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# The longer the first stimulus is explored in softness discrimination the longer it can be compared to the second one*

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*Abstract*— **In haptic perception information is often sampled serially over a certain interval of time. For example, a stimulus is repeatedly indented to repeatedly estimate its softness. Albeit such redundant estimates are equally reliable, they seem to contribute differently to the overall haptic percept in a comparison task. When comparing the softness of two silicon rubber stimuli, the within-stimulus weights of estimates of the second stimulus' softness decrease during the exploration. Here we test the hypothesis that such decrease of weights depends on the representation strength of the first stimulus' softness. We varied the length of the first stimulus' exploration. Participants subsequently explored two silicon rubber stimuli by indenting the first stimulus (comparison) 1 or 5 times and the second stimulus (standard) always 3 times. We assessed the weights of indentation-specific estimates from the second stimulus by manipulating perceived softness during single indentations. Our results show that the longer the first stimulus is explored the more estimates of the second stimulus' softness can be included in the comparison of the two stimuli. This suggests that the exploration length of the first stimulus determines the strength of its representation which influences the decrease of weights of indentation-specific estimates of the second stimulus.**

## I. INTRODUCTION

When redundant sensory signals on an environmental property are simultaneously available from different senses (e. g. haptic and visual signals to an object's size [1]) or from different cues in a single sense (e. g. different visual depth cues [2]), they are usually integrated in a statistically optimal fashion (maximum likelihood estimation, MLE, [1]). This is done by averaging over the $n$ available signals $s_i$ weighted by their relative reliabilities $r_i$ (defined as the inverse of variance $r_i = 1/\sigma_i^2$):

$$\hat{S} = \sum_{i=1}^{n} w_i s_i, \text{with } w_i = \frac{\sigma_i^{-2}}{\sum_{i=1}^{n} \sigma_i^{-2}},\ w_i \geq 0 \text{ and} \sum_{i=1}^{n} w_i = 1 \quad (1)$$

This kind of integration is considered to be statistically optimal because it maximizes the reliability of the combined estimate $\hat{S}$ [3]. However, research did not yet reveal a concordant model for the integration of redundant signals which are serially available, e. g. from the repeated indentations used to estimate the softness of a stimulus. There is evidence that visual information sampled serially across saccades (i.e. with relatively short time intervals in-between) is integrated according to the MLE model [4][5]. However, the integration of serially acquired information in haptic perception where the sampling of information spans longer time intervals seems not to be consistent with the commonly used model for the integration of redundant information (MLE). Several studies show that the reliability of haptic percepts of various object properties increases with additional redundant sensory information obtained from prolonged explorations [6][7][8][9][10][11], but much less than would be predicted by the MLE model (1) [6][7][11].

The question arises, why the integration of redundant serially sampled information in haptic perception is not consistent with the MLE model. [6] studied how the estimates from single exploratory segments in a multi-segmented haptic exploration of virtual gratings are weighted in texture discrimination. In that experiment participants subsequently explored 2 virtual gratings (a standard and a comparison) by striking 2-5 times across each grating. A movement segment was defined by a unidirectional stroke across the entire grating. To assess the weights of texture estimates gathered from single strokes the spatial frequency of the standard grating was slightly changed during one of the strokes across that grating. Perceived spatial frequency was measured as the Point of Subjective Equality (PSE) between the manipulated standard stimulus and the not manipulated comparison stimuli. The extent to which the PSE was shifted from the standard's basic frequency towards the changed spatial frequency corresponded to the weight of the stroke during that spatial frequency had been changed. [6] showed that the weights of estimates from single strokes decreased with the distance of the stroke to the other stimulus. For example, the estimates from the last segment on the first stimulus and the first segment on the second stimulus obtained the highest weights. The MLE model had predicted equal weights for the segment-specific estimates because the estimates were based on highly similar sensory information and should have the same reliability. [6] hypothesized that unequal weighting of the estimates is the reason for the "suboptimal" integration in haptic perception.

In a recent study [7] we extended the investigations in [6] to the haptic perception of softness. Softness is a psychological correlate of compliance, which is defined as the ratio between the displacement of an object's surface and the associated force applied to this object (it is measured in mm/N). In our experiment participants subsequently explored two silicon rubber stimuli by indenting both 2-5 times using their bare index finger. A movement segment was defined as a single indentation of a stimulus consisting of a force increase and a subsequent force decrease by which the finger moved into and then out of the stimulus. To assess the weights of the estimates from different indentations we had to manipulate perceived softness during a single indentation.

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Participants explored real stimuli the compliance of which was given. However, we were able to manipulate their perceived softness by applying subtle external forces to the exploring finger of the participant [12]. External forces that pushed the index finger into the stimulus resulted in a softer percept and forces that pulled the finger out of the stimulus, resulted in a harder percept of the same stimulus. The forces were calculated as a fixed fraction $\alpha$ of the force applied by the participant. Perceived softness changed proportional to $\alpha$ [12]. We used this manipulation to change perceived softness during single indentations in the multi-segmented softness exploration [7]. We showed that the weights of the segment-specific estimates from the first stimulus were rather equal, whereas weights of the estimates from the second stimulus decreased with an increasing distance of the segment from the first stimulus. This is in line with the results in [6], where the authors did not distinguish between weights from the first and the second stimulus. We further found that the decrease of the segment-specific weights on the second stimulus differed depending on the length of the exploration, with the shortest exploration yielding the steepest decrease [7]. We hypothesized that the decrease in weights on the second stimulus is due to memory effects: The task requires to remember the softness of the first stimulus, and to compare this representation to softness estimates from the second stimulus. Short-term memory is commonly modeled as short-term synaptic facilitation of neurons [13], resulting from Calcium influx during the firing of a cell and increasing its excitability shortly after the excitation. In a perceptual decision making task, the memory of the first stimulus might be implemented as synaptic facilitation of selective neurons, which are activated again when the second stimulus is presented, in order to retrieve the memory and compare the two stimuli [14][15]. Assuming that the memory of the first stimulus in our experiment is also implemented as synaptic facilitation of selective neurons, we suggested that it fades over sequential indentations of the second stimulus, which interferes with the comparison to later estimates from the second stimulus. We further speculated that the first stimulus' representation is the weaker the shorter that stimulus has been explored. A weak representation fades quickly during the exploration of the second stimulus, explaining a steep decrease of weights. In contrast after a longer exploration, the representation of the first stimulus should be stronger, fade slowly and can be still reliably compared to later estimates from the second stimulus. But please note that data from [7] do not clearly support the latter speculation, because in [7] the exploration length of the first stimulus was not independently varied, but the first and the second stimuli were always explored with equal number of indentations.

In the present study we directly tested the hypothesis that the decrease of weights of estimates gathered during the exploration of the second stimulus depends on the length of the exploration of the first stimulus. In [7] the exploration length of the first stimulus was varied together with the exploration length of the second stimulus. Hence, in that study the decrease of weights during the exploration of the second stimulus could not be directly compared between conditions because these differed in the number of exploratory segments and effects could not be unequivocally led back to the length of the exploration of the first stimulus. In the present experiment participants subsequently explored two silicon rubber stimuli and decided which one felt softer. Here we systematically varied the length of the exploration of the first stimulus keeping the length of the exploration of the second stimulus constant. The first stimulus was explored by indenting it either one or five times and the second stimulus was always indented three times. For the short exploration of the first stimulus, we expected the weights to rapidly decrease. In the extreme case only the estimate from the first indentation of the second stimulus can be reliably compared with the representation of the first stimulus and thus only the first indentation receives a high weight. In contrast after the long exploration, the representation of the first stimulus should persist longer and it should be possible to reliably compare estimates from later indentations of the second stimulus to this representation - resulting in less decrease of weights for later estimates. Because weights sum up to 1 and are predicted to decrease more steeply after short as compared to long exploration, we also expected that the first segment-specific weight should be higher, but the second and third segment-specific weights should be lower after short as compared to long exploration. To assess the segment-specific weights of the estimates from the exploration of the second stimulus we manipulated perceived softness during each single indentation by transmitting subtle external forces to the exploring index finger of the participant [7][12].

## II. METHODS

### A. Participants

14 students (naïve to the purpose of the experiment, 7 females, 20 to 29 years old, average 23.2 years) volunteered to participate in the experiment. They were reimbursed for their participation (8€/h). All participants were right-handed and did not reported any sensory or motor impairment at the right hand. The study was approved by the local ethics committee LEK FB06 at Giessen University and was in line with the declaration of Helsinki from 2008. Written informed consent was obtained from each participant.

### B. Apparatus and setup

The experimental setup (visuo-haptic workbench, Figure 1) comprised a PHANToM 1.5A haptic force feedback device (finger position measurement and force transmission), a 22"-computer screen (120 Hz, 1280x1024 pixel), a force sensor (measuring beam LCB 130 and measuring amplifier GSV-2AS, resolution 0.05 N, temporal resolution 682 Hz), a mirror, stereo glasses and headphones. The silicon rubber stimuli were placed on the force sensor in front of the participant. The mirror prevented direct view of the stimuli and the participant's hand. Instead participants viewed (40 cm viewing distance, fixated by a chin rest) via stereo glasses a virtual 3D representation of the real scene (finger and stimuli). Importantly, the visual representation of the finger (sphere of 8 mm diameter) was hidden during the exploration of the stimuli (force > 0.1 N), so that no visual information of the indentation of the stimuli was available. The virtual scene was displayed on the screen and reflected by the mirror, inclined to spatially align the virtual and the real scenes. The participant's index finger was connected to the PHANToM with a custom-made gimbal-like adapter as described in [12] allowing relatively free exploration of the silicon rubber stimuli with the bare finger pad (only rotation around the x-axis was blocked) and simultaneous transmission of external forces by the PHANToM. White noise played via headphones covered sounds of the PHANToM engines when transmitting external forces. A custom-made software controlled the experiment, collected responses and recorded relevant parameters (finger position and force) every 3 ms.

External forces were transmitted vertically to the index finger of the participant. We used downwards and upwards directed forces which either pushed the finger into or pulled it out of the rubber stimulus. The amount of external force was a fixed fraction $\alpha$ of $\pm 0.16$ of the force participants applied themselves. For more detail on the force manipulation see [12]. External forces were applied only during one of the indentations of the second stimulus. The algorithm to detect and count the indentations is described in detail in [7].

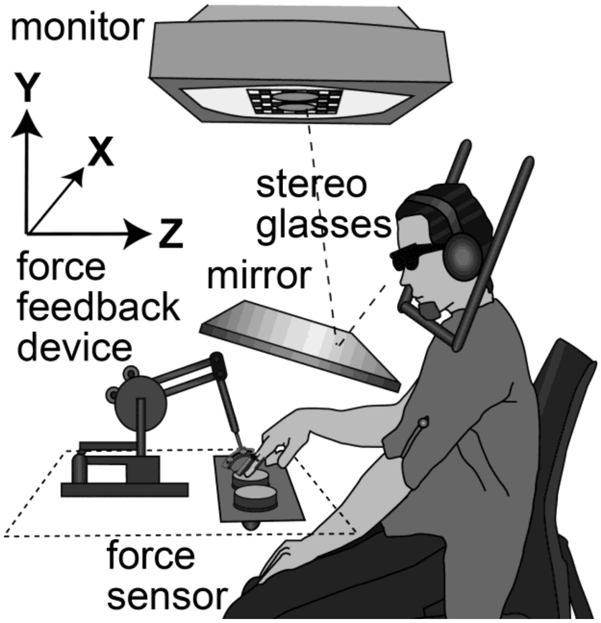

Figure 1. Visuo-haptic workbench.

### C. Softness stimuli

We used a two-component silicon rubber solution (Alpa Sil EH 10:1) to create silicon rubber stimuli. To obtain different compliances we varied the amount of a diluent (silicone oil, viscosity 50 mPa·s) which was added to the silicon. The silicon and oil mixtures were poured in cylindrical plastic dishes (75 mm diameter, 38 mm height). After the stimuli cured we measured the compliance using our experimental setup but exchanging the adapter by a flat–ended cylindrical probe of 1 $cm^2$ area ('standard finger'). The probe was manually pressed into the stimulus 5 times exceeding a force of 15 N. Compliance was determined as the slope in the linear function fitted to the force-displacement traces in the range of 0-9 N. For more details on the compliance measurement see [16]. For this study only data from the increase of force (pressing into the stimulus) was analyzed to exclude hysteresis effects.

We created a series of 12 stimuli consisting of one standard stimulus and 11 comparison stimuli. The comparison stimuli spanned a range of 2.5 Weber fractions to each side (lower and higher compliance) around the standard stimulus. The value for the Weber fraction in softness perception of 20% is taken from [16]. Two neighboring comparison stimuli differed by 1/2 Weber fraction (0.03 mm/N). The compliance of the standard was 0.32 mm/N, for the comparisons it was 0.16, 0.19, 0.23, 0.26, 0.29, 0.32, 0.36, 0.39, 0.43, 0.46 and 0.49 mm/N.

### D. Design & Procedure

The experimental design comprised two within-participant variables: *Exploration length of the first stimulus* (1 vs. 5) and *Indentation Nr on the second stimulus* (1, 2, 3) resulting in 6 experimental conditions. For each *Exploration length of the first stimulus* condition we had a control condition in which no external forces were transmitted during the exploration of the second stimulus. For every participant and experimental condition we measured the PSEs of the standard stimulus manipulated with pulling and pushing forces as compared to non-manipulated comparison stimuli, using a two-interval forced-choice task (2IFC) combined with 1-Up-1-Down staircases. A 2IFC task is a commonly used psychophysical method to measure the subjective experience of a certain stimulus (standard). It consists of two sequential intervals in which participants are presented with two alternative stimuli (standard and comparison) between which they have to choose according to the task instruction. When combined with a staircase, the values of the comparison stimuli are adaptively varied depending on the responses of the participant. The 1-Up-1-Down staircase determines the stimulus level at which the standard is chosen 50% of times (PSE). The *Exploration length of the first stimulus* conditions were presented during two different sessions (half of participants started with 1 and the other half started with 5 indentations).

In every trial participant explored first the comparison stimulus and afterwards the standard stimulus and decided which one felt softer. The beginning of a trial was indicated by a signal tone and the appearance of the comparison stimulus on the screen. The position (left vs. right) was randomly chosen. Participants explored the comparison stimulus by indenting it 1 or 5 times with the index finger of their dominant hand. After participants had completed the exploration of the comparison stimulus the standard stimulus was displayed on the screen and was explored by indenting it 3 times. Subsequently participants indicated which stimulus felt softer by tapping one of the two virtual decision buttons located above the stimuli. The stimuli were changed manually between the trials by the experimenter. Meanwhile participants moved their index finger to the indicated corner of the workspace. Participants did not receive any feedback on their performance. The number of indentations allowed to explore the first stimulus (comparison, 1x or 5x) and the second stimulus (standard, 3x) was instructed before the experimental session. Trials in which the number of indentations was incorrect were repeated later in a block.

Every PSE was measured using two staircases. One staircase started with the softest comparison stimulus (downwards-directed staircase) and the other with the hardest comparison (upwards-directed staircase). The next comparison stimulus in the staircase was determined by the response of the participant. If the comparison felt softer than the standard, a harder comparison was presented in the next trial of this staircase. In the opposite case the comparison of the next trial was softer. In the cases the softest comparison felt harder or the hardest comparison felt softer to the participants the same stimulus was presented in the corresponding next trial. Each staircase terminated after participants changed the direction in this staircase 15 times by changing their judgment from harder to softer and vice versa.

The experiment consisted of two sessions each of an average duration of 2.7 h and were completed on two separate days within one week. Every session was split in blocks in which the current step of each staircase was presented in a randomized order, balancing the effects of fatigue or inattention between conditions. Sessions were interspersed with 1 minute pauses about every 15 min (not in phase with the change of the blocks).

### *E. Data analysis*

The PSEs were estimated as the average over all comparisons at which a reversal occurred (30 per PSE). To test whether the manipulation of perceived softness was successful and whether there were differences in average PSEs between the *Exploration length of the first stimulus* conditions we performed a repeated measures ANOVA on the PSEs with the two factors *Fraction of external force* (-0.16,0,+0.16) and *Exploration length of the first stimulus*. To assess the weights we calculated for every participant and every condition the relative change in the PSE with the pulling and the pushing external force as compared to the control condition, in which perceived softness was not manipulated ($[PSE_{manipulated} - PSE_{control}]/PSE_{control}$) [7][12]. We performed a linear regression of the relative PSE change on the fraction of external force $\alpha$ [-0.16, 0, +0.16]. In [12] we found that the fraction of external force $\alpha$ is related to the change in the PSE by the factor $w_k = 0.26$ when the external force is transmitted during the whole exploration. Hence to calculate the weight of an estimate from a single exploratory segment we had to divide the slope obtained in the regression function by $w_k$.

We conducted a limited number of planned comparisons to test our directional hypotheses on weights. To compare the decrease of the weights between neighbored segments (equivalent to the slope) after short versus long exploration, we calculated the two pairwise interaction contrasts *Exploration length of the first stimulus* X [1st vs. 2nd indentation on second stimulus] and *Exploration length of the first stimulus* X [2nd vs. 3rd indentation on second stimulus]. Because we expected for the first two segments a steeper decrease after short exploration, this test was conducted one-sided. Further we tested our segment-wise hypotheses on the differences between weights in the two *Exploration length of the first stimulus* by segment-wise one-sided *t*-tests. Finally, to determine which estimates contributed to the estimation of the second stimulus softness, we tested each single weight in each of the two *Exploration length of the first stimulus* conditions against zero using one-sided *t*-tests. As a sanity check we calculated for each participant the sum of the within-stimulus weights for the second stimulus and tested the averages with a *t*-test against the predicted sum of weights (1).

## III. RESULTS

The PSEs with pulling, pushing and no forces are plotted in Figure 2 as a function of the indentation Nr on the second stimulus separately for the two exploration lengths of the first stimulus. Overall pushing forces resulted in a PSE shift to higher values, indicating that the standard was perceived softer in this case, whereas pulling forces caused a PSE shift to lower values, indicating a harder percept of the standard. Comparing the two *Exploration length of the first stimulus* conditions, there was a general offset in the PSEs. Repeated measures ANOVA on the PSEs revealed a significant main effect of *Fraction of external force*, $F(2,26) = 9.7$, $p < .001$, confirming that the manipulation of perceived softness was (as expected) successful. Also the main effect of *Exploration length of the first stimulus* was significant, $F(1,13) = 12.01$, $p = 0.004$, indicating that the second stimulus was perceived differently after long as compared to short exploration of the first stimulus. This could be also confirmed by comparing only the baseline data between the two conditions, $t(13) = -2.56$, $p = 0.024$. However, only the baseline in the condition in which the first stimulus was explored with five indentations, was significantly shifted to higher values from the physical value of the standard (0.32 mm/N), 5 indentations: $t(13) = 2.24$, $p = 0.043$, 1 indentation: $t(13) = -1.45$, $p = 0.170$, indicating that only with the longer exploration the perception of the standard was changed (shifted a softer percept). The interaction of the two factors *Fraction of external force* and *Exploration length of the first stimulus* was not significant, $F(2,26) = 0.29$, $p = 0.752$.

In Figure 3 the weights of the estimates gathered from indentations on the second stimulus are plotted as a function of the indentation number on the second stimulus. After a short exploration (1 indentation) of the first stimulus the indentation-specific weights of the estimates of the second stimulus' softness decreased steeper as compared to a longer exploration of the first stimulus. The interaction contrast between the weights of the first two indentations was significant $t(13) = 1.99$, $p = 0.034$ (one-sided), confirming the prediction of a steeper increase of weights after short as compared to long explorations. The interaction contrast between the weights of the last two indentations was not significant $t(13) = 1.51$, $p = 0.153$. The indentation-wise comparisons of the weights between the *Exploration length of the first stimulus* conditions revealed the predicted significant difference for the second segment, $t(13) = -2.02$, $p = 0.033$, but not for the first, $t(13) = 0.62$, $p = 0.274$, and third, $t(13) = -0.05$, $p = 0.481$ segments (all tests one-sided).

Finally, we conducted *t*-tests of the single weights against zero (one-sided): When the first stimulus was explored with one indentation, only the weight of the first estimate was significantly larger than zero, $t(13) = 4.75$, $p < 0.001$ (2. segment: $t(13) = 0.25$, $p = 0.402$; 3. segment $t(13) = 1.0$, $p = 0.168$). In contrast when the first stimulus was indented five times the weights of the first two estimates were significantly larger than zero (1. segment: $t(13) = 2.45$, $p = 0.015$, 2. segment: $t(13) = 3.08$, $p = 0.004$) whereas the weight of the

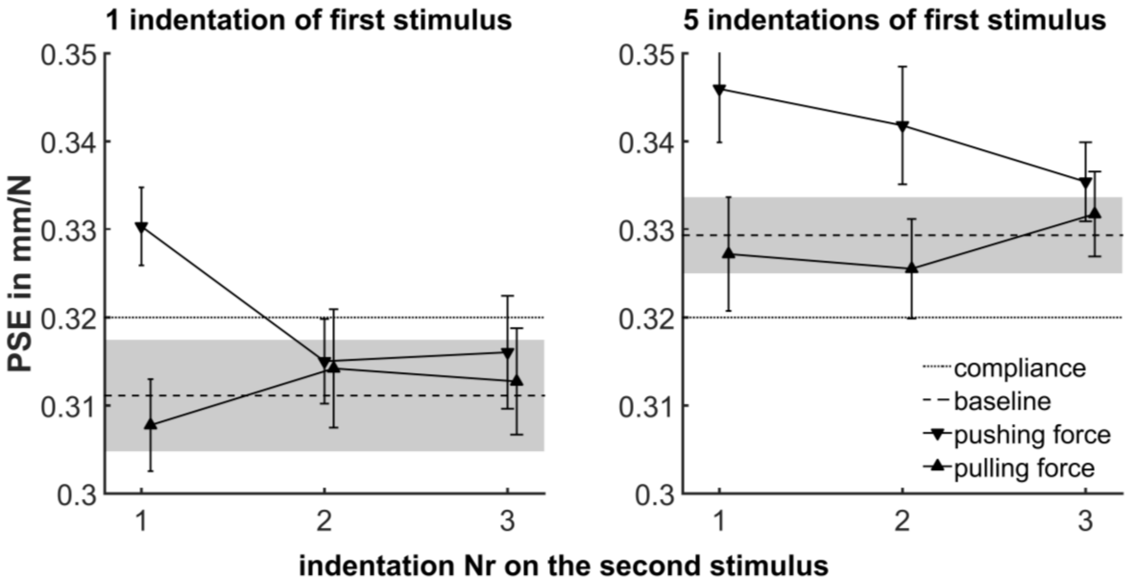

Figure 2. Average PSEs with pulling and pushing forces (downwards and upward pointing triangles, respectively) and their standard errors are plotted separately for the two lengths of the exploration of the first stimulus (1 and 5 indentations) as a function of the indentation Nr on the second stimulus. The average PSEs in the condition without external forces are plotted as a dashed line. The respective standard error is indicated by a gray area. Additionally the physical value of the standard is indicated by a dotted line.

third segment was not, $t(13) = 0.72$, $p = 0.243$. The sums of weights were as predicted not different from 1 in both conditions, 1 indentation: $t(13) = 0.17$, $p = 0.866$, 5 indentions: $t(13) = 0.68$, $p = 0.506$.

## IV. DISCUSSION

In a recent study [7] we showed that in a comparison of the softness of two silicon rubber stimuli, estimates of the first stimulus' softness were weighted relatively equal, whereas the weights on the second stimulus decreased during the exploration, possibly due to memory effects. In the present study, we tested the prediction that the decrease of the weights depends on the length of the exploration of the first stimulus. We systematically varied the length of the exploration of the first stimulus (1 vs. 5 indentations) keeping the length of the exploration of the second stimulus constant (3 indentations) and assessed segment-specific weights by selectively manipulating perceived softness during single indentations of the second stimulus [12]. We replicated the finding from [7] that the weights on the second stimulus overall decreased over the sequential indentations of the second stimulus. More importantly, for the different exploration lengths of the first stimulus we found that with a short exploration (1 indentation) the weights on the second stimulus decreased more rapidly than when the first stimulus was explored longer (5 indentations). With a short exploration only the first estimate contributed to the estimation of the second stimulus' softness, whereas with a longer exploration the first two estimates had weights larger than zero. In line, we found that after a long exploration of the first stimulus the estimate from the second indentation of the second stimulus was weighted significantly higher than after the shorter exploration of the first stimulus. This suggests that a longer exploration of the first stimulus resulted in a longer-lasting representation of this stimulus, so that it could be reliably compared to the second estimate of the second stimulus' softness. Overall, our results indicate that the decrease of the weights on the second stimulus depends on the length of the exploration of the first stimulus, which confirms our hypothesis that the representation of the first stimulus is stronger the longer this stimulus is explored and that it decreases the less over the sequential indentations of the second stimulus. Still the decay of the representation of the first stimulus limits the possibility to reliably compare it with the second stimulus.

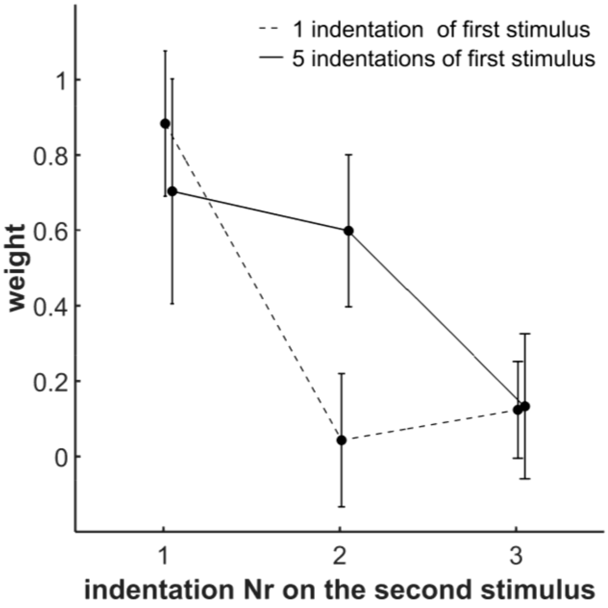

Figure 3. Average weights and their standard errors of the estimates from single indentations on the second stimulus as a function of the indentation Nr on the second stimulus plotted separately for each length of exploration of the first stimulus (1 and 5 indentations).

Our findings are well in line with the neurophysiological model of sensory decisions proposed in [14].

In contrast, the findings are not in agreement with the MLE model (1), which predicts equal weights if redundant estimates are gathered from equally reliable sensory information, because equal weights would maximize overall reliability under these conditions. Such integration has been referred to as being “optimal”. However integration with unequal weights might also represent an optimal integration that maximizes perceptual reliability - under conditions of information processing that violate implicit assumptions underlying the MLE model. In particular, when applying the MLE model to processes of perceptual integration, it is often implicitly assumed that the all gathered sensory information is available during the entire process. However, in a sequential comparison task, in particular if it spans a longer interval of time, the representation of the first stimulus might fade over the gathering of information from the second stimulus, which decreases the reliability of the information from the first stimulus’ during the perceptual process. Hence, the MLE model may not provide an appropriate benchmark to judge the optimality of such tasks. Instead a model is required that can also account for effects of memory decay.

Further, we observed that the perceived softness of the second stimulus (standard) depended on the length of the exploration of the first stimulus: It was higher after long as compared to short exploration. This likely indicates stronger adaptation to softness after 5 indentations than after 1 indentation of the first stimulus [17]. In [17] we found that a standard is perceived to be softer after adaptation to stimuli that are harder than the standard, and vice versa for softer adaptation stimuli. That study also showed that the PSE shift is larger for harder as compared to softer adaptation stimuli. Furthermore we found that when participants adapted to a stimulus with the same compliance as the standard the PSE was shifted to a softer percept when the standard was relatively hard (0.32 mm/N) and to a harder percept when the standard was rather soft (0.67 mm/N). In the present study the standard stimulus was relatively hard (0.32 mm/N) and the number of comparison stimuli explored before that were harder than the standard was the same as the number of softer comparisons. Both the larger PSE shifts after harder as compared to softer adaptation stimuli, and the fact that the standard was relatively hard predict that in the present experiment adaptation should induce an overall shift of the standard towards a softer percept, in particular after a longer adaptation phase, i.e. after five indentations. This was indeed what we observed in the present study.

Taken together our results confirm that when the softness of two real stimuli is compared haptically, the information gathered about the softness of the second stimulus is weighted unequally, with the later estimates being weighted less than the first ones [7]. Moreover our results suggest that, the unequal weighting is due to the fading representation of the first stimulus, which depends on the exploration length of the first stimulus. More precisely, it seems that with a longer exploration information gathered from more indentations of the second stimulus can be integrated in the comparison of the two stimuli, because the representation of the first stimulus lasts longer. We further argue that the MLE model might need to be extended for modeling serial integration of redundant signals in perceptual comparison tasks, because it cannot account for memory effects.

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## REFERENCES

[1] M. O. Ernst and M. S. Banks, “Humans integrate visual and haptic information in a statistically optimal fashion.,” *Nature*, vol. 415, no. 6870, pp. 429–433, 2002.

[2] I. Oruç, L. T. Maloney, and M. S. Landy, “Weighted linear cue combination with possibly correlated error,” *Vision Res.*, vol. 43, no. 23, pp. 2451–2468, 2003.

[3] W. G. Cochran, “Problems arising in the analysis of a series of similar experiments,” *Suppl. to J. R. Stat. Soc.*, vol. 4, no. 1, pp. 102–118, 1937.

[4] L. Oostwoud Wijdenes, L. Marshall, and P. M. Bays, “Evidence for optimal integration of visual feature representations across saccades,” *J. Neurosci.*, vol. 35, no. 28, pp. 10146–53, 2015.

[5] C. Wolf and A. C. Schütz, “Trans-saccadic integration of peripheral and foveal feature information is close to optimal,” *J. Vis.*, vol. 15, no. 16, pp. 1-18, 2015.

[6] A. Lezkan and K. Drewing, “Unequal but fair? Weights in the serial integration of haptic texture information,” in *International Conference on Human Haptic Sensing and Touch Enabled Computer Applications*, 2014, pp. 386–392.

[7] A. Metzger, A. Lezkan, and K. Drewing, “Integration of serial sensory information in haptic perception of softness,” *Not yet Publ. Manuscr.*

[8] G. A. Gescheider, S. J. Bolanowski, J. V Pope, and R. T. Verrillo, “A four-channel analysis of the tactile sensitivity of the fingertip: frequency selectivity, spatial summation, and temporal summation,” *Somatosens. Mot. Res.*, vol. 19, no. 2, pp. 114–24, 2002.

[9] C. D. Giachritsis, A. M. Wing, and P. G. Lovell, “The role of spatial integration in the perception of surface orientation with active touch.,” *Attention, Perception, Psychophys.*, vol. 71, no. 7, pp. 1628–1640, 2009.

[10] S. Louw, A. M. L. Kappers, and J. J. Koenderink, “Haptic detection of sine-wave gratings,” *Perception*, vol. 34, no. 7, pp. 869–885, 2005.

[11] K. Drewing, A. Lezkan, and S. Ludwig, “Texture discrimination in active touch: Effects of the extension of the exploration and their exploitation,” *2011 IEEE World Haptics Conf. WHC 2011*, pp. 215–220, 2011.

[12] A. Metzger and K. Drewing, “Haptically perceived softness of deformable stimuli can be manipulated by applying external forces during the exploration,” in *IEEE World Haptics Conference, WHC 2015*, 2015, pp. 75–81.

[13] G. Mongillo, O. Barak, and M. Tsodyks, “Synaptic theory of working memory,” *Science*, vol. 319, no. 5869, pp. 1543–1546, 2008.

[14] G. Deco, E. T. Rolls, and R. Romo, “Synaptic dynamics and decision making,” *Proc. Natl. Acad. Sci. U. S. A.*, vol. 107, no. 16, pp. 7545–7549, 2010.

[15] R. Romo, A. Hernandez, and A. Zainos, “Neuronal correlates of perceptual decision in ventral premotor cortex,” *Neuron*, vol. 41, pp. 165–173, 2004.

[16] L. Kaim and K. Drewing, “Exploratory strategies in haptic softness discrimination are tuned to achieve high levels of task performance,” *IEEE Trans. Haptics*, vol. 4, no. 4, pp. 242–252, 2011.

[17] A. Metzger and K. Drewing, “Haptic aftereffect of softness,” in F. Bello, H. Kajimoto & Y. Visell (Eds.), *Haptics: Perception, Devices, Control, and Applications: 10th International Conference, EuroHaptics 2016, Proceedings, Part I*, 2016, pp. 23-32.