Dynamics Synchronization of the Running of Planar Biped Robots with SLIP Model in Stance Phase

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***Abstract*—In this work a control law is derived to synchronize dynamics of multibody biped robots and the spring loaded inverted pendulum (SLIP) model in stance phase of running. The goal is to use the vast literature on the SLIP model dynamics for the control of real multibody robots. Three kneed biped robot models are considered in this work: with springs parallel to motors, with springs series to motors, and without springs. Dynamic equations of the multibody biped models are derived using Lagrange equation and then the applicability of the derived control law to these models are investigated using simulation. The initial state of the multibody robot is found such that its center of mass (CoM) has the same initial condition as SLIP model. Then the trajectory of its CoM is compared to SLIP model. Also motor torque profiles are compared for the models with and without springs and also for the motors with and without rotor inertias. The results show that the effect of rotor inertia is a big challenge in implementing fast biped running on real robots.**

Keywords—Bipedal Running, SLIP Model, Dynamics Synchronization

I. INTRODUCTION

Biped robot research field has not been able yet to develop a real robot to run as efficient and as fast as animals. Bipedal running dynamic model is highly nonlinear with discontinuities and open loop unstable system. In real robots we have actuating system limitations and also existence of compliant elements which are necessary in running robots makes the dynamic model and the control system more complicated. There are two main practical compliant actuating systems for biped robots: springs parallel to motors for example RABBIT [7] and MABEL [8], and springs series with motors for example ATRIAS [10]. Systems with the number of actuators less than the degrees of freedom are called underactuated systems. While fully actuated biped robots mainly controlled by ZMP (Zero Moment Point) method show unnatural and slow gaits 0, underactuated robots show more natural looking and faster walking and running gaits [2]. Biped robots with no motors in ankle joint constitute a main category of underactuated biped robots. Raibert's robot is an important example of this type of robot. He divided the control action to three parts: hopping height, touch-down angle and torso angle, and did successful experiments on 2D and 3D biped running [3]. McGeer's passive biped walker [4] encouraged many researchers to work on passive robots dynamics and stability [5], and passivity-based active controllers [6]. The robot RABBIT using a time-invariant controller could walk stably and run several steps [7]. The robot MABEL showed stable walking and running by hybrid zero dynamics control method [9]. In this method, a lower dimensional sub-system is defined using virtual constraints to describe the main system. The robot ATRIAS has been designed to have a mechanical structure near to SLIP (Spring Loaded Inverted Pendulum) model, aiming to walk and run outdoors on uneven surfaces [10].

The role of compliant elements for energy efficiency of biped running has been shown [11]. The SLIP model [12], consisting of a point mass as torso and a massless spring as leg, describes the basic dynamics of human walking and running. The CoM (Center of Mass) trajectory and the ground reaction force are used to show analogy of the dynamics of the two systems. This passive theoretical model generates stable running for a narrow range of spring stiffness and attack angle that covers a wide range of funning speed [13], i.e. a SLIP model with properly chosen parameters can reject disturbances and return to the stable limit cycle of running passively. Similar results have been reported for 3D SLIP model [14]. Therefore from self-stability and energy efficiency point of view, SLIP model can be considered as a useful pattern to control biped robots. Furthermore, several control strategies have been proposed to raise the settling rate and increase the basin of attraction for the SLIP mode. For instance, dead-beat controllers by adjusting attack angle or spring stiffness can reject disturbances during a single step of running [15]. Swing leg retraction is a bio-inspired strategy for flight phase in which the front leg starts to rotate backwards with a constant angular velocity from the apex instance [16]. A similar strategy is utilized by animals during running to synchronize front toe and ground speed. An important and simple control strategy for flight phase is to update each touch-down angle equal to minus of the previous take-off angle. This strategy stabilizes the overall SLIP running gait [17].

Regardless of advantages of SLIP model to describe bipedal running dynamics and to stabilize running motion, it's still a challenge to use SLIP model dynamics to control real biped robots. This is due to higher degrees of freedom, more complicated dynamics, and energy losses and discontinuities at touch-down. The CoM trajectory of SLIP has been used as hybrid zero dynamics for an asymmetric hopper and generated stable hopping for it [18]. A controller has been proposed in [19] for a kneed hopper to track SLIP dynamics. Feedback linearization has been used in [20] to track SLIP CoM trajectory by a three link rigid hopper. Inheritance of SLIP running stability by a biped model whose legs have mass was discussed in [21].

This paper aims to answer a question: Is it possible to make multibody biped robots to have exactly the same dynamics as SLIP model? To do so, a stance phase controller is derived to synchronize the dynamics of multibody robot CoM with the SLIP model. The investigated model consists of kneed legs with masses and moments of inertia and a point mass at the hip. This model is considered in three cases: with no springs, with springs parallel to motors and with springs series to motors. The possibility of dynamics synchronization of these models with SLIP model is discussed.

II. Compliant Structures & Dynamic Modeling

Having the clear role of elastic elements in the energy efficiency of biped walking and running proven [22], in recent years biped robot research has moved towards the use of the compliant elements in robot actuation system. Also due to the impact forces in running gaits to prevent damage to the robot motors and mechanisms, the use of elastic elements in the legs seems inevitable. Despite complex neural and muscle - tendon systems, human leg shows a simple spring-like behavior in running and some walking speeds [23]. Animals' leg can be modeled using Hill-type muscle model, but it adds unnecessary complexities to the robot model and controller. Empirical kneed biped robots have two basic compliant structures: springs parallel with the motors and series with motors. As two typical robots MABEL has springs parallel with motors [8] and ATRIAS has springs series with motors [10]. The parallel compliant mechanism has the advantage that it makes the controller design easier and in other side the series compliant mechanism isolates motors from legs and no impact moment at touch-down is transferred to the motor and gearbox. In this paper three biped models with springs parallel and series to the motors and without springs are considered and possibility of their dynamics synchronization with SLIP model is investigated.

A. Robot Model with Parallel Compliant Mechanism

We consider a kneed biped model in stance phase as shown in figure 1a. Leg segments thigh and shin have mass and moment of inertia and the mass of torso is modeled as a point mass at hip. There are three motors parallel with rotational springs at the hip and the knees joints. Stance phase generalized coordinates $[\boldsymbol{q}_s]_{4\times1}$ has four components including absolute angles of thigh links BH and DH with respect to vertical and relative angles of shin links AB and CD with respect to thigh links. All angles are assumed positive in counter-clockwise direction. Masses and dimensions of the robot model a typical human. The robot has point feet with no motors at ankle joints, so the system is underactuated. Free angle of the knee springs are selected at $q_2 = -\pi/4$ and $q_4 = -\pi/4$ and for the hip spring at $q_3 = 0$. Inspired by bio-mechanisms, we use two different spring stiffness for the stance and swing knees. It has been shown that because the stance and swing knees have different loads, their different spring stiffness is necessary for energy efficiency and natural looking motion [24]. So, the knee spring stiffness is switched between K_{st} and K_{sw} at the end of each running phase. The values for springs stiffness are selected to generate an appropriate running gait with the speed of 10 *m/s*. Due to high nominal speed range of electric motors, using them as bipedal robot actuators requires the use of a speed reduction system like a gearbox or a cable differential drive system. Consequently, the kinetic energy of rotors has a considerable effect at the dynamics of the system. The rotor moment of inertia and the gear reduction ratio has been selected according to the real robot ATRIAS [10]. In this section, we model the stance phase in which the stance foot is assumed as an ideal pivot on the ground and the swing leg is swinging forward.

TABLE I. Nomenclature

Parameter	Value (SI units)	Description
$m_1, l_1, \bar{I}_1$	8, 0.45, 0.135	Mass, length and moment of inertia of thigh BH, DH
$m_2, l_2, \bar{I}_2$	7, 0.5, 0.146	Mass, length and moment of inertia of shank AB, CD
m_h	50	Point mass of hip at H
q_1	variable	Angle of thigh BH with respect to vertical
q_2	variable	Angle of shank AB with respect to thigh BH
q_3	variable	Angle of thigh CH with respect to vertical
q_4	variable	Angle of shank CD with respect to thigh CH
u_1	variable	Torque of hip motor in the direction of $q_3 - q_1$
u_2	variable	Torque of motor in knee B in the direction of q_2
u_3	variable	Torque of motor in knee C in the direction of q_4
K_h	200	Torsion spring stiffness in hip
K_{st}	1000	Torsion spring stiffness in the knee of stance leg
K_{sw}	500	Torsion spring stiffness in the knee of swing leg

To derive equations of motion we use Lagrange equation,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \quad (1)$$

in which, Lagrangian function $L(\mathbf{q},\dot{\mathbf{q}}) = T(\mathbf{q},\dot{\mathbf{q}}) - V(\mathbf{q})$ is the subtraction of kinetic and potential energy and Q_i are the generalized forces that are calculated using virtual work. The stance phase dynamic equations become

$$\left[\mathbf{D}_s\left(q_s\right)\right]_{4\times4}\cdot\left[\ddot{\mathbf{q}}_s\right]_{4\times1}+\left[\mathbf{C}_s\left(\mathbf{q}_s,\dot{\mathbf{q}}_s\right)\right]_{4\times1}=\left[\mathbf{B}_s\right]_{4\times3}\cdot\left[\mathbf{u}_s\right]_{3\times1} \tag{2}$$

in which, input matrix and input vector are

$$\mathbf{B}_s=K_g\begin{bmatrix}-1&0&0\\0&1&0\\1&0&0\\0&0&1\end{bmatrix},\mathbf{u}=\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix}. \tag{3}$$

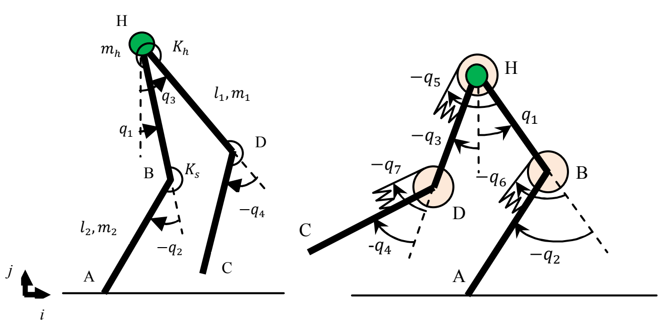
Here, symbolic manipulation software (Maple/Matlab toolbox) can be used to derive the analytical dynamic equations. First, x and y components of the joints positions are written as functions of generalized coordinates. Kinetic and potential energies are then found using these functions and their symbolic time derivatives, and substituted in equation 1 to derive equations 3. By defining the stance phase state vector as $\boldsymbol{x}_s=\left[\mathbf{q}_s;\dot{\mathbf{q}}_s\right]$ the four second-order differential equations 3 turn into eight first order state equations.

$$\dot{\mathbf{x}}_s=\mathbf{f}_s\left(\mathbf{x}_s\right)+\mathbf{g}_s\left(\mathbf{x}_s\right).u \tag{4}$$

in which,

$$\mathbf{f}_s\left(\mathbf{x}_s\right)=\begin{bmatrix}\dot{\mathbf{q}}_s\left(\mathbf{x}_s\right)\\-\mathbf{D}_s^{-1}\left(\mathbf{x}_s\right).\mathbf{C}_s\left(\mathbf{x}_s\right)\end{bmatrix},\ \mathbf{g}_s\left(\mathbf{x}_s\right)=\begin{bmatrix}\mathbf{0}_{3\times1}\\\mathbf{D}_s^{-1}\left(\mathbf{x}_s\right).\mathbf{B}_s\end{bmatrix} \tag{5}$$

More details about the dynamic equations can be found in [25].



(a) (b)

Fig. 1- Stance phase generalized coordinates for the kneed biped model (a) with parallel compliant mechanism (b) with series compliant mechanism

B. Robot Model with No Springs

Dynamic model of the robot with no springs is similar to the previous case with parallel springs. The robot in stance phase is again a 4 DOF (Degrees of Freedom) system and its dynamic model is obtained from equation 3 by putting the springs stiffness equal to zero.

C. Robot Model with Series Compliant Mechanism

In this model as shown in figure 1b rotational springs are located between output shaft of the gearboxes and the relevant links of the robot. The deflection of the spring is equal to the difference of the motor output shaft angle (load-side) and the angle of the relevant link of the robot. The spring torque which is also equal to the motor output torque is equal to the spring deflection multiplied by its stiffness. Because of series springs between the rotors and their relevant links, three generalized coordinates are added to the system and the robot has 7 DOF in stance phase. Stance phase generalized coordinates $[\boldsymbol{q}_s]_{7\times1}$ consists of four components q_1 to q_4 as links angles identically similar to the previous model, and three components q_5 to q_7 as motors output angles at hip joint H and knee joints B and D, respectively. Stance phase dynamic equations for this model are written as:

$$\left[\mathbf{D}_s\left(\mathbf{q}_s\right)\right]_{7\times7}\cdot\left[\ddot{\mathbf{q}}_s\right]_{7\times1}+\left[\mathbf{C}_s\left(\mathbf{q}_s,\dot{\mathbf{q}}_s\right)\right]_{7\times1}=\left[\mathbf{B}_s\right]_{7\times3}\cdot\left[\mathbf{u}_s\right]_{3\times1} \tag{6}$$

in which, input matrix is

$$\mathbf{B}_s=K_g\begin{bmatrix}0&0&0&0&1&0&0\\0&0&0&0&0&1&0\\0&0&0&0&0&0&1\end{bmatrix}^T \tag{7}$$

It is noticeable that in this case the robot links angles and the motor angles are dynamically isolated by the springs, i.e. torques of the motors do not have direct influence to the links of the robot.

D. Spring Loaded Inverted Pendulum Model

The SLIP model, according to figure 2, consists of a point mass as torso and a massless spring as leg. The system is 2 DOF and stance phase generalized coordinates $[\boldsymbol{q}_s]_{2\times1}=[l,\alpha]^T$ consists of the spring length and the leg angle. Dynamic model of the system is written as:

$$\left[\mathbf{D}_s\left(\mathbf{q}_s\right)\right]_{2\times2}\cdot\left[\ddot{\mathbf{q}}_s\right]_{2\times1}+\left[\mathbf{C}_s\left(\mathbf{q}_s,\dot{\mathbf{q}}_s\right)\right]_{2\times1}=0 \tag{8}$$

in which, l_0 is free length of the spring and the dynamic matrices are written as

$$\mathbf{D}_{\text{slip}}=\begin{bmatrix}m&0\\0&mq_1\end{bmatrix} \tag{9}$$

$$\mathbf{C}_{\text{slip}}=\begin{bmatrix}-mq_1\dot{q}_2^2+mg\cos q_2+k\left(q_1-l_0\right)\\2m\dot{q}_1\dot{q}_2-mg\sin q_2\end{bmatrix}. \tag{10}$$

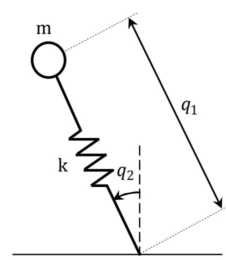


Fig. 2- Stance phase generalized coordinates for the SLIP model

III. Controller Design for Dynamics Synchronization

In this section we aim to find a control law for systems 3 and 7 to make their CoM have the same dynamics as system 9. To do so we need to derive dynamic equations for CoM of the multibody robots. We define the generalized coordinate of the

CoM of the multibody robot by polar coordinates with radial coordinate ρ and angular coordinate γ with respect to the stance foot A. the polar coordinates of CoM is written in terms of its Cartesian coordinates $[x_c, y_c]$ as

$$\rho = \sqrt{x_c^2 + y_c^2} = \rho(q) \qquad (11)$$

$$\gamma = \tan^{-1}\left(\frac{x_c}{y_c}\right) = \gamma(q)\ . \qquad (12)$$

and their time derivatives are written as

$$\dot{\rho} = \frac{\partial \rho}{\partial q}\dot{q} \ \rightarrow \ \ddot{\rho} = \dot{q}^T \frac{\partial^2 \rho}{\partial q^2}\dot{q} + \frac{\partial \rho}{\partial q}\ddot{q} \qquad (13)$$

$$\dot{\gamma} = \frac{\partial \gamma}{\partial q}\dot{q} \ \rightarrow \ \ddot{\gamma} = \dot{q}^T \frac{\partial^2 \gamma}{\partial q^2}\dot{q} + \frac{\partial \gamma}{\partial q}\ddot{q}\ . \qquad (14)$$

Substituting $\ddot{\mathbf{q}}$ from equations 3 or 7 into 14 and 15, functions f_ρ, $\boldsymbol{g}_\rho$, f_γ and $\boldsymbol{g}_\gamma$ are defined as follows.

$$\begin{aligned}\ddot{\rho} &= \dot{\mathbf{q}}^T \frac{\partial^2 \rho}{\partial \mathbf{q}^2}\dot{\mathbf{q}} + \frac{\partial \rho}{\partial \mathbf{q}}\mathbf{D}_s^{-1}\left(-\mathbf{C}_s + \mathbf{B}_s \cdot \mathbf{u}_s\right) \\ &= \left(\dot{\mathbf{q}}^T \frac{\partial^2 \rho}{\partial \mathbf{q}^2}\dot{\mathbf{q}} - \frac{\partial \rho}{\partial \mathbf{q}}\mathbf{D}_s^{-1}\mathbf{C}_s\right) + \left(\frac{\partial \rho}{\partial \mathbf{q}}\mathbf{D}_s^{-1}\mathbf{B}_s\right)\mathbf{u}_s \\ &= f_\rho(\mathbf{q}, \dot{\mathbf{q}}) + \boldsymbol{g}_\rho(\mathbf{q}) \cdot \mathbf{u}_s\end{aligned} \qquad (15)$$

$$\begin{aligned}\ddot{\gamma} &= \dot{\mathbf{q}}^T \frac{\partial^2 \gamma}{\partial \mathbf{q}^2}\dot{\mathbf{q}} + \frac{\partial \gamma}{\partial \mathbf{q}}\mathbf{D}_s^{-1}\left(-\mathbf{C}_s + \mathbf{B}_s \cdot \mathbf{u}_s\right) \\ &= \left(\dot{\mathbf{q}}^T \frac{\partial^2 \gamma}{\partial \mathbf{q}^2}\dot{\mathbf{q}} - \frac{\partial \gamma}{\partial \mathbf{q}}\mathbf{D}_s^{-1}\mathbf{C}_s\right) + \left(\frac{\partial \gamma}{\partial \mathbf{q}}\mathbf{D}_s^{-1}\mathbf{B}_s\right)\mathbf{u}_s \\ &= f_\gamma(\mathbf{q}, \dot{\mathbf{q}}) + \boldsymbol{g}_\gamma(\mathbf{q}) \cdot \mathbf{u}_s\end{aligned} \qquad (16)$$

So, dynamic equations for the CoM of the multibody robot is written as

$$\begin{bmatrix}\ddot{\rho} \\ \ddot{\gamma}\end{bmatrix} + [C']_{2\times1} = [B']_{2\times3} \cdot [u]_{3\times1} \qquad (17)$$

in which,

$$C' = \begin{bmatrix}-f_\rho(q, \dot{q}) \\ -f_\gamma(q, \dot{q})\end{bmatrix}, B' = \begin{bmatrix}g_\rho(q) \\ g_\gamma(q)\end{bmatrix}. \qquad (18)$$

We rewrite the dynamic equations of the SLIP model as

$$[D]_{2\times2}\begin{bmatrix}\ddot{l} \\ \ddot{\alpha}\end{bmatrix} + [C]_{2\times1} = 0 \qquad (19)$$

To synchronize the dynamics of the systems 18 and 20, i.e. $l \equiv \rho, \alpha \equiv \gamma$, it is needed to have

$$B'u = C' - D^{-1}C \qquad (20)$$

Since $\mathbf{B}'$ is a 2×3 matrix, equations 21 are a set of 2 equations and 3 unknowns and have countless answers. A minimum norm answer for these equations can be calculated using pseudo-inverse operator:

$$\mathbf{u} = pinv(\mathbf{B}')(\mathbf{C}' - \mathbf{D}^{-1}\mathbf{C})\ . \qquad (21)$$

This control law synchronizes dynamics of the CoM of multibody robots with SLIP model.

IV. Simulation Results and Discussion

We use Matlab command ode45 to solve the dynamic equations 5 numerically. Absolute and relative tolerances are set 1×10^{-9} and maximum step size 0.002 *s*. To test dynamic synchronization the two systems should have the same initial condition. To find an appropriate initial condition, we use dynamic model of a complete SLIP running step including stance phase and flight phase, and then find a periodic orbit for this system. According to dimensions of the kneed robot model we choose the free length of the spring of SLIP model 69 *cm*, the spring stiffness 100 *kN/m* and the point mass 80 *kg*. Since length of the spring at the beginning of stance phase is its free length, state of the robot at the beginning of each stance phase is defined by $[\alpha, \dot{l}, \dot{\alpha}]$ which is used as Poincare section. Poincare map relates state of the robot at the beginning of each step to the beginning of the next step.

$$x(k+1) = P\big(x(k)\big) \qquad (22)$$

Any fixed point of this map defines an initial condition of a periodic orbit or a limit cycle for SLIP running. At least one fixed point can be found for any running speed. We choose the running speed of 10 *m/s* that is roughly the maximum speed of human running. Having defined the initial state of the SLIP model, we calculate an initial condition for the kneed model so that its COM has the same state as SLIP model. Stance phase dynamic equations 3 or 7 are solved with theses initial condition and the control law 22. The moment of ground reaction force approaching zero defines the take-off event and the end of the stance phase.

A. Robot Model with Parallel Compliant Mechanism

Simulation results for the kneed biped model with parallel compliant mechanism show that its CoM dynamics is totally synchronized with SLIP model and the two systems undergo the same trajectory starting from the same initial condition. Figure 3a shows CoM trajectories of the two systems during on complete running step. The thick line shows CoM trajectory of the kneed robot and the thin line show SLIP model. Dashed lines show touch-down and take-off angles which show also simultaneous take-off events for the two systems. Although no control action has been applied for the flight phase, the two systems have the same trajectories in flight phase as well. This is because they are projectiles with the same initial condition undergoing ballistic motions. The end of flight phase is not simultaneous for the two systems that is due to zero motor torques during flight and unadjusted attack angle for the kneed biped model. Stick diagram of the robot in stance phase (from touch-down to take-off) has been shown in figure 3b which reveals a reasonable and natural looking motion.

Motor torques exerted to rotors and motor powers during one stance phase are depicted in figures 4a and 4b which show big values so that providing them for a human scale biped robot using available electric motors is very hard. For example, a Toyota Brushless AC motor with peak power output of 50 *kW* weighs 36.3 *kg* [27]. In these figures the maximum torque and power are needed by motor 2 which is located in stance knee. Although human has the ability to run with the speed of 10 m/s, it is almost impossible for a human scale biped robot to run with this speed using available electric motors mounted on the robot. Motor torques for this biped model with no rotor inertias are shown in figure 4c that have maximum value of about %80 lower than the case with inertias. This proves that the existence of rotor inertia is a big challenge to use electric motors as the actuating system of running biped robots.

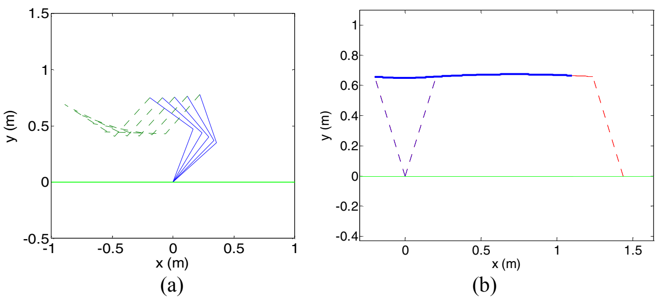


Fig. 3- (a) CoM trajectories for the SLIP model and the kneed robot with springs parallel to motors (b) Stick diagram of the kneed robot with parallel compliant mechanism in stance phase

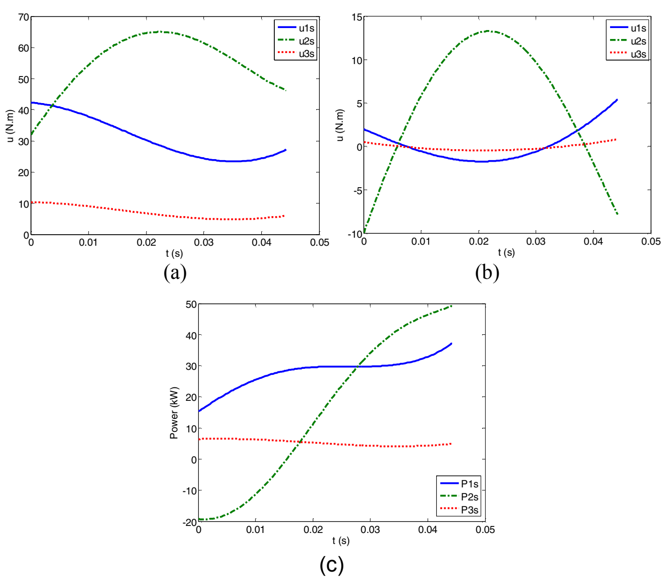


Fig. 4- (a) Motor torques in stance phase for the kneed robot with parallel compliant mechanism and with rotor inertia (b) Motor torques in stance phase for the kneed robot with parallel compliant mechanism and without rotor inertia (c) Motor powers in stance phase for the kneed robot with parallel compliant mechanism and with rotor inertia

B. *Robot Model with No Springs*

Dynamics of the biped model with no springs is the same as the biped model with parallel compliant mechanism. The only difference is that there is no spring to save and retrieve energy, and the motors have to supply the entire needed torques. Dynamics of this model is synchronized with SLIP model by control law (21), as well. Trajectory of the robot CoM coincides with SLIP model similar to figure 3a. Motor torques for this model with and without rotor inertias are shown in figures 5a and 5b, respectively. Maximum torque for the biped model with rotor inertia is 74 Nm and without rotor inertia is 27 Nm, which shows again that rotor inertia plays a very important role in the needed motor torques in bipedal running. It is observable from figures 4a and 5a that the peak torque for the robot without springs is %11 higher than the robot with springs which shows the positive effect of the springs in running efficiency. Furthermore, by optimizing springs stiffness the energy efficiency of compliant biped robot can be improved more.

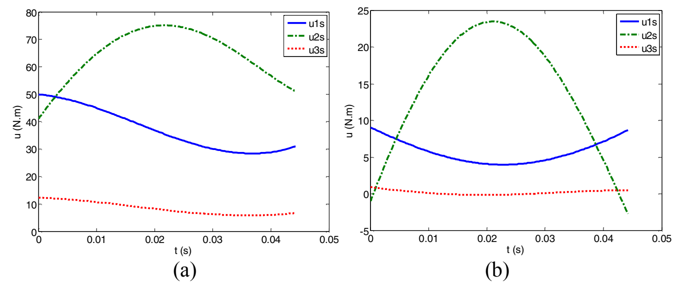


Fig. 5- (a) Motor torques in stance phase for the kneed robot with no springs and with rotor inertia (b) Motor torques in stance phase for the kneed robot with no springs and without rotor inertia

C. *Robot Model with Series Compliant Mechanism*

Simulations show that the control law 22 for the robot model with series compliant mechanism leads to zero moments and the dynamic synchronization does not take place. According to the dynamic modeling section we know that the generalized coordinate $\mathbf{q}$ for this model is 7 dimensional whose components 1 to 4 are relevant to links angles and components 5 to 7 are relevant to motors angles. From equation 16 we have $\boldsymbol{g}_\rho = \frac{\partial \rho}{\partial \mathbf{q}} \mathbf{D}_s^{-1} \mathbf{B}_s$. Since variation of motors angles does not affect the robot CoM, $\frac{\partial \rho}{\partial \mathbf{q}}$ is a row vector with components 5 to 7 equal to zero. Also according to equation 8 rows 1 to 4 of matrix $\mathbf{B}$ are zero. So, $\boldsymbol{g}_\rho$ and in a similar way $\boldsymbol{g}_\gamma$ become a zero row vector. Then according to equation 19, $\mathbf{B}'$ becomes a zero matrix and equations 21 and 22 become invalid. Therefore for a biped model with springs series to motors it is impossible to synchronize its CoM dynamics with SLIP model.

By observing this result we tried to control this robot model using the method proposed in [20]. In this method a controller is designed using feedback linearization to have CoM of the robot track SLIP trajectory. It is observed that the same zero matrix $\mathbf{B}'$ appears in this method, as well, making the control law invalid. So, we have a serious challenge to design a SLIP based controller for biped robots with series compliant mechanism. One solution for this problem is to track SLIP toe force profile by the main robot. We have applied this method in simulation for running control of ATRIAS robot, successfully [26].

V. Conclusions

In this paper, a control law was derived for dynamic synchronization of the CoM of multibody robots with SLIP model in stance phase. This control law does not guarantee stability of the whole running gait but it shows basic properties of SLIP based controllers for biped robots. The control law was applied to three biped models: with parallel compliant mechanism, with series compliant mechanism and with no springs. These models are theoretical and relatively simple models but they have the basic elements of real biped robots models. Simulation results showed that dynamics of the robot with parallel compliant mechanism and the robot with no springs are synchronized well with SLIP model. It was proved that dynamics of a robot with series compliant mechanism cannot be synchronized with SLIP model. Parallel compliant mechanism adds a helpful torque to motor torques and improves energy efficiency of running but it allows touch-down impacts to be transferred to the motors and gearbox and likely to damage them. Series compliant mechanism isolates impacts from the motors and gearbox but it adds extra degrees of freedom to the system and makes the controller design difficult. Also simulation results showed that the rotor inertia contributes considerably to the required motor torque and makes a major limitation for the use of electric motors in running biped robots. It is possible to reduce rotor inertia effect on the robot dynamics by reducing gear reduction ratio, but it reduces the output torque as well. We took these values from ATRIAS human size real robot and using specifications of a typical AC motor, it was shown that it is impossible for a human scale biped robot to run with human running high speeds using available electric motors mounted on the robot.

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