rate volatilities.[6]

A first observation that can be made from Figures 9 and 10 is that there are no significant differences between the time-varying correlations of the export and import currency volatilities or between the two oil benchmarks. One notable exception is the correlation between oil and the INR volatility that exhibits significantly more abrupt swings, compared to the other currencies. The correlations are indeed time-varying and fluctuate in both positive and negative values. Nevertheless, to analyse in greater detail the time-varying correlations, it is important to examine to what extent geopolitical and economic-driven events can alter the correlations in the volatility of different currencies and crude oil. To do so, we first turn our attention to the economic-driven events.

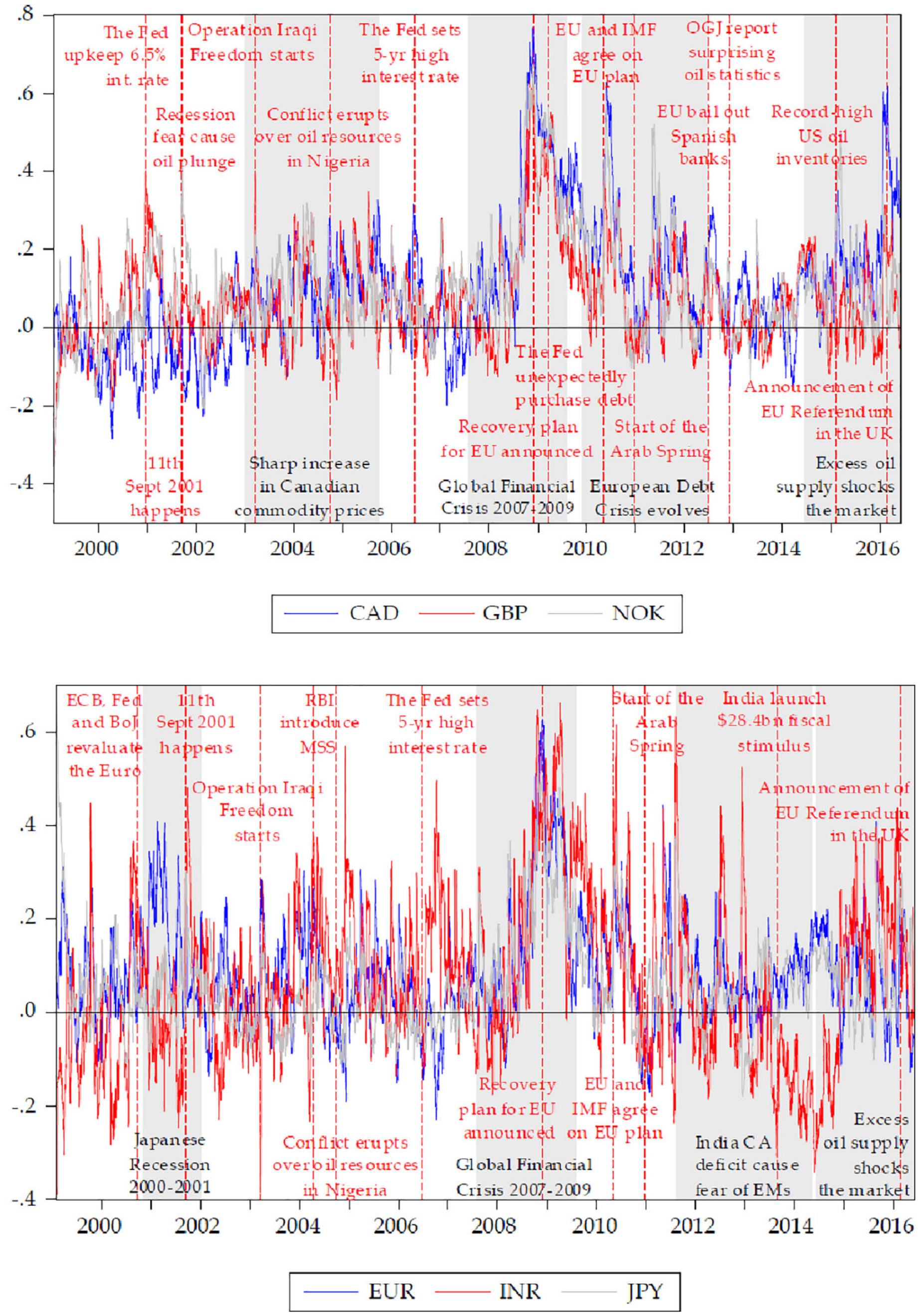



**FIGURE 9** Dynamic correlations for Brent and export/import currencies. *Note*: Black text relates to grey-shaded areas, red text relates to vertical red lines. Top (bottom) panel refers to export (import) currencies [Colour figure can be viewed at wileyonlinelibrary.com]

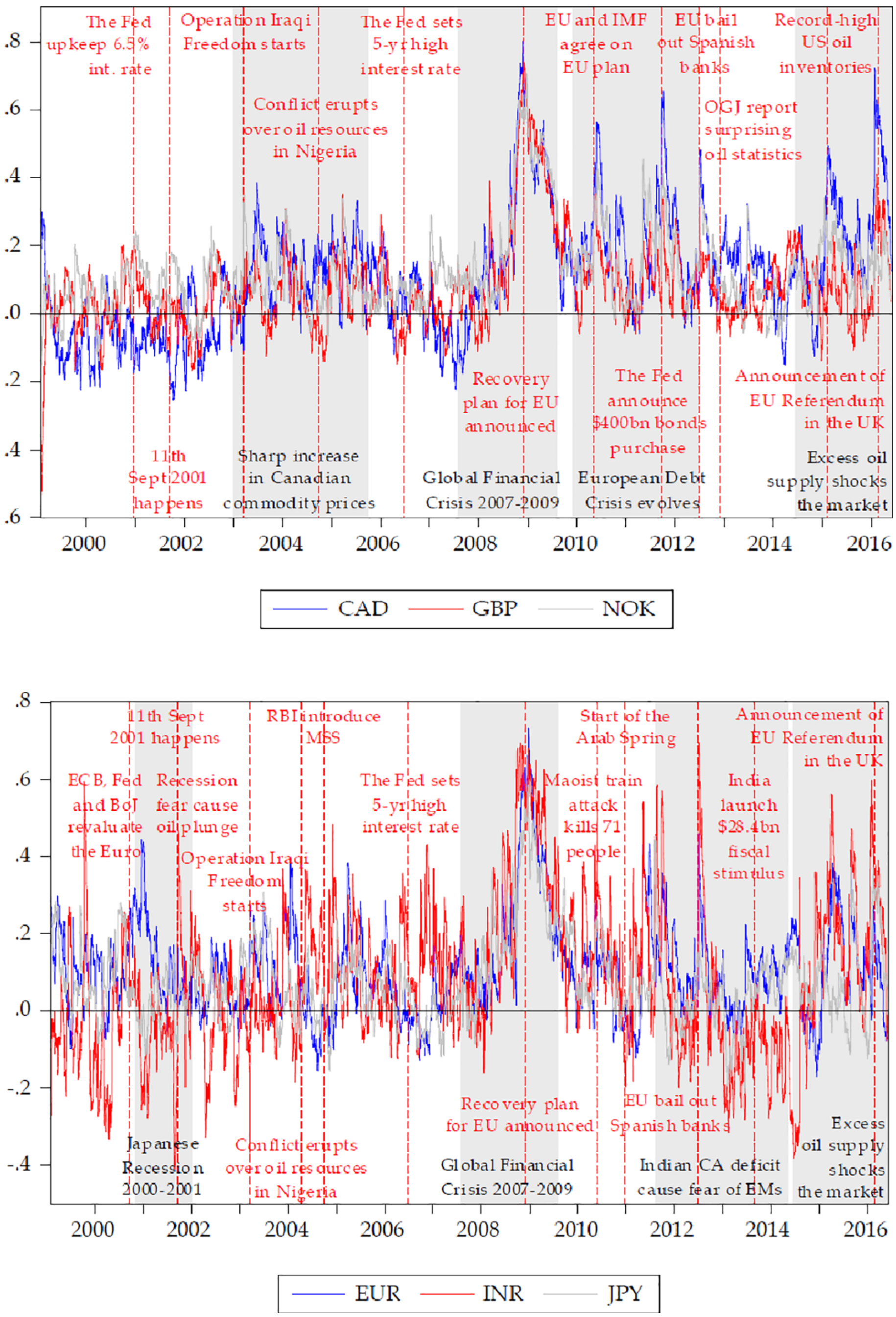



**FIGURE 10** Dynamic correlations for WTI and export/import currencies. *Note*: Black text relates to grey-shaded areas, red text relates to vertical red lines. Top (bottom) panel refers to export (import) currencies [Colour figure can be viewed at wileyonlinelibrary.com]

The sample period covers three major economic recessions, which are the early-2000 recession, the Global Financial Crisis of 2007–2009 and the European Debt crisis of 2010–2013. Interestingly enough we observe correlation heterogeneity in terms of both the oil status of the economy and the different recessions. More specifically, we find that correlations during recessionary periods are not necessarily positive. For example, during the early-2000 recession correlations are mainly negative, especially for the USD/CAD volatility. On the other hand, correlations between the oil-importing currencies and oil volatilities during the early-2000 recession are positive, with the exception the USD/INR volatility. By contrast, during the European Debt crisis, correlations are low positive for all oil-exporting currencies, whereas fluctuations in both the positive and the negative values are observed for the oil-importing currency volatilities. The event though that triggers homogeneous correlations for all currency volatilities is the Global Financial Crisis of 2007–2009, where correlations reach unprecedent highs.

These unprecedented positive volatility correlations observed during the Global Financial Crisis of 2007–2009 across all currencies, along with positive correlation levels during the European debt crisis, can be partly attributed to what Todea (2016) argue to be a stronger integration of individual markets with the global market, and as such higher degree of volatility correlation. Sensoy, Yuksel, and Erturk (2013) also suggests that during times of high volatility in financial markets, the degree to which different asset classes are correlated increases substantially and theoretically allows for volatility spillovers to occur immediately.

Apart from these global economic events, there are other notable regional economic events that could explain heterogeneous fluctuations in the time-varying correlations of the different currencies. For example, on the 19th December 2000, the Federal Reserve decided to maintain its high federal funds rate of 6.5% (FED, 2000) to the surprise of investors, in which the volatility correlation of the Canadian Dollar to Brent decreased substantially, generating a material correlation spread relative to GBP and NOK given their increase in volatility correlation. A plausible explanation to the divergence is the impact of the monetary policy decision upon the regional economic conditions to a greater extent than that upon economic conditions of distant countries, for example, UK and Norway in this particular case.

Furthermore, the joint efforts of the European Central Bank (ECB), the Federal Reserve, Bank of Japan (BoJ) and Bank of Canada (BoC) on the September 22, 2000 to appreciate the Euro currency as a response to its sustained depreciation (ECB, 2000; The Financial Times, 2010) resulted in a substantial increase in the correlation of EUR to oil. However, in general, market disequilibria and the economic policy responses to market conditions along with distressed time periods yield uniform developments in volatility correlations for EUR and JPY.

With respect to the Indian Rupee volatility, we observe that its correlation with the crude oil volatility exhibits significantly more abrupt changes. The Indian economy has experienced severe difficulties with respect to its terms of trade (The Financial Times, 2013d), in which the currency volatility correlation with crude oil volatility diverge materially from those of EUR and JPY. Further, upon the Federal funds rate increase on the June 29, 2006 (FED, 2006), the Rupee responded with a material increase in its volatility correlation, diverging from the uniform decrease in correlation experienced by the EUR and the JPY. Even more, upon the launch of a fiscal stimulus worth USD $28.4bn by the Indian Government on 27th August 2013 (BBC, 2013a, 2013b), the volatility correlation decreased into a significant negative range whilst the general trend for EUR and JPY over the equivalent time period was for its correlation to increase. The fiscal stimulus was launched during a period of financial distress in India following concerns of unsustainable (yet growing) Current Account deficits (The Financial Times, 2013d), in which the volatility correlation ranged between −0.32 and 0.67 relative to those of EUR (−0.05 and 0.43) and JPY (−0.21 and 0.44). Overall, these findings suggests how the volatility for the Indian Rupee is primarily a mere function of geopolitical and economic events taking place in India and its immediate geographical region.

Turing to the geopolitical events, we observe events that trigger significant turbulence in both the foreign exchange market and crude oil and yield volatility correlations that are uniform across all currencies. Such events are the terrorist attack in the US on September 2001 and the political upheaval in Lybia, Yemen and Bahrain during 2011–2012, as well as, the escalation of the Syrian civil war. Positive correlations for all currency volatilities are observed during these events, although this does not hold true for the INR volatility correlation. A further event that triggers homogeneous positive correlations is the sharp decline in the oil prices during 2014–2015, which is partly attribute to the excess supply of oil, as well as, the anaemic global aggregate demand.

Even though we observe that geopolitical events trigger similar correlations, regardless the oil status of the country or the timing of the event, there are events that could lead to heterogeneous correlation fluctuations. This primarily holds for the INR volatility. For example, the suspected Maoist train attack on the 28th May 2010 in the Jhargram area of West Midnapore in India (BBC, 2010; CNN, 2010; The New York Times, 2010) caused a dramatic increase in the correlation of volatilities of oil and INR, whilst other importing and exporting currencies exerted little, if any, reaction to the event, as expected, due to the regional character of the event.

Overall, we show that correlations are driven by global economic and geopolitical events, which can trigger similar correlation movements irrespectively of the oil status of the economy. On the other hand, there are regional events which could result in divergence in correlations. These findings hold for currency volatility correlations with both Brent and WTI crude oil volatility and they advocate in favour of active risk management which can accommodate the inter-temporal character of the aforementioned correlations.

## 5.2 | Optimal portfolio weights

Table 4 reports the average weights recommended for an optimally designed portfolio, which is comprised by oil and currency volatility assets. It is evident that the highest proportion is allocated to the currency volatility asset (between 92 and 96%), regardless to whether the

**TABLE 4** Optimal portfolio weights for oil and currency volatilities portfolios

| | Export currency volatilities | | | Import currency volatilities | | |
|---|---|---|---|---|---|---|
| | $w_{CAD}$ | $w_{GBP}$ | $w_{NOK}$ | $w_{EUR}$ | $w_{INR}$ | $w_{JPY}$ |
| *Brent* | | | | | | |
| Mean | 0.958 | 0.951 | 0.920 | 0.940 | 0.960 | 0.933 |
| Minimum | 0.782 | 0.864 | 0.716 | 0.797 | 0.364 | 0.781 |
| Maximum | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| *SD* | 0.033 | 0.030 | 0.052 | 0.038 | 0.073 | 0.038 |
| 1 − Min.[a] | 0.218 | 0.136 | 0.284 | 0.203 | 0.636 | 0.219 |
| 1 − Max.[b] | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| *WTI* | | | | | | |
| Mean | 0.960 | 0.954 | 0.929 | 0.947 | 0.961 | 0.937 |
| Minimum | 0.780 | 0.843 | 0.746 | 0.804 | 0.326 | 0.703 |
| Maximum | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| *SD* | 0.035 | 0.029 | 0.052 | 0.037 | 0.075 | 0.046 |
| 1 − Min.[a] | 0.220 | 0.157 | 0.254 | 0.196 | 0.674 | 0.297 |
| 1 − Max.[b] | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

*Note:* Portfolio weights are estimated following Equation (8).
[a]The maximum weight of Brent/WTI volatility given by 1 − Min. is warranted.
[b]The minimum weight of Brent/WTI volatility given by 1 − Max. is warranted.

currency belongs to an oil-exporter or oil-importer. This is explained by the significantly higher volatility of the crude oil relative to currencies[7] . Furthermore, from Table 4 we can notice that there are cases where the optimal weights are 100% in the currency volatility asset and 0% in the oil volatility asset. By contrast the maximum weight that can be allocated to the oil volatility asset is about 20–30%. Notable exceptions are the portfolios with the GBP and INR volatilities. In the first case, we observe that the maximum weight allocation of the oil volatility does not exceed the 13.6% or the 15.7% (for the Brent and WTI, respectively). On the other hand, we show that investors could allocate up to 67.4% (63.6%) of their funds in the WTI (Brent) oil volatility, when they combine it with the INR volatility.

Figure 11 exhibits the time series plots of the optimal portfolio weights for currency in the two-asset volatility portfolios, in which the data confirms a dynamic asset allocation strategy to be more appropriate. A constant asset allocation strategy with fixed weights would not allow for a minimization of variance in the portfolio, and as such it cannot provide the investor with an optimal portfolio design as a means of minimizing risk and maximize returns. However, such a dynamic portfolio design strategy arguably impairs portfolio returns, as continuous rebalancing of the full two-asset portfolio is required.

It is clear that the currency allocation is more stable over time for the more heavily traded currencies, which is rather expected, whereas this does not hold for INR and NOK. INR, for instance, appears to have significantly lower minimum asset allocations in its portfolio with both Brent and WTI volatilities. The higher instability in the currency weights for INR and NOK is evident in periods when key events for the two currencies take place.

For instance, NOK has been subject to significant speculation from market participants, along with a high sensitivity to announcements of disappointing economic data for the Norwegian economy for the period 2009–2012 (Wall Street Journal, 2013), in which the portfolio weight experience large declines as a result of increased volatility in the currency. For INR, significant reallocation into crude oil is suggested during the second half 2012 and the first half of 2015, where the previously discussed Indian economic issues and policy responses dominated market behaviour and facilitated volatility in the Rupee (The Financial Times, 2013b). Furthermore, investors' concerns over emerging markets, hereunder India, increased following the US Federal Reserve's announcement of reduction in quantitative easing (The Financial Times, 2013a, 2013c), with its implications for the valuation of the Indian Rupee. The subsequent depreciation in the Rupee, which led to an increase of its volatility, could plausibly explain the suggested reallocation of funds from IND to Brent and WTI.

### 5.3 | Optimal hedge ratios

Tables 5 and 6 present brief descriptive statistics of the hedge ratios for a long position in the oil and currency volatility, respectively, as estimated by Equation (12).

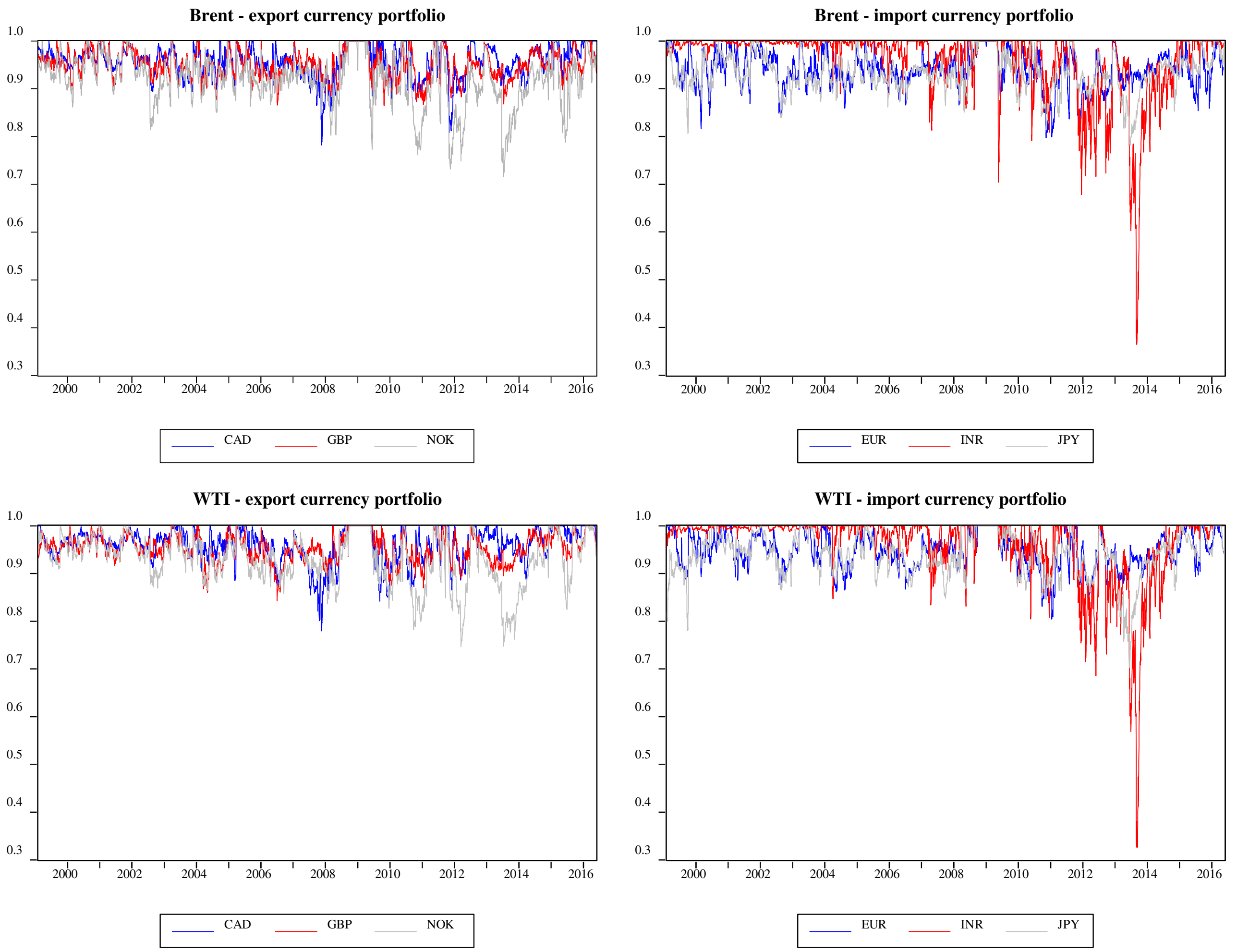



**FIGURE 11** Optimal portfolio weights for currencies in oil-currency volatility portfolios [Colour figure can be viewed at wileyonlinelibrary.com]

**TABLE 5** Optimal hedge ratios for long positions in oil volatility

| | **Export currencies** | | | **Import currencies** | | |
|---|---|---|---|---|---|---|
| | $\gamma_{CAD}$ | $\gamma_{GBP}$ | $\gamma_{NOK}$ | $\gamma_{EUR}$ | $\gamma_{INR}$ | $\gamma_{JPY}$ |
| *Brent* | | | | | | |
| Mean | 0.304 | 0.270 | 0.284 | 0.314 | 0.390 | 0.236 |
| Minimum | −2.649 | −5.993 | −5.277 | −1.157 | −10.978 | −2.666 |
| Maximum | 3.667 | 2.301 | 2.919 | 2.750 | 12.276 | 4.536 |
| *SD* | 0.697 | 0.547 | 0.484 | 0.447 | 1.979 | 0.473 |
| *WTI* | | | | | | |
| Mean | 0.400 | 0.314 | 0.411 | 0.434 | 0.445 | 0.348 |
| Minimum | −3.182 | −5.062 | −1.247 | −0.985 | −14.886 | −0.701 |
| Maximum | 4.700 | 3.051 | 2.764 | 3.421 | 13.896 | 3.584 |
| *SD* | 0.838 | 0.621 | 0.447 | 0.591 | 2.432 | 0.572 |

Table 5 shows that, on average, a USD $1 portfolio of oil volatility can be hedged with about USD ¢ 30 cents in currency volatility. Nevertheless, we observe that the most expensive hedge is achieved using the INR volatility (i.e., USD ¢ 39 cents), whereas, by contrast the cheapest hedge is obtained using the JPY (USD ¢ 23.6 cents).

**TABLE 6** Optimal hedge ratios for long positions in currency volatility

| | Export currencies | | | Import currencies | | |
|---|---|---|---|---|---|---|
| | *CAD* | *GBP* | *NOK* | *EUR* | *INR* | *JPY* |
| $\gamma_{Brent}$ | | | | | | |
| Mean | 0.033 | 0.022 | 0.037 | 0.025 | 0.014 | 0.018 |
| Minimum | −0.042 | −0.043 | −0.051 | −0.072 | −0.277 | −0.057 |
| Maximum | 0.386 | 0.256 | 0.297 | 0.178 | 0.299 | 0.241 |
| *SD* | 0.055 | 0.038 | 0.053 | 0.034 | 0.045 | 0.033 |
| $\gamma_{WTI}$ | | | | | | |
| Mean | 0.033 | 0.021 | 0.041 | 0.028 | 0.015 | 0.021 |
| Minimum | −0.073 | −0.061 | −0.038 | −0.052 | −0.211 | −0.126 |
| Maximum | 0.328 | 0.195 | 0.215 | 0.161 | 0.327 | 0.141 |
| *SD* | 0.049 | 0.035 | 0.043 | 0.033 | 0.045 | 0.029 |

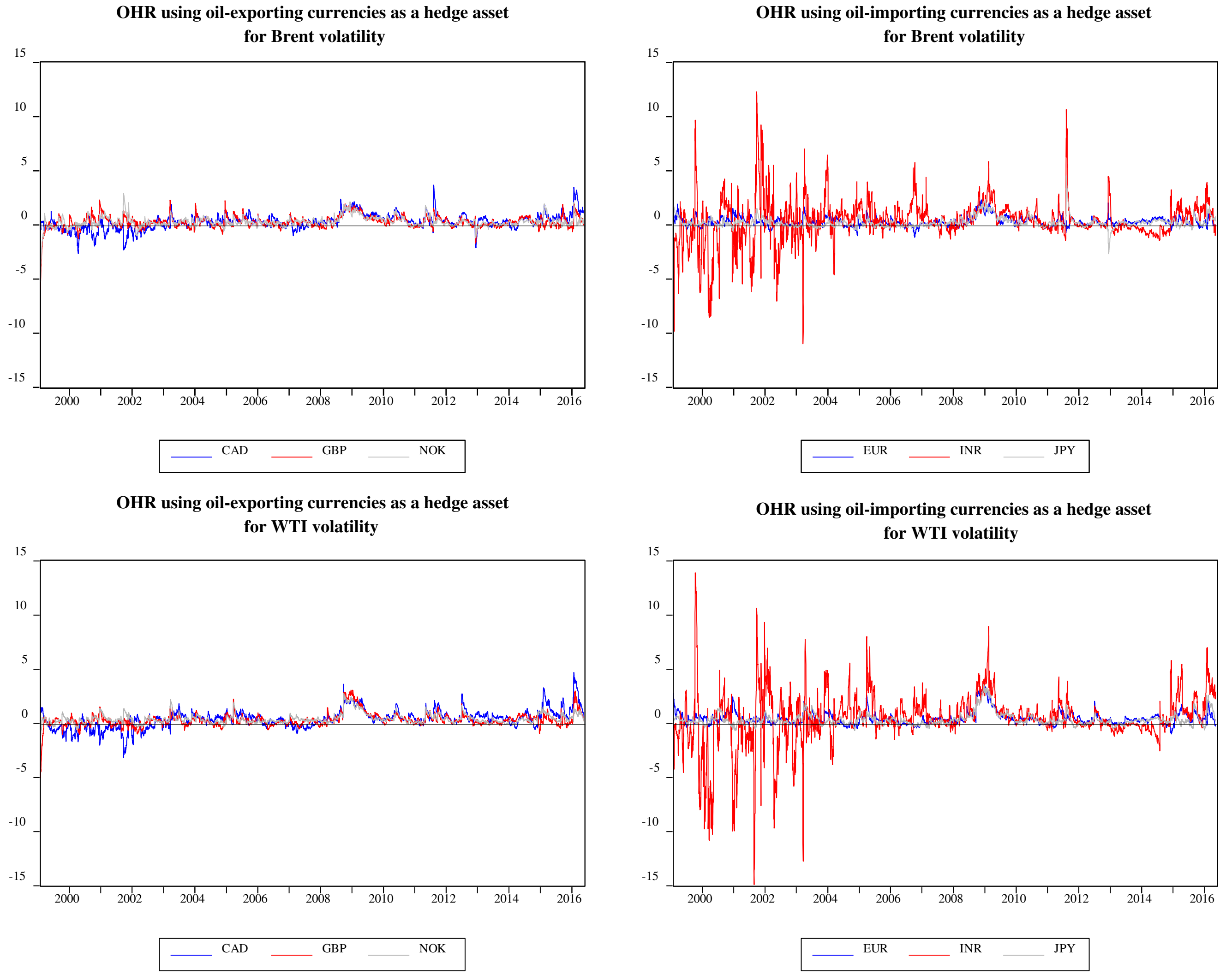



**FIGURE 12** Optimal hedge ratios for a long position in oil volatility hedged with currency volatility [Colour figure can be viewed at wileyonlinelibrary.com]

Interstingly, we also observe that hedging is more expensive for the WTI volatility compared to the Brent volatility, as evident by the mean values of the hedge ratios. Furthermore, the minimum and maximum values in Table 5 show a substantial fluctuation in hedge ratios, suggesting that hedging can become significantly more

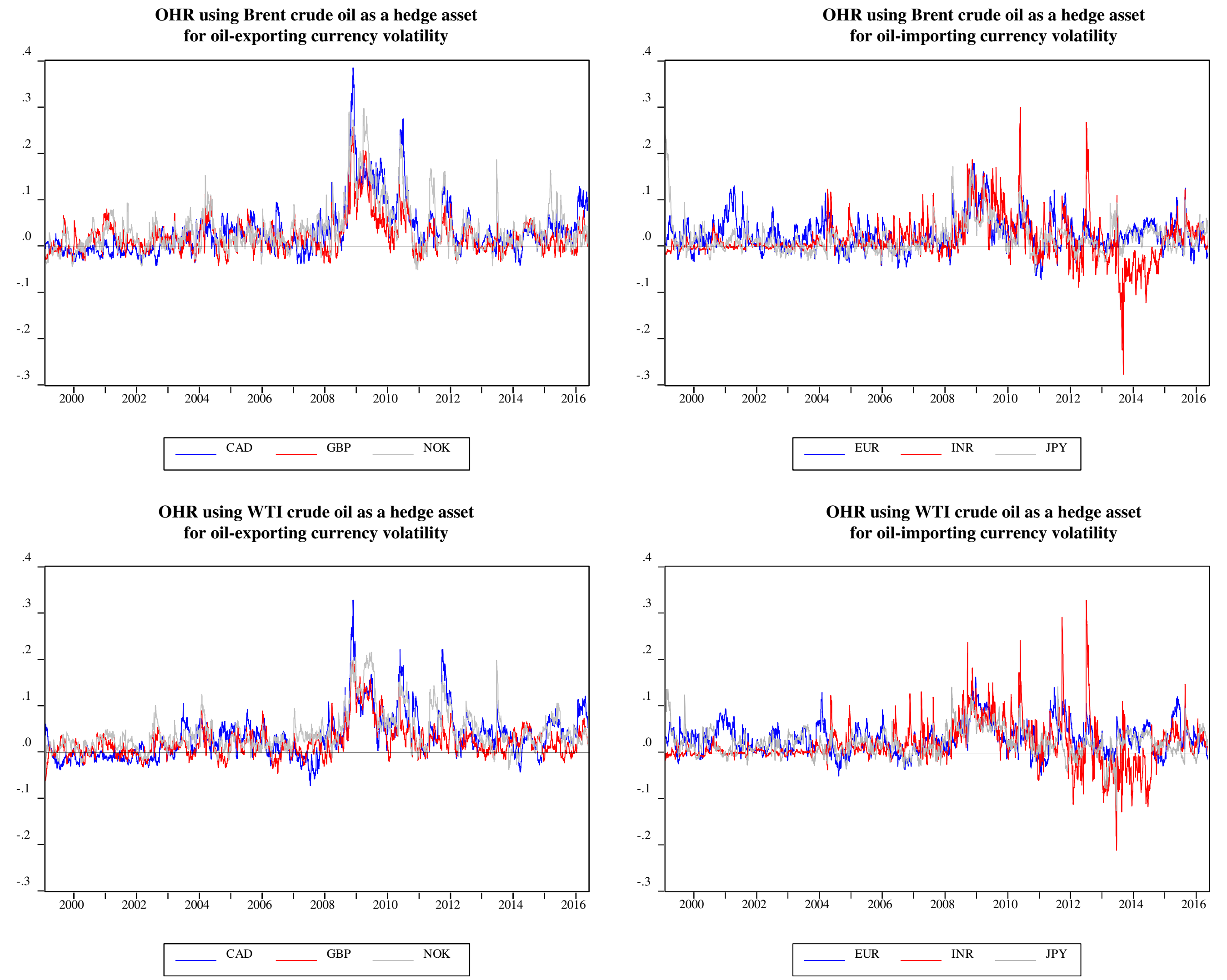



**FIGURE 13** Optimal hedge ratios for a long position in currency volatility hedged with oil volatility [Colour figure can be viewed at wileyonlinelibrary.com]

expensive. We should also highlight here that the dispersion in the daily hedge ratios are much higher for the volatilities of the CAD and INR than for the other currencies. Such higher dispersion is primarily driven by the higher volatility that is observed in the USD/INR currency in the post-2009 period, as well as, by the sudden increase in the USD/CAD volatility during the global financial crisis period. Thus, given that the volatilities of these two currencies exhibit different behaviour over time, the cost of hedging oil volatility will also exhibit higher variability.

On the other hand, the hedge ratios for a long position in the currency volatility using oil volatility are significantly cheaper, given that a USD $1 portfolio of currency volatility can be hedged with about USD ¢ 3 cents in oil volatility (see Table 6). Table 6 also shows that INR and JPY can be hedged more cheaply using oil volatility. Finally, we cannot observe any notable differences between the Brent and WTI volatility.

The results from Tables 5 and 6 suggest that there should be a substantial time-varying nature of the optimal hedge ratios for the bivariate volatility portfolios. This is confirmed by plotting the daily dynamic conditional optimal hedge ratios $\gamma_{c,t}$ and $\gamma_{o,t}$ for all currency and oil volatilities in Figures 12 and 13. Overall, the plots provide evidence in favour of a dynamic hedging strategy, which is particularly necessary for the INR volatility.

## 5.4 | Performance evaluation of the alternative risk management strategies

Table 7 reports the average dynamic optimal portfolio weights of crude oil and currency volatilities, the portfolio variance of the optimally weighted portfolio, the minimum variance of an unhedged single asset portfolio (either oil or currency volatility), as well as, the hedging

**TABLE 7** Performance of an optimal portfolio weights strategy

| | Portfolio weights | | Portfolio variance | | |
|---|---|---|---|---|---|
| **Portfolio** | **Oil**[b] | **Currency**[c] | **Optimal weighted port** | **Single-asset port**[d] | **HE**[a ,e] **(%)** |
| *Export currencies* | | | | | |
| Brent, CAD | 0.042 | 0.958 | 0.164 | 2.661, 0.169 | 93.83, 2.95 |
| Brent, GBP | 0.049 | 0.951 | 0.152 | 2.584, 0.158 | 94.11, 3.95 |
| Brent, NOK | 0.080 | 0.920 | 0.252 | 2.673, 0.268 | 90.57, 6.34 |
| WTI, CAD | 0.040 | 0.960 | 0.163 | 2.864, 0.168 | 94.30, 2.97 |
| WTI, GBP | 0.046 | 0.954 | 0.150 | 2.810, 0.155 | 94.66, 3.22 |
| WTI, NOK | 0.071 | 0.929 | 0.252 | 2.917, 0.265 | 91.36, 4.91 |
| *Import currencies* | | | | | |
| Brent, EUR | 0.060 | 0.940 | 0.179 | 2.650, 0.188 | 93.24, 4.79 |
| Brent, INR | 0.040 | 0.960 | 0.125 | 3.216, 0.145 | 96.11, 13.79 |
| Brent, JPY | 0.067 | 0.933 | 0.184 | 2.776, 0.195 | 93.01, 5.64 |
| WTI, EUR | 0.053 | 0.947 | 0.176 | 2.865, 0.184 | 93,85, 4.34 |
| WTI, INR | 0.039 | 0.961 | 0.120 | 3.525, 0.138 | 96.59, 13.04 |
| WTI, JPY | 0.063 | 0.937 | 0.182 | 3.104, 0.193 | 94.13, 5.70 |

*Note:* All values are average daily values.
[a]HE = Hedging Effectiveness compared to the minimum variance of the one-asset portfolio.
[b]The number represents the average daily value of $w_{o,\ t}$ (see Equation 8).
[c]The number represents the average daily value of $w_{c,\ t}$ (see Equation 8).
[d]The variance of the single-asset portfolio refers to the oil and currency volatility, respectively.
[e]The HE refers to the comparison against the single-asset portfolio of oil and currency volatility, respectively.

effectiveness. We find that the optimal portfolio weights are always performing better in terms of variance reduction, compared to the variance of the single asset portfolio. Nevertheless, risk reduction is particularly beneficial for oil volatility investors, given that the hedging effectiveness against the unhedged oil volatility portfolio ranges between 90.57 and 96.59%. This holds for both oil benchmarks and all currencies. Furthermore, we report that the most effective hedging is primarily achieved with the INR volatility. Furthermore, the risk reduction for a currency volatility investor that is achieved using an optimal portfolio strategy is at the levels of about 4% (on average) for the oil-exporting currencies, whereas more than 5% risk reduction is reported for the oil-importing currencies.

We further evaluate the hedging effectiveness of the optimal hedge ratio strategy. Table 8 reports the average performance of the hedge ratios for the two crude oil volatilities, as well as, the six currency volatilities. The results show that hedging is more effective when the hedge asset is the oil volatility, as opposed to the currency volatility. In particular, when a USD $1 long position in the Brent or WTI volatility is optimally hedged with a position in currency volatility, investors' can obtain a risk reduction between 3.12 and 9.97%. By contrast, the risk reduction that is achieved when the hedge asset is the oil volatility ranges between 3 and 12.01%. We further show that hedging is more effective for the export-currencies.

Comparing the hedging effectiveness between the two risk management strategies we notice that optimal hedging is preferred across all currency volatilities, whereas the optimal portfolio weights strategy is preferred for oil volatility investors.

## 6 | CONCLUSION

This paper investigates the time-varying correlations between oil and currency volatilities, focusing on both oil-importing and oil-exporting economies, employing a Diag-BEKK framework. Furthermore, the study addresses the issue of risk exposure of portfolios comprised by oil and currency volatilities through the evaluation of risk management strategies focused on optimal portfolio weights and hedge ratios. The period of the study spans from February 1, 1999 to May 30, 2016 and we focus on two oil benchmarks (Brent and WTI) and six highly tradeable exchange rates, namely, USD/CAD, USD/GBP, USD/K, USD/EUR, USD/INR and USD/JPY.

**TABLE 8** Performance of optimal hedge ratio strategy for a volatility portfolio

| | Average OHR | Variance of portfolios | | |
|---|---|---|---|---|
| | $\gamma$ | Unhedged long position | Optimal[a] | HE (%) |
| *Export currencies (long/short)* | | | | |
| BRENT/CAD | 0.304 | 2.6610 | 2.4755 | 6.96 |
| BRENT/GBP | 0.270 | 2.5838 | 2.4830 | 3.90 |
| BRENT/NOK | 0.284 | 2.6735 | 2.5382 | 5.06 |
| WTI/CAD | 0.400 | 2.8636 | 2.5891 | 9.59 |
| WTI/GBP | 0.314 | 2.8100 | 2.6098 | 7.13 |
| WTI/NOK | 0.411 | 2.9166 | 2.7005 | 7.41 |
| CAD/BRENT | 0.033 | 0.1693 | 0.1498 | 11.55 |
| GBP/BRENT | 0.022 | 0.1580 | 0.1489 | 5.71 |
| NOK/BRENT | 0.037 | 0.2685 | 0.2507 | 6.62 |
| CAD/WTI | 0.033 | 0.1683 | 0.1481 | 12.01 |
| GBP/WTI | 0.021 | 0.1551 | 0.1433 | 7.60 |
| NOK/WTI | 0.041 | 0.2650 | 0.2456 | 7.31 |
| *Import currencies (long/short)* | | | | |
| BRENT/EUR | 0.314 | 2.6503 | 2.5573 | 3.51 |
| BRENT/INR | 0.390 | 3.2158 | 3.0049 | 6.56 |
| BRENT/JPY | 0.236 | 2.7757 | 2.6891 | 3.12 |
| WTI/EUR | 0.434 | 2.8655 | 2.6657 | 6.97 |
| WTI/INR | 0.445 | 3.5253 | 3.1737 | 9.97 |
| WTI/JPY | 0.348 | 3.1039 | 2.9584 | 4.69 |
| EUR/BRENT | 0.025 | 0.1879 | 0.1810 | 3.66 |
| INR/BRENT | 0.013 | 0.1448 | 0.1357 | 6.28 |
| JPY/BRENT | 0.018 | 0.1952 | 0.1893 | 3.00 |
| EUR/WTI | 0.028 | 0.1839 | 0.1743 | 5.21 |
| INR/WTI | 0.015 | 0.1377 | 0.1258 | 8.66 |
| JPY/WTI | 0.021 | 0.1932 | 0.1873 | 3.02 |

*Note:* All values are average daily values.

Abbreviations: HE, hedging effectiveness; OHR, optimal hedge ratio.

[a]Oil (Currency) volatility portfolio hedged with currency (oil) volatility with an optimal hedge ratio ($\gamma|\Omega_{t-1}$, see Equation 12).

The findings suggest that major geopolitical and economic events impact upon the dynamic conditional correlation of the volatilities of crude oil and currencies. Geopolitical events such as the terrorist attacks on the World Trade Center in New York on September 21, 2001 and the rise to the Middle East unrest in 2012–2013 appear to yield similar correlations across all currencies and oil benchmarks. On the other hand, we report that domestic events tend to trigger heterogeneous correlations, such as the train attack in India in 2010. Interestingly enough, we show that even though the behaviour of correlation tends to be homogeneous during economic-driven events, we observe that this behaviour is also event-specific. For example, the Global Financial Crisis of 2007–2009 led to a significant increase in the correlations, whereas the early-2000 recession results in both positive and negative correlations. These findings apply to both import- and export-currencies, with the exception of the Indian Rupee that is found to be influenced more by domestic developments.

The computation of optimal portfolio weights and optimal hedge ratios demonstrate that a dynamic risk management strategy is required for optimal results. Furthermore, we show that the risk reduction effectiveness of the optimal portfolio weight strategy, compared to the effectiveness of the optimal hedge ratios, is found to be materially higher in the case of an oil volatility investor. By comparison, a currency volatility investor should prefer to engage in an optimal hedge ratio strategy, as opposed to an optimal portfolio weights strategy.

Further research should aim to incorporate alternative measures for investor endowment maximization, such as utility maximization and Value-at-Risk that may focus on alternative aims of the investor rather than minimization of variance. The employment of transaction cost modelling in line with Wu et al. (2012) assuming a break-even transaction cost as a fixed proportion of the value of each asset traded may also prove beneficial as it would allow for the recommendations to be subject to fewer restrictive assumptions that may otherwise impair their validity. Finally, similar empirical designs could be employed for other financial assets' volatility that could form portfolios with oil volatility, such as stocks and bonds.

## ORCID

*George Filis* https://orcid.org/0000-0002-4912-0973

## ENDNOTES

[1] The price of tradable goods increases due to the increase in the price of oil, which is considered a major production input.

[2] The data are available upon request from the corresponding author.

[3] We have excluded China from our sample despite the fact that it is one the major oil importers, given that the trading activity of its currency is extremely thin.

[4] We acknowledge that our measure of volatility is not tradeable. However, we maintain that this measure can well approximate tradeable volatility products, such as EFTs (e.g the CBOE Crude Oil ETF Volatility Index or the CBOE/CME FX Yen Volatility Index), or volatility trading through straddle strategies using option contracts for the series under examination.

[5] The hypothesis of the variables following a Gaussian normal distribution has been rejected in favour of Student-*t* distribution when subjected to Likelihood ratio testing (Power, Vedenov, Anderson, & Klose, 2013).

[6] Estimation results for the Diag-BEKK models are presented in Tables A1 and A2.

[7] This finding is in line with the findings of Chkili (2016).

## DATA AVAILABILITY STATEMENT

The data are available upon request from the corresponding author.

## ORCID

*George Filis* https://orcid.org/0000-0002-4912-0973

## REFERENCES

Alizadeh, A. H., Nomikos, N. K., & Pouliasis, P. K. (2008). A markov regime switching approach for hedging energy commodities. *Journal of Banking & Finance*, *32*(9), 1970–1983.

Aloui, R., Aïssa, M. S. B., & Nguyen, D. K. (2013). Conditional dependence structure between oil prices and exchange rates: a copula-GARCH approach. *Journal of International Money and Finance*, *32*, 719–738.

Amano, R. A., & Van Norden, S. (1998a). Exchange rates and oil prices. *Review of International Economics*, *6*(4), 683–694.

Amano, R. A., & Van Norden, S. (1998b). Oil prices and the rise and fall of the US real exchange rate. *Journal of International Money and Finance*, *17*(2), 299–316.

Antonakakis, N., Cunado, J., Filis, G., Gabauer, D., & De Gracia, F. P. (2018). Oil volatility, oil and gas firms and portfolio diversification. *Energy Economics*, *70*, 499–515.

Antonakakis, N., & Kizys, R. (2015). Dynamic spillovers between commodity and currency markets. *International Review of Financial Analysis*, *41*, 303–319.

Arouri, M. E. H., Jouini, J., & Nguyen, D. K. (2012). On the impacts of oil price fluctuations on European equity markets: Volatility spillover and hedging effectiveness. *Energy Economics*, *34*(2), 611–617.

Baillie, R. T., & Myers, R. J. (1991). Bivariate GARCH estimation of the optimal commodity futures hedge. *Journal of Applied Econometrics*, *6*(2), 109–124.

Bank of International Settlements. (2016a). Triennial OTC derivatives statistics: D11.3 Foreign exchange turnover, by currency, 1995–2016. Retrieved from http://www.bis.org/statistics/derstats3y.htm?m=6%7C32%7C617

Bank of International Settlements. (2016b). Triennial central bank survey: Foreign exchange turnover in April 2016. Retrieved from http://www.bis.org/publ/rpfx16fx.pdf

Basher, S. A., & Sadorsky, P. (2016). Hedging emerging market stock prices with oil, gold, VIX, and bonds: A comparison between DCC, ADCC and GO-GARCH. *Energy Economics*, *54*, 235–247.

BBC. (2010). India 'Maoist' train attack kills more than 100. Retrieved from http://www.bbc.co.uk/news/10178967

BBC. (2013a). India infrastructure projects set to boost economy. Retrieved from http://www.bbc.co.uk/news/business-23847059

BBC. (2013b). Indian rupee falls to new low against US dollar. Retrieved from http://www.bbc.co.uk/news/business-23860458

Beckmann, J., & Czudaj, R. (2013a). Is there a homogeneous causality pattern between oil prices and currencies of oil importers and exporters? *Energy Economics*, *40*, 665–678.

Beckmann, J., & Czudaj, R. (2013b). Oil prices and effective dollar exchange rates. *International Review of Economics & Finance*, *27*, 621–636.

Blomberg, S. B., & Harris, E. S. (1995). The commodity-consumer price connection: fact or fable? *Economic Policy Review*, *1*(3).21–38.

Bodart, V., Candelon, B., & Carpantier, J.-F. (2015). Real exchanges rates, commodity prices and structural factors in developing countries. *Journal of International Money and Finance*, *51*, 264–284.

Boldanov, R., Degiannakis, S., & Filis, G. (2016). Time-varying correlation between oil and stock market volatilities: Evidence from oil-importing and oil-exporting countries. *International Review of Financial Analysis*, *48*, 209–220.

Brahmasrene, T., Huang, J.-C., & Sissoko, Y. (2014). Crude oil prices and exchange rates: Causality, variance decomposition and impulse response. *Energy Economics*, *44*, 407–412.

Broadstock, D., & Filis, G. (2014). Oil price shocks and stock market returns: New evidence from the United States and China.

*Journal of International Financial Markets, Institutions and Money*, *33*, 417–433.

Brooks, C., Henry, O. T., & Persand, G. (2002). The effect of asymmetries on optimal hedge ratios. *The Journal of Business*, *75*(2), 333–352.

Chakraborty, A., & Barkoulas, J. T. (1999). Dynamic futures hedging in currency markets. *The European Journal of Finance*, *5* (4), 299–314.

Chang, C.-L., McAleer, M., & Tansuchat, R. (2011). Crude oil hedging strategies using dynamic multivariate GARCH. *Energy Economics*, *33*(5), 912–923.

Chen, S.-S., & Chen, H.-C. (2007). Oil prices and real exchange rates. *Energy Economics*, *29*(3), 390–404.

Chen, W.-P., Choudhry, T., & Wu, C.-C. (2013). The extreme value in crude oil and US dollar markets. *Journal of International Money and Finance*, *36*, 191–210.

Cheung, Y.-W., Chinn, M. D., & Marsh, I. W. (2004). How do UK-based foreign exchange dealers think their market operates? *International Journal of Finance & Economics*, *9*(4), 289–306.

Chkili, W. (2016). Dynamic correlations and hedging effectiveness between gold and stock markets: Evidence for BRICS countries. *Research in International Business and Finance*, *38*, 22–34.

Ciner, C., Gurdgiev, C., & Lucey, B. M. (2013). Hedges and safe havens: An examination of stocks, bonds, gold, oil and exchange rates. *International Review of Financial Analysis*, *29*, 202–211.

CNN. (2010). At least 73 killed in India train crash. Retrieved from http://edition.cnn.com/2010/WORLD/asiapcf/05/27/india.train.crash/

Cotter, J., & Hanly, J. (2012). Hedging effectiveness under conditions of asymmetry. *The European Journal of Finance*, *18*(2), 135–147.

Ding, L., & Vo, M. (2012). Exchange rates and oil prices: A multivariate stochastic volatility analysis. *The Quarterly Review of Economics and Finance*, *52*(1), 15–37.

ECB. (2000). The ECB announces joint intervention in the exchange markets. Retrieved from https://www.ecb.europa.eu/press/pr/date/2000/html/pr000922.en.html

Engle, R. F., & Kroner, K. F. (1995). Multivariate simultaneous generalized ARCH. *Econometric Theory*, *11*(01), 122–150.

Fan, R., Li, H., & Park, S. Y. (2015). Estimation and hedging effectiveness of time-varying hedge ratio: Nonparametric approaches. *Journal of Futures Markets*, *36*(10), 968–991.

Fattouh, B., Kilian, L., & Mahadeva, L. (2013). The role of speculation in oil markets: What have we learned so far? *The Energy Journal*, *34*(3), 7–33.

FED. (2000). Federal reserve release—19th December Press Release. Retrieved from https://www.federalreserve.gov/boarddocs/press/general/2000/20001219/default.htm

FED. (2006). Federal reserve release—29th June Press Release. Retrieved from https://www.federalreserve.gov/newsevents/press/monetary/20060629a.htm

Filis, G., Degiannakis, S., & Floros, C. (2011). Dynamic correlation between stock market and oil prices: The case of oil-importing and oil-exporting countries. *International Review of Financial Analysis*, *20*(3), 152–164.

Forsberg, L., & Ghysels, E. (2007). Why do absolute returns predict volatility so well? *Journal of Financial Econometrics*, *5*(1), 31–67.

Galati, G., & Heath, A. (2007). What drives the growth in FX activity? Interpreting the 2007 triennial survey. *BIS Quarterly Review*.63–72.

Golub, S. S. (1983). Oil prices and exchange rates. *The Economic Journal*, *93*(371), 576–593.

Habib, M. M., Bützer, S., & Stracca, L. (2016). Global exchange rate configurations: Do oil shocks matter? *IMF Economic Review*, *64* (3), 443–470.

Haigh, M. S. (2005). Conditional volatility forecasting in a dynamic hedging model. *Journal of Forecasting*, *24*(3), 155–172.

Hammoudeh, S. M., Yuan, Y., McAleer, M., & Thompson, M. A. (2010). Precious metals–exchange rate volatility transmissions and hedging strategies. *International Review of Economics & Finance*, *19*(4), 633–647.

Juvenal, L., & Petrella, I. (2015). Speculation in the oil market. *Journal of Applied Econometrics*, *30*(4), 621–649.

Kavussanos, M. G., & Visvikis, I. D. (2008). Hedging effectiveness of the Athens stock index futures contracts. *European Journal of Finance*, *14*(3), 243–270.

Krugman, P. (1983). Oil shocks and exchange rate dynamics. In *Exchange rates and international macroeconomics* (pp. 259–284). Chicago: University of Chicago Press.

Krugman, P. R. (1980). Oil and the dollar. NBER Working Paper Series, 554.

Lee, H.-T., & Yoder, J. (2007). Optimal hedging with a regime-switching time-varying correlation GARCH model. *Journal of Futures Markets*, *27*(5), 495–516.

Lee, H.-T., Yoder, J. K., Mittelhammer, R. C., & McCluskey, J. J. (2006). A random coefficient autoregressive markov regime switching model for dynamic futures hedging. *Journal of Futures Markets*, *26*(2), 103–129.

Lin, B., Wesseh, P. K., & Appiah, M. O. (2014). Oil price fluctuation, volatility spillover and the Ghanaian equity market: Implication for portfolio management and hedging effectiveness. *Energy Economics*, *42*, 172–182.

Lizardo, R. A., & Mollick, A. V. (2010). Oil price fluctuations and US dollar exchange rates. *Energy Economics*, *32*(2), 399–408.

Lowes, J., & Nenova, T. (2013). The foreign exchange and over-the-counter interest rate derivatives market in the United Kingdom. *Bank of England Quarterly Bulletin*.*53*(4), 394–404.

McGuirk, A. K. (1983). Oil price changes and real exchange rate movements among industrial countries. *Staff Papers—International Monetary Fund*, *30*(4), 843–884.

OPEC. (2015). World Oil Outlook 2015. Retrieved from http://www.opec.org/opec_web/static_files_project/media/downloads/publications/WOO%202015.pdf

Pan, Z., Wang, Y., & Yang, L. (2014). Hedging crude oil using refined product: A regime switching asymmetric DCC approach. *Energy Economics*, *46*, 472–484.

Powell, J. (2005). A history of the Canadian dollar. Technical Report. Bank of Canada.

Power, G. J., Vedenov, D. V., Anderson, D. P., & Klose, S. (2013). Market volatility and the dynamic hedging of multi-commodity price risk. *Applied Economics*, *45*(27), 3891–3903.

Razgallah, B., & Smimou, K. (2011). Oil prices and the greenback: It takes two to tango. *Applied Financial Economics*, *21*(8), 519–528.

Reboredo, J. C., Rivera-Castro, M. A., & Zebende, G. F. (2014). Oil and us dollar exchange rate dependence: A detrended cross-correlation approach. *Energy Economics*, *42*, 132–139.

Sadorsky, P. (2000). The empirical relationship between energy futures prices and exchange rates. *Energy Economics*, *22*(2), 253–266.

Sadorsky, P. (2014). Modeling volatility and correlations between emerging market stock prices and the prices of copper, oil and wheat. *Energy Economics*, *43*, 72–81.

Sensoy, A., Yuksel, S., & Erturk, M. (2013). Analysis of cross-correlations between financial markets after the 2008 crisis. *Physica A: Statistical Mechanics and its Applications*, *392*(20), 5027–5045.

Syriopoulos, T., Makram, B., & Boubaker, A. (2015). Stock market volatility spillovers and portfolio hedging: BRICS and the financial crisis. *International Review of Financial Analysis*, *39*, 7–18.

Tang, K., & Xiong, W. (2012). Index investment and the financialization of commodities. *Financial Analysts Journal*, *68*(5), 54–74.

The Financial Times. (2010). A quick guide to ECB intervention. Retrieved from http://ftalphaville.ft.com/2010/05/21/238771/a-quick-guide-to-ecb-intervention/

The Financial Times. (2013a). Emerging markets: Volatility is part opportunity and part trap. Retrieved from https://www.ft.com/content/ac75ce5a-47a1-11e3-9398-00144feabdc0

The Financial Times. (2013b). India current account gap at record high. Retrieved from https://www.ft.com/content/6338d094-97a6-11e2-97e0-00144feabdc0

The Financial Times. (2013c). India is out of the woods but a long way from safe. Retrieved from https://www.ft.com/content/e06fff6a-5c54-11e3-931e-00144feabdc0

The Financial Times. (2013d). India's current account deficit: smoke and golden mirrors? Retrieved from http://blogs.ft.com/beyond-brics/2013/12/05/indias-current-account-deficit-smoke-and-golden-mirrors/

The New York Times. (2010). Maoist rebels suspected as Indian train derails. Retrieved from http://www.nytimes.com/2010/05/29/world/asia/29india.html?_r=1

The Wall Street Journal. (2015). Norway keeps rates unchanged. Retrieved from http://www.wsj.com/articles/norway-keeps-rates-unchanged-1426762028

Todea, A. (2016). Cross-correlations between volatility, volatility persistence and stock market integration: the case of emergent stock markets. *Chaos, Solitons & Fractals*, *87*, 208–215.

Toyoshima, Y., Nakajima, T., & Hamori, S. (2013). Crude oil hedging strategy: new evidence from the data of the financial crisis. *Applied Financial Economics*, *23*(12), 1033–1041.

Wall Street Journal. (2013). Norway's krone losing its crown. Retrieved from http://blogs.wsj.com/moneybeat/2013/10/03/norways-krones-losing-its-crown/

Wu, C.-C., Chung, H., & Chang, Y.-H. (2012). The economic value of co-movement between oil price and exchange rate using copula-based GARCH models. *Energy Economics*, *34*(1), 270–282.

Xu, W., & Yang, L. (2009). Hedging effectiveness with S&P00 index futures under different volatility regimes. In P. Catlere (Ed.), *Financial hedging* (pp. 95–117). New York: Nova Science Publishers.

Yousefi, A., & Wirjanto, T. S. (2004). The empirical role of the exchange rate on the crude-oil price formation. *Energy Economics*, *26*(5), 783–799.

Zhang, B., & Wang, P. (2014). Return and volatility spillovers between China and world oil markets. *Economic Modelling*, *42*, 413–420.

**How to cite this article:** Olstad A, Filis G, Degiannakis S. Oil and currency volatilities: Co-movements and hedging opportunities. *Int J Fin Econ*. 2021;26:2351–2374. https://doi.org/10.1002/ijfe.1911

## APPENDIX: ESTIMATIONS OF THE DIAG-BEKK COEFFICIENTS

**TABLE A1** Diag-BEKK coefficients for Brent-currency volatility estimations

| | **Brent, CAD** | | **Brent, GBP** | | **Brent, NOK** | |
|---|---|---|---|---|---|---|
| **Parameter** | **Coeff.** | ***SE*** | **Coeff.** | ***SE*** | **Coeff.** | ***SE*** |
| $\mu_1$ | 1.1778 | (0.0195)*** | 1.2246 | (0.0199)*** | 1.1942 | (0.0192)*** |
| $\mu_2$ | 0.3046 | (0.0044)*** | 0.3272 | (0.0050)*** | 0.4261 | (0.0062)*** |
| $\omega_{1,\ 1}$ | 0.0595 | (0.0090)*** | 0.0695 | (0.0104)*** | 0.0749 | (0.0111)*** |
| $\omega_{2,\ 2}$ | 0.0011 | (0.0002)*** | 0.0019 | (0.0004)*** | 0.0027 | (0.0005)*** |
| $\tilde{a}_{1,1}$ | 0.2043 | (0.0112)*** | 0.2106 | (0.0116)*** | 0.2202 | (0.0124)*** |
| $\tilde{a}_{2,2}$ | 0.1691 | (0.0099)*** | 0.1577 | (0.0118)*** | 0.1458 | (0.0098)*** |
| $\tilde{b}_{1,1}$ | 0.9638 | (0.0037)*** | 0.9595 | (0.0042)*** | 0.9571 | (0.0044)*** |
| $\tilde{b}_{2,2}$ | 0.9820 | (0.0021)*** | 0.9809 | (0.0028)*** | 0.9835 | (0.0022)*** |
| $\lambda$ | 4.6388 | (0.2275)*** | 5.0039 | (0.2442)*** | 4.5441 | (0.2098)*** |
| | Brent, EUR | | Brent, INR | | Brent, JPY | |
| $\mu_1$ | 1.2244 | (0.0194)*** | 1.1807 | (0.0200)*** | 1.2155 | (0.0193)*** |
| $\mu_2$ | 0.3686 | (0.0056)*** | 0.0639 | (0.0015)*** | 0.3734 | (0.0056)*** |
| $\omega_{1,\ 1}$ | 0.0771 | (0.0119)*** | 0.0698 | (0.0121)*** | 0.0747 | (0.0120)*** |
| $\omega_{2,\ 2}$ | 0.0024 | (0.0006)*** | 0.0002 | (0.0000)*** | 0.0031 | (0.0007)*** |
| $\tilde{a}_{1,1}$ | 0.2183 | (0.0127)*** | 0.1956 | (0.0122)*** | 0.2098 | (0.0128)*** |
| $\tilde{a}_{2,2}$ | 0.1517 | (0.0121)*** | 0.3616 | (0.0150)*** | 0.1292 | (0.0102)*** |
| $\tilde{b}_{1,1}$ | 0.9571 | (0.0046)*** | 0.9703 | (0.0035)*** | 0.9615 | (0.0044)*** |
| $\tilde{b}_{2,2}$ | 0.9819 | (0.0029)*** | 0.9443 | (0.0030)*** | 0.9825 | (0.0027)*** |
| $\lambda$ | 4.5831 | (0.2188)*** | 3.4298 | (0.1447)*** | 4.2060 | (0.1828)*** |

***Denotes statistical significance at the 0.01% level.

**TABLE A2** Diag-BEKK coefficients for WTI-currency volatility estimations

| Parameter | WTI, CAD Coeff. | WTI, CAD SE | WTI, GBP Coeff. | WTI, GBP SE | WTI, NOK Coeff. | WTI, NOK SE |
|---|---|---|---|---|---|---|
| $\mu_1$ | 0.3047 | (0.0044)*** | 0.3299 | (0.0051)*** | 0.4240 | (0.0063)*** |
| $\mu_2$ | 1.2515 | (0.0199)*** | 1.2804 | (0.0200)*** | 1.2544 | (0.0195)*** |
| $\omega_{1,\,1}$ | 0.0009 | (0.0002)*** | 0.0016 | (0.0003)*** | 0.0019 | (0.0004)*** |
| $\omega_{2,\,2}$ | 0.0368 | (0.0064)*** | 0.0377 | (0.0066)*** | 0.0341 | (0.0062)*** |
| $\tilde{a}_{1,1}$ | 0.1718 | (0.0096)*** | 0.1456 | (0.0103)*** | 0.1277 | (0.0081)*** |
| $\tilde{a}_{2,2}$ | 0.1932 | (0.0103)*** | 0.1928 | (0.0104)*** | 0.1875 | (0.0105)*** |
| $\tilde{b}_{1,1}$ | 0.9817 | (0.0020)*** | 0.9836 | (0.0023)*** | 0.9875 | (0.0016)*** |
| $\tilde{b}_{2,2}$ | 0.9722 | (0.0028)*** | 0.9718 | (0.0029)*** | 0.9745 | (0.0027)*** |
| $\lambda$ | 5.0609 | (0.2607)*** | 5.3478 | (0.2782)*** | 4.7540 | (0.2289)*** |
| | WTI, EUR | | WTI, INR | | WTI, JPY | |
| $\mu_1$ | 0.3693 | (0.0058)*** | 0.0644 | (0.0015)*** | 0.3675 | (0.0056)*** |
| $\mu_2$ | 1.2779 | (0.0198)*** | 1.2199 | (0.0205)*** | 1.2611 | (0.0191)*** |
| $\omega_{1,\,1}$ | 0.0020 | (0.0005)*** | 0.0002 | (0.0000)*** | 0.0030 | (0.0006)*** |
| $\omega_{2,\,2}$ | 0.0385 | (0.0069)*** | 0.0355 | (0.0071)*** | 0.0396 | (0.0075)*** |
| $\tilde{a}_{1,1}$ | 0.1409 | (0.0106)*** | 0.3558 | (0.0141)*** | 0.1318 | (0.0099)*** |
| $\tilde{a}_{2,2}$ | 0.1926 | (0.0107)*** | 0.1923 | (0.0112)*** | 0.2015 | (0.0119)*** |
| $\tilde{b}_{1,1}$ | 0.9844 | (0.0024)*** | 0.9433 | (0.0030)*** | 0.9822 | (0.0026)*** |
| $\tilde{b}_{2,2}$ | 0.9722 | (0.0030)*** | 0.9777 | (0.0025)*** | 0.9723 | (0.0031)*** |
| $\lambda$ | 4.9132 | (0.2470)*** | 3.6350 | (0.1564)*** | 4.2318 | (0.1880)*** |

***Denotes statistical significance at the 0.01% level.