A MATHEMATICAL MODEL TO SIMULATE SMALL BOAT BEHAVIOUR

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Submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy under the conditions for the award of higher degrees of the Council for National Academic Awards.

October 1990

BOURNEMOUTH POLYTECHNIC

In collaboration with:

CETREK, (MARINEX INDUSTRIES LIMITED), POOLE, DORSET

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Acknowledgements

The author wishes to express his thanks and gratitude to a number of people who have given of their time and resources to support the research of this thesis.

To Mr Alan Potton, Director of Studies, and Mr John Reynolds, Project Supervisor, both of the Dorset Institute, for their aid in academic matters relating to the presentation and content of the thesis.

To Dr Walter Randolph, Technical Director at Cetrek, and Mr Peter Thomasson, of Cranfield College of Aeronautics, both of whom are Project Supervisors, for their invaluable contributions and suggestions throughout the research.

To the Directors at Cetrek for their financial support.

To Mr F.D.Hudson, Chief Technical Officer, and Mr Stuart Welford, Research and Development Manager, both of the Headquarters of the Royal National Lifeboat Institution in Poole, for their generosity and aid in recording sea trials.

To Mr David Winwood, Technical Manager at Cetrek for his aid in matters concerning the autopilot.

To Mr Sa'ad Medhat, Manager of the Electronics Industrial Unit at the Dorset Institute, for the use of computer equipment and work space.

The author's greatest thanks go, of course, to his parents, parents-in-law and especially his wife Alison, for their support, encouragement and understanding especially during the writing up of the thesis.

Abstract

A Mathematical Model To Simulate Small Boat Behaviour

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The use of mathematical models and associated computer simulation is a well established technique for predicting the behaviour of large marine vessels. For a variety of reasons, mainly related to effects of scale, existing models are unable to adequately predict the manoeuvring characteristics of smaller vessels. The accuracy with which the performance of a boat under autopilot control can be predicted leaves much to be desired. The thesis provides a mathematical model to simulate small boat behaviour and so can assist with the design and testing of marine autopilots.

The boat model is presented in six degrees-of-freedom, which, with suitable wave disturbance terms, allows motions such as broaching to be analysed. Instabilities in the performance of an autopilot arising from such sea induced yaw motions can be assessed with a view to improving the control algorithms and methodology.

The traditional "regressional" style models used for large ships are not suitable for a small boat model since there exist numerous small boat types and diverse hull shapes. Instead, a modular approach has been adopted where individual forces and moments are categorised in separate sections of the model. This approach is still in its infancy in the field of marine simulation. The modular concept demands a clearer understanding of the physical hydrodynamic processes involved in the boat system, and the formulation of equations which do not rely solely upon approximations to, or multiple regression of, data from sea trials. Although many hydrodynamic coefficients have been introduced into the model, a multi-variable Taylor series expansion of the states about some equilibrium condition has been avoided, since this would infer an approximation to have been made, and the higher order terms rapidly become abstract in their nature and difficult to relate to the real world.

The research rectifies the glaring omission of a small boat mathematical model, the framework of which could be expanded to encompass other marine vehicles. Additional forces and moments can be appended to the model in new modules, or existing modules modified to suit new applications. Much more work, covering a greater range and fidelity, is required in order to provide equations which accurately describe the true physical situation.

CHAPTER 1

INTRODUCTION

1.01 The Thesis

With the ever expanding small boat market and specialised interest in racing boats coupled with the increase in the navigation and marine control electronics industry which supports this market, there is now a need for a suitable mathematical model capable of simulating small boat (those with a length of less than 30 metres) manoeuvres. The requirement for a facility to assess the performance of autopilots used to guide small boats has been identified since the accuracy of prediction so far leaves much to be desired.

Due mainly to the effects of scale and to simplifying assumptions incorporated into large ship models, these prove incapable of accurately simulating small boat manoeuvres, especially in a seaway. A literature search revealed that whilst there is a wealth of large ship information, there is a disturbing absence of small boat publications and trials data. At present virtually all large ship simulators are restricted to the horizontal motions of surge, sway and yaw, though there are a few instances where roll motions are considered in tight turns to form a four degrees-of-freedom model. Without the coupled effects of pitch and roll in a seaway it is not possible to study effects such as broaching which can cause the autopilot great problems. It has been subjectively suggested by boat owners that the performance of autopilots diminishes in following seas and theoretically it is possible, under conditions of synchronism, that the rudder commands from the autopilot can become out of phase with the position that the rudder should attain to reduce the yaw created.

No large ship simulators to date are capable of predicting such sea induced motions. Some simulators, where complex graphics and motion platforms exist, incorporate sea conditions as cues to the mariner only. Most ship simulators include tidal effects, but otherwise tend to concentrate on shallow water effects for berthing manoeuvres within port limits. Since the draft of small boats is far smaller and the turning ability much greater than that of large ships, these effects are of much less importance to a small boat model. Instead it is the motion in and induced by a seaway that requires analysis and inclusion in a small boat model, especially as the final simulation is to be used under autopilot control.

Large ship simulator models are based upon a multi-variable Taylor series expansion of the forces and moments about some initial equilibrium condition. Such an approach is deemed unacceptable for small boat modelling for the following reasons:

1) Deviations from the equilibrium condition introduce inaccuracies or require complete re-computation of the model;

2) Theoretically an infinite number of terms will be generated, although in practice higher order terms will be discarded;

3) Relating the coefficients produced to the physical characteristics of the vessel becomes extremely abstract in nature for most terms above the second order;

4) Assessing or evaluating the coefficients is far from easy since many of the terms are difficult to isolate during tests. Instead multiple regression techniques are often employed on ship trials data. This provides a model which fits the data well, but will the parameters take on their true values?

5) It is difficult to apply this method on a general basis to small craft where there is an enormous number of different hull shapes and types. Unlike the large ship situation where models are often designed for specific ship types and since most ships tend to have high block coefficients at their midships section.

6) Theoretical and empirical formulae based on experimental tests can usually provide sufficient accuracy, as demonstrated by Japanese researchers (Ref.75).

Instead of the Taylor series expansion, a more reasoned approach will be adopted in an attempt to provide equations which contain terms which have conceptual meanings.

Since all three rotations of roll, pitch and yaw are to be considered, it is necessary to determine the attitude of the boat at any given time interval. This thesis draws upon techniques utilised in aircraft simulators to monitor the Euler angles based upon the boat's angular velocities. The particular method can also provide time savings as well as remove the possibility of a singularity.

With sufficient data pertaining to the righting moments of a boat, in particular the locus of the centre of buoyancy, there is no reason why this mathematical model cannot be used, if so desired, to assess the capsizing of a boat. This is particularly relevant since the boat used for validation purposes happens to be an Arun class Royal National Lifeboat. For any other type of boat, or ship, additions may have to be made in order to account for flooding of decks et cetera.

This research will help rectify the glaring omission of a small boat mathematical model, the framework of which could be used to provide ship models with more meaningful and accurate equations. The modular approach will enable additional forces and moments due to, for example, thrusters on tugs, to be easily incorporated within the model without the need to recompute all the other modules. The performance of small boat autopilots can be assessed at the development stage and improvements made to counteract instabilities such as broaching.

The novelty and originality of this project lies in the following facts:

1) There is no established model capable of accurately simulating small boat manoeuvres;

2) Virtually all present ship models assume calm water conditions; the remainder consider only wave drifting and tidal effects. Since small boats are much more affected by sea conditions than large vessels, much finer detail on wave forces and moments is required for small boat modelling;

3) Of all the ship models based upon traditional methods none include equations for all six degrees-of-freedom and most are limited to four degrees-of-freedom. Whilst the horizontal motions of a ship, plus the effects of roll in tight turns, are sufficient for large vessel simulation, pitch must be included in order to model wave induced motions like broaching and trim due to the use of trim tabs;

4) The boat model will be used as a tool to highlight areas of poor autopilot performance. Improvements made to the autopilot control system can then be re-assessed at the development stage.

It is worth noting that many of the additions proposed for the small boat model will remain valid for the large ship counterparts and can provide additional accuracy to such models. The inclusion of heave, for instance, could aid the study of squat on encountering shallow water and sinkage on entering less saline water. The framework of the model is designed to be of a flexible nature so that upgrades, additions or alterations can be easily made by tackling the individual module concerned rather than the entire model. This sort of approach is still in its infancy in the field of marine simulation.

CHAPTER 2

LITERATURE REVIEW

2.01 Historical Background

The origins of ships and the science of sailing was first founded by the ancient Egyptian civilisation who built boats for the purposes of trade and travel up and down the Nile river. The variety of craft at this early stage was based upon the owner's requirements and little thought was given to their stability and handling.

The Phoenicians advanced the art of shipbuilding as they explored beyond the horizon to fetch cargoes back to Tyre and Sidon. By Roman times ships were designed with holds to store cargo capacities of 250 tons and ship length increased to 30 metres. Different designs emerged from other parts of the world dependent on the local civilisation and sea conditions. The Vikings, for instance, who were perpetually at war with the North Sea, built their beamy longships.

Through the centuries ships, with their sails and rigs, evolved. As the European countries expanded their empires and established long distance trade routes spanning the globe, so bigger and faster ships were built. It became possible to furnish ships with weighty cannons and navies wrested for sea-power.

Technology marched on and by the end of the nineteenth century wooden sailing ships began to give way to iron and steel hulls and steam engines. This new era in ship history provided greater cargo capacities and required less manning. With the dramatic increase in ship length that the strength of steel allowed, was born the quest for an understanding and eventual prediction of the characteristics, manoeuvrability and design criteria of ships.

2.02 The Advent Of The Digital Computer

The introduction of the computer, with its insatiable appetite for processing large quantities of numerical data, allowed theoretical equations of ship dynamics to be implemented in prediction models. The possibilities that the computer opened up created the need to model ship motions. Initially entire mainframes were given over to providing ship simulators used almost entirely for training purposes. As computing power became compressed into desktop units and work stations, so mathematical models grew in complexity and their role changed to include research work.

The Japanese used computers extensively to optimise the performance of onboard control systems. Their "efficient ship" programme launched, in August 1980, the 1600 tons "Shinaitoku Maru" with, among other things, its hinged, rigid, computer controlled sail.

The scope and usage of the computer for marine training and research has expanded in all directions. Many countries and institutions are currently involved in a variety of different projects incorporating ship simulation and automatic guidance control. The following literature review is aimed at giving the reader a taste of some of the establishments and work ongoing in the marine field. The author apologises for the brevity of the description given for many of the texts reviewed, but, as will soon be appreciated, the number of institutions and areas of research are extensive.

2.03 The Literature

There exist numerous maritime research establishments and societies in many countries of the world, especially those steeped in marine history. A great wealth of books, technical papers, transcripts and so on provide hydrodynamists and those involved in ship simulation with a vast quantity and variety of reference material. However, virtually all of this information is concerned with large ocean-going vessels. Literature searches conducted at the start of this project showed a disturbing lack of published data on small boat modelling, simulation and parameter measurement.

The literature within the marine simulation field covers all aspects of the ship system. Some papers present mathematical models to predict ship manoeuvres in the horizontal plane, incorporating the motions of surge, sway and yaw. Later texts have also included effects of roll, whilst some research has investigated the combination of roll and pitch or roll and heave motions as separate entities. Other material tackles and concentrates on a specific section of the ship system, for example the characteristics of the rudder, propeller, stabilising fins or other appendages.

The dynamics of a variety of ship hull types is well documented and research into the effects of wind and wave drifting forces and moments also appear in publication. A few technical papers describe a "one off" or specific type of marine vehicle, such as hydrofoils, ROVs (Remotely Operated underwater Vehicles) and oil rigs. From the results, each model would seem to satisfactorily suit its application.

Another area of research, particularly in the Netherlands and Scandinavian countries, is the investigation into improved automatic control of ships, especially roll reduction on warships by use of the rudder and fins.

The majority of papers which describe the manoeuvrability of large ships are limited to the three degrees-of-freedom in the horizontal plane, namely: surge, sway and yaw. These provide adequate models in open, calm water since large vessels are little affected by small seas. Later extensions to such models has meant the inclusion of roll motions. The primary reason for this is the ability to study high speed container carriers, roll-on/roll-off ships et cetera which exhibit large angles of heel during turns.

The major application for ship mathematical models is for incorporation within some form of simulator. The requirements of simulators fall under the two headings of training and research.

Training purposes include: shipboard training for masters tickets et cetera, ship handling appreciation and familiarisation, passage planning and bridge team work, practising port approach manoeuvres and system failure procedures.

Research purposes include: determining changes or additions to the collision avoidance rules, assessing the effectiveness of existing or proposed vessel traffic systems or traffic routing schemes, marine law and policy research including allocation of blame in marine catastrophes, performing ship manoeuvrability studies at the initial design stage prior to commencing construction, analysing the behaviour of ships in canals and other confined waterways, in conjunction with fluid flow models to assess changes to port design and layout before dredging, and human factors research.

Simulators such as those at the University of Wales Institute of Science and Technology and at Plymouth Polytechnic provide complete bridge layout facilities for use in both training courses and research projects. The visual displays and bridge instruments respond to the computed ship motion giving fully interactive systems.

Additional uses of mathematical models of ships include adaptive, model reference, control algorithms which are designed to either achieve accurate course-keeping when, for example, within port approaches, or to optimise fuel usage.

2.04 The Initiators Of Ship Modelling

Lamb, 1879 (Ref.86), is regarded as the "classical" text on hydrodynamics and as this field has widened in its practical application, so Lamb has revised and extended his book a number of times. In the sixth edition it is chapter six which deals specifically with the motion of solids through a liquid. Here the theory treats the solids and the fluid as forming a single dynamical system, thus avoiding the troublesome calculation of the effect of the fluid pressures on the surfaces of the solids.

Lamb expresses the motion of the fluid as a velocity potential, ϕ, which by adopting Euler's axes system, and after Kirchoff, can be written:

$$\phi = u\phi_1 + v\phi_2 + w\phi_3 + p\chi_1 + q\chi_2 + r\chi_3$$

where u, v, w are the translational velocities, p, q, r are the rotational velocities and ϕ_1, ϕ_2, ϕ_3, χ_1, χ_2, χ_3 are functions determined by the surface of the solid. The mathematical language of dynamics is not always easy to follow, but Lamb's development of the equations of motion of solids is reflected in many subsequent texts.

Lockwood-Taylor, 1930 (Ref.93), examines the question of "virtual inertia" of a body immersed in fluid. The paper is sub-divided into four parts. Part one gives solutions for motion in two dimensions due to the translation of cylinders. Cylinders with a variety of cross-sections are presented, giving the inertia coefficients for each. Part two considers the free surface condition for the case of horizontal vibration. Part three deals with the effect of rigid boundaries, for example, shallow water or canals. Finally part four considers motion in three-dimensions and of compressibility of the fluid as applied to the particular case of a circular cylinder.

The equations are developed in a general form and can be readily applied to ship motions where the hull is approximated by a cylinder with an appropriate cross-section. The paper is perhaps a little removed from a practical application, but serves to demonstrate the functionality of equations for regular geometrical shapes, such as ellipsoids.

Weinblum & St Denis, 1950 (Ref.137), provide in their introduction a good outlined premise for research into ship motion prediction. The approach they adopt is essentially analytic, but making reference to experimental and empirical results when these are available. With the exception of roll, the equations are based upon second order linear differential equations and coupled motions are not treated. The basis of the equations of motion are remarked upon and a note on the expression to be used to define hull geometry is given.

The inertia forces are tackled first, and for a vessel moving in calm water are presented thus:

$$F_s = \rho\nabla \frac{d^2s}{dt^2} + m_s \frac{d^2s}{dt^2}$$

$$M_s = I_s \frac{d^2s}{dt^2} + I_{ss} \frac{d^2s}{dt^2}$$

where the first term is due to the ship and the second term due to the surrounding water. The symbols m_s and I_{ss} are the added mass and added moment of inertia. Since exact mathematical solutions for the ships moving in unbounded fluid are not available, the general ellipsoid is considered in its stead. Additionally free-surface effects are considered.

The damping forces are the topic of the second section. Up until the time of this document the only damping motion which had been seriously investigated was that of rolling. This paper considers also heave, pitch and pounding or slamming in addition to roll. It is noted that damping motion of surge, sway and yaw, because of their lesser importance, have hardly been investigated.

A short section on the restoring forces of heave, pitch and roll is presented before examining the concept of the seaway. Assumptions of regular wave trains are discussed and in addition to wave period, length and velocity, observations on wave height or steepness and wave profile are made. A number of pages are given to the exciting forces in all six degrees of freedom. Then additional sections deal with secondary effects, free oscillations, forced oscillations and the stabilisation of motions. The text is very readable and gives a complete overview of the motion of ships at sea.

Weinblum, 1952 (Ref.138), presented the original paper in German at Hamburg and this reference is the English version as a DTMB report. Prior to this report, the subject of hydrodynamic mass (or added mass) had been somewhat neglected and this study gives a review of the extent of the understanding of this topic at that point in time. The paper begins with the assumption of a "Kelvin flow field" that the fluid is ideal and extends infinitely in all directions. Much of Lamb is recapitulated and the notation of a hydrodynamic mass tensor is used. In section two on free surface, the addition of boundary conditions are recounted when the fluid cannot be assumed to extend to infinity. Vertical translations, horizontal translations and oscillations, such as roll, of a body floating in the fluid are considered. Finally the influence of viscosity is mentioned briefly. The report concludes that only few solutions for some simple kinds of motion are known.

Other earlier texts include Milne-Thomson, 1955 (Ref.109), which can be regarded as a parallel text to Lamb. John, 1949 (Refs.78 & 79), who presents two papers on the motion of floating bodies. However, as demonstrated by the periodical they appeared in, the mathematics is pure and often beyond the ability of many. Peters & Stoker, 1957 (Ref.119) who develop mathematical theories for three basic hull forms. Kaplan, 1966 (Ref.81), who considers the problem of non-linear ship rolling motion in a random seaway.

Abkowitz, 1964 (Ref.2), forms a firm foundation for the study of ship hydrodynamics, steering and manoeuvrability. The forces and moments acting on a body are presented as functions of the properties of the body, the properties of the motion and the properties of the fluid. Although the concept of six degrees of freedom is discussed, only the three horizontal motions of surge, sway and yaw are presented. The equations are designed to describe the "shape" of a ship and use a dimensionalising (or scaling) term which is a function of the length, breadth, draft and block coefficient of the vessel.

A Taylor series expansion of a function of several variables about a chosen initial equilibrium condition is used to express the forces and moments. A set of terms known as the hydrodynamic coefficients results, many of which, especially the dynamic response terms of second order smallness, are sufficiently small that they are either assumed zero or neglected. The number of terms in the expansion determines the accuracy and by limiting it to the first order terms, the well known linearised expansion is obtained. Strom-Tejsen, 1965 (Ref.127) shows that the linearised equation of motion for surge using straight ahead motion at constant speed with rudder amidships as the equilibrium condition can be written:

$$X = X_{*} + X_{u}\Delta u + X_{v}v + X_{r}r + X_{\dot{u}}\dot{u} + X_{\dot{v}}\dot{v} + X_{\dot{r}}\dot{r} + X_{\delta}\delta$$

Linear or Quasi-Non-Linear models perform adequately for small perturbations from the equilibrium state, but deviate from the true ship motion when larger variations occur. In order to overcome the limitations of the linear model when performing manoeuvres such as tight turns with large angles of rudder (typically greater than 10°), it is necessary to include higher order terms from the Taylor series. The non-linear surge equation, including terms of third order, is of the form:

$$
\begin{aligned}
X = X_* &+ [X_u \Delta u + X_v v + X_r r + X_{\dot{u}} \dot{u} + X_{\dot{v}} \dot{v} + X_{\dot{r}} \dot{r} + X_\delta \delta] \\
&+ \frac{1}{2!} [X_{uu} \Delta u^2 + X_{vv} v^2 + \ \ldots\ + X_{\delta\delta} \delta^2 + \\
&\qquad 2X_{uv} \Delta uv + 2X_{ur} \Delta ur + \ldots + 2X_{\dot{r}\delta} \dot{r}\delta] \\
&+ \frac{1}{3!} [X_{uuu} \Delta u^3 + X_{vvv} v^3 + \ldots + X_{\delta\delta\delta} \delta^3 + \\
&\qquad 3X_{uuv} \Delta u^2 v + 3X_{uur} \Delta u^2 r + \ldots + 3X_{\dot{r}\delta\delta} \dot{r}\delta^2 + \\
&\qquad 6X_{uvr} \Delta uvr + 6X_{uv\dot{u}} \Delta uv\dot{u} + \ldots + 6X_{\dot{v}\dot{r}\delta} \dot{v}\dot{r}\delta]
\end{aligned}
$$

where the dots indicate similar terms in functions of u, v, r and δ.

Although, as Abkowitz demonstrates, many of these terms can be neglected, due to symmetry about the xz-plane et cetera, it soon becomes obvious that producing a Taylor series expansion leads to an enormous number of coefficients which are neither easy to relate to the physical characteristics of the ship nor isolate for evaluation purposes when conducting model tests.

Before the powerful digital microcomputers had fully established themselves, research work by Bech & Wagner-Smitt, (Ref.23) 1969, used analogue simulation to model ship manoeuvres in response to rudder action and external disturbances. This provided a useful tool to study ship manoeuvrability and autopilot development.

2.05 UK Establishments

BMT (British Maritime Technology) at Teddington is among the leading UK centres for mathematical modelling of ship manoeuvres for use in simulators. BMT originated as the Ship Division of the NPL (National Physical Laboratory) and is most often recalled for Barnes Wallis' bouncing bomb experiments. Early in the 1970's the NPL Ship Division moved into the area of ship manoeuvring simulation with the express desire to produce a mathematical model capable of representing a wide range of ship types. The NPL Ship Division later became NMI (National Maritime Institute) and subsequently, due to privatisation, NMI Ltd. in October 1982. A final name change to BMT occurred after 1984.

The simulation models developed were designed to utilise the experimental facilities which exist at BMT, thus assuring that model parameters can be extracted from tank test measurements. Lewison, 1973 (Ref.91), formulated the initial ship manoeuvring model which, although taking account of speed loss in turns and non-linearities in the motions, only allowed for conditions where the ship has a forward speed with its propeller in the ahead regime and was restricted to comparatively small drift angles ($\pm 20^\circ$).

Gill, 1976 (Refs.60 & 61), suggested additions to the model to remedy the deficiencies. By 1980 the so-called "cruising speed" model was emended to the point where only the limitation of ±20° drift angle existed. Until at least 1984 these equations were used in all of the major UK simulators.

Coupled with the success of this model came a demand for simulations capable of predicting berthing and other low speed manoeuvres where no drift angle limitations could be accepted. A further model named the "low-speed" model was developed by Barratt, 1981 (Ref.22). The forces and moments had to be non-dimensionalised in a different manner from the previous models in order to avoid the low speed instabilities at zero speed. Three additional algorithms were added to the model and as a result shallow water effects, ship-to-ship attractions, wave induced drift and forces from tugs and mooring lines could be incorporated in simulations.

Due to the troublesome nature of switching between the low speed and cruising speed models when going between port and sea, research at BMT since 1982 has centred on the requirement for a single modular model where the rudder, propeller and hull each form their own separate model rather than being part of an overall global model. The approach allowed the large amount of hull and propeller data already collected at BMT to be exploited and catered for both cruising speeds and large drift angle manoeuvring regimes.

Other work, in conjunction with the College of Aeronautics at Cranfield and initiated by the UK Department of Energy (Ref.89), has branched into the realm of ROV's (Remotely Operated underwater Vehicles) with their increased use in the offshore industry for surveying of the seabed. Here all six degrees of freedom will be required with modules for umbilical hydrodynamics which, on long umbilicals, can cause performance inhibiting drag. A more detailed history of BMT can be gleaned from Dand & Reynolds, 1984 (Ref.42).

More recently, Khattab, 1987 (Ref.82), presented the latest developments to the BMT model. A real time simulation of ship handling in harbours is implemented on a hybrid computer and provides a facility to investigate manoeuvring capabilities of ships in a given harbour configuration and under specified environmental conditions. The model consists of a set of modules which allow changes in vessel type and harbour layout to be implemented. A good agreement between estimation data and real data is shown and the simulation of M.V.Belard in Ardrossan harbour is demonstrated.

BSRA (British Ship Research Association) at Wallsend, Tyne and Wear (which recently became part of BMT) is another long established UK centre for research into all aspects of ship design, materials and manoeuvring models. Technical reports produced at BSRA range from studies of marine fouling organisms through the welding of higher tensile shipbuilding steels to simulation studies of autopilot performance. The spectrum of work undertaken covers virtually all components of the ship system and is closely allied to the once great shipyards of the north-east of England.

Clarke, who has been working at BSRA on problems associated with ship dynamics, posed the question, 1982 (Ref.39), "Do autopilots save fuel?". This paper points out that whilst automatic course control of ships has been possible for many years, the availability of small, powerful microcomputers has given rise to a new class of adaptive autopilots. Their principal feature is the optimisation of a cost function which can be used to minimise fuel consumption, time of the voyage, speed losses or rudder wear.

Based on observations of Nomoto and Motoyama about the magnitude of drag forces on the ship due to disturbances and rudder usage, and utilising a performance indices proposed by Koyama and Norrbin, Clarke developed a series of cost functions of the form:

$$F = a\bar{\psi}^2 + b\bar{r}^2 + c\bar{\delta}^2$$

where: $\bar{\psi}^2$, $\bar{r}^2$, $\bar{\delta}^2$ are the mean square heading error, mean square rate of turn and mean square rudder angle respectively. The desire for more efficient autopilot systems for ocean-going ships, has meant a shift away from the classical PID (Proportional Integral Derivative) controllers to adaptive optimal controllers using state space techniques. Clarke concludes that properly controlled tests need to be performed in order to assess the effectiveness of such controllers. Unfortunately, there are very few published results of trials which detail ship speed and fuel consumption figures.

In the same year Clarke, Gedling & Hine, 1982 (Ref.40), produced three criteria concerned with turning and course changing ability, dynamic stability and course keeping ability, and manual steering ability. The purpose for such work stems from the reason that although resolutions adopted by the IMO (International Marine Organisation) recommend that each ship over 50000 dwt should carry information about its manoeuvring particulars, there are still no manoeuvrability standards for ships.

The criteria were developed in terms of linear theory, based on the linearised equations of motion given as:

$$(X_{\dot{u}}-m)\dot{u} + X_u\Delta u = 0$$
$$(Y_{\dot{v}}-m)\dot{v} + Y_v v + (Y_{\dot{r}}-mx_g)\dot{r} + (Y_r-mu_0)r = 0$$
$$(N_{\dot{v}}-mx_g)\dot{v} + N_v v + (N_{\dot{r}}-I_z)\dot{r} + (N_r-mx_g u_0)r = 0$$

Ignoring the surge since it has no effect on the transverse motion of the ship and adding the rudder terms, the dimensionless form of the equations, obtained by dividing the sway forces by $\frac{1}{2}\rho u_0^2 L^2$ and the yaw moments by $\frac{1}{2}\rho u_0^2 L^3$, are:

$$(Y_{\dot{v}}'-m')\dot{v}' + Y_v'v' + (Y_{\dot{r}}'-m'x_g')\dot{r}' + (Y_r'-m')r' + Y_\delta'\delta = 0$$
$$(N_{\dot{v}}'-m'x_g')\dot{v}' + N_v'v' + (N_{\dot{r}}'-I_z')\dot{r}' + (N_r'-m'x_g')r' + N_\delta'\delta = 0$$

However, it is possible to reduce the number of variables required to describe the ship's behaviour and provide other advantages, by expressing the coefficients in terms of time constants and system gains. This approach was first used by Nomoto and yields a pair of decoupled second order equations:

$$T_1'T_2'\ddot{r}' + (T_1' + T_2')\dot{r}' + r' = K_r'\delta + K_r'T_3'\dot{\delta}'$$
$$T_1'T_2'\ddot{v}' + (T_1' + T_2')\dot{v}' + v' = K_v'\delta + K_v'T_4'\dot{\delta}'$$

The turning ability criterion was expressed in terms of a turning index, after Norrbin, which represents the heading change per unit rudder angle in one ship length travelled and is given by:

$$\frac{\psi(t)}{\delta} = K' \left[1 - (T_1'+T_2'-T_3') + \frac{(T_1'-T_3')}{(T_1'-T_2')} T_1' e^{-1/T_1'} + \frac{(T_2'-T_3')}{(T_1'-T_2')} T_2' e^{-1/T_2'} \right]$$

It is desirable for a ship to be dynamically stable, therefore, the dynamic stability criterion is satisfied if the time constants T_1' and T_2' are positive. Referring to Abkowitz (Ref.2), the condition for stability can also be written as:

$$Y_v' \, (N_r' - m'x_g') - N_v' \, (Y_r' - m') > 0$$

The third criterion, that of manual control, is not easily defined in terms of mathematics since it involves the helmsman's behaviour in the control loop. As a reasoned rule, the phase margin of the ship should be greater than -30° to allow satisfactory manual steering.

The paper also presents methods of determining values for the acceleration and velocity derivatives using strip theory, semi-empirical methods and multiple regression analysis. The criteria have been thoroughly researched and even if not implemented as standards, they have shown how ship manoeuvrability can be quantitatively assessed using simple linear theory.

Virtually the only six degrees of freedom model to simulate ships appearing in published literature is presented by Matthews, 1984 (Ref.101). This work, at Maritime Dynamics Limited of Llantrisant, describes a model based on identifying force terms as a foil in the fluid. Central to this formulation is the concept of drift angle since all non re-entrant moving bodies exhibit the phenomenon of progressive non-alignment. That is to say, when perturbed a body will depart from its original alignment. This particular methodology has allowed a relatively compact simulation model capable of performing in a wide range of manoeuvring regimes.

Work at UWIST (University of Wales Institute of Science and Technology) by McCallum, 1976 (Ref.102 & 103), has taken a direct approach to the problem of simulating manoeuvring ships, by considering the hull as a hydrofoil surface inclined at a draft angle α to the incoming stream of water. The three basic equations of motion in surge, sway and yaw are expressed as:

$$m_1\dot{u} = [\ \ L_H\sin\alpha - D_H\cos\alpha - L_R\sin\alpha_e - D_R\cos\alpha_e + T + m_2vr]$$

$$m_2\dot{v} = [\ -L_H\cos\alpha - D_H\sin\alpha + L_R\cos\alpha_e - D_R\sin\alpha_e + F_p - m_1ur]$$

$$I_z\dot{r} = [-d_1L_H\cos\alpha - d_1D_H\sin\alpha + d_2L_R\cos\alpha_e - d_2D_R\sin\alpha_e + d_3F_p - N_v]$$

where L_H is the hull hydrodynamic lift, L_R is the rudder hydrodynamic lift, D_H is the hull hydrodynamic drag, D_R is the rudder hydrodynamic drag, T is the propeller thrust and F_p is the propeller sideways force.

McCallum concludes that this model is capable of simulating the behaviour of a variety of ships operating in a wide range of regimes. Changes in operating conditions can be simulated by simple alterations to those parameters logically associated with the new conditions. However, the model is not intended to be fully rigorous in its approach. A number of empirical relationships have been used to overcome the complexities of the hydrodynamic behaviour around the stern of the ship, for instance. Furthermore, entirely accurate results cannot be expected when a wide range of operating conditions have been specified and the model suffers from weaknesses in the area of the dynamic relationship between the hull, rudder and propeller.

In 1979 Cardiff acquired Europe's first CGI (Computer Generated Imagery) simulator. It is operated jointly by UWIST and SGIHE (South Glamorgan Institute of Higher Education) and was the result of successful collaboration between the UK Government's Department of Industry. The design features and operational philosophy of CASSIM (CArdiff Ship SIMulator) are described by McCallum & Rawson, 1981 (Ref.104). The basic design consists of a visual scene, as observed from the bridge structure through one or more forward looking windows. Three projectors give 120° horizontal field of view and 30° in the vertical plane. This can be extended to a 200° horizontal view with the addition of two extra projectors.

The visual scene is fully interactive with bridge commands, so that engine or rudder changes are fed to the motion computer which contains the manoeuvring equations of a range of ships. Bridge instruments are similarly updated in response to the computed ship motions. The Controller is also able to alter the scene by introducing different visibility conditions or other ships.

The visual side of the simulator was developed by Marconi Radar Systems Ltd. under the auspices of the Tepigen trademark. The principal characteristics being:

a 625 line colour television, back projection onto a four metres radius cylindrical screen, 1000 "faces" or lights and marks, four additional instructor controlled pre-programmed ships and potentially unlimited area of data base. The significant features include: atmospheric scattering (land which is further away appears to fade), edge smoothing (so that sloping lines do not appear as a number of steps), sea texture (ripples appear to move in the direction of apparent water flow), distance effects (lights are given perspective so they get smaller and less intense at greater ranges), land texture (enables woodland and walls to be presented at close range) and data base preparation (additional charts, photographs, drawings et cetera of different ports can be added).

The bridge design is supplied by Racal Decca Systems and Simulators Limited. It measures four metres wide by five metres deep and is mounted on a large vibrating platform. Ship's officers rely on propeller induced vibrations as an important cue and all UK simulators are fitted with this feature. A set of instruments, similar to those found on most ship's bridges, includes: steering pedestal with autopilot and manual wheel, sixteen inch radar display, engine telegraph, intercoms, VHF radio, chart table, heading repeater, log, rate of turn indicator, RPM indicator, depth repeater, wind speed and direction, Decca navigator and ship's sounds.

McCallum, 1983 (Ref.105), discusses how operational criteria can influence ship simulator design. With simulators costing from £3K to over £3M it is important to select one to match the needs. The paper identifies ship simulator development trends and uses and applies the law of diminishing returns to simulator realism versus expense of complexity. An extremely important point made by McCallum about simulators is that:

> "... *adequate provision must be made for in life updates,* ..."

Continuing the simulator theme, McCallum, 1984 (Ref.106), presents a critical survey of three specific ship simulator mathematical manoeuvring models. Each is implemented on the CASSIM in a port evaluation study. Measures of performance were related in terms of three performance indices, namely the mariner, ship and port performance indices. The conclusion drawn was that for most simulation tasks it is quite feasible to use mathematical models which have been produced relatively cheaply. However, fine detailed close manoeuvres require higher fidelity models and simulation of smaller ships down heavy quartering seas are beyond the scope of any simulator in service today.

Joint work between the Royal Naval Engineering College at Manadon in Plymouth and UWIST by Fuzzard & Towill, 1982 (Ref.56), has been aimed at investigating the possibility of using PNS (Pseudo-Noise Sequences) injection and cross-correlation techniques to produce transfer functions to model non-linear ship dynamics about a given steady state condition. Where linearised small perturbations about a specific operating condition are sufficient, this method dispenses with the need for costly and often time consuming tank testing of physical models.

Other research between these two establishments has included the implementation of fuzzy sets to control algorithms, Sutton & Towill, 1985 (Ref.128 & 129). These papers provide a straight-forward introduction to fuzzy sets, discussing the concepts of "linguistic hedges", "fuzzy relations" and "composite rule of inference". The fuzzy controller, as developed, is used as an autopilot to control the non-linear yaw dynamics model of a Royal Navy frigate.

Dove, 1974 (Ref.45), surveyed the methods of pilotage and berthing, including statistics of collisions and groundings. The development of shipborne automatic control devices is suggested for the berthing of large vessels. A move from Southampton College of Technology to Plymouth Polytechnic (now known as Polytechnic South West), allowed these ideas to become reality. Burns, Dove, Bouncer & Stockel, 1985 (Ref.35), who form part of the Ship Dynamics and Control Research Group at Polytechnic South West, firstly developed a discrete, time-varying, non-linear mathematical model to simulate ship response to demanded rudder and engine speed plus wind and current. The second phase of the research, which relied on an accurate model, entailed the construction of a digital filter/estimator, for use with an optimal controller, capable of navigating large ships in port approaches (Refs.46 & 47).

The model is based on state space methods with eight system states and two deterministic inputs. It simulates the horizontal motions of surge, sway and yaw. The best estimate of each state is passed to an adaptive optimal controller to compute those inputs which minimise a given performance criterion. A further consideration of the work at Polytechnic South West, is that of integration of navigational data so that deficiencies in one navigational system can be offset be those in another by use of minimum variance or Kalman-Bucy filters.

Mikelis, 1983 (Ref.107), of Lloyd's Register of Shipping, observed from model experiments and full scale operations that a ship's handling behaviour changes when moving from deep to shallow water or into canals. A review of existing manoeuvring theories is presented and a simplified simulation model is developed. The aim of the work at Lloyd's Register of Shipping is to arrive at a method of predicting ship handling at the design stage. With this in mind it is desirable that such a formulation does not rely on data from model experiments.

For the type of manoeuvres being considered at Lloyd's Register the linearised equations appear acceptable and the acceleration coefficients can be adequately calculated from lines plans. However, parametric equations for the velocity coefficients proved less accurate and experimentally derived values had to be utilised. Where ship's propulsion characteristics are known at the design stage, they should be included in the model instead of empirical constants.

Mikelis, Clarke, Roberts & Jackson, 1985 (Refs.108 & 72), have assembled a mathematical model consisting of coupled equations of surge, sway, yaw, roll and propeller revolutions. The method employs two computer programs. The first is a pre-simulation routine which generates resistance, propulsion and hydrodynamic coefficients from ship geometry and other readily accessed data. The second program performs the simulation using the data made available by the first routine.

The simultaneous equations are solved using an IBM mainframe computer and provide up to 500 times faster than real time simulations. A simulator designed to study and analyse safety aspects during ship handling operations has also be implemented at Lloyd's Register using a VAX/11-780 and is known as the Multi-Ship Manoeuvring Simulator (MSMS). It is expected to have a wide future use as it provides radar view graphics, interactive input of rudder and engine commands, real-time or fast simulation and multi-ship simulation.

The approach to the mathematical model has been to use the classical Taylor series expansion to describe the hydrodynamic reactions on the hull, but the rudder and propeller forces and moments follow the Japanese treatment. The model was verified, as are many other ship models, by comparison to full scale manoeuvring tests carried out for the 278000 dwt tanker "Esso Osaka". Numerous applications are envisaged for the MSMS, especially in the field of maritime safety studies.

Broome, 1982 (Ref.34), has conducted a series of tests using computer simulation and radio controlled scale models to investigate the effect of ship autopilot tuning on course keeping efficiency. Fast Fourier transform techniques on the non PRBS (Pseudo Random Binary Sequences) yaw and rudder signals have been used to assess the dominant free body natural frequencies of the ship. A program has been written to perform ARMA (Auto-Regressive Moving Average) identifications of linear system mathematical models, based on least squares of maximum likelihood algorithms. The aim is to provide a self-tuning adaptive autopilot which has reference to the roll dynamics of the ship.

A suite of programs developed at Southampton University and the University of Newcastle Upon Tyne by Wellicome & Mirza, 1987 (Ref.139), use slender body theory to predict the course keeping and steady rate of turn of a ship. Slender body techniques have been successfully used in the past to determine ship response to waves. This paper shows a method for finding the forces and moments arising from a ship manoeuvring in calm water. The motive behind such work is to provide an inexpensive tool for predicting ship manoeuvring characteristics.

2.06 Scandinavian And European Research

The SSPA (Statens Skeppsprovningsanstalt or Swedish State Shipbuilding Experimental Tank) at Göteborg have over the past 35 years or so regularly published a wide range of texts on ship theory and research. Report number 68 by Norrbin, 1971 (Ref.112), provides an exceedingly useful description of mathematical modelling of ship manoeuvres in both deep and confined waters. The non-dimensionalising of hydrodynamic coefficients uses the so-called "Bis" system, which differs from the method used by Abkowitz et al. Topics discussed in this report include: the kinematics of fixed and moving systems, calculations of hull forces, modelling deep water horizontal manoeuvres, free water and confined water flow phenomena and model tests.

Norrbin, 1972 (Ref.113), also provides an introduction to ship manoeuvring with application to shipborne predictors and real-time simulators. Details of simulator models and man-machine interface are discussed. Records of helm manoeuvres on board large tankers in harbour approaches revealed the need for predictor assistance. The resulting simulator produces electronically generated symbols to be projected in a "predictor window" to show predicted path information by perspective line tracks.

As with many other establishments, researchers at SSPA have tackled the subject of system identification as applied to the determination of steering dynamics of ships. Byström & Källström, 1978 (Ref.36), evaluate full scale experiments using the identification program LISPID which contains the output error method, the maximum likelihood method and the prediction error method. Since free steering experiments may be performed both in model and full scale, the identification technique offers the added attraction of analysing the effects of scaling.

Also at SSPA Källström & Ottosson, 1982 (Ref.80), have carried out investigations into regulators to reduce roll motions by use of rudder and active fins. Some types of modern fast ships exhibit a severe tendency to heel significantly during turns. High super-structures and a relatively low density cargo produce small metacentric heights and consequently poor dynamical stability and great sensitivity for disturbances from wind and waves. A non-linear mathematical model for a ship moving in wind and irregular waves is developed and three differing regulators are presented. Results and a large portion of the mathematics are included in the paper.

Berg & Flobakk, 1979 (Ref.25), of the Norwegian Institute of Technology and Hydrodynamic Laboratories, firstly present a non-linear mathematical model of a ship, and then show methods to determine the coefficients in the manoeuvring equations. Since the

Ocean Environmental Basin was not available at the time of the research, coefficients had to be determined theoretically by choosing a suitable description of the forces acting on the ship. Planar motion mechanism tests are thus avoided. Initially a simulation study is made to establish a procedure for generating quasi-optimal rudder control signals for free-sailing tests in a towing tank.

Work at the Norwegian Marine Technology Research Institute (MARINTEK) by Martinussen & Linnerud, 1987 (Ref.100), has similar goals to those of Lloyd's Register. The aim being to provide a prediction simulation of the manoeuvring characteristics of ships at the design stage. A discussion on the model tests is given and the applicability of free running model tests as a prediction method is presented. The simulation gives sufficiently accurate results for hulls within the range of existing empirical data.

Bech & Chislett, 1980 (Ref.24), who are associated with the DSRL (Danish Ship Research Laboratory), statistically investigate the invariant coefficients of the ship's equation of response to steering. The aim being to provide an improved non-dimensionalisation of the transfer function of ship heading response to rudder action. Traditionally, constants have been non-dimensionalised with ship length to speed ratio, but by including Froude number, block coefficient, ship length to beam ratio, length to draft ratio, trim, rudder chord and propeller diameter, it is hoped that considerably better results will be obtained.

The DMI (Danish Maritime Institute) has been assessing the special berthing and navigational needs of cruise liners. The requirement for cabin space has led to large superstructures which suffer from windage problems. Tersløv, 1985 (Ref.130), describes the capabilities of a simulator developed at DMI which allows the skills of navigators and masters of these ships to be enhanced.

Research at the Control Laboratory in the Electrical Engineering Department of Delft University in the Netherlands over the past 20 years has been directed towards automatic steering control of ships. Much of the work has been in association with the Royal Netherlands Navy and the principal researchers are: van Amerongen, van den Bosch, Goeij, Hoogenraad, Keizer, van der Klugt, Leeuwen, Moraal, van Nauta Lemke, Ort, Postuma, Schouten and Verhage (Refs.5-13 & 29-30 & 88).

One of the earlier papers describes a method of accurately determining the speed of a ship during manoeuvres based on accurate position fixes and using automated Snellius techniques. Usual speed measuring devices are only capable of determining accurate results when running straight line courses and become unreliable when the ship undergoes a manoeuvre. The principle of the Snellius method is that given the two bearings between three known points, it is possible to derive the position of the ship. Sextants are used to provide the angular information. However, the known points must be selected so that the angle of the intersection of the two circles formed when determining position provide a good cut (that is, it is not a shallow angle).

The majority of the remainder of the papers concentrate on the design of an autopilot which uses the rudder not only for course keeping, but also for roll stabilisation, where stabilising fins have been used up to now. A simple mathematical model of a ship describing the transfer between the rudder and yaw motions was obtained from modelling experiments. The model was used within the design of the controller utilising the concepts of model reference adaptive systems and Kalman filtering. An additional computer aided design package PSI (Interactive Simulation Program) has been developed which provides an optimisation facility based upon a fast hill-climbing algorithm. This is capable of computing a "best-fit" model of a system by means of simulation and optimisation.

de Vries of Delft Hydraulics Laboratory, 1984 (Ref.135), has developed a special model testing technique for determination of manoeuvring coefficients, which is used in combination with straight line towing. The manoeuvring simulator uses equations composed of a number of terms with unknown coefficients and it is these coefficients which are determined by curve fitting of data from systematic model experiments. Two air propellers are fitted to model ships to exert lateral forces, and other measuring techniques are applied to provide a facility at a lower cost than using planar motion mechanisms.

More recent work in the Netherlands includes further considerations on mathematical models as presented by Hooft, 1987 (Ref.73), of MARIN (MAritime Research Institute Netherlands). Here investigations have been directed toward improving the assessment of hydrodynamic coefficients in non-linear ship models, principally because empirical methods to date have proved insufficiently accurate.

Brard, 1951 (Ref.31), provides an earlier French text on the manoeuvring of ships in deep water, in shallow water and in canals. A large number of experiments have been carried out utilising both the large turning basin, installed at the BEC (Bassin d'Essais des Carènes or

Paris Model Basin) in 1945, and the rectilinear basin designed for shallow water testing, installed five years later. The paper presents some of the principal test results obtained from these experiments.

An overview of the ship's bridge simulator designed by LMT Simulator and Electronic System Division of France is given by Martin, 1978 (Ref.99). It includes a diagram showing the important features of the ship handling simulator. Another French aid for port design and training of captains and pilots has been developed by Sogreah and is presented by Demenet, Garraud & Graff, 1984 (Ref.44).

Thöm, 1980 (Ref.131), has carried out theoretical and experimental modelling of ship dynamics in West Germany. Theoretical modelling by application of physical laws yields a general understanding of the model structure and of the influencing factors, but is impractical without approximations. Experimental modelling by evaluation of full scale ship trials or of model tests produces realistic results, but the proper design of experiments have to be based on theoretical considerations.

At the Technical University of Gdansk in Poland, Dziedzic & Morawski, 1980 (Ref.48), have been developing algorithms to control ship's motion according to desired trajectory. The paper presents a model of the process and control algorithms which minimise lateral deviations of the ship from a desired path or trajectory. An ε-optimal controller is applied to the problem, which is sub-divided into a kinematic and dynamic problem.

Computerised estimation of ship manoeuvrability at the design stage is also taking place in Bulgaria at the BSHC (Bulgarian Ship Hydrodynamics Centre) in Varna by Bogdanov & Milanov, 1987 (Ref.28). This paper discusses the SIMP software system which has been developed as a design tool for stern counter and rudder blade form, taking into account requirements for adequate ship manoeuvrability. By way of an example, results from an application of the system to the design of the after-body hull section of a real ship.

2.07 America

The DTMB (David Taylor Model Basin) at Washington DC and the DTMBRDC (David Taylor Model Basin Research and Development Centre) at Bethesda form the principal sites of this long-established institution for ship research. Strom-Tejsen, 1965 (Ref.127), although an earlier text, provides a good introduction to mathematical models based upon Taylor series expansion techniques. The report presents a non-linear mathematical model representing the motion of a surface ship. The associated computer program is written in FORTRAN II for the IBM 7090 computer. The sample calculations are based on the hydrodynamic coefficients of the "Mariner" hull form.

A mathematical presentation of shallow water flows past slender bodies is given by Tuck, 1965 (Ref.134). The problem solved concerns the disturbance to a stream of shallow water due to an immersed slender body, with particular reference to steady motions of ships in shallow water. The analysis assumes a ship to be slender in the sense that it is longer than it is broad or deep, and uses the technique of matched asymptotic expansions to construct approximate solutions.

Further research at DTMB has theoretically investigated the prediction of the motions of high-speed planing boats in waves. In his paper Martin, 1978 (Ref.98), compares the theoretical predictions with existing experimental data and obtains reasonably good agreement. Non-linear terms are required for speed-to-length ratios greater than about 6, otherwise linear theory is capable of providing a simple and fast means of determining the effect of various parameters such as trim, deadrise, loading and so on.

Lee, O'Dea & Meyers, 1983 (Ref.87), describe more recent work at DTMB on the prediction of relative motion of ships in waves. An analytical method is developed for predicting the vertical motion of a point on a ship relative to the free surface. The method is based on a two-dimensional approximation within the context of strip theory. The two-dimensional approximation simplifies the process of incorporating it into an existing ship motion computer program and enables the validity of the relative motion prediction to be checked based entirely upon strip theory. The paper compares computed results with experimental data for two hull forms.

MIT (Massachusetts Institute of Technology) is one of several institutes of technology involved in ship research. Newman, 1959 (Ref.111), considers the damping and wave resistance of a thin ship which is moving in calm conditions with constant velocity and oscillating in pitch and heave. Green's theorem is used to obtain the velocity potential. The coefficients of damping and increased wave resistance are found by separation of the energy components after an asymptotic expansion of the Green's function. Calculations are given for a polynomial hull and compared with experimental data.

Abkowitz's lecture notes, 1964 (Ref.2), have been previously mentioned, but in a later paper, 1983 (Ref.3), roll damping at forward speed is considered. Conflict exists between three-dimensional body theories and strip-slender body theories about the magnitude of the effect of forward speed on the linear roll damping coefficient. Forced rolling tests on models indicate that forward speed does in fact have a significant effect on roll damping which confirms the three-dimensional theory trend.

Abkowitz, 1984 (Ref.4), also demonstrates methods of measuring ship hydrodynamic coefficients by performing simple trials during regular operations. System identification techniques are applied to measurements of forward speed (u), sway (v), yaw velocity (r) and heading (ψ). The paper concludes that results indicate that the coefficients can be successfully identified from simple trials conducted during routine voyages using a minimum of the two measurements of u and ψ. Clearly, whilst it appears possible to produce a reasonable working model for ocean-going vessels, the accuracy for finer detailed manoeuvring must leave much to be desired.

One of the principal researchers at SIT (Stevens Institute of Technology) is Eda, 1965 onwards (Refs.49-52). Earlier work considered the steering characteristics of ships in calm water and waves, which led to yaw control in waves. A method to predict ship's yawing motion in following and quartering seas has been developed and used to study control system characteristics. During the 1970's digital simulations of standard manoeuvres were carried out to analyse manoeuvring performance and the effects of roll motions with respect to steering control were studied.

At the University of California Fukino & Tomizuka, 1986 (Ref.55), describe an adaptive time optimal control autopilot for ship steering. The so-called SOHM (Successive Order Heightening Method), is used to obtain the time optimal control law based on the solution of a third order differential equation model. It is combined with a least squares type parameter estimation algorithm and the scheme is evaluated by a computer simulation study.

Work at Hydronautics by Goodman, Gertler & Kohl, 1976 (Ref.62), and Ankudinov, Miller, Alman & Jakobsen, 1987 (Ref.14), has been directed at analysing, predicting and assessing surface ship manoeuvrability at the design stage. The experimental techniques and methods of analysis are described in the first paper and include reference to the use of a LAHPMM (Large Amplitude Horizontal Planar Motion Mechanism). While the second paper has advanced to using numerical simulation techniques to determine ship manoeuvrability performance.

Joint collaboration between Asinovsky, Landsburg & Hagen, 1987 (Ref.17), has also been analysing ship manoeuvrability, but using a differential approach. Mathematical representations of the hydrodynamic forces and moments are here based on the separate determination and analysis of the hydrodynamic characteristics for the hull, rudder and propeller and on the hydrodynamic interactions in the hull/propeller/rudder system.

The IMO (International Maritime Organisation) has moved towards the implementation of standards for ship manoeuvring in full scale trials, model tests and simulator performance. A number of papers suggest approaches to achieving standardisation, one such paper is that by Cojeen, Landsburg & MacFarlane, 1987 (Ref.41), of the US and Canadian Coast Guards. They anticipate that ship owners will establish preliminary manoeuvring performance by submitting lines plans to design simulators. Final manoeuvring performance capabilities could then be determined from trials conducted in conjunction with the shipbuilder's trials.

An extremely good text introducing many concepts in the dynamics of marine vehicles is provided by Bhattacharyya who is the Director of Naval Architecture at the US Naval Academy in Annapolis, 1978 (Ref.26). Much of the text deals with the seaway and motion due to waves. It begins with theories of sinusoidal water waves and progresses to an irregular seaway.

2.08 Japan

Nomoto provides the basis for much of the work as regards modelling the hydrodynamics of ships in the design of autopilots. This approach uses transfer functions to express the coefficients of the states and their derivatives in terms of time constants and systems gains (consistent with control engineering practice). The number of variables required to describe the behaviour of a ship is reduced by this method, however, each time constant can be related to several of the hydrodynamic coefficients. The time constants can often be extracted from plots of manoeuvres, but relate to autopilot control of models rather than the model itself.

Ohtagaki & Tanaka, 1984 (Ref.116), describe how, since its installation in 1975, the IHI (Ishikawajima-Harima heavy Industries) man-in-the-loop ship manoeuvring simulator has served the needs in ship design work and ship handling training. Applications of the simulator are presented and, due to increasing requirements for greater sophistication, it is mentioned that the facility is to be revamped.

Ongoing work between Kyushu University and Mitsui Engineering has led to the development of a practical calculation method of ship manoeuvring motion. The principal researchers are: Fukushima, Hirano, Inoue, Kijima, Moriya, Saruta, Takaishi and Takashina, (Refs.68-71,75-76,83). This method uses the principal particulars of the ship hull, rudder and propeller as basic input data. The mathematical model employs the coupled equations of surge, sway, yaw and roll. Initial papers present a simplistic model with only the fundamental manoeuvring terms; later papers deal with the inclusion of the effects of shallow water, banks, lateral thrust units, wind and wave. Computed results satisfactorily agree with full scale trial data.

Another joint endeavour exists between the Ship Research Institute and Mitsubishi Heavy Industries. The initial mathematical model developed by Ogawa & Kasai, 1978 (Ref.115), is similar to that of Hirano et al described earlier. However, Baba, Asai & Toki, 1982 (Ref.18), have deviated from the usual ocean-going ship models to investigate sway-roll-yaw coupled instability of semi-displacement type high-speed ships. Round bilge and hard chine type hulls are considered, as are variations in metacentric height and the effect of spray strips. Further study is envisaged to include non-linear terms in drift angle and yaw rate.

More recently Kobayashi & Asai, 1987 (Ref.84), have expanded the basic simulation model to cater for low speeds and astern manoeuvres. Four models have resulted for the following regimes:

1	Ordinary advance speed model	$Fn >= Fn_{min1}$
2	Average model of 1 & 3	$Fn_{min1} > Fn >= Fn_{min2}$
3	Low advance speed model	$Fn_{min2} > Fn >= 0$
4	Astern model	$0 > Fn$

where Fn is the Froude number and Fn_{min1} and Fn_{min2} are specified minimum limits. Model validation was made by comparison to free running model tests. The four models should be capable of evaluating operations both approaching and within harbour limits.

Recent research between the Yokogawa Hokushin Electric Corporation, Nippon Kokan K.K. and Nagoya Institute of Technology has closely followed Dutch work on MARC (Model Reference Adaptive Control). Arie, Itoh, Senoh, Takahashi, Fujii & Mizuno present a paper, 1985 (Ref.15), which uses "hill-climbing" techniques to achieve automatic steering control in course-keeping or course-changing modes.

2.09 Other Countries

Most of the countries which have some form of merchant navy are engaged, to a certain degree, in either mathematically modelling ship manoeuvres or autopilot control theory research. In Brazil, for instance, Rios-Neto & Da Cruz, 1985 (Ref.123), describe a heuristic stochastic rudder control law for ship automatic path-following in restricted waters. An extended Kalman filter is combined with a dynamical model compensation technique and a state noise adjustment procedure. The performance of the controller is illustrated with results obtained by digital simulation. After further feasibility studies it is anticipated that the autopilot could be implemented in an onboard minicomputer.

2.10 Additional

Many texts deal with particular aspects relevant to ship simulation. Hirano, Takashina, Takaishi & Saruta (Ref.67) present the results of a study on the turning trajectory of ships under the influence of regular waves. Norrbin (Ref.114) discusses the generation of a lateral force (known as the rudder normal force) due to the flow past a rudder. Estimates for the rudder derivatives and recommendations of minimum rudder area for certain ship types plus stability derivatives formed by regressional analysis are included. Aage (Ref.1) provides four-component (surge, sway, yaw and roll) wind coefficients for nine ship models.

The list of technical publications is almost limitless, and there are further texts written in languages other than English which the author is unable to review. However, it is hoped that a reasonable cross-section of material has been presented in order to give the reader an indication of the scope of the work being carried out within maritime research establishments.

CHAPTER 3

AUTOMATIC CONTROL OF BOATS

3.01 Introduction

The research covered in this thesis was inspired by the requirement for a facility to assess the performance of small boat autopilots. It, therefore, seems pertinent to include an introduction to the subject of automatic control of boats and examine the functionality of the autopilot.

The two main products of interest manufactured by Cetrek are the Fluxgate Compass Sensor and the Automatic Pilot (Fig.3.1).

The fluxgate sensor is essentially a magnetometer which can detect the magnitude and direction of a magnetic field. It is a "second harmonic" device since the output signal voltage will have a frequency twice that of the driving or exciting frequency. The sensor will give a maximum output when it is aligned exactly with the direction of the measured magnetic field and a minimum when lying at 90° to the field. Two sensing coils, set at right angles to each other and known as a sine/cosine system, are used to generate angular position information without ambiguity.

The autopilot control unit requires heading information which is supplied by the fluxgate sensor. This, along with the drive unit, consisting of a hydraulic drive

Typical Cetrek System Layout

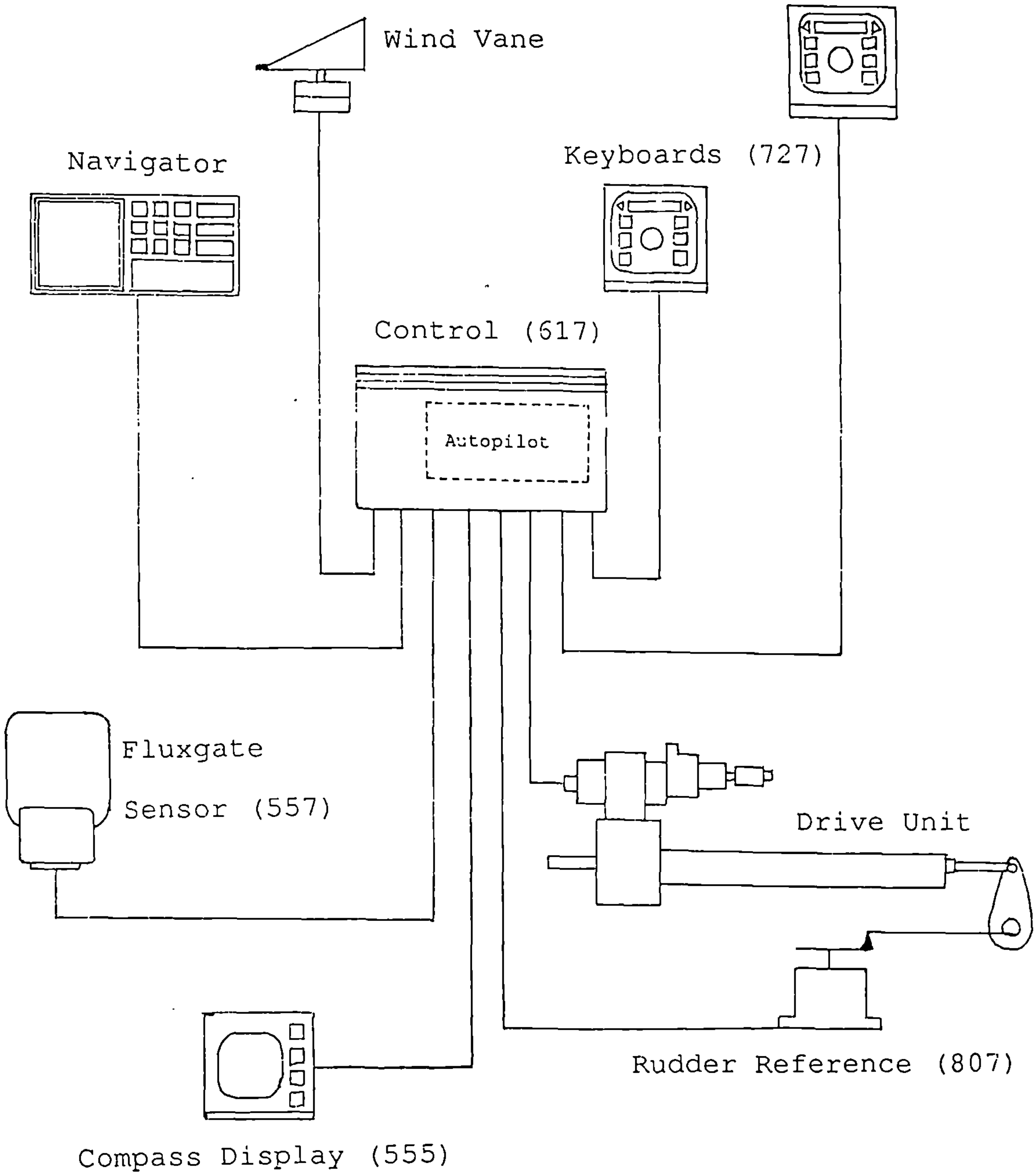


Fig.3.1

system, and the rudder reference or feedback unit, make up the important components necessary to achieve automatic control. In addition, the Cetrek autopilot can be combined with radio navigation systems, that is Satnav and Hyperbolic navigators, thus allowing adherence to track between user defined waypoints. A wind vane can be attached, for yachts, in order that the autopilot can keep the boat on a prescribed relative heading to the wind direction. Also, extra keyboards and compass display units may be added as repeaters to secondary helm positions, such as the fly bridge.

The fundamental principles of an autopilot will now be discussed since this unit is closely allied to the mathematical model project.

3.02 PID Control Theory

An autopilot is designed to steer a boat by manipulating the rudder in such a way as to reduce the difference between the desired and actual heading of a vessel. Conventional autopilots rely on the three term Proportional, Integral and Derivative (PID) control (Fig.3.2).

Proportional Control

This causes rudder to be applied in proportion to the

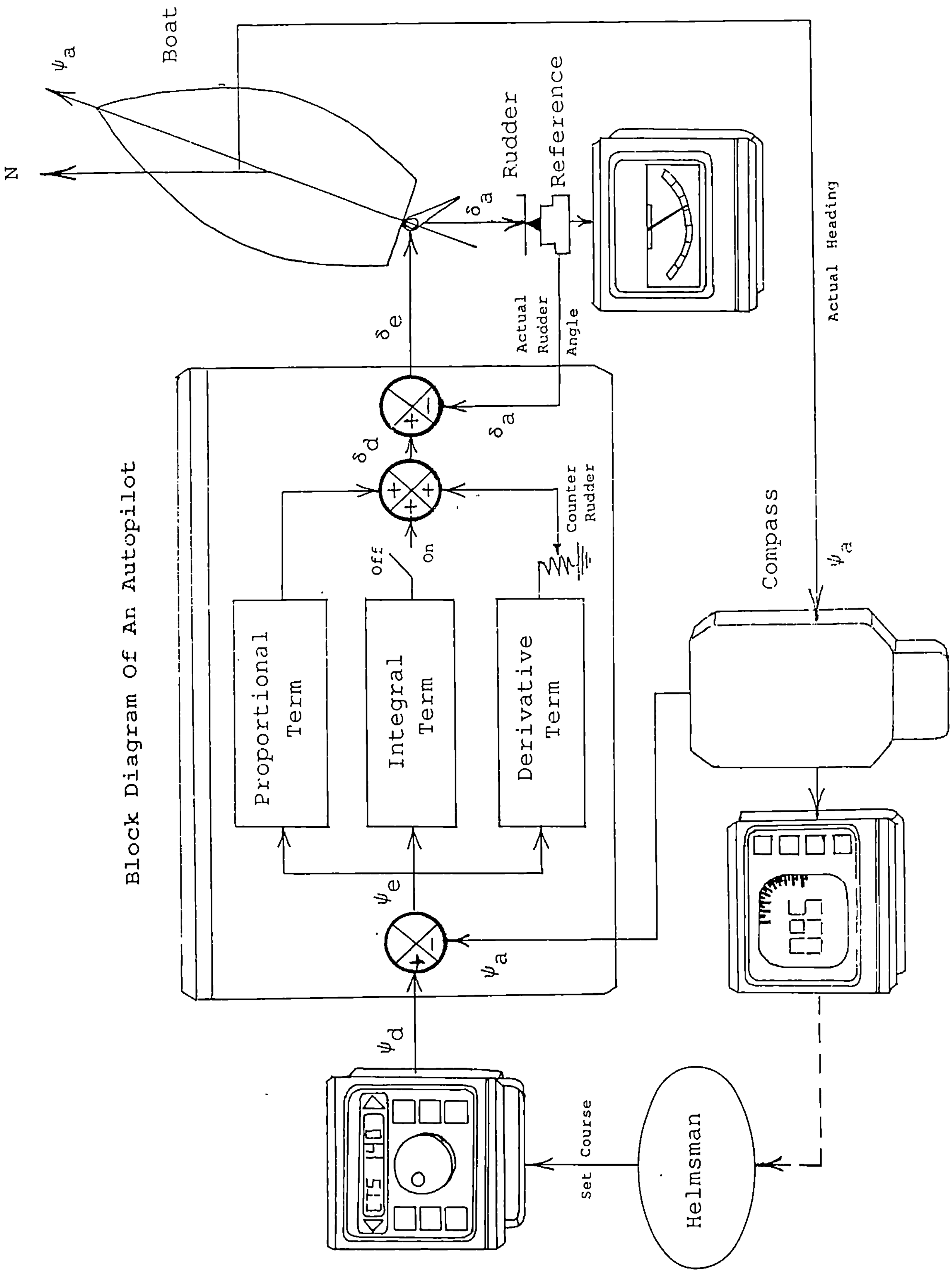

Block Diagram Of An Autopilot
Boat
N
ψ_a
Rudder
Reference
δ_a
δ_e
Actual Rudder Angle
Actual Heading
δ_d
Counter Rudder
Off
On
Proportional Term
Integral Term
Derivative Term
ψ_e
ψ_d
Compass
CTS 140
095
Set Course
Helmsman

Fig.3.2

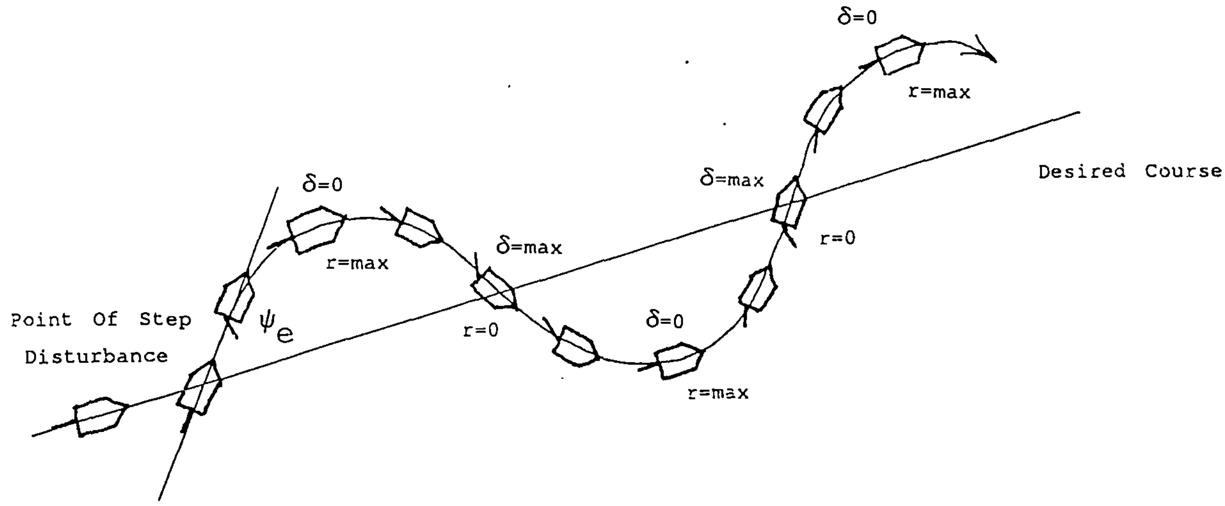
heading error, that is the difference between the desired and actual heading. At the instant an error is detected, rudder is applied, the yaw rate increases, bringing the boat back towards the desired course. However, when the heading error, and hence the rudder angle, reach zero, the yaw rate is at its maximum and the desired course is overshot. Corrective rudder, in the opposite sense is now applied and hence the oscillatory motion is continued (Fig.3.3a). This motion leads to additional fuel and time expense, down-graded efficiency and unacceptable rudder component wear.

Derivative Control

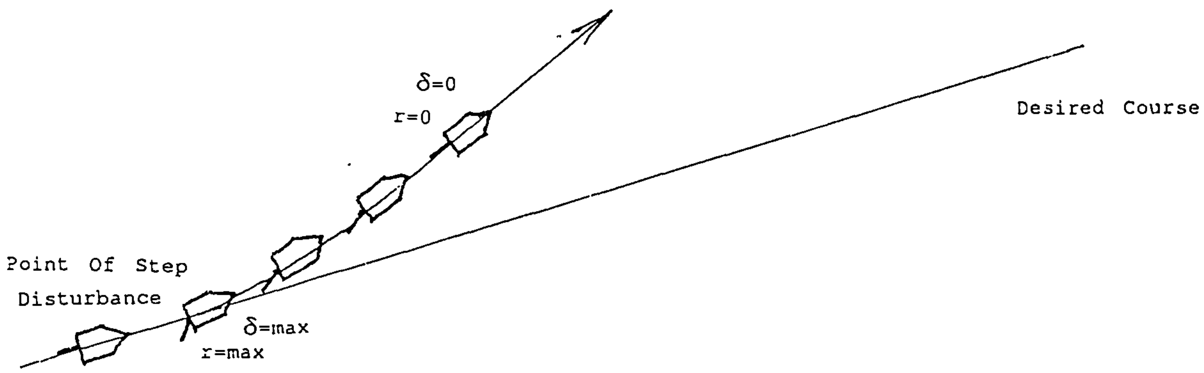
This causes rudder to be applied in direct opposition to the rate of change of heading error (which is the same as the rate of change of the actual heading). The applied rudder will act against the direction of turn and provides "counter rudder" to decrease the rate of turn of the boat (Fig.3.3b). Pure derivative control is applied only while the boat is yawing.

PD Combination

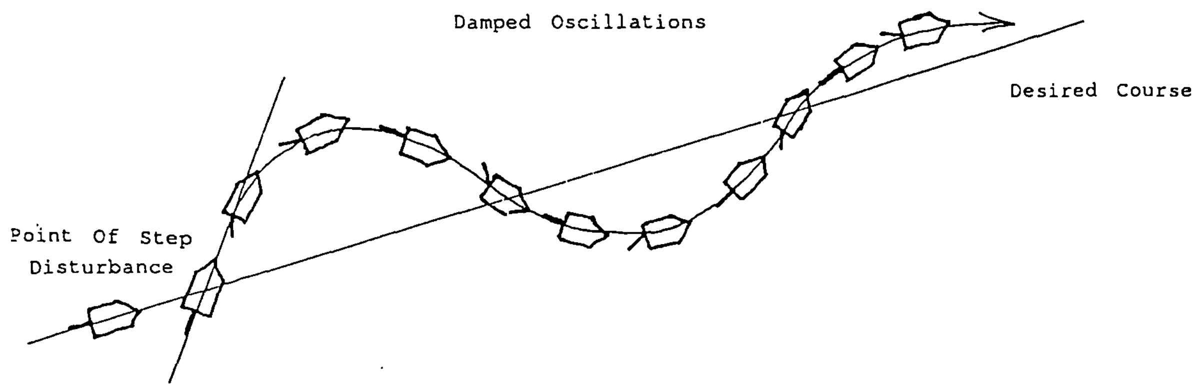
Proportional rudder is used to reduce the heading error, whilst derivative rudder reduces the yaw rate when nearing the desired course, thus damping the oscillatory motion (Fig.3.3c). High gains can cause



Proportional Control Only Fig.3.3a



Derivative Control Only Fig.3.3b



Combined Proportional And Derivative Control Fig.3.3c severe oscillations before the boat settles on course, and low gains can cause little overshoot with a sluggish return to course. The correct balance is necessary.

For most small boat purposes, a slightly under-damped setting of the gains is desirable as this ensures that the desired course is actually reached and achieved relatively quickly. A large overshoot is to be avoided as this introduces excess rudder movement and a longer course settling time. However, when manoeuvring in reasonably unrestricted areas, the condition of critical damping can be used to execute a smooth alteration of course when approaching a waypoint.

Integral Control

This causes rudder to be applied in direct relation to the sum of the heading errors (determined by integrating the heading error). It is used to provide a permanent helm or rudder offset (false centre) in order to combat the effects of wind and tidal disturbances which continually move the boat off course. This is often referred to as a weather helm.

PID Combination

It is possible that a disturbance acting continuously, for example, a wind or wave drifting moment, can balance the moment generated by the demanded rudder position calculated by the PD controller. The course error cannot therefore be reduced. The addition of the integral term effectively adds a little more desired rudder at each iteration of the control loop all the while the disturbance is acting. This term gradually mounts up until the effect of the disturbing moment is neutralised.

Ghost Rudder

Currently available autopilots have additional functions peripheral to the PID controller. When the rudder reference is unavailable a "Ghost Rudder" often provides a replacement for the feedback signal. This can be based on a knowledge of the steering gear response and the signals sent by the autopilot to the rudder.

Rudder Limit

The desired rudder angle, computed by the autopilot, is given a finite limit to ensure the system does not attempt to drive the rudder beyond physical limits.

Course Deadband

This deadband is designed to avoid the condition where the rudder is continually subjected to rapid port and starboard commands in quick succession. This occurs when the boat is approximately on course but oscillating slightly either side of the desired course. By setting a high deadband the amplitude of yaw that can be tolerated before the steering gear is enabled is increased. In heavy weather the course deadband would be opened out, and reduced again in calmer conditions.

Rudder Deadband

Small rudder errors that fall within the rudder deadband are not implemented. This is used to avoid continuous rudder action.

3.03 Autopilot Gains

Selection of the gains for each of the PID terms depends on the type of boat under control and the sea conditions in which it is operating. Additionally the deadbands will be expanded or contracted as conditions dictate. While adaptive autopilots exist for large ships which have the equivalent of mini-computers on board, there are, at present, no such facilities on small boat autopilots.

The current procedure is to determine appropriate settings for the gains by conducting simple sea trials when the autopilot is installed. These settings are intuitive values which "feel" right for the particular boat and will provide a reference from which the gains can be adjusted.

Ideally automatic selection of the gains is desirable. A model reference system could be used to assess boat response to the gain settings and alter them to suit the prevailing conditions. This could be incorporated within the autopilot as a complete model or as a set of pre-computed gains in tabular form.

A mathematical model of a small boat, incorporating forcing functions to represent the effects of wind and waves, would provide a development tool capable of assessing autopilot performance in a range of pseudo sea conditions. Furthermore, it will enable criteria for optimising autopilot gains to be established. It was with this view in mind that the construction of a simulation model was first proposed.

CHAPTER 4

MOTION OF A BOAT IN A SEAWAY

"There be three things which are too wonderful
for me, yea, four which I know not:
The way of an eagle in the air;
the way of a serpent upon a rock;
the way of a ship in the midst of the sea;
and the way of a man with a maid"

Proverbs 30 v18&19 (KJV)

4.01 Introduction

Many terms are used to express different aspects of the motion of ships and boats in a seaway. It is generally agreed that a boat must be seaworthy and have satisfactory stability, but exactly what constitutes these qualities is still very much a matter of the judgement and experience of the individual naval architect. The general field of boat motion can be divided under two headings, "manoeuvrability" and "seakeeping".

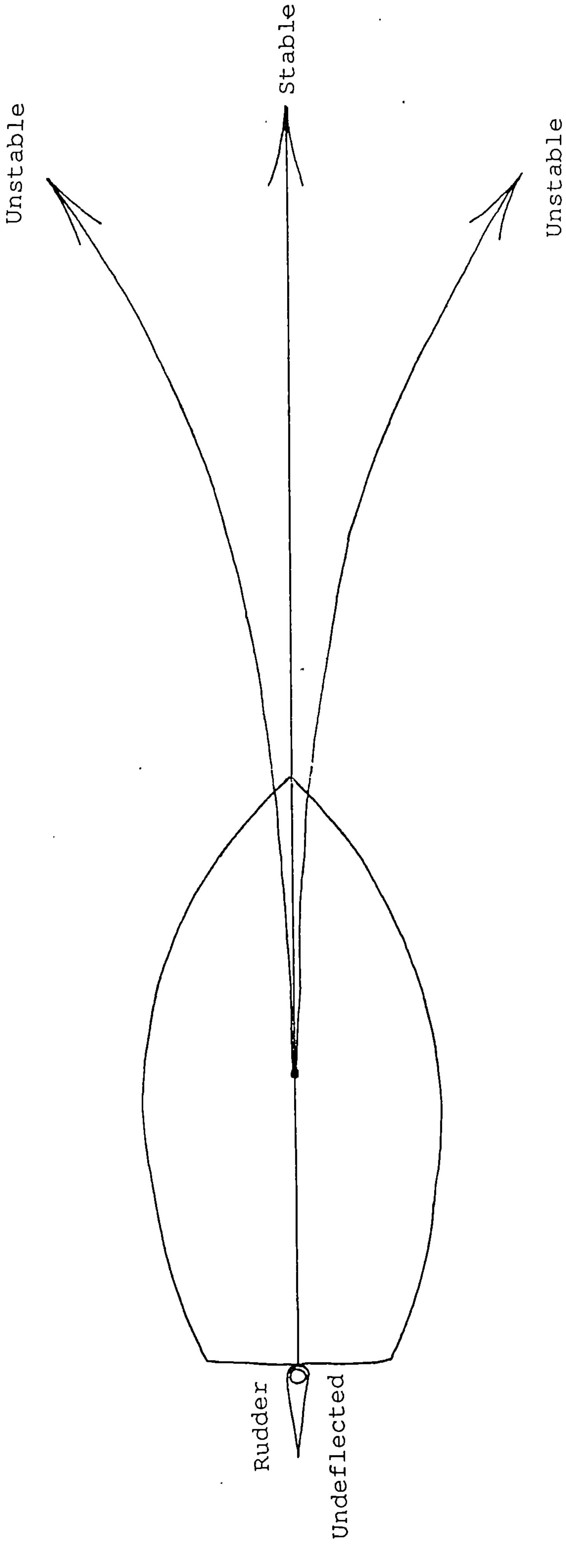
Manoeuvrability refers to those motions which result solely from the excitation forces and moments due to application of control surfaces, such as rudders, in the absence of disturbing forces and moments due to external influences, such as wind and waves. Seakeeping, however, deals with the motion of a boat resulting from the external disturbing forces and moments of the sea and wind. When the control surfaces are used to effect a manoeuvre in the presence of excitations due to the sea, either to maintain a desired course or perform a course change, there is a combination of the two areas which is referred to as manoeuvring in a seaway.

Both of these areas are additionally concerned with two further concepts: "motion stability", which is a measure of the boat's ability to maintain a prescribed motion without the use of the control surfaces and in the absence of external disturbances, and "motion control", which deals with the ability of the control surfaces, applied either manually or automatically, to achieve the desired motion or compensate for disturbing forces and moments.

The easiest test for motion stability is to impose the equilibrium condition of straight ahead motion at constant speed with no rudder deflection. A boat which is dynamically unstable will be incapable of maintaining a straight line and will deviate to either port or starboard (Fig.4.1).

A boat is also expected to be "seakindly", which is a term that is important to the comfort of those onboard. Again, it is an arbitrary term which implies that the boat will behave in a manner so as to minimise the requirement for expert boat-handling ability. A seakindly boat should not exhibit heavy rolling motions, have excessive accelerations or produce rapid oscillations of small amplitude, but rather have good steering response and be free from spray and green water.

The motions of a boat in a seaway can be formulated



Course Stability

Fig.4.1

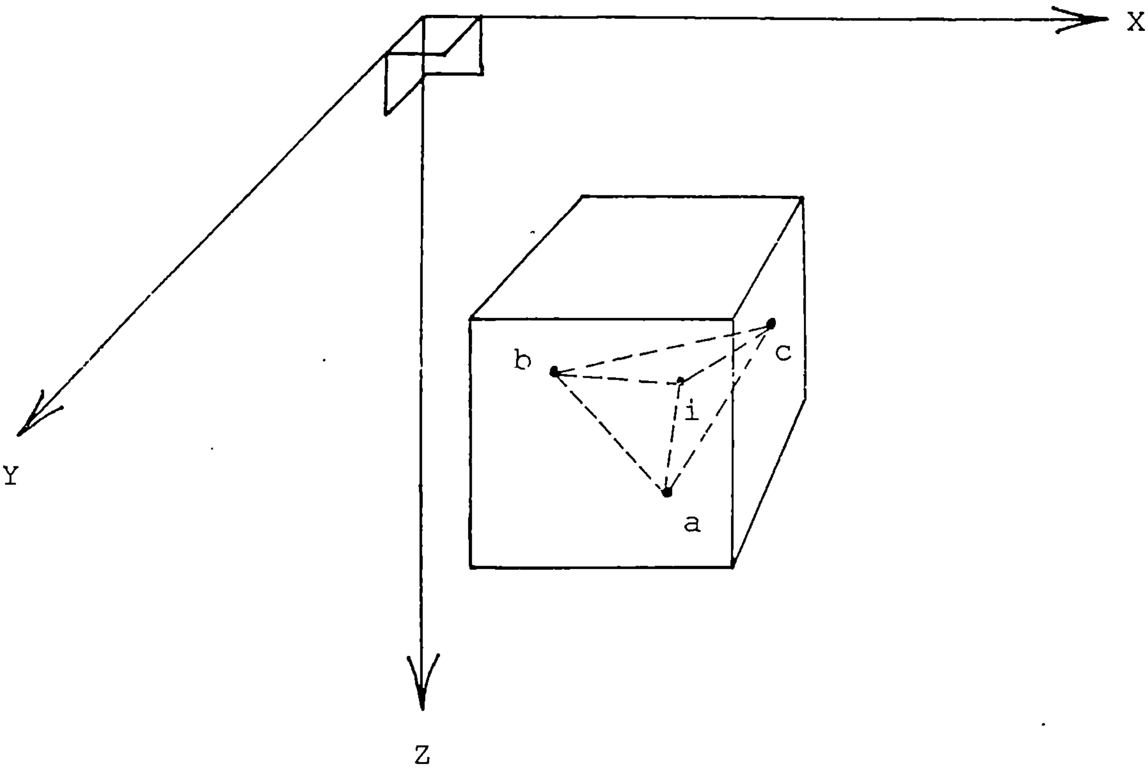
according to rigid body dynamics and will be shown to possess six degrees-of-freedom.

4.02 Definition of a Rigid Body

In this analysis, as with submarines, aircraft, missiles, spacecraft et cetera, a small boat is said to exhibit the properties of a "rigid body". The term "rigid" can be thought of thus: assume that a body is composed of a large, but finite, number of elementary particles. If the separation between each particle remains constant (that is all the particles are unable to move relative to each other) then the body is said to be rigid.

Strictly speaking, the atoms of any natural body will always be undergoing some microscopic relative motion, but for the purposes of describing the macroscopic motion of a body this can be ignored. Compressions and stresses et cetera can cause elastic deformations within a body, but again these geometrical shape changes can be neglected with only minimal loss of accuracy.

If all of the n particles within the body were independent of one another, it would require three cartesian coordinates to fix each of the n points. The particles are not, however, all independent and may be



A Rigid Body

Fig.4.2

specified by distances to any three non-colinear points in the body. From (Fig.4.2) it can be seen that the distances r_{ia}, r_{ib} and r_{ic} specify any point i relative to the three non-colinear points a, b and c.

Since the condition for a rigid body is that the distances r_{ia}, r_{ib} and r_{ic} are constant, then the position of the particle i is fixed once the locations of the three non-colinear points are known. Each of the n particles in the body can be specified in the same manner. In other words, the position of the body as a whole can be determined by the positions of these three points.

Nine cartesian coordinates would be required to specify three independent points, but since the distances r_{ab}, r_{bc} and r_{ca} are also constant, only six coordinates are necessary to locate the rigid body and determine its orientation.

The rigid body is thus said to have six degrees-of-freedom and Chasles' theorem, which states:

> *"any arbitrary finite motion of a body may be considered to be the sum of two independent motions - a linear translation of some point of the body plus a rotation about that point."*

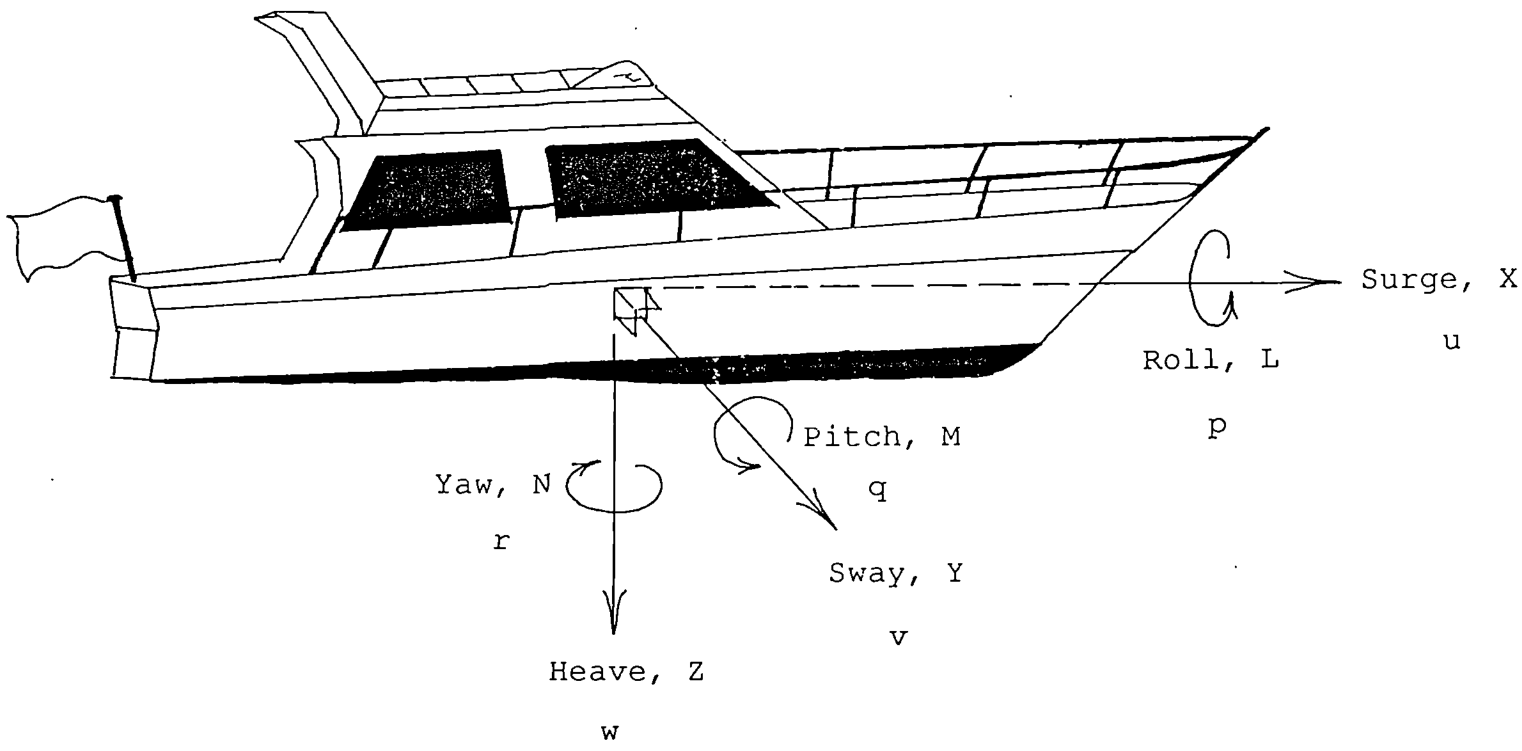
allows the problem to be divided into two parts, one involving only translation and the other only rotation. The three translational motions, described by the three coordinates required to fix the position of one of the particles, can therefore be treated separately from the three rotational motions, described by the remaining three coordinates necessary to determine the relative orientation of the other two points. This type of separation is essential for a relatively uncomplicated description of rigid-body motion.

4.03 The Six Degrees-Of-Freedom Of A Boat

The six degrees-of-freedom of a boat (Fig.4.3) are represented in an orthogonal coordinate system having the centre of gravity as its origin. A description of these six quantities now follows.

Surge

Surge is the forward and aft translation of the boat directed along the X-axis. This not only includes the propelled movement, but also the tendency for a boat to move forward on a wave crest, known as surfing, and backward in a trough. Naturally, a boat under way through a swell will not actually move backwards and forwards but will be alternately accelerated and retarded according to the relative direction of travel



Six Degrees Of Freedom Of A Boat

Fig.4.3

to the waves.

Sway

Sway is the transverse translational motion of the boat along the Y-axis. As well as sideslip due to centripetal forces when executing a turn, this includes the effect of successive wave crests providing a series of "pushes" which amount to drift. Most vessels, if not under way, tend to become orientated broadside on to the swell.

Heave

Heaving is the vertical bodily translation of the whole boat upwards (which in this analysis is negative motion) as well as the reverse motion of dipping. This motion is due to the change in buoyancy as each wave passes the boat. Heaving is periodic and in this respect associated with rolling and pitching.

Roll

Roll is the rotation about the longitudinal X-axis of the boat and is treated with consideration by naval architects since it affects the comfort of those onboard. Unlike a pendulum, a boat has no fixed axis of rotation, but has what is termed the *instantaneous axis* which is located near the centre of gravity. The centre of gravity will therefore describe a path in space as the boat rotates, even though relative to the boat as a whole it remains fixed.

Pitch

Pitch is the rotation about the transverse Y-axis. Pitching is in fact the bow down motion (which in this analysis is negative motion) while the bow up motion is known as 'scending. Pitching tends to be heaviest when heading into a sea and increases when conditions of synchronism occur.

Yaw

Yawing is the rotation about the vertical Z-axis. It is the tendency for a boat to veer off course. Unlike rolling and pitching there is no restorative moment; this must be applied by use of external control surfaces such as rudders.

4.04 The Axes Systems

Conventionally, and for convenience and simplicity, the motion of a rigid body is described with respect to a coordinate system fixed within the body. By choosing the centre of gravity to be its origin, the equations of motion can be reduced, as will become apparent in later chapters. However, a second axes system must be specified which forms the reference or inertial frame. This can be viewed as a coordinate system fixed with respect to the Earth.

It is necessary to transform various vector quantities between these two axes systems. Wind and tidal information, for example, needs to be converted into functions of apparent angles. Whereas, the velocity or displacement of the rigid body is required in the inertial frame in order to determine the position of the vessel at sea. It is also of primary importance to know the orientation of the boat, in other words the angles of roll, pitch and yaw, with respect to the inertial frame. This is often referred to as kinematics.

4.05 Transformation Between Orthogonal Axes

By keeping track of the orientation of the moving axes system, vectors can be readily transformed between systems. Three methods for specifying the relative orientation of the two axes systems are: Direction cosines, Euler angles and Quaternions.

Direction Cosines

The principal tool which provides the means for performing axes transformations is the direction cosine matrix. This matrix contains nine elements which are the cosines of the base angles between each of the axes of the moving system with each of the axes of the fixed system.

Consider a vector $\underset{\sim}{r}$ with components x_0, y_0 and z_0 in the inertial frame. Let the unit vectors along the X_0, Y_0 and Z_0 axes be $\underset{\sim}{i}_1$, $\underset{\sim}{i}_2$ and $\underset{\sim}{i}_3$ then:

$$\underset{\sim}{r} = \underset{\sim}{i}_1 \, x_0 + \underset{\sim}{i}_2 \, y_0 + \underset{\sim}{i}_3 \, z_0$$

Now assume another coordinate system X, Y, Z with the same origin as X_0, Y_0, Z_0 but having some arbitrary orientation with respect to it. If the components of $\underset{\sim}{r}$ in the new axes system are x, y and z and the unit vectors are $\underset{\sim}{a}_1$, $\underset{\sim}{a}_2$ and $\underset{\sim}{a}_3$ then:

$$\underset{\sim}{r} = \underset{\sim}{a}_1 \, x + \underset{\sim}{a}_2 \, y + \underset{\sim}{a}_3 \, z$$

In order to determine the orthogonal transformation between the two coordinate systems, it is necessary to express x, y and z in terms of x_0, y_0, z_0 and the relative orientation.

$\underset{\sim}{a}_1$ can be written in terms of its components in the X_0, Y_0, Z_0 system:

$$\underset{\sim}{a}_1 = (\underset{\sim}{a}_1 \cdot \underset{\sim}{i}_1)\underset{\sim}{i}_1 + (\underset{\sim}{a}_1 \cdot \underset{\sim}{i}_2)\underset{\sim}{i}_2 + (\underset{\sim}{a}_1 \cdot \underset{\sim}{i}_3)\underset{\sim}{i}_3$$

Since all these vectors have unit magnitude, the dot product of two of them is simply the cosine of the angle between them:

$$\begin{aligned}
\underset{\sim}{a}_1 \cdot \underset{\sim}{i}_1 &= \cos(\underset{\sim}{a}_1 \angle \underset{\sim}{i}_1) = \lambda_{11} \\
\underset{\sim}{a}_1 \cdot \underset{\sim}{i}_2 &= \cos(\underset{\sim}{a}_1 \angle \underset{\sim}{i}_2) = \lambda_{12} \\
\underset{\sim}{a}_1 \cdot \underset{\sim}{i}_3 &= \cos(\underset{\sim}{a}_1 \angle \underset{\sim}{i}_3) = \lambda_{13}
\end{aligned}$$

the same process applies to $\underset{\sim}{a}_2$ and $\underset{\sim}{a}_3$ and the resulting relationships can be written:

$$\begin{aligned}
\underset{\sim}{a}_1 &= \lambda_{11}\,\underset{\sim}{i}_1 + \lambda_{12}\,\underset{\sim}{i}_2 + \lambda_{13}\,\underset{\sim}{i}_3 \\
\underset{\sim}{a}_2 &= \lambda_{21}\,\underset{\sim}{i}_1 + \lambda_{22}\,\underset{\sim}{i}_2 + \lambda_{23}\,\underset{\sim}{i}_3 \\
\underset{\sim}{a}_3 &= \lambda_{31}\,\underset{\sim}{i}_1 + \lambda_{32}\,\underset{\sim}{i}_2 + \lambda_{33}\,\underset{\sim}{i}_3
\end{aligned}$$

where: λ_{jk} $(j,k=1,2,3)$ are the nine direction cosines or in matrix form:

$$\begin{bmatrix} \underset{\sim}{a}_1 \\ \underset{\sim}{a}_2 \\ \underset{\sim}{a}_3 \end{bmatrix} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix} \begin{bmatrix} \underset{\sim}{i}_1 \\ \underset{\sim}{i}_2 \\ \underset{\sim}{i}_3 \end{bmatrix}$$

exactly the same procedure can be applied for transformations in a reverse direction and the equations become:

$$\begin{bmatrix} \underset{\sim}{i}_1 \\ \underset{\sim}{i}_2 \\ \underset{\sim}{i}_3 \end{bmatrix} = \begin{bmatrix} \lambda_{11} & \lambda_{21} & \lambda_{31} \\ \lambda_{12} & \lambda_{22} & \lambda_{32} \\ \lambda_{12} & \lambda_{23} & \lambda_{33} \end{bmatrix} \begin{bmatrix} \underset{\sim}{a}_1 \\ \underset{\sim}{a}_2 \\ \underset{\sim}{a}_3 \end{bmatrix}$$

this gives rise to the relationship that the inverse of the direction cosine matrix is its transpose:

$$\begin{bmatrix} \mathrm{DCM} \end{bmatrix}^{-1} = \begin{bmatrix} \mathrm{DCM} \end{bmatrix}^{T}$$

the components of the vector $\underset{\sim}{r}$ in the inertial frame can thus be written:

$$x = \underset{\sim}{r} \cdot \underset{\sim}{i}_1 = \lambda_{11}\, x_0 + \lambda_{12}\, y_0 + \lambda_{13}\, z_0$$
$$y = \underset{\sim}{r} \cdot \underset{\sim}{i}_2 = \lambda_{21}\, x_0 + \lambda_{22}\, y_0 + \lambda_{23}\, z_0$$
$$z = \underset{\sim}{r} \cdot \underset{\sim}{i}_3 = \lambda_{31}\, x_0 + \lambda_{32}\, y_0 + \lambda_{33}\, z_0$$

or in matrix form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

The nine direction cosines are not independent but must satisfy six algebraic relationships which exist between them. Since the length of the vector $\underset{\sim}{r}$ must remain unchanged, that is:

$$x^2 + y^2 + z^2 = x_0^2 + y_0^2 + z_0^2$$

then the six constraint equations for orthogonal transformations are:

$$\begin{bmatrix} \lambda_{1j} & \lambda_{2j} & \lambda_{3j} \end{bmatrix} \begin{bmatrix} \lambda_{1j} \\ \lambda_{2j} \\ \lambda_{3j} \end{bmatrix} = 1 \qquad (j=1,2,3)$$

and:

$$\begin{bmatrix} \lambda_{j1} & \lambda_{j2} & \lambda_{j3} \end{bmatrix} \begin{bmatrix} \lambda_{k1} \\ \lambda_{k2} \\ \lambda_{k3} \end{bmatrix} = 0 \qquad (j=1,2,3; \; k=2,3,1)$$

The direction cosine matrix will be time dependent during simulations since the boat is unlikely to maintain the same orientation. The direction cosine matrix must be re-evaluated at each sample interval and is dependent upon the angular velocities p, q and r about each of the three boat fixed axes. The rate of change of the elements of the direction cosine matrix can be determined as follows:

if the angular velocity of the rotating axes system X, Y, Z is:

$$\underset{\sim}{\omega} = \underset{\sim}{a}_1 p + \underset{\sim}{a}_2 q + \underset{\sim}{a}_3 r$$

and recall:

$$\underset{\sim}{i}_j = \lambda_{1j}\, \underset{\sim}{a}_1 + \lambda_{2j}\, \underset{\sim}{a}_2 + \lambda_{3j}\, \underset{\sim}{a}_3 \qquad (j=1,2,3)$$

then:

$$\frac{\partial \underset{\sim}{i}_j}{\partial t} = \frac{d\underset{\sim}{i}_j}{dt} - \underset{\sim}{\omega} \times \underset{\sim}{a}_j \qquad (j=1,2,3)$$

but, since $\underset{\sim}{i}_j$ is the fixed coordinate system:

$$\frac{d\underset{\sim}{i}_j}{dt} = 0 \qquad (j=1,2,3)$$

hence:

$$\frac{\partial \underset{\sim}{i}_j}{\partial t} = -\underset{\sim}{\omega} \times \underset{\sim}{a}_j = \dot{\lambda}_{1j}\underset{\sim}{a}_1 + \dot{\lambda}_{2j}\underset{\sim}{a}_2 + \dot{\lambda}_{3j}\underset{\sim}{a}_3$$

$$= \begin{vmatrix} \underset{\sim}{a}_1 & \underset{\sim}{a}_2 & \underset{\sim}{a}_3 \\ -p & -q & -r \\ \lambda_{1j} & \lambda_{2j} & \lambda_{3j} \end{vmatrix}$$

which when expanded gives:

$$\dot{\lambda}_{1j} = - q\,\lambda_{3j} + r\,\lambda_{2j}$$
$$\dot{\lambda}_{2j} = p\,\lambda_{3j} - r\,\lambda_{1j}$$
$$\dot{\lambda}_{3j} = - p\,\lambda_{2j} + q\,\lambda_{1j}$$

(j=1,2,3)

or in matrix form:

$$\begin{bmatrix} \dot{\lambda}_{1j} \\ \dot{\lambda}_{2j} \\ \dot{\lambda}_{3j} \end{bmatrix} = \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix} \begin{bmatrix} \lambda_{1j} \\ \lambda_{2j} \\ \lambda_{3j} \end{bmatrix}$$

(j=1,2,3)

these can be integrated to determine the subsequent direction cosine matrix elements.

Euler Angles

The most familiar method of describing the orientation of one axes system with respect to another is by use of the Euler angles. This is the only three parameter method in common use and it provides an easily understood representation of the relative orientation of two axes systems. The orientation is expressed by three successive rotations, which must be performed in a specific order, about each of the three axes. For the purposes of this analysis, and in keeping with established convention, the order for transformation from the inertial frame to the moving coordinate system is through an angle of yaw ψ (psi) about the Z-axis, an angle of pitch θ (theta) about the new position of the Y-axis and an angle of roll ϕ (phi) about the new position of the X-axis.

The successive rotations can be represented by a matrix which is the product of the transformation matrices for each individual rotation. Since matrix multiplication is not commutative, a different order of rotations would yield an entirely different final orientation. Since the components along the axis of rotation remain unchanged, each individual rotation can be viewed as a two-dimensional transformation. From a knowledge of simple trigonometry the three individual transformation matrices can be deduced.

1) A rotation ψ about the Z_0-axis (yaw) (Fig.4.4a):
   the components of a vector $\underset{\sim}{r}$ in X_1, Y_1, Z_1 can be expressed in terms of its components in X_0, Y_0, Z_0:

$$
\begin{aligned}
x_1 &= x_0 \cos\psi + y_0 \sin\psi \\
y_1 &= -x_0 \sin\psi + y_0 \cos\psi \\
z_1 &= \qquad\qquad\qquad\qquad z_1
\end{aligned}
$$

or:

$$T_\psi = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2) A rotation θ about the Y_1-axis (pitch) (Fig.4.4b): the components of a vector $\underset{\sim}{r}$ in X_2, Y_2, Z_2 can be expressed in terms of its components in X_1, Y_1, Z_1:

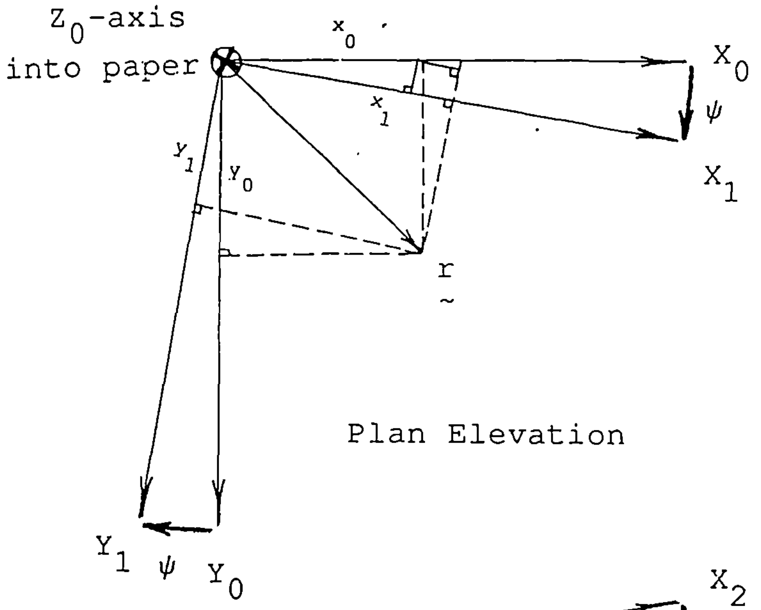
$$\begin{aligned} x_2 &= x_1 \cos\theta - z_1 \sin\theta \\ y_2 &= y_1 \\ z_2 &= x_1 \sin\theta + z_1 \cos\theta \end{aligned}$$

or:

$$S_\theta = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

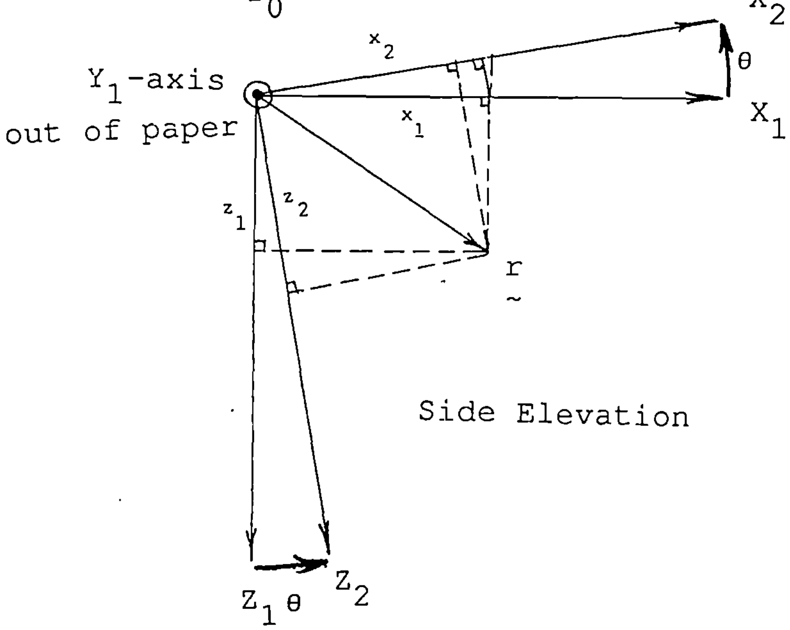
3) A rotation ϕ about the X_2-axis (roll) (Fig.4.4c): the components of a vector $\underset{\sim}{r}$ in X_3, Y_3, Z_3 can be expressed in terms of its components in X_2, Y_2, Z_2:

$$\begin{aligned} x_3 &= x_2 \\ y_3 &= y_2 \cos\phi + z_2 \sin\phi \\ z_3 &= -y_2 \sin\phi + z_2 \cos\phi \end{aligned}$$



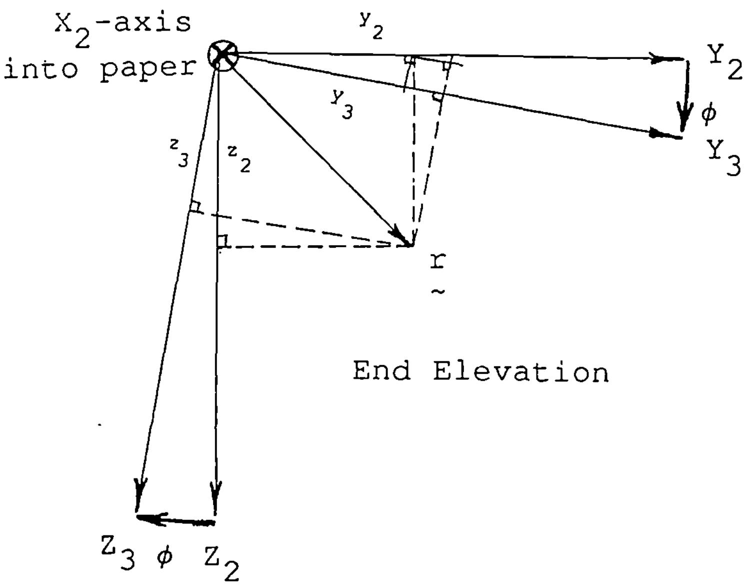
Plan Elevation

Fig.4.4a



Side Elevation

Fig.4.4b



End Elevation

Fig.4.4c

Two-Dimensional Transformations

or:

$$R_\phi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$$

The product of these three matrices will give the total transformation matrix from the inertial frame to the moving axes system and is equivalent to the direction cosine matrix.

$$R_\phi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$$

$$R_\phi S_\theta = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ \sin\phi\sin\theta & \cos\phi & \sin\phi\cos\theta \\ \cos\phi\sin\theta & -\sin\phi & \cos\phi\cos\theta \end{bmatrix}$$

$$R_\phi S_\theta T_\psi = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \begin{array}{c}\sin\theta\sin\phi\cos\psi \\ -\cos\phi\sin\psi\end{array} & \begin{array}{c}\sin\theta\sin\phi\sin\psi \\ +\cos\phi\cos\psi\end{array} & \cos\theta\sin\phi \\ \begin{array}{c}\sin\theta\cos\phi\cos\psi \\ +\sin\phi\sin\psi\end{array} & \begin{array}{c}\sin\theta\cos\phi\sin\psi \\ -\sin\phi\cos\psi\end{array} & \cos\theta\cos\phi \end{bmatrix}$$

The complete transformation is thus specified by the three independent parameters ϕ, θ and ψ (Fig.4.5).

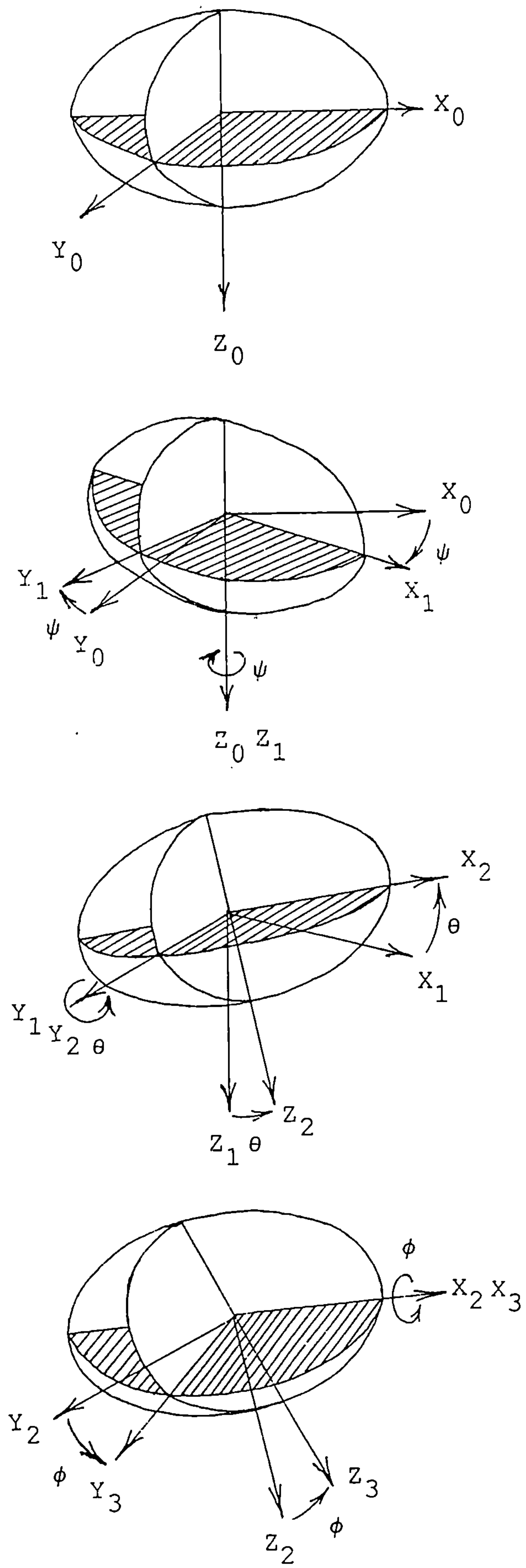


Fig.4.5

Axes Transformation

The direction cosine matrix for transformations in the opposite direction is simply:

$$[T_{-\psi}S_{-\theta}R_{-\phi}] = [R_{\phi}S_{\theta}T_{\psi}]^{-1} = [R_{\phi}S_{\theta}T_{\psi}]^{T}$$

hence the matrix for transformations from the body fixed axes system to the inertial frame, $T_{-\psi}S_{-\theta}R_{-\phi}$, is:

$$\begin{bmatrix} \cos\theta\cos\psi & \begin{matrix}\sin\theta\sin\phi\cos\psi \\ -\cos\phi\sin\psi\end{matrix} & \begin{matrix}\sin\theta\cos\phi\cos\psi \\ +\sin\phi\sin\psi\end{matrix} \\ \cos\theta\sin\psi & \begin{matrix}\sin\theta\sin\phi\sin\psi \\ +\cos\phi\cos\psi\end{matrix} & \begin{matrix}\sin\theta\cos\phi\sin\psi \\ -\sin\phi\cos\psi\end{matrix} \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix}$$

As before, the direction cosine matrix will be time dependent and it is necessary to determine the rate of change of the Euler angles $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$ in terms of the angular velocities p, q and r. Each of the Euler angle rates can be associated with a vector along the axis of rotation. In other words the associated vector for:

$\dot{\psi}$ is along the Z_0-axis (downward when positive),
$\dot{\theta}$ is along the Y_1-axis (starboard when positive),
$\dot{\phi}$ is along the X_2-axis (forward when positive).

These vectors must be summed, using the laws of vector addition, in order to obtain the overall rate of rotation of the system. The vectors are not all mutually orthogonal. The $\dot{\psi}$ vector is normal to the $\dot{\theta}$ vector and the $\dot{\theta}$ vector is normal to the $\dot{\phi}$ vector, but the $\dot{\phi}$ vector is not normal to the $\dot{\psi}$ vector.

In order to sum the vectors it is necessary to first transform them all into the X_3, Y_3, Z_3 axes system.

The $\dot{\psi}$ vector, being associated with the X_0, Y_0, Z_0 system, must have the full transformation matrix $(R_\phi S_\theta T_\psi)$ applied:

$$\begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\theta\sin\phi\cos\psi - \cos\phi\sin\psi & \sin\theta\sin\phi\sin\psi + \cos\phi\cos\psi & \cos\theta\sin\phi \\ \sin\theta\cos\phi\cos\psi + \sin\phi\sin\psi & \sin\theta\cos\phi\sin\psi - \sin\phi\cos\psi & \cos\theta\cos\phi \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} -\dot{\psi}\sin\theta \\ \dot{\psi}\cos\theta\sin\phi \\ \dot{\psi}\cos\theta\cos\phi \end{bmatrix}$$

if $\underset{\sim}{a}_1$, $\underset{\sim}{a}_2$ and $\underset{\sim}{a}_3$ are the unit vectors in the X_3, Y_3, Z_3 axes system then:

$$\underset{\sim}{\dot{\psi}} = - \underset{\sim}{a}_1 \dot{\psi} \sin\theta + \underset{\sim}{a}_2 \dot{\psi} \cos\theta \sin\phi + \underset{\sim}{a}_3 \dot{\psi} \cos\theta \cos\phi$$

The $\dot{\theta}$ vector, being associated with the X_1, Y_1, Z_1 system, must be transformed through the last two rotations $(R_\phi S_\theta)$:

$$\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ \sin\phi\sin\theta & \cos\phi & \sin\phi\cos\theta \\ \cos\phi\sin\theta & -\sin\phi & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{\theta}\cos\phi \\ -\dot{\theta}\sin\phi \end{bmatrix}$$

hence:

$$\dot{\underset{\sim}{\theta}} = \underset{\sim}{a}_2\, \dot{\theta} \cos\phi - \underset{\sim}{a}_3\, \dot{\theta} \sin\phi$$

The $\dot{\phi}$ vector, being associated with the X_2, Y_2, Z_2 system, only needs to be transformed through the last rotation (R_ϕ):

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix}$$

hence:

$$\dot{\underset{\sim}{\phi}} = \underset{\sim}{a}_1\, \dot{\phi}$$

Adding these three equations gives the entire velocity vector equivalent to a single rotation rate about some instantaneous axis of rotation:

$$\underset{\sim}{\omega} = \dot{\underset{\sim}{\psi}} + \dot{\underset{\sim}{\theta}} + \dot{\underset{\sim}{\phi}}$$

$$\begin{aligned} \omega = \; & \underset{\sim}{a}_1(\dot{\theta}-\dot{\psi}\sin\theta) \\ + \; & \underset{\sim}{a}_2(\dot{\theta}\cos\phi+\dot{\psi}\cos\theta\sin\phi) \\ + \; & \underset{\sim}{a}_3(\dot{\psi}\cos\theta\cos\phi-\dot{\theta}\sin\phi) \end{aligned}$$

or:

$$\underset{\sim}{\omega} = \underset{\sim}{a}_1 p + \underset{\sim}{a}_2 q + \underset{\sim}{a}_3 r$$

with:

$$\begin{aligned} p &= \dot{\theta} - \dot{\psi}\sin\theta \\ q &= \dot{\theta}\cos\phi + \dot{\psi}\cos\theta\sin\phi \\ r &= \dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi \end{aligned}$$

which can be solved for $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$ giving:

$$\begin{aligned} \dot{\phi} &= p + q\tan\theta\sin\phi + r\tan\theta\cos\phi \\ \dot{\theta} &= q\cos\phi - r\sin\phi \\ \dot{\psi} &= r\sec\theta\cos\phi + q\sec\theta\sin\phi \end{aligned}$$

These equations can also be obtained by equating the elements of the full Euler angle transformation matrix $(R_\phi S_\theta T_\psi)$ with the rate of change of the elements of the direction cosine matrix (see earlier), so that:

1) $\frac{d(-\sin\theta)}{dt} = \dot{\lambda}_{13} = -q\,\lambda_{33} + r\,\lambda_{23}$

$-\dot{\theta}\cos\theta = -q\cos\theta\cos\phi + r\cos\theta\sin\phi$

$\dot{\theta} = q\cos\phi - r\sin\phi$

2) $\frac{d(\cos\theta\sin\phi)}{dt} = \dot{\lambda}_{23} = -r\,\lambda_{13} + p\,\lambda_{33}$

$-\dot{\theta}\sin\theta\sin\phi + \dot{\phi}\cos\theta\cos\phi = r\sin\theta + p\cos\theta\cos\phi$

substitute for $\dot{\theta}$ (from above):

$\dot{\phi} = p + q\tan\theta\sin\phi + r\tan\theta\cos\phi$

3) $\frac{d(\cos\theta\cos\psi)}{dt} = \dot{\lambda}_{11} = r\,\lambda_{21} - q\,\lambda_{31}$

$$-\dot{\theta}\sin\theta\cos\psi - \dot{\psi}\cos\theta\sin\psi = r(\sin\theta\sin\phi\cos\psi-\cos\phi\sin\psi) - q(\sin\theta\cos\phi\cos\psi+\sin\phi\sin\psi)$$

substitute for $\dot{\theta}$ (from above):

$\dot{\psi} = r\sec\theta\cos\phi + q\sec\theta\sin\phi$

These can be collected together in matrix form:

40

$$
\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sec\theta\sin\phi & \sec\theta\cos\phi \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}
$$

for infinitesimal angles:

$$
\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}
$$

The advantages of the Euler angles are their concise form and readily visible meaning. However, the direction cosine matrix constructed using Euler angles consists of numerous trigonometric functions which are time consuming to compute. It is also worth noting that when $\theta = \pm 90^\circ$ the equations experience a singularity. As θ tends towards $\pm 90^\circ$ so $\sec\theta$ and $\tan\theta$ approach infinity. Both $\dot{\phi}$ and $\dot{\psi}$ will be infinite at $\theta = \pm 90^\circ$, even though $\dot{\theta}$ encounters no such anomaly. In fact, the equations begin to present numerical problems when θ is less than 30° from either $\pm 90^\circ$. Therefore, Euler angles are unsuitable for simulations where the angle of pitch is expected to be large (hopefully this is not very likely in the case of small boats).

Quaternions (Euler Parameters)

The four parameter transformation method was first introduced by Euler in 1776 (the parameters are often referred to as the Euler parameters and denoted by the letter 'e'), as a result of spherical trigonometry considerations. The method was subsequently improved upon by Hamilton in 1843 and the parameters have become known as quaternions.

This alternative approach of representing the relative orientation of two axes systems relies on Euler's theorem which states:

> *"The rotation of any axes system from one arbitrary orientation to some other arbitrary orientation may be expressed by a single rotation about some fixed axis"*

Consider, as in the development of the DCM (Direction Cosine Matrix), two orthogonal coordinate systems X_0,Y_0,Z_0 (the inertial axes) and X,Y,Z (the moving axes) having the same origin and initially coincident. Suppose the X,Y,Z system is then rotated through an angle μ about some instantaneous axis $\underset{\sim}{r}$ which is inclined at the angles α_1, α_2 and α_3 from the X_0, Y_0 and Z_0 axes respectively (and as it happens from the X, Y and Z axes also). It is then possible to determine the four Euler symmetric parameters from these three angles and single rotation.

The rotation of the X,Y,Z coordinate system through the angle μ about the $\underset{\sim}{r}$ axis can be viewed as three separate rotations, namely:

1) A rotation of the X,Y,Z axes which causes the X-axis to become coincident with the $\underset{\sim}{r}$ axis and the new position of the Y-axis to lie in the X_0Y_0-plane.

2) A rotation of the new position of the X,Y,Z axes through an angle μ about the $\underset{\sim}{r}$ axis (presently also the X-axis)

3) A rotation which is the reverse of the first. This restores the original angular separation of the X-axis and $\underset{\sim}{r}$ axis.

By determining the matrix for each separate transformation, the total transformation is then obtained from the product of the three.

The first transformation matrix can be deduced by examining the direction cosines and applying the conditions of orthogonality, and is:

$$
T_1 = \begin{bmatrix} \cos\alpha_1 & \cos\alpha_2 & \cos\alpha_3 \\ \pm\cos\alpha_2\csc\alpha_3 & \pm\cos\alpha_1\csc\alpha_3 & 0 \\ \pm\cos\alpha_1\cot\alpha_3 & \pm\cos\alpha_2\cot\alpha_3 & \pm\sin\alpha_3 \end{bmatrix}
$$

the sign ambiguities can be resolved by making use of the requirement that the matrix must reduce to the identity matrix when α_1 is zero, thus:

$$
T_1 = \begin{bmatrix} \cos\alpha_1 & \cos\alpha_2 & \cos\alpha_3 \\ -\cos\alpha_2\csc\alpha_3 & \cos\alpha_1\csc\alpha_3 & 0 \\ -\cos\alpha_1\cot\alpha_3 & -\cos\alpha_2\cot\alpha_3 & \sin\alpha_3 \end{bmatrix}
$$

The second rotation of μ about $\underset{\sim}{r}$ is simply the two dimensional transformation matrix:

$$
T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\mu & \sin\mu \\ 0 & -\sin\mu & \cos\mu \end{bmatrix}
$$

The third rotation is the inverse of T_1. In other words $[T_3] = [T_1]^{-1}$. The overall transformation is:

$$
[E] = [T_1]^{-1}[T_2][T_1]
$$

and E, for transformations from the inertial frame to the moving body axes, comes out to be:

$$\begin{bmatrix} 1-2\sin^2\frac{\mu}{2}\sin^2\alpha_1 & 2(\sin^2\frac{\mu}{2}\cos\alpha_1\cos\alpha_2 + \sin\frac{\mu}{2}\cos\frac{\mu}{2}\cos\alpha_3) & 2(\sin^2\frac{\mu}{2}\cos\alpha_1\cos\alpha_3 - \sin\frac{\mu}{2}\cos\frac{\mu}{2}\cos\alpha_2) \\ 2(\sin^2\frac{\mu}{2}\cos\alpha_1\cos\alpha_2 - \sin\frac{\mu}{2}\cos\frac{\mu}{2}\cos\alpha_3) & 1-2\sin^2\frac{\mu}{2}\sin^2\alpha_2 & 2(\sin^2\frac{\mu}{2}\cos\alpha_2\cos\alpha_3 + \sin\frac{\mu}{2}\cos\frac{\mu}{2}\cos\alpha_1) \\ 2(\sin^2\frac{\mu}{2}\cos\alpha_1\cos\alpha_3 + \sin\frac{\mu}{2}\cos\frac{\mu}{2}\cos\alpha_2) & 2(\sin^2\frac{\mu}{2}\cos\alpha_2\cos\alpha_3 - \sin\frac{\mu}{2}\cos\frac{\mu}{2}\cos\alpha_1) & 1-2\sin^2\frac{\mu}{2}\sin^2\alpha_3 \end{bmatrix}$$

This is a similarity transformation and therefore the sum of the elements on the leading diagonal are invariant and obey the constraint equation:

$$E_{11} + E_{22} + E_{33} = 1 + 2\cos\mu$$

The Euler symmetric parameters are given as:

$$e_0 = \cos\frac{\mu}{2}$$

$$e_1 = \cos\alpha_1\sin\frac{\mu}{2}$$

$$e_2 = \cos\alpha_2\sin\frac{\mu}{2}$$

$$e_3 = \cos\alpha_3 \sin\frac{\mu}{2}$$

and the transformation matrix simplifies to:

$$E = \begin{bmatrix} e_0^2+e_1^2-e_2^2-e_3^2 & 2(e_0e_3+e_1e_2) & 2(e_1e_3-e_0e_2) \\ 2(e_1e_2-e_0e_3) & e_0^2-e_1^2+e_2^2-e_3^2 & 2(e_0e_1+e_2e_3) \\ 2(e_0e_2+e_1e_3) & 2(e_2e_3-e_0e_1) & e_0^2-e_1^2-e_2^2+e_3^2 \end{bmatrix}$$

and, since only three of the Euler parameters are independent, the constraint equation becomes:

$$e_0^2 + e_1^2 + e_2^2 + e_3^2 = 1$$

furthermore, they are all restricted within the range of ±1.

Again, the inverse of this matrix will be its transpose, thus the matrix for transformations from the moving axes system to the inertial frame using Euler parameters is:

$$\begin{bmatrix} e_0^2+e_1^2-e_2^2-e_3^2 & 2(e_1e_2-e_0e_3) & 2(e_0e_2+e_1e_3) \\ 2(e_0e_3+e_1e_2) & e_0^2-e_1^2+e_2^2-e_3^2 & 2(e_2e_3-e_0e_1) \\ 2(e_1e_3-e_0e_2) & 2(e_0e_1+e_2e_3) & e_0^2-e_1^2-e_2^2+e_3^2 \end{bmatrix}$$

The relationship between the Euler parameters and the direction cosines can be determined by equating the transformation matrices formed from each method. The magnitude of the Euler parameters is obtained from:

$$4e_0^2 = 1 + \lambda_{11} + \lambda_{22} + \lambda_{33}$$

$$4e_1^2 = 1 + \lambda_{11} - \lambda_{22} - \lambda_{33}$$

$$4e_2^2 = 1 - \lambda_{11} + \lambda_{22} - \lambda_{33}$$

$$4e_3^2 = 1 - \lambda_{11} - \lambda_{22} + \lambda_{33}$$

whilst the sign is determined by comparing terms in the transformation matrix, from which it is possible to show that:

$$4e_0e_1 = \lambda_{23} - \lambda_{32}$$

$$4e_0e_2 = \lambda_{31} - \lambda_{13}$$

$$4e_0e_3 = \lambda_{12} - \lambda_{21}$$

by assuming that e_0 is always positive, the signs of the other Euler parameters can then be deduced.

By equating the elements of the Euler angle transformation matrix with the Euler parameter transformation matrix, it is possible to express their relationship as:

$$\tan\phi = \frac{\lambda_{23}}{\lambda_{33}} = 2(e_0e_1+e_2e_3) \;/\; (e_0^2-e_1^2-e_2^2+e_3^2)$$

$$\sin\theta = -\lambda_{13} = -2(e_1e_3-e_0e_2)$$

$$\tan\psi = \frac{\lambda_{12}}{\lambda_{11}} = 2(e_0e_3+e_1e_2) \;/\; (e_0^2+e_1^2-e_2^2-e_3^2)$$

which provides a meaningful output for the user. Through trigonometric manipulation, not shown here, the Euler parameters can be expressed in terms of the Euler angles. This allows the Euler parameters to be initialised if the original orientation of the moving axes system is known. Thus:

$$e_0 = \cos\frac{\psi}{2}\cos\frac{\theta}{2}\cos\frac{\phi}{2} + \sin\frac{\psi}{2}\sin\frac{\theta}{2}\sin\frac{\phi}{2}$$

$$e_1 = \cos\frac{\psi}{2}\cos\frac{\theta}{2}\sin\frac{\phi}{2} - \sin\frac{\psi}{2}\sin\frac{\theta}{2}\sin\frac{\phi}{2}$$

$$e_2 = \cos\frac{\psi}{2}\sin\frac{\theta}{2}\cos\frac{\phi}{2} + \sin\frac{\psi}{2}\cos\frac{\theta}{2}\sin\frac{\phi}{2}$$

$$e_3 = \sin\frac{\psi}{2}\cos\frac{\theta}{2}\cos\frac{\phi}{2} - \cos\frac{\psi}{2}\sin\frac{\theta}{2}\sin\frac{\phi}{2}$$

The rate of change of the Euler parameters can be expressed in terms of the spin component velocities p, q and r by differentiating the Euler parameters and equating the resultant elements with those in the rate of change of the direction cosines. A similar matrix results:

$$\begin{bmatrix} \dot{e}_0 \\ \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix} \begin{bmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

alternatively the spin components can be given by:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = 2 \begin{bmatrix} -e_1 & +e_0 & +e_3 & -e_2 \\ -e_2 & -e_3 & +e_0 & +e_1 \\ -e_3 & +e_2 & -e_1 & +e_0 \end{bmatrix} \begin{bmatrix} \dot{e}_0 \\ \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix}$$

The direction cosine matrix constructed using the Euler parameters does not suffer from any singularities and the constraint equation is only required to prevent drift in the parameters due mainly to the slight computational inaccuracies inherent in extended calculations performed on a computer. There are no trigonometric functions to be evaluated (with the exception of determining the Euler angles for user interaction purposes). The Euler parameters therefore allow an overall reduction in the number of arithmetic operations required and can provide time savings during simulation.

CHAPTER 5

APPROACH TO THE MATHEMATICAL MODEL

5.01 Modularity

Mathematical models of marine vehicles have been revolutionised over the past few years by modular techniques and new models are moving away from the regressional methodology. Dand, 1987 (Ref.43), indicates how regressional models are being superseded by modular models, with particular reference to work carried out at BMT (British Maritime Technology).

In the past, simulator models have been constructed using regressional techniques, especially as the equations of motion are usually based upon a multi-variable Taylor series expansion of the states about some initial equilibrium condition. The hydrodynamic forces and moments acting on a vessel are therefore presented in terms which combine the motion variables u, v and r and some regression coefficients.

Global multi-variable regression is applied to ship trials data or to scaled model test data to assign values to these coefficients. Consequently, the propeller and rudder coefficients will be drawn into the regression analysis along with all the other terms, and they will not specifically incorporate information on propeller or rudder geometry. Instead, the coefficients will relate to the particular data from whence they are derived and will assume, or take on, values which generate the correct states at given propeller revolutions and rudder angles. Naturally the characteristics of the rudder and propeller are subsumed into the regression coefficients since the model apes the real situation.

Dand defines the pure regressional model as:

> *'A model which performs satisfactorily when taken as a whole, but which does not allow individual elements to be changed readily as the design is changed'*

The modular technique is to describe the individual elements, such as hull, rudder, propeller and so forth, as separate modules which will be incorporated within the overall system. In principle, it is possible to alter a single module without affecting any of the other modules, that is any given module does not require a knowledge of the contents of the remaining modules.

This concept potentially provides an extremely flexible design tool, for example simulations can be performed with different rudder designs in order to assess the effect on manoeuvrability. The forces and moments contained within each module will be constructed with reference to the particular physical processes involved; this provides a far more rigorous structure than a regressional model.

The modular approach does suffer from problems related to the connections and data pathways between the modules. The behaviour of one module will inevitably affect that of another. Furthermore, a number of variables will end up being global to the whole simulation, especially when duplicate calculations are to be removed in order to save time. Other problems arise from the general shortage of data with which to construct the separate modules, especially as most experimental does not cover the complete operating environment usually encountered.

The author believes that, despite any drawbacks, the future of marine simulation models lies within the realms of the modular format and consequently this thesis will adopt such an approach.

5.02 The Division Of The Forces

The forces and moments acting on a boat can be broadly grouped into four categories, namely the inertial forces, the damping forces, the restoring forces and the exciting forces. The inertial forces produce a resistance when the boat is set in motion. The damping forces act in opposition to the boat's motion in such a manner that they always tend to reduce the motion. The restoring forces act to always bring the boat back to its equilibrium position. The exciting forces can be sub-divided further into those due to the control surfaces, such as the rudder and propeller, which provide manoeuvrability and those due to disturbances, such as the wind and waves, which cause unwanted external forces.

CHAPTER 6

EQUATIONS OF MOTION OF A SMALL BOAT

6.01 Newton's Law Of Motion

The equations of motion of a body moving in a fluid can be summarised by Newton's second law of motion:

$$F = m\,a$$

where F is the total external force applied to the body, m is the actual mass of the body, and a is the acceleration in the direction of F. This requires no further explanation here, but since the mathematical model includes moments, it seems appropriate to digress to a discussion of moments of inertia.

6.02 Inertia Tensor

The general equation, which corresponds to Newton's second law, used to express the relationship of a moment and rotational acceleration is:

$$M = I\,\dot{\omega}$$

where M is the moment, I is a tensor quantity called the moment of inertia and $\dot{\omega}$ is the rotational acceleration.

The inertia tensor can be represented in the form of a

3 x 3 matrix. The components of this tensor in the boat fixed axes system can be represented thus:

$$I = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$$

The diagonal elements, I_{xx}, I_{yy} and I_{zz}, are called the moments of inertia about the x-axis, y-axis and z-axis respectively. The negatives of the off-diagonal elements I_{xy}, I_{xz} et cetera are termed the products of inertia.

The elements of I are given as:

$$I = \begin{bmatrix} \sum_i m_i(y_i^2+z_i^2) & -\sum_i m_i x_i y_i & -\sum_i m_i x_i z_i \\ -\sum_i m_i y_i x_i & \sum_i m_i(x_i^2+z_i^2) & -\sum_i m_i y_i z_i \\ -\sum_i m_i z_i x_i & -\sum_i m_i z_i y_i & \sum_i m_i(x_i^2+y_i^2) \end{bmatrix}$$

The inertia tensor is clearly symmetric about the leading diagonal and, therefore, only six independent elements are required to construct I.

Birbanescu-Biran, 1987 (Ref.27), presents a general method of calculating the mass, centre of gravity and the inertia tensor for subdivisions of a ship, and subsequently the whole ship, given the mass data of all items within the ship. A simplifying assumption that the mass distribution of any ship item is a function of the longitudinal coordinate x only. The formulae for the mass moments of inertia of the *i*th ship item are given as:

$$I_{xx_i}(x) = \begin{cases} 0 & , \quad x < X_{AFT_i} \\ \int_{X_{AFT_i}}^{x} u^2 dW_i(u) & , X_{AFT_i} <= x < X_{FWD_i} \\ \int_{X_{AFT_i}}^{X_{FWD_i}} u^2 dW_i(u) & , X_{FWD_i} <= x \end{cases}$$

$$I_{yy_i}(x) = \begin{cases} 0 & , \quad x < X_{AFT_i} \\ Y_{CG_i}^2 W_i(x) + I'_{yy_i}(x) & , X_{AFT_i} <= x < X_{FWD_i} \\ Y_{CG_i}^2 W_i + I'_{yy_i} & , X_{FWD_i} <= x \end{cases}$$

$$I_{zz_i}(x) = \begin{cases} 0 & , \quad x < X_{AFT_i} \\ Z^2_{CG_i} W_i(x) + I'_{zz_i}(x) & , X_{AFT_i} <= x < X_{FWD_i} \\ Z^2_{CG_i} W_i + I'_{zz_i} & , X_{FWD_i} <= x \end{cases}$$

$$I_{xy_i}(x) = \begin{cases} 0 & , \quad x < X_{AFT_i} \\ Y_{CG_i} M_{yz_i}(x) + I_{\xi\eta_i}(x) & , X_{AFT_i} <= x < X_{FWD_i} \\ X_{CG_i} Y_{CG_i} W_i + I_{\xi\eta_i} & , X_{FWD_i} <= x \end{cases}$$

$$I_{yz_i}(x) = \begin{cases} 0 & , \quad x < X_{AFT_i} \\ Y_{CG_i} Z_{CG_i} W_i(x) + I_{\eta\zeta_i}(x) & , X_{AFT_i} <= x < X_{FWD_i} \\ Y_{CG_i} Z_{CG_i} W_i + I_{\eta\zeta_i} & , X_{FWD_i} <= x \end{cases}$$

$$
I_{zx_i}(x) = \begin{cases} 0 & , \quad x < X_{AFT_i} \\ Z_{CG_i} M_{yz_i}(x) + I_{\zeta\xi_i}(x) & , X_{AFT_i} <= x < X_{FWD_i} \\ X_{CG_i} Z_{CG_i} W_i + I_{\zeta\xi_i} & , X_{FWD_i} <= x \end{cases}
$$

The total distribution function for the whole ship is therefore:

$$I_{xx}(x) = \sum_{i=1}^{n} I_{xx_i}(x)$$

$$I_{yy}(x) = \sum_{i=1}^{n} I_{yy_i}(x)$$

$$I_{zz}(x) = \sum_{i=1}^{n} I_{zz_i}(x)$$

$$I_{xy}(x) = \sum_{i=1}^{n} I_{xy_i}(x)$$

$$I_{yz}(x) = \sum_{i=1}^{n} I_{yz_i}(x)$$

$$I_{zx}(x) = \sum_{i=1}^{n} I_{zx_i}(x)$$

where: i is a suffix meaning the *i*th ship item; I'_{yy} and I'_{zz} are moments of inertia about planes parallel to the

ZX-plane and XY-plane respectively; $I_{\xi\eta}$, $I_{\eta\zeta}$ and $I_{\zeta\xi}$ are products of inertia about the corresponding ξ, η and ζ axes, which are parallel to the X, Y and Z axes respectively; X_{CG}, Y_{CG} and Z_{CG} are the coordinates of the centre of gravity; W is the mass distribution function; M is the mass moment distribution function; and u is a dummy variable of integration.

However, calculation of the integrals or summations is involved and requires a large computational overhead. The design of such a program is beyond the scope of this thesis. The mathematics is included for reference purposes, in practice the products of inertia will be assumed small enough to be neglected and approximate empirical formulae will be deemed sufficient to compute the moments of inertia on the leading diagonal of I.

The yaw moment is usually assumed to be about the same order as the pitch moment, which in naval architecture is given as:

$$I_{zz} \cong I_{yy} = m\ (0.25\ L_{BP})^2$$

In the case of the Arun, the length factor turns out to be more like 0.23 rather than 0.25.

6.03 Expanding Newton's Second Law

Expanding the mass acceleration product of Newton's second law in terms of the six degrees-of-freedom allows the complete equations describing the motion of a rigid body to be written thus:

$$X = m[\dot{u}+wq-vr-x_g(qq+rr)+y_g(pq-\dot{r})+z_g(rp+\dot{q})]$$

$$Y = m[\dot{v}+ur-wp-y_g(rr+pp)+z_g(rq-\dot{p})+x_g(pq+\dot{r})]$$

$$Z = m[\dot{w}+vp-uq-z_g(pp+qq)+x_g(rp-\dot{q})+y_g(rq+\dot{p})]$$

$$L = I_{xx}\dot{p}-I_{xy}\dot{q}-I_{xz}\dot{r}+I_{yx}pr-I_{yy}qr+I_{yz}rr-I_{zx}pq-I_{zy}qq+I_{zz}rq$$
$$+m[y_g(\dot{w}+vp-uq)-z_g(\dot{v}+ur-wp)]$$

$$M = I_{xx}pr-I_{xy}qr-I_{xz}rr-I_{yx}\dot{p}+I_{yy}\dot{q}-I_{yz}\dot{r}+I_{zx}pp+I_{zy}qp-I_{zz}rp$$
$$+m[z_g(\dot{u}+wq-vr)-x_g(\dot{w}+vp-uq)]$$

$$N = -I_{xx}pq+I_{xy}qq+I_{xz}rq-I_{yx}pp+I_{yy}qp-I_{yz}rp-I_{zx}\dot{p}-I_{zy}\dot{q}+I_{zz}\dot{r}$$
$$+m[x_g(\dot{v}+ur-wp)-y_g(\dot{u}+wq-vr)]$$

The left-hand side represents the forces and moments along and about the coordinate axes, whilst the right-hand side shows the corresponding dynamic response terms.

At this point, most existing ship models perform a Taylor series expansion of the forces and moments as a function of the properties of the body, the properties of the fluid and the properties of the motion. The majority of ship models restrict the motions considered to the horizontal plane and therefore represent the forces and moments with respect to the three degrees-of-freedom motion parameters and the rudder deflection, thus:

$$\left.\begin{array}{c} X \\ Y \\ Z \end{array}\right\} \quad f(\dot{u},\dot{v},\dot{r},u,v,r,\delta)$$

A Taylor series expansion of a function of several variables about an initial equilibrium condition is performed. Using the straight ahead motion at constant speed with rudder amidships as the chosen initial conditions, gives a Taylor series expansion, with terms up to and including the third order, of:

$$\begin{aligned} X = X_* &+ [X_u\Delta u + X_v v + X_r r + X_{\dot{u}}\dot{u} + X_{\dot{v}}\dot{v} + X_{\dot{r}}\dot{r} + X_\delta\delta] \\ &+ \frac{1}{2!}\ [X_{uu}\Delta u^2 + X_{vv}v^2 + \ \ldots. + X_{\delta\delta}\delta^2 \ + \\ &\qquad 2X_{uv}\Delta uv + 2X_{ur}\Delta ur + \ldots. + 2X_{\dot{r}\delta}\dot{r}\delta] \\ &+ \frac{1}{3!}\ [X_{uuu}\Delta u^3 + X_{vvv}v^3 + \ldots. + X_{\delta\delta\delta}\delta^3 + \\ &\qquad 3X_{uuv}\Delta u^2 v + 3X_{uur}\Delta u^2 r + \ldots. + 3X_{\dot{r}\delta\delta}\dot{r}\delta^2 + \\ &\qquad 6X_{uvr}\Delta uvr + 6X_{uv\dot{u}}\Delta uv\dot{u} + \ldots. + 6X_{\dot{v}\dot{r}\delta}\dot{v}\dot{r}\delta] \end{aligned}$$

where the dots indicate similar terms in functions of u, v, r and δ. Similar expressions are developed for sway and yaw.

The number of hydrodynamic terms produced is large and they begin to bear little clearly defined relation to the physical ship. For this reason, many modellers content themselves with the linearised equations by limiting the expansion to the first order terms, where the surge force becomes:

$$X = X_{*} + X_u \Delta u + X_v v + X_r r + X_{\dot{u}} \dot{u} + X_{\dot{v}} \dot{v} + X_{\dot{r}} \dot{r} + X_\delta \delta$$

Quite often some of the higher order terms which are found to have a significant effect are re-introduced into the model to produce a Quasi-linear or non-linear model.

In this analysis the author follows the basic approach of Japanese researchers and splits the forces and moments into separate categories so that each component contributing to the motion of the boat is contained within an individual module. The categories chosen can be represented as follows:

$$
\begin{aligned}
&\text{Surge:}\quad X = X_D + X_H + X_G + X_R + X_P + X_T + X_W + X_S + X_V \\
&\text{Sway:}\quad Y = Y_D + Y_H + Y_G + Y_R + Y_P + Y_T + Y_W + Y_S + Y_V \\
&\text{Heave:}\quad Z = Z_D + Z_H + Z_G + Z_R + Z_P + Z_T + Z_W + Z_S + Z_V \\
&\text{Roll:}\quad L = L_D + L_H + L_G + L_R + L_P + L_T + L_W + L_S + L_V \\
&\text{Pitch:}\quad M = M_D + M_H + M_G + M_R + M_P + M_T + M_W + M_S + M_V \\
&\text{Yaw:}\quad N = N_D + N_H + N_G + N_R + N_P + N_T + N_W + N_S + N_V
\end{aligned}
$$

where the subscripts D, H, G, R, P, T, W, S, V denote the dynamic, hydrodynamic, gravity, rudder, propeller, trim tab, wind, sea inertia and wave forces and moments respectively.

Each of the nine categories will be described by a 6x1 matrix or vector of the form:

$$
\begin{bmatrix}
\text{Surge Force} & X \\
\text{Sway Force} & Y \\
\text{Heave Force} & Z \\
\text{Roll Moment} & L \\
\text{Pitch Moment} & M \\
\text{Yaw Moment} & N
\end{bmatrix}_i
$$

where i = D,H,G,R,P,T,W,S,V

This will relate to Newton's second law thus:

$$\begin{array}{ccccc} \text{Vector Sum of Forces \& Moments} & = & \text{Mass Matrix} & \times & \text{Acceleration Vector} \end{array}$$

$$\text{Sum} \begin{bmatrix} \text{Surge} \\ \text{Sway} \\ \text{Heave} \\ \text{Roll} \\ \text{Pitch} \\ \text{Yaw} \end{bmatrix}_i = \begin{bmatrix} \\ 6 \times 6 \\ \text{Mass} \\ \text{Matrix} \\ \\ \end{bmatrix} \times \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}$$

To give an indication of the variables influencing the various forces and moments, this can be generally represented after Thomasson et al, 1984 (Ref.89), as:

$$\text{Mass} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{aligned} & \text{Dynamic}(u,v,w,p,q,r,m,C_g,I,H) \\ & + \text{Hydrodynamic}(u_r,v_r,w_r,p,q,r,C_b,\bar{m},H) \\ & + \text{Gravity}(\lambda_{ij},g,\rho_w,\nabla,m,C_b,C_g) \\ & + \text{Rudder}(F_n,\delta,x_R,y_R,z_R) \\ & + \text{Propeller}(\rho_w,D,u,v,w,n,K_T,C_{PU},w_P) \\ & + \text{Tabs}(u,v,w,\alpha_T) \\ & + \text{Wind}(\rho_a,L_{OA},A_{TS},A_{LS},H_{WL},\psi,\gamma,V_w) \\ & + \text{Sea}(\dot{u}_s,\dot{v}_s,\dot{w}_s,u_s,v_s,w_s,p,q,r,C_b,\bar{m},H) \\ & + \text{Wave}(\rho_w,g,L_{BP},\psi,\beta_v,\zeta,\lambda_v) \end{aligned}$$

where Mass is the Mass matrix.

The forces and moments will be functions of the state variables, plus some other values, which in turn generate the accelerations or state derivatives. Therefore, in order to simultaneously solve these equations in terms of the state variables at discrete time intervals, it becomes necessary to write the equations in such a manner that the state derivatives or accelerations appear on the left-hand side of the equation thus:

$$a = m^{-1} F$$

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \\ & 6 \text{ x } 6 & \\ & \text{Mass} & \\ & \text{Matrix} & \\ \\ \end{bmatrix}^{-1} \times \begin{bmatrix} X_{Total} \\ Y_{Total} \\ Z_{Total} \\ L_{Total} \\ M_{Total} \\ N_{Total} \end{bmatrix}$$

6.04 Added Mass

The general equations of motion of a ship partially immersed in water contain terms which are due to the added mass effect. The concept of added mass is well known and its effects have been included in all accurate ship simulation models. However, most ship modellers are content to make only a vague reference to this phenomenon. What it actually is can be described thus: A body moving in a fluid behaves as if it has more mass than is actually the case; this apparent increase is termed added mass.

Various names are attributed to this phenomenon and they include: "virtual mass", "ascension to mass", apparent mass" and "hydrodynamic mass". Equally there exist a number of different definitions, as Motora, 1960 (Ref.110), demonstrates with these examples:

a) Added mass m_1 is defined as the difference between the moment of inertia in a vacuum $m\dot{v}$ and that in a fluid $(m+m_1)\dot{v}$;

b) Added mass m' is defined as the difference between the period of oscillation in a vacuum $2\pi\sqrt{m/k}$ and that in a fluid $2\pi\sqrt{(m+m')/k}$;

c) Added mass m'' is defined as the difference between the momentum in a vacuum mv and that in a fluid $(m+m'')v$;

d) Added mass $\hat{m}$ is defined as the difference between the kinetic energy in a vacuum $\frac{1}{2}mv^2$ and that in a fluid $\frac{1}{2}(m+\hat{m})v^2$.

With the exception of the case of motion in an ideal fluid without free surfaces effects, these different definitions will not always coincide. Many treatments of the topic of added mass are biased towards a specific application and therefore present equations with only those terms which are required for that use.

In this analysis the derivation of the added mass expressions will be based upon the kinetic energy approach. Lamb, 1879 (Ref.86), was perhaps the first to document this method, but Imlay, 1961 (Ref.74), gives a clearer text for those first entering the subject.

The presence of a fluid surrounding a boat introduces the phenomenon of added mass. If the boat is moving then it will induce a motion in the otherwise stationary fluid. This is because the fluid has to move out of the way and then close in behind the boat in order that the boat may itself make headway through the fluid. Consequently, the fluid will possess kinetic energy imparted by the boat doing work on the fluid. It is therefore necessary that the equations of motion take into account the kinetic energy given to the fluid, and this is performed through the added mass terms.

If the boat motion is steady, then the corresponding fluid motion will be steady and the kinetic energy in the fluid is constant. The added mass terms can therefore be omitted from the equations if, and only if, the boat motion is steady.

When the motion is accelerated, this implies that the motion of the boat is in a state of change and there must be an associated change in the kinetic energy of the surrounding fluid. That is to say, the boat must do work on the fluid in order to accelerate. In applying a force to the fluid the boat will experience an opposite reaction force. Therefore, the force required to accelerate the boat must be greater than the reaction force.

Lamb's approach to obtaining an equation for this kinetic energy given to the fluid begins by supposing the motion of the fluid to be characterised by a single valued velocity potential ϕ which satisfies the equation of continuity:

$$\nabla^2\phi = 0$$

Then if the motion of a body through the fluid at any instant is defined by the translational velocities u, v and w and the angular velocities p, q and r, it is possible to describe the velocity potential with six components:

$$\phi = u\phi_1 + v\phi_2 + w\phi_3 + p\chi_1 + q\chi_2 + r\chi_3$$

The kinetic energy **T** of the fluid is written:

$$2\mathbf{T} = -\rho \int\int \phi \frac{\partial\phi}{\partial n} dS$$

where the integration will extend over the surface of the moving solid. Substituting for ϕ gives:

$$\begin{aligned}2\mathbf{T} = \; & \mathbf{A}u^2 + \mathbf{B}v^2 + \mathbf{C}w^2 + 2\mathbf{A}'vw + 2\mathbf{B}'wu + 2\mathbf{C}'uv \\ & + \mathbf{P}p^2 + \mathbf{Q}q^2 + \mathbf{R}r^2 + 2\mathbf{P}'qr + 2\mathbf{Q}'rp + 2\mathbf{R}'pq \\ & + 2p(\mathbf{F}u + \mathbf{G}v + \mathbf{H}w) + 2q(\mathbf{F}'u + \mathbf{G}'v + \mathbf{H}'w) \\ & + 2r(\mathbf{F}''u + \mathbf{G}''v + \mathbf{H}''w)\end{aligned}$$

where the twenty-one coefficients are constants determined by the form and shape of the surface relative to the coordinate axes. Lamb gives as examples:

$$\begin{aligned}\mathbf{A} &= -\rho \int\int \phi_1 \frac{\partial\phi_1}{\partial n} dS \\ &= \rho \int\int \phi_1 \, l \, dS\end{aligned}$$

$$\begin{aligned}\mathbf{A}' &= -\frac{\rho}{2} \int\int \left(\phi_2 \frac{\partial\phi_3}{\partial n} + \phi_3 \frac{\partial\phi_2}{\partial n} \right) dS \\ &= \rho \int\int \phi_2 \, n \, dS \\ &= \rho \int\int \phi_3 \, m \, dS\end{aligned}$$

$$P = -\rho \int\int \chi_1 \frac{\partial \chi_1}{\partial n} dS$$

$$= \rho \int\int \chi_1 \ (ny - mz) \ dS$$

where l, m and n denote the direction cosines of the normal, drawn towards the fluid, at any point of this surface.

The general expression for kinetic energy in the fluid can be written in modern notation as:

$$\begin{aligned} 2T = & - X_{\dot{u}}u^2 - Y_{\dot{v}}v^2 - Z_{\dot{w}}w^2 - 2Y_{\dot{w}}vw - 2X_{\dot{w}}wu - 2X_{\dot{v}}uv \\ & - L_{\dot{p}}p^2 - M_{\dot{q}}q^2 - N_{\dot{r}}r^2 - 2M_{\dot{r}}qr - 2L_{\dot{r}}rp - 2L_{\dot{q}}pq \\ & - 2p(X_{\dot{p}}u + Y_{\dot{p}}v + Z_{\dot{p}}w) - 2q(X_{\dot{q}}u + Y_{\dot{q}}v + Z_{\dot{q}}w) \\ & - 2r(X_{\dot{r}}u + Y_{\dot{r}}v + Z_{\dot{r}}w) \end{aligned}$$

Lamb's coefficients, summarised in matrix form:

$$\begin{bmatrix} A & C' & B' & F & F' & F'' \\ & B & A' & G & G' & G'' \\ & & C & H & H' & H'' \\ & & & P & R' & Q' \\ & & & & Q & P' \\ & & & & & R \end{bmatrix}$$

relate to the presently established symbolism and those used by the author as follows:

$$\begin{bmatrix} -X_{\dot{u}} & -X_{\dot{v}} & -X_{\dot{w}} & -X_{\dot{p}} & -X_{\dot{q}} & -X_{\dot{r}} \\ & -Y_{\dot{v}} & -Y_{\dot{w}} & -Y_{\dot{p}} & -Y_{\dot{q}} & -Y_{\dot{r}} \\ & & -Z_{\dot{w}} & -Z_{\dot{p}} & -Z_{\dot{q}} & -Z_{\dot{r}} \\ & & & -L_{\dot{p}} & -L_{\dot{q}} & -L_{\dot{r}} \\ & & & & -M_{\dot{q}} & -M_{\dot{r}} \\ & & & & & -N_{\dot{r}} \end{bmatrix}$$

where $X_{\dot{u}} = \dfrac{\partial X}{\partial \dot{u}}$

and is the partial derivative of the surge force with respect to the forward acceleration. It is known as a hydrodynamic derivative. The other coefficients have similarly implied meanings.

From Newton's second law:

$$F = m\ a$$

now if the reaction force is denoted F_1 and the remainder of the contributions to F are denoted F_2 then this becomes:

$$F_1 + F_2 = m\ a$$

where F_1 is the general force with the components X_1, Y_1 and Z_1 along the X, Y and Z axes and L_1, M_1 and N_1 about those axes.

The force components due to the kinetic energy can then be written:

$$X_1 = -\frac{d}{dt}\frac{\partial T}{\partial u} - q\frac{\partial T}{\partial w} + r\frac{\partial T}{\partial v}$$

$$Y_1 = -\frac{d}{dt}\frac{\partial T}{\partial v} - r\frac{\partial T}{\partial u} + p\frac{\partial T}{\partial w}$$

$$Z_1 = -\frac{d}{dt}\frac{\partial T}{\partial w} - p\frac{\partial T}{\partial v} + q\frac{\partial T}{\partial u}$$

$$L_1 = -\frac{d}{dt}\frac{\partial T}{\partial p} - q\frac{\partial T}{\partial r} + r\frac{\partial T}{\partial q} - v\frac{\partial T}{\partial w} + w\frac{\partial T}{\partial v}$$

$$M_1 = -\frac{d}{dt}\frac{\partial T}{\partial q} - r\frac{\partial T}{\partial p} + p\frac{\partial T}{\partial r} - w\frac{\partial T}{\partial u} + u\frac{\partial T}{\partial w}$$

$$N_1 = -\frac{d}{dt}\frac{\partial T}{\partial r} - q\frac{\partial T}{\partial q} + r\frac{\partial T}{\partial p} - u\frac{\partial T}{\partial v} + v\frac{\partial T}{\partial u}$$

Six partial derivatives must be obtained from the equation for twice the total kinetic energy in order to expand the previous set of equations. These are:

$$\frac{\partial T}{\partial u} = - X_{\dot{u}}u - X_{\dot{v}}v - X_{\dot{w}}w - X_{\dot{p}}p - X_{\dot{q}}q - X_{\dot{r}}r$$

$$\frac{\partial T}{\partial v} = - X_{\dot{v}}u - Y_{\dot{v}}v - Y_{\dot{w}}w - Y_{\dot{p}}p - Y_{\dot{q}}q - Y_{\dot{r}}r$$

$$\frac{\partial T}{\partial w} = - X_{\dot{w}}u - Y_{\dot{w}}v - Z_{\dot{w}}w - Z_{\dot{p}}p - Z_{\dot{q}}q - Z_{\dot{r}}r$$

$$\frac{\partial T}{\partial p} = - X_{\dot{p}}u - Y_{\dot{p}}v - Z_{\dot{p}}w - L_{\dot{p}}p - L_{\dot{q}}q - L_{\dot{r}}r$$

$$\frac{\partial T}{\partial q} = - X_{\dot{q}}u - Y_{\dot{q}}v - Z_{\dot{q}}w - L_{\dot{q}}p - M_{\dot{q}}q - M_{\dot{r}}r$$

$$\frac{\partial T}{\partial r} = - X_{\dot{r}}u - Y_{\dot{r}}v - Z_{\dot{r}}w - L_{\dot{r}}p - M_{\dot{r}}q - N_{\dot{r}}r$$

By substituting these partial derivatives into the equations for the components of F_1 yields the complete expressions for added mass with reference to a set of orthogonal axes fixed in the boat moving in a frictionless fluid. After Imlay, 1961 (Ref.74), these can be represented thus:

$$\begin{aligned} X_1 = {} & X_{\dot{u}}\dot{u} + X_{\dot{v}}\dot{v} + X_{\dot{w}}\dot{w} + X_{\dot{p}}\dot{p} + X_{\dot{q}}\dot{q} + X_{\dot{r}}\dot{r} \\ & - X_{\dot{v}}ur - Y_{\dot{v}}vr - Y_{\dot{w}}wr - Y_{\dot{p}}pr - Y_{\dot{q}}qr - Y_{\dot{r}}rr \\ & + X_{\dot{w}}uq + Y_{\dot{w}}vq + Z_{\dot{w}}wq + Z_{\dot{p}}pq + Z_{\dot{q}}qq + Z_{\dot{r}}rq \end{aligned}$$

$$\begin{aligned} Y_1 = {} & X_{\dot{u}}ur + X_{\dot{v}}vr + X_{\dot{w}}wr + X_{\dot{p}}pr + X_{\dot{q}}qr + X_{\dot{r}}rr \\ & + X_{\dot{v}}\dot{u} + Y_{\dot{v}}\dot{v} + Y_{\dot{w}}\dot{w} + Y_{\dot{p}}\dot{p} + Y_{\dot{q}}\dot{q} + Y_{\dot{r}}\dot{r} \\ & - X_{\dot{w}}up - Y_{\dot{w}}vp - Z_{\dot{w}}wp - Z_{\dot{p}}pp - Z_{\dot{q}}qp - Z_{\dot{r}}rp \end{aligned}$$

$$
\begin{aligned}
Z_1 = \; & - X_{\dot u}uq - X_{\dot v}vq - X_{\dot w}wq - X_{\dot p}pq - X_{\dot q}qq - X_{\dot r}rq \\
& + X_{\dot v}up + Y_{\dot v}vp + Y_{\dot w}wp + Y_{\dot p}pp + Y_{\dot q}qp + Y_{\dot r}rp \\
& + X_{\dot w}\dot u + Y_{\dot w}\dot v + Z_{\dot w}\dot w + Z_{\dot p}\dot p + Z_{\dot q}\dot q + Z_{\dot r}\dot r
\end{aligned}
$$

$$
\begin{aligned}
L_1 = \; & - X_{\dot v}uw - Y_{\dot v}vw - Y_{\dot w}ww - Y_{\dot p}pw - Y_{\dot q}qw - Y_{\dot r}rw \\
& + X_{\dot w}uv + Y_{\dot w}vv + Z_{\dot w}wv + Z_{\dot p}pv + Z_{\dot q}qv + Z_{\dot r}rv \\
& + X_{\dot p}\dot u + Y_{\dot p}\dot v + Z_{\dot p}\dot w + L_{\dot p}\dot p + L_{\dot q}\dot q + L_{\dot r}\dot r \\
& - X_{\dot q}ur - Y_{\dot q}vr - Z_{\dot q}wr - L_{\dot q}pr - M_{\dot q}qr - M_{\dot r}rr \\
& + X_{\dot r}uq + Y_{\dot r}vq + Z_{\dot r}wq + L_{\dot r}pq + M_{\dot r}qq + N_{\dot r}rq
\end{aligned}
$$

$$
\begin{aligned}
M_1 = \; & X_{\dot u}uw + X_{\dot v}vw + X_{\dot w}ww + X_{\dot p}pw + X_{\dot q}qw + X_{\dot r}rw \\
& - X_{\dot w}uu - Y_{\dot w}vu - Z_{\dot w}wu - Z_{\dot p}pu - Z_{\dot q}qu - Z_{\dot r}ru \\
& + X_{\dot p}ur + Y_{\dot p}vr + Z_{\dot p}wr + L_{\dot p}pr + L_{\dot q}qr + L_{\dot r}rr \\
& + X_{\dot q}\dot u + Y_{\dot q}\dot v + Z_{\dot q}\dot w + L_{\dot q}\dot p + M_{\dot q}\dot q + M_{\dot r}\dot r \\
& - X_{\dot r}up - Y_{\dot r}vp - Z_{\dot r}wp - L_{\dot r}pp - M_{\dot r}qp - N_{\dot r}rp
\end{aligned}
$$

$$
\begin{aligned}
N_1 = \; & - X_{\dot u}uv - X_{\dot v}vv - X_{\dot w}wv - X_{\dot p}pv - X_{\dot q}qv - X_{\dot r}rv \\
& + X_{\dot v}uu + Y_{\dot v}vu + Y_{\dot w}wu + Y_{\dot p}pu + Y_{\dot q}qu + Y_{\dot r}ru \\
& - X_{\dot p}uq - Y_{\dot p}vq - Z_{\dot p}wq - L_{\dot p}pq - L_{\dot q}qq - L_{\dot r}rq \\
& + X_{\dot q}up + Y_{\dot q}vp + Z_{\dot q}wp + L_{\dot q}pp + M_{\dot q}qp + M_{\dot r}rp \\
& + X_{\dot r}\dot u + Y_{\dot r}\dot v + Z_{\dot r}\dot w + L_{\dot r}\dot p + M_{\dot r}\dot q + N_{\dot r}\dot r
\end{aligned}
$$

The general expressions for added mass contain 21 different constants, however, theoretically there are 36 constants which relate the six components of force and moment to the accelerations in the six degrees-of-freedom, these can be depicted as an array:

$$\begin{bmatrix} X_{\dot{u}} & X_{\dot{v}} & X_{\dot{w}} & X_{\dot{p}} & X_{\dot{q}} & X_{\dot{r}} \\ Y_{\dot{u}} & Y_{\dot{v}} & Y_{\dot{w}} & Y_{\dot{p}} & Y_{\dot{q}} & Y_{\dot{r}} \\ Z_{\dot{u}} & Z_{\dot{v}} & Z_{\dot{w}} & Z_{\dot{p}} & Z_{\dot{q}} & Z_{\dot{r}} \\ L_{\dot{u}} & L_{\dot{v}} & L_{\dot{w}} & L_{\dot{p}} & L_{\dot{q}} & L_{\dot{r}} \\ M_{\dot{u}} & M_{\dot{v}} & M_{\dot{w}} & M_{\dot{p}} & M_{\dot{q}} & M_{\dot{r}} \\ N_{\dot{u}} & N_{\dot{v}} & N_{\dot{w}} & N_{\dot{p}} & N_{\dot{q}} & N_{\dot{r}} \end{bmatrix}$$

In a real fluid all 36 coefficients may well be distinct, but in an ideal (frictionless) fluid the coefficients which are symmetrical with the respect to the leading diagonal will be equal. It is therefore sufficient to only retain the coefficients on and above the leading diagonal, thus:

$$\begin{bmatrix} X_{\dot{u}} & X_{\dot{v}} & X_{\dot{w}} & X_{\dot{p}} & X_{\dot{q}} & X_{\dot{r}} \\ & Y_{\dot{v}} & Y_{\dot{w}} & Y_{\dot{p}} & Y_{\dot{q}} & Y_{\dot{r}} \\ & & Z_{\dot{w}} & Z_{\dot{p}} & Z_{\dot{q}} & Z_{\dot{r}} \\ & & & L_{\dot{p}} & L_{\dot{q}} & L_{\dot{r}} \\ & & & & M_{\dot{q}} & M_{\dot{r}} \\ & & & & & N_{\dot{r}} \end{bmatrix}$$

These 21 added mass derivatives are functions of the shape of the boat and density of the fluid only. They are necessary and sufficient to completely describe the added mass properties of a boat moving in any manner in an ideal fluid. The values for the added mass coefficients in a real fluid have been found, by other researchers, to be in good agreement to the ideal fluid coefficients.

6.05 Added Mass Coefficients For An Ellipsoid

The equation of an ellipsoid is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

where a, b and c are the semi-major, semi-minor and semi-vertical axes of the ellipsoid respectively.

Lamb demonstrates that due to the symmetry within an ellipsoid, only the added mass coefficients on the leading diagonal will have non-zero values, thus:

$$X_{\dot{u}} = -\frac{\alpha_0}{2 - \alpha_0}\frac{4}{3}\pi\rho abc$$

$$Y_{\dot{v}} = -\frac{\beta_0}{2-\beta_0}\frac{4}{3}\pi\rho abc$$

$$Z_{\dot{w}} = -\frac{\gamma_0}{2-\gamma_0}\frac{4}{3}\pi\rho abc$$

$$L_{\dot{p}} = -\frac{1}{5}\frac{(b^2-c^2)^2(\gamma_0-\beta_0)}{2(b^2-c^2)+(b^2+c^2)(\beta_0-\gamma_0)}\frac{4}{3}\pi\rho abc$$

$$M_{\dot{q}} = -\frac{1}{5}\frac{(c^2-a^2)^2(\alpha_0-\gamma_0)}{2(c^2-a^2)+(c^2+a^2)(\gamma_0-\alpha_0)}\frac{4}{3}\pi\rho abc$$

$$N_{\dot{r}} = -\frac{1}{5}\frac{(a^2-b^2)^2(\beta_0-\alpha_0)}{2(a^2-b^2)+(a^2+b^2)(\alpha_0-\beta_0)}\frac{4}{3}\pi\rho abc$$

and:

$$X_{\dot{v}} = X_{\dot{w}} = X_{\dot{p}} = X_{\dot{q}} = X_{\dot{r}} = Y_{\dot{w}} = Y_{\dot{p}} =$$
$$Y_{\dot{r}} = Z_{\dot{p}} = Z_{\dot{q}} = Z_{\dot{r}} = L_{\dot{q}} = L_{\dot{r}} = M_{\dot{r}} = 0$$

where α_0, β_0 and γ_0 are purely numerical quantities that describe the relative proportions of the ellipsoid.

Going to a further stage of simplification a prolate ellipsoid will be formed if b = c, therefore $\beta_0 = \gamma_0$, and a > b. The off-diagonal added mass coefficients will still be zero, and the remainder reduce to:

$$X_{\dot{u}} = -\frac{\alpha_0}{2-\alpha_0}\,\frac{4}{3}\pi\rho ab^2$$

$$Y_{\dot{v}} = Z_{\dot{w}} = -\frac{\beta_0}{2-\beta_0}\,\frac{4}{3}\pi\rho ab^2$$

$$L_{\dot{p}} = 0$$

$$M_{\dot{q}} = N_{\dot{r}} = -\frac{1}{5}\,\frac{(b^2-a^2)^2(\alpha_0-\beta_0)}{2(b^2-a^2)+(b^2+a^2)(\beta_0-\alpha_0)}\,\frac{4}{3}\pi\rho ab^2$$

where:

$$\alpha_0 = \frac{2(1-e^2)}{e^3}\left(\frac{1}{2}\log\frac{1+e}{1-e} - e\right)$$

$$\beta_0 = \frac{1}{e^2} - \frac{1-e^2}{2e^3}\log\frac{1+e}{1-e}$$

and the eccentricity of the meridian elliptical section is:

$$e^2 = 1 - (b/a)^2$$

Lamb gives a set of k factors, which for a prolate spheroid are:

$$k_1 = \frac{\alpha_0}{2 - \alpha_0}$$

$$k_2 = \frac{\beta_0}{2 - \beta_0}$$

$$k' = \frac{e^4(\beta_0 - \alpha_0)}{(2 - e^2)[2e^2 - (2 - e^2)(\beta_0 - \alpha_0)}$$

so the non-zero added mass coefficients can be written:

$$X_{\dot{u}} = - k_1 \tfrac{4}{3}\pi\rho ab^2$$

$$Y_{\dot{v}} = Z_{\dot{w}} = - k_2 \tfrac{4}{3}\pi\rho ab^2$$

$$M_{\dot{q}} = N_{\dot{r}} = - k' \frac{4}{15}\pi\rho ab^2(a^2 + b^2)$$

It is worth noting that the factor $\frac{4}{3}\pi\rho ab^2$ is the mass of the volume of fluid displaced by the ellipsoid, and the factor $\frac{4}{15}\pi\rho ab^2(a^2+b^2)$ is the moment of inertia about the y-axis or z-axis of the same volume of fluid.

The authors of most ship mathematical models do not explain how the values for the added mass coefficients were obtained. For this analysis, as undoubtedly for many others, the expressions derived for the prolate ellipsoid are assumed to give values of the correct order.

6.06 Methods Of Determining The Added Mass Coefficients

Motora, 1960 (Ref.110), proposes methods to determine the surge, sway and yaw added mass effects from model tests. Three methods were studied, namely:

1) **The Vibration Method:** This is based upon the fact that the limit of the added mass obtained from the prolongation of the period of vibration occurs when the period becomes infinitesimal. However, this method proved unsatisfactory because it was impossible to generate vibrations with sufficiently short period.

2) **The Acceleration Method:** This method is intended to measure the resultant acceleration when a known force is applied to the ship model. The added mass is then extracted from this acceleration. However, it was found difficult to measure the acceleration with the required degree of accuracy.

3) **The Impact Method:** This is intended to measure the initial velocity caused when a known impact is applied to the ship model. The added mass is then extracted from this velocity information. This method proved to give results of the required accuracy.

The impact method was used for measuring the surge and yaw added mass coefficients, but with the greater damping in sway it was possible to use the acceleration method.

Generally, the added mass in fore-aft motion (surge) is fairly small, but plays a significant role in sway motions. Theory suggests that the added mass coefficients change with water depth and can increase sharply to exceed the vessel's inherent mass as the water becomes shallow. While the equations for added mass are complete and sufficient for mathematical application, there is still adequate uncertainty in establishing the value of the coefficients to justify further research.

6.07 The Mass Equations

The mass matrix will contain all the terms which are multiplied by the accelerations or state derivatives. These terms are to be found in the mass acceleration product of Newton's second law and the added mass equations. If X_1 is used to represent the surge added mass terms and X_2 is used to represent the remaining surge forces then referring to the expanded equations of Newton's second law:

$$X = m[\dot{u}+wq-vr-x_g(qq+rr)+y_g(pq-\dot{r})+z_g(rp+\dot{q})]$$

but:

$$X = X_1 + X_2$$

and:

$$\begin{aligned} X_1 = \; & X_{\dot{u}}\dot{u} + X_{\dot{v}}\dot{v} + X_{\dot{w}}\dot{w} + X_{\dot{p}}\dot{p} + X_{\dot{q}}\dot{q} + X_{\dot{r}}\dot{r} \\ & - X_{\dot{v}}ur - Y_{\dot{v}}vr - Y_{\dot{w}}wr - Y_{\dot{p}}pr - Y_{\dot{q}}qr - Y_{\dot{r}}rr \\ & + X_{\dot{w}}uq + Y_{\dot{w}}vq + Z_{\dot{w}}wq + Z_{\dot{p}}pq + Z_{\dot{q}}qq + Z_{\dot{r}}rq \end{aligned}$$

therefore:

$$
\begin{aligned}
X_2 = {} & m[\dot{u}+wq-vr-x_g(qq+rr)+y_g(pq-\dot{r})+z_g(rp+\dot{q})] \\
& - X_{\dot{u}}\dot{u} - X_{\dot{v}}\dot{v} - X_{\dot{w}}\dot{w} - X_{\dot{p}}\dot{p} - X_{\dot{q}}\dot{q} - X_{\dot{r}}\dot{r} \\
& + X_{\dot{v}}ur + Y_{\dot{v}}vr + Y_{\dot{w}}wr + Y_{\dot{p}}pr + Y_{\dot{q}}qr + Y_{\dot{r}}rr \\
& - X_{\dot{w}}uq - Y_{\dot{w}}vq - Z_{\dot{w}}wq - Z_{\dot{p}}pq - Z_{\dot{q}}qq - Z_{\dot{r}}rq
\end{aligned}
$$

collecting the acceleration terms on the right-hand side of the equation, and adopting a similar procedure for sway, heave, roll, pitch and yaw, gives:

$$
\begin{aligned}
& X_2 - Y_{\dot{u}}ur - Y_{\dot{w}}wr - Y_{\dot{p}}pr - Y_{\dot{q}}qr - Y_{\dot{r}}rr \\
& \quad + Z_{\dot{u}}uq + Z_{\dot{v}}vq + Z_{\dot{p}}pq + Z_{\dot{q}}qq + Z_{\dot{r}}rq \\
& \quad - (m-Z_{\dot{w}})wq + (m-Y_{\dot{v}})vr + m[x_g(qq+rr) - y_g pq - z_g rp] \\
& = (m-X_{\dot{u}})\dot{u} - X_{\dot{v}}\dot{v} - X_{\dot{w}}\dot{w} - X_{\dot{p}}\dot{p} + (mz_g-X_{\dot{q}})\dot{q} + (-my_g-X_{\dot{r}})\dot{r}
\end{aligned}
$$

$$
\begin{aligned}
& Y_2 + X_{\dot{v}}vr + X_{\dot{w}}wr + X_{\dot{p}}pr + X_{\dot{q}}qr + X_{\dot{r}}rr \\
& \quad - Z_{\dot{u}}up - Z_{\dot{v}}vp - Z_{\dot{p}}pp - Z_{\dot{q}}qp - Z_{\dot{r}}rp \\
& \quad - (m-X_{\dot{u}})ur + (m-Z_{\dot{w}})wp + m[y_g(rr+pp) - z_g rq - x_g pq] \\
& = - Y_{\dot{u}}\dot{u} + (m-Y_{\dot{v}})\dot{v} - Y_{\dot{w}}\dot{w} + (-mz_g-Y_{\dot{p}})\dot{p} - Y_{\dot{q}}\dot{q} + (mx_g-Y_{\dot{r}})\dot{r}
\end{aligned}
$$

$$
\begin{aligned}
& Z_2 - X_{\dot{v}}vq - X_{\dot{w}}wq - X_{\dot{p}}pq - X_{\dot{q}}qq - X_{\dot{r}}rq \\
& \quad + Y_{\dot{u}}up + Y_{\dot{w}}wp + Y_{\dot{p}}pp + Y_{\dot{q}}qp + Y_{\dot{r}}rp \\
& \quad - (m-Y_{\dot{v}})vp + (m-X_{\dot{u}})uq + m[z_g(pp+qq) - x_g rp - y_g rq] \\
& = - Z_{\dot{u}}\dot{u} - Z_{\dot{v}}\dot{v} + (m-Z_{\dot{w}})\dot{w} + (my_g-Z_{\dot{p}})\dot{p} + (-mx_g-Z_{\dot{q}})\dot{q} - Z_{\dot{r}}\dot{r}
\end{aligned}
$$

$$
\begin{aligned}
L_2 & - Y_{\dot{u}}uw - Y_{\dot{v}}vw - Y_{\dot{w}}ww - Y_{\dot{p}}pw - Y_{\dot{q}}qw - Y_{\dot{r}}rw \\
& + Z_{\dot{u}}uv + Z_{\dot{v}}vv + Z_{\dot{w}}wv + Z_{\dot{p}}pv + Z_{\dot{q}}qv + Z_{\dot{r}}rv \\
& - M_{\dot{u}}ur - M_{\dot{v}}vr - M_{\dot{w}}wr + N_{\dot{u}}uq + N_{\dot{v}}vq + N_{\dot{w}}wq \\
& + [(I_{yy}-M_{\dot{q}}) - (I_{zz}-N_{\dot{r}})]rq + (I_{zy}+N_{\dot{q}})qq \\
& - (I_{yz}+M_{\dot{r}})rr + (I_{zx}+N_{\dot{q}})pq - (I_{yx}+M_{\dot{p}})pr \\
& + m[z_g(ur-wp) - y_g(vp-uq)] \\
= & - L_{\dot{u}}\dot{u} + (-mz_g-L_{\dot{v}})\dot{v} + (my_g-L_{\dot{w}})\dot{w} + (I_{xx}-L_{\dot{p}})\dot{p} \\
& - (I_{xy}+L_{\dot{q}})\dot{q} - (I_{xz}+L_{\dot{r}})\dot{r}
\end{aligned}
$$

$$
\begin{aligned}
M_2 & + X_{\dot{u}}uw + X_{\dot{v}}vw + X_{\dot{w}}ww + X_{\dot{p}}pw + X_{\dot{q}}qw + X_{\dot{r}}rw \\
& - Z_{\dot{u}}uu - Z_{\dot{v}}vu - Z_{\dot{w}}wu - Z_{\dot{p}}pu - Z_{\dot{q}}qu - Z_{\dot{r}}ru \\
& + L_{\dot{u}}ur + L_{\dot{v}}vr + L_{\dot{w}}wr - N_{\dot{u}}up - N_{\dot{v}}vp - N_{\dot{w}}wp \\
& + [(I_{zz}-N_{\dot{r}}) - (I_{xx}-L_{\dot{p}})]pr + (I_{xz}+L_{\dot{r}})rr \\
& - (I_{zx}+N_{\dot{p}})pp - (I_{zy}+N_{\dot{q}})qp + (I_{xy}+L_{\dot{q}})qr \\
& + m[x_g(vp-uq) - z_g(wq-vr)] \\
= & \quad (mz_g-M_{\dot{u}})\dot{u} - M_{\dot{v}}\dot{v} + (-mx_g-M_{\dot{w}})\dot{w} - (I_{yx}+M_{\dot{p}})\dot{p} \\
& + (I_{yy}-M_{\dot{q}})\dot{q} - (I_{yz}+M_{\dot{r}})\dot{r}
\end{aligned}
$$

$$
\begin{aligned}
N_2 & - X_{\dot{u}}uv - X_{\dot{v}}vv - X_{\dot{w}}wv - X_{\dot{p}}pv - X_{\dot{q}}qv - X_{\dot{r}}rv \\
& + Y_{\dot{u}}uu + Y_{\dot{v}}vu + Y_{\dot{w}}wu + Y_{\dot{p}}pu + Y_{\dot{q}}qu + Y_{\dot{r}}ru \\
& - L_{\dot{u}}uq - L_{\dot{v}}vq - L_{\dot{w}}wq + M_{\dot{u}}up + M_{\dot{v}}vp + M_{\dot{w}}wp \\
& + [(I_{xx}-L_{\dot{p}}) - (I_{yy}-M_{\dot{q}})]pq + (I_{yx}+M_{\dot{p}})pp \\
& - (I_{xy}+L_{\dot{q}})qq + (I_{yz}+M_{\dot{p}})rp - (I_{xz}+L_{\dot{r}})rq \\
& + m[y_g(wq-vr) - x_g(ur-wp)] \\
= & \quad (-my_g-N_{\dot{u}})\dot{u} + (mx_g-N_{\dot{v}})\dot{v} - N_{\dot{w}}\dot{w} - (I_{zx}+N_{\dot{p}})\dot{p} \\
& - (I_{zy}+N_{\dot{q}})\dot{q} + (I_{zz}-N_{\dot{r}})\dot{r}
\end{aligned}
$$

The left-hand side of these equations have been arranged for later use where they appear in the dynamic and hydrodynamic vectors.

The Mass Matrix

Collecting the right-hand side of the equations together and extracting the six accelerations leaves the elements of the mass matrix which can now be written:

$$\begin{bmatrix} m-X_{\dot{u}} & -X_{\dot{v}} & -X_{\dot{w}} & -X_{\dot{p}} & mz_g-X_{\dot{q}} & -my_g-X_{\dot{r}} \\ -Y_{\dot{u}} & m-Y_{\dot{v}} & -Y_{\dot{w}} & -mz_g-Y_{\dot{p}} & -Y_{\dot{q}} & mx_g-Y_{\dot{r}} \\ -Z_{\dot{u}} & -Z_{\dot{v}} & m-Z_{\dot{w}} & my_g-Z_{\dot{p}} & -mx_g-Z_{\dot{q}} & -Z_{\dot{r}} \\ -L_{\dot{u}} & -mz_g-L_{\dot{v}} & my_g-L_{\dot{w}} & I_{xx}-L_{\dot{p}} & -I_{xy}-L_{\dot{q}} & -I_{xz}-L_{\dot{r}} \\ mz_g-M_{\dot{u}} & -M_{\dot{v}} & -mx_g-M_{\dot{w}} & -I_{yx}-M_{\dot{p}} & I_{yy}-M_{\dot{q}} & -I_{yz}-M_{\dot{r}} \\ -my_g-N_{\dot{u}} & mx_g-N_{\dot{v}} & -N_{\dot{w}} & -I_{zx}-N_{\dot{p}} & -I_{zy}-N_{\dot{q}} & I_{zz}-N_{\dot{r}} \end{bmatrix}$$

Since the added mass coefficients show symmetry about the leading diagonal in their matrix, so the mass matrix is also symmetric about its leading diagonal.

6.08 The Dynamic Forces And Moments

This will be made up of the dynamic terms associated with the inertial velocities u, v, w, p, q and r. Some of the terms on the left-hand side of the equations developed for the mass matrix will make up the dynamic vector. The remainder will appear in the hydrodynamic vector.

Hydrodynamic Coefficients

The Dynamic and Hydrodynamic vectors contain a number of hydrodynamic coefficients which are actually entered in terms of their perfect fluid component. A list of these hydrodynamic coefficients and their related perfect fluid components appears below, where H are the hydrodynamic coefficients and P are the perfect fluid components.

H	P	H	P	H	P	H	P	H	P	H	P
X_{uq}	$Z_{\dot{u}}$	Y_{up}	$-Z_{\dot{u}}$	Z_{up}	$Y_{\dot{u}}$	L_{uv}	$Z_{\dot{u}}$	M_{uu}	$-Z_{\dot{u}}$	N_{uu}	$Y_{\dot{u}}$
X_{vq}	$Z_{\dot{v}}$	Y_{vp}	$-Z_{\dot{v}}$	Z_{vp}	$Y_{\dot{v}}$	L_{vv}	$Z_{\dot{v}}$	M_{vu}	$-Z_{\dot{v}}$	N_{vu}	$Y_{\dot{v}}$
X_{wq}	$Z_{\dot{w}}$	Y_{wp}	$-Z_{\dot{w}}$	Z_{wp}	$Y_{\dot{w}}$	L_{wv}	$Z_{\dot{w}}$	M_{wu}	$-Z_{\dot{w}}$	N_{wu}	$Y_{\dot{w}}$
X_{pq}	$Z_{\dot{p}}$	Y_{pp}	$-Z_{\dot{p}}$	Z_{pp}	$Y_{\dot{p}}$	L_{pv}	$Z_{\dot{p}}$	M_{pu}	$-Z_{\dot{p}}$	N_{pu}	$Y_{\dot{p}}$
X_{qq}	$Z_{\dot{q}}$	Y_{qp}	$-Z_{\dot{q}}$	Z_{qp}	$Y_{\dot{q}}$	L_{qv}	$Z_{\dot{q}}$	M_{qu}	$-Z_{\dot{q}}$	N_{qu}	$Y_{\dot{q}}$
X_{rq}	$Z_{\dot{r}}$	Y_{rp}	$-Z_{\dot{r}}$	Z_{rp}	$Y_{\dot{r}}$	L_{rv}	$Z_{\dot{r}}$	M_{ru}	$-Z_{\dot{r}}$	N_{ru}	$Y_{\dot{r}}$
X_{ur}	$-Y_{\dot{u}}$	Y_{ur}	$X_{\dot{u}}$	Z_{uq}	$-X_{\dot{u}}$	L_{uw}	$-Y_{\dot{u}}$	M_{uw}	$X_{\dot{u}}$	N_{uv}	$-X_{\dot{u}}$
X_{vr}	$-Y_{\dot{v}}$	Y_{vr}	$X_{\dot{v}}$	Z_{vq}	$-X_{\dot{v}}$	L_{vw}	$-Y_{\dot{v}}$	M_{vw}	$X_{\dot{v}}$	N_{vv}	$-X_{\dot{v}}$
X_{wr}	$-Y_{\dot{w}}$	Y_{wr}	$X_{\dot{w}}$	Z_{wq}	$-X_{\dot{w}}$	L_{ww}	$-Y_{\dot{w}}$	M_{ww}	$X_{\dot{w}}$	N_{wv}	$-X_{\dot{w}}$
X_{pr}	$-Y_{\dot{p}}$	Y_{pr}	$X_{\dot{p}}$	Z_{pq}	$-X_{\dot{p}}$	L_{pw}	$-Y_{\dot{p}}$	M_{pw}	$X_{\dot{p}}$	N_{pv}	$-X_{\dot{p}}$
X_{qr}	$-Y_{\dot{q}}$	Y_{qr}	$X_{\dot{q}}$	Z_{qq}	$-X_{\dot{q}}$	L_{qw}	$-Y_{\dot{q}}$	M_{qw}	$X_{\dot{q}}$	N_{qv}	$-X_{\dot{q}}$
X_{rr}	$-Y_{\dot{r}}$	Y_{rr}	$X_{\dot{r}}$	Z_{rq}	$-X_{\dot{r}}$	L_{rw}	$-Y_{\dot{r}}$	M_{rw}	$X_{\dot{r}}$	N_{rv}	$-X_{\dot{r}}$
						L_{uq}	$N_{\dot{u}}$	M_{up}	$-N_{\dot{u}}$	N_{up}	$M_{\dot{u}}$
						L_{vq}	$N_{\dot{v}}$	M_{vp}	$-N_{\dot{v}}$	N_{vp}	$M_{\dot{v}}$
						L_{wq}	$N_{\dot{w}}$	M_{wp}	$-N_{\dot{w}}$	N_{wp}	$M_{\dot{w}}$
						L_{pq}	$N_{\dot{p}}$	M_{pp}	$-N_{\dot{p}}$	N_{pp}	$M_{\dot{p}}$
						L_{qq}	$N_{\dot{q}}$	M_{qp}	$-N_{\dot{q}}$	N_{qp}	$M_{\dot{q}}$
						L_{rq}	$N_{\dot{r}}$	M_{rp}	$-N_{\dot{r}}$	N_{rp}	$M_{\dot{r}}$
						L_{ur}	$-M_{\dot{u}}$	M_{ur}	$L_{\dot{u}}$	N_{uq}	$-L_{\dot{u}}$
						L_{vr}	$-M_{\dot{v}}$	M_{vr}	$L_{\dot{v}}$	N_{vq}	$-L_{\dot{v}}$
						L_{wr}	$-M_{\dot{w}}$	M_{wr}	$L_{\dot{w}}$	N_{wq}	$-L_{\dot{w}}$
						L_{pr}	$-M_{\dot{p}}$	M_{pr}	$L_{\dot{p}}$	N_{pq}	$-L_{\dot{p}}$
						L_{qr}	$-M_{\dot{q}}$	M_{qr}	$L_{\dot{q}}$	N_{qq}	$-L_{\dot{q}}$
						L_{rr}	$-M_{\dot{r}}$	M_{rr}	$L_{\dot{r}}$	N_{rq}	$-L_{\dot{r}}$

The Dynamic Vector

The dynamic vector will contain the following coefficients:

H	P	H	P	H	P	H	P	H	P	H	P
X_{wq}	$Z_{\dot{w}}$	Y_{wp}	$-Z_{\dot{w}}$	Z_{vp}	$Y_{\dot{v}}$	L_{pq}	$N_{\dot{p}}$	M_{pp}	$-N_{\dot{p}}$	N_{pp}	$M_{\dot{p}}$
X_{vr}	$-Y_{\dot{v}}$	Y_{ur}	$X_{\dot{u}}$	Z_{uq}	$-X_{\dot{u}}$	L_{qq}	$N_{\dot{q}}$	M_{qp}	$-N_{\dot{q}}$	N_{qp}	$M_{\dot{q}}$
						L_{rq}	$N_{\dot{r}}$	M_{rp}	$-N_{\dot{r}}$	N_{rp}	$M_{\dot{r}}$
						L_{pr}	$-M_{\dot{p}}$	M_{pr}	$L_{\dot{p}}$	N_{pq}	$-L_{\dot{p}}$
						L_{qr}	$-M_{\dot{q}}$	M_{qr}	$L_{\dot{q}}$	N_{qq}	$-L_{\dot{q}}$
						L_{rr}	$-M_{\dot{r}}$	M_{rr}	$L_{\dot{r}}$	N_{rq}	$-L_{\dot{r}}$

The dynamic vector, D, is thus written:

$$
\begin{bmatrix}
(m-Y_{\dot v})vr - (m-Z_{\dot w})wq + m[x_g(qq+rr) - y_g pq - z_g rp] \\[2ex]
(m-Z_{\dot w})wp - (m-X_{\dot u})ur + m[y_g(rr+pp) - z_g rq - x_g pq] \\[2ex]
(m-X_{\dot u})uq - (m-Y_{\dot v})vp + m[z_g(pp+qq) - x_g rp - y_g rq] \\[2ex]
[(I_{yy}-M_{\dot q}) - (I_{zz}-N_{\dot r})]rq + (I_{zy}+N_{\dot q})qq - (I_{yz}+M_{\dot r})rr \\
+ (I_{zx}+N_{\dot q})pq - (I_{yx}+M_{\dot p})pr + m[z_g(ur-wp) - y_g(vp-uq)] \\[2ex]
[(I_{zz}-N_{\dot r}) - (I_{xx}-L_{\dot p})]pr + (I_{xz}+L_{\dot r})rr - (I_{zx}+N_{\dot p})pp \\
- (I_{zy}+N_{\dot q})qp + (I_{xy}+L_{\dot q})qr + m[x_g(vp-uq) - z_g(wq-vr)] \\[2ex]
[(I_{xx}-L_{\dot p}) - (I_{yy}-M_{\dot q})]pq + (I_{yx}+M_{\dot p})pp - (I_{xy}+L_{\dot q})qq \\
+ (I_{yz}+M_{\dot p})rp - (I_{xz}+L_{\dot r})rq + m[y_g(wq-vr) - x_g(ur-wp)]
\end{bmatrix}
$$

6.09 The Hydrodynamic Forces And Moments

This vector contains the hydrodynamic coefficients which are functions of the relative velocities u_r, v_r, w_r, p, q and r. The remaining hydrodynamic coefficients which are neither in the mass matrix nor the dynamic vector reside within the hydrodynamic vector. These are:

H	P	H	P	H	P	H	P	H	P	H	P
X_{uq}	$Z_{\dot{u}}$	Y_{up}	$-Z_{\dot{u}}$	Z_{up}	$Y_{\dot{u}}$	L_{uv}	$Z_{\dot{u}}$	M_{uu}	$-Z_{\dot{u}}$	N_{uu}	$Y_{\dot{u}}$
X_{vq}	$Z_{\dot{v}}$	Y_{vp}	$-Z_{\dot{v}}$			L_{vv}	$Z_{\dot{v}}$	M_{vu}	$-Z_{\dot{v}}$	N_{vu}	$Y_{\dot{v}}$
				Z_{wp}	$Y_{\dot{w}}$	L_{wv}	$Z_{\dot{w}}$	M_{wu}	$-Z_{\dot{w}}$	N_{wu}	$Y_{\dot{w}}$
X_{pq}	$Z_{\dot{p}}$	Y_{pp}	$-Z_{\dot{p}}$	Z_{pp}	$Y_{\dot{p}}$	L_{pv}	$Z_{\dot{p}}$	M_{pu}	$-Z_{\dot{p}}$	N_{pu}	$Y_{\dot{p}}$
X_{qq}	$Z_{\dot{q}}$	Y_{qp}	$-Z_{\dot{q}}$	Z_{qp}	$Y_{\dot{q}}$	L_{qv}	$Z_{\dot{q}}$	M_{qu}	$-Z_{\dot{q}}$	N_{qu}	$Y_{\dot{q}}$
X_{rq}	$Z_{\dot{r}}$	Y_{rp}	$-Z_{\dot{r}}$	Z_{rp}	$Y_{\dot{r}}$	L_{rv}	$Z_{\dot{r}}$	M_{ru}	$-Z_{\dot{r}}$	N_{ru}	$Y_{\dot{r}}$
X_{ur}	$-Y_{\dot{u}}$					L_{uw}	$-Y_{\dot{u}}$	M_{uw}	$X_{\dot{u}}$	N_{uv}	$-X_{\dot{u}}$
		Y_{vr}	$X_{\dot{v}}$	Z_{vq}	$-X_{\dot{v}}$	L_{vw}	$-Y_{\dot{v}}$	M_{vw}	$X_{\dot{v}}$	N_{vv}	$-X_{\dot{v}}$
X_{wr}	$-Y_{\dot{w}}$	Y_{wr}	$X_{\dot{w}}$	Z_{wq}	$-X_{\dot{w}}$	L_{ww}	$-Y_{\dot{w}}$	M_{ww}	$X_{\dot{w}}$	N_{wv}	$-X_{\dot{w}}$
X_{pr}	$-Y_{\dot{p}}$	Y_{pr}	$X_{\dot{p}}$	Z_{pq}	$-X_{\dot{p}}$	L_{pw}	$-Y_{\dot{p}}$	M_{pw}	$X_{\dot{p}}$	N_{pv}	$-X_{\dot{p}}$
X_{qr}	$-Y_{\dot{q}}$	Y_{qr}	$X_{\dot{q}}$	Z_{qq}	$-X_{\dot{q}}$	L_{qw}	$-Y_{\dot{q}}$	M_{qw}	$X_{\dot{q}}$	N_{qv}	$-X_{\dot{q}}$
X_{rr}	$-Y_{\dot{r}}$	Y_{rr}	$X_{\dot{r}}$	Z_{rq}	$-X_{\dot{r}}$	L_{rw}	$-Y_{\dot{r}}$	M_{rw}	$X_{\dot{r}}$	N_{rv}	$-X_{\dot{r}}$
						L_{uq}	$N_{\dot{u}}$	M_{up}	$-N_{\dot{u}}$	N_{up}	$M_{\dot{u}}$
						L_{vq}	$N_{\dot{v}}$	M_{vp}	$-N_{\dot{v}}$	N_{vp}	$M_{\dot{v}}$
						L_{wq}	$N_{\dot{w}}$	M_{wp}	$-N_{\dot{w}}$	N_{wp}	$M_{\dot{w}}$
						L_{ur}	$-M_{\dot{u}}$	M_{ur}	$L_{\dot{u}}$	N_{uq}	$-L_{\dot{u}}$
						L_{vr}	$-M_{\dot{v}}$	M_{vr}	$L_{\dot{v}}$	N_{vq}	$-L_{\dot{v}}$
						L_{wr}	$-M_{\dot{w}}$	M_{wr}	$L_{\dot{w}}$	N_{wq}	$-L_{\dot{w}}$

Since the fluid is not ideal, in other words the boat will experience resistance in the form of friction and damping terms, the hydrodynamic vector will contain additional terms to describe these retarding forces and moments.

The only terms which remain after the linearisation of a Taylor series expansion are X_u, Y_v, Y_r, N_v and N_r. The latter four are important since they lend themselves to the criterion for dynamic stability of the linear model, which is:

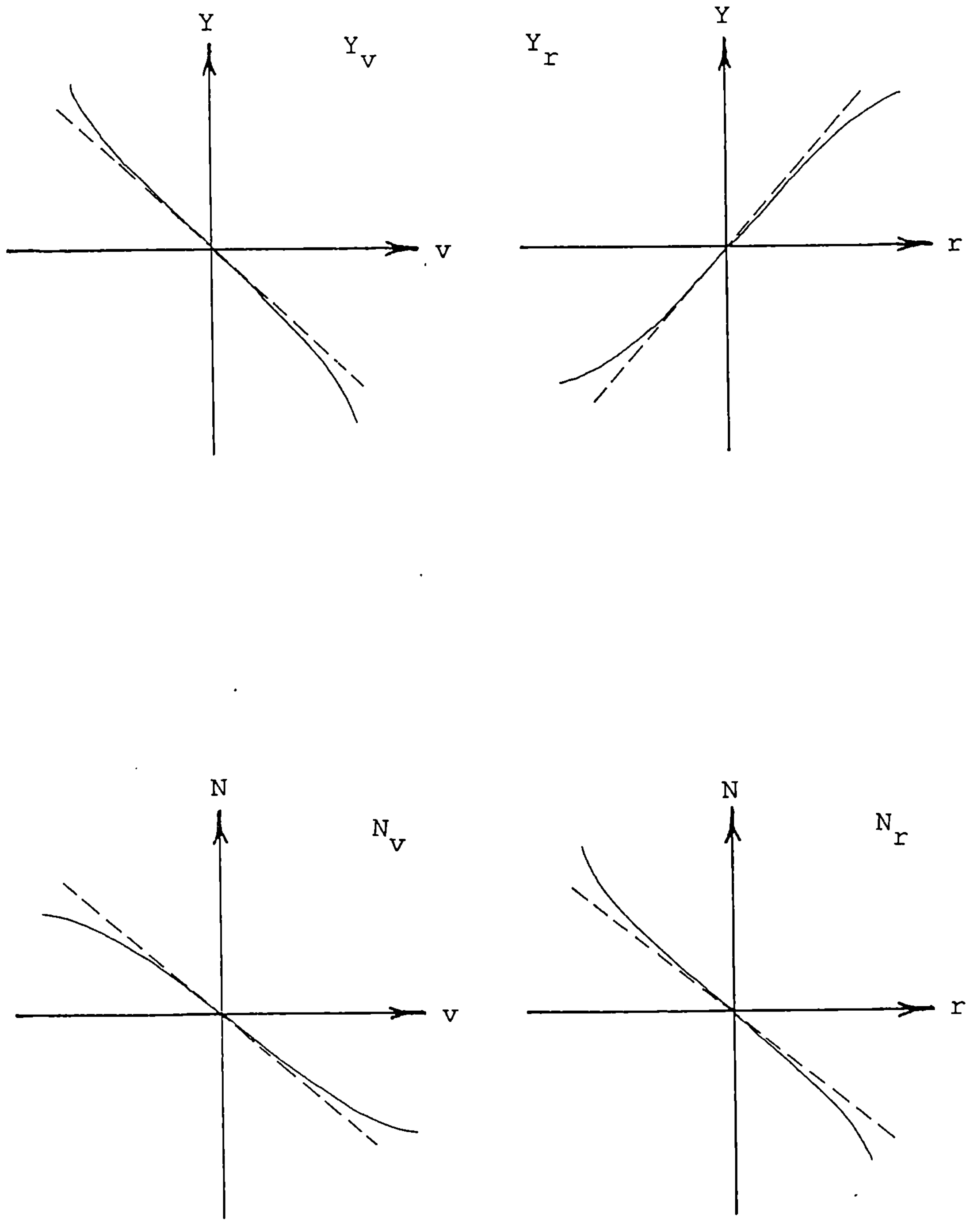
$$C = Y_v(N_r - mx_g u) - N_v(Y_r - mu)$$

where $C > 0$ for a dynamically stable boat.

In the non-linear equations of motion proposed by Abkowitz, 1964 (Ref.2), and Strom-Tejsen, 1965 (Ref.127), higher order terms in the Taylor series expansion are included. After some simplification, the hydrodynamic coefficients appearing in the surge equation will be X_u, X_{uu}, X_{uuu}, X_{vv}, X_{rr}, X_{vvu}, X_{rru} and X_{vru}. Similar terms exist in the sway and yaw equations, however, the author feels that these could be better represented in more meaningful equations.

Japanese researchers adopt hydrodynamic coefficients which can be related to the real ship, with terms such as: Y_v, Y_r, $Y_{v|v|}$, $Y_{v|r|}$, $Y_{r|r|}$, N_v, N_r, N_ϕ, N_{vrr}, N_{vrr}, $N_{r|r|}$, $N_{v|\phi|}$ and $N_{r|\phi|}$. The number of terms to be included is not obvious without the ability to test models and assess the magnitude of all the pertinent coefficients. Most researchers choose varying terms to suit their own models and the application involved. Källström and Ottosson, 1982 (Ref.80), go so far as to include terms such as X_{uuuu}, X_{up}, $X_{uvv|v|}$, Y_{uur}, Y_{uuv}, $Y_{uu\phi}$, Y_{up}, $Y_{v|v|}$, $Y_{r|r|}$, $Y_{v|r|}$, L_{up}, $L_{p|p|}$, $L_{v|v|}$, $L_{r|r|}$, $L_{v|r|}$, N_{uur}, $N_{uv|\phi|}$, N_{uuv}, $N_{uu\phi}$, N_{up}, $N_{v|v|}$, $N_{r|r|}$ and $N_{v|r|}$.

In some respects, the selection of terms to be included and those to be neglected seems more than a little haphazard! In fact it becomes easier to prove a term's existence rather than its non-existence. The Taylor series expansions can become bewildering and the author believes that simplification of the small boat model is desirable at this stage. Therefore, a minimal number of non-perfect fluid hydrodynamic coefficients will be assumed to provide sufficient accuracy for the purposes of this thesis. Since standard plots of Y versus v, Y versus r, N versus v and N versus r, (Fig.6.1), show that while the gradient can be assumed linear in the region close to the origin, further terms will be required for greater accuracy, especially away from the origin. Therefore, in addition to the linear terms Y_v, Y_r, N_v and N_r, the author proposes including terms in $X_{u|u|}$, $Y_{v|v|}$, $Z_{w|w|}$, $L_{p|p|}$, $M_{q|q|}$ and $N_{r|r|}$ which amount to damping coefficients working in opposition to the boat's motion.



Hydrodynamic Coefficients Fig.6.1

It is customary to express such hydrodynamic coefficients in terms of a non-dimensional coefficient and a dimensionalising factor. These afore-proposed ten terms can be so expressed thus:

$$Y_v = Y'_v \frac{\rho L^2 u}{2}$$

$$Y_r = Y'_r \frac{\rho L^3 u}{2}$$

$$N_v = N'_v \frac{\rho L^3 u}{2}$$

$$N_r = N'_r \frac{\rho L^4 u}{2}$$

$$X_{u|u|} = X'_{u|u|} \frac{\rho L^2}{2}$$

$$Y_{v|v|} = Y'_{v|v|} \frac{\rho L^2}{2}$$

$$Z_{w|w|} = Z'_{w|w|} \frac{\rho L^2}{2}$$

$$L_{p|p|} = L'_{p|p|} \frac{\rho L^5}{2}$$

$$M_{q|q|} = M'_{q|q|} \frac{\rho L^5}{2}$$

$$N_{r|r|} = N'_{r|r|} \frac{\rho L^5}{2}$$

The length squared in the last six coefficients will be replaced by areas for more meaningful terms, therefore:

$$X_{u|u|} = X'_{u|u|} \frac{\rho}{2} A_{UT}$$

$$Y_{v|v|} = Y'_{v|v|} \frac{\rho}{2} A_{UL}$$

$$Z_{w|w|} = Z'_{w|w|} \frac{\rho}{2} A_{WL}$$

$$L_{p|p|} = L'_{p|p|} \frac{\rho L^3}{2} A_{UL}$$

$$M_{q|q|} = M'_{q|q|} \frac{\rho L^3}{2} A_{WL}$$

$$N_{r|r|} = N'_{r|r|} \frac{\rho L^3}{2} A_{UL}$$

where: A_{UT} is the underwater transverse cross-sectional area, A_{UL} is the underwater longitudinal centreline area, and A_{WL} is the area of the waterplane.

The Hydrodynamic Vector

The hydrodynamic vector, H, is thus written:

$$
\begin{bmatrix}
-Y_{\dot u}u_r r \qquad\qquad - Y_{\dot w}w_r r - Y_{\dot p}pr - Y_{\dot q}qr - Y_{\dot r}rr \\
+Z_{\dot u}u_r q + Z_{\dot v}v_r q \qquad\qquad + Z_{\dot p}pq + Z_{\dot q}qq + Z_{\dot r}rq \\
-X_{u|u|}u|u| \\
\\
X_{\dot v}v_r r + X_{\dot w}w_r r + X_{\dot p}pr + X_{\dot q}qr + X_{\dot r}rr \\
-Z_{\dot u}u_r p - Z_{\dot v}v_r p \qquad\qquad - Z_{\dot p}pp - Z_{\dot q}qp - Z_{\dot r}rp \\
-Y_{v|v|}v|v| + Y_v v + Y_r r \\
\\
-X_{\dot v}v_r q - X_{\dot w}w_r q - X_{\dot p}pq - X_{\dot q}qq - X_{\dot r}rq \\
+Y_{\dot u}u_r p \qquad\qquad + Y_{\dot w}w_r p + Y_{\dot p}pp + Y_{\dot q}qp + Y_{\dot r}rp \\
-Z_{w|w|}w|w| \\
\\
-Y_{\dot u}u_r w_r - Y_{\dot v}v_r w_r - Y_{\dot w}w_r w_r - Y_{\dot p}pw_r - Y_{\dot q}qw_r - Y_{\dot r}rw_r \\
+Z_{\dot u}u_r v_r + Z_{\dot v}v_r v_r + Z_{\dot w}w_r v_r + Z_{\dot p}pv_r + Z_{\dot q}qv_r + Z_{\dot r}rv_r \\
-M_{\dot u}u_r r - M_{\dot v}v_r r - M_{\dot w}w_r r + N_{\dot u}u_r q + N_{\dot v}v_r q + N_{\dot w}w_r q \\
-L_{p|p|}p|p| \\
\\
+X_{\dot u}u_r w_r + X_{\dot v}v_r w_r + X_{\dot w}w_r w_r + X_{\dot p}pw_r + X_{\dot q}qw_r + X_{\dot r}rw_r \\
-Z_{\dot u}u_r u_r - Z_{\dot v}v_r u_r - Z_{\dot w}w_r u_r - Z_{\dot p}pu_r - Z_{\dot q}qu_r - Z_{\dot r}ru_r \\
+L_{\dot u}u_r r + L_{\dot v}v_r r + L_{\dot w}w_r r - N_{\dot u}u_r p - N_{\dot v}v_r p - N_{\dot w}w_r p \\
-M_{q|q|}q|q| \\
\\
-X_{\dot u}u_r v_r - X_{\dot v}v_r v_r - X_{\dot w}w_r v_r - X_{\dot p}pv_r - X_{\dot q}qv_r - X_{\dot r}rv_r \\
+Y_{\dot u}u_r u_r + Y_{\dot v}v_r u_r + Y_{\dot w}w_r u_r + Y_{\dot p}pu_r + Y_{\dot q}qu_r + Y_{\dot r}ru_r \\
-L_{\dot u}u_r q - L_{\dot v}v_r q - L_{\dot w}w_r q + M_{\dot u}u_r p + M_{\dot v}v_r p + M_{\dot w}w_r p \\
-N_{r|r|}r|r| + N_v v + N_r r
\end{bmatrix}
$$

where: $u_r = u - u_s$

$v_r = v - v_s$

$w_r = w - w_s$

and u_s, v_s and w_s describe the motion of the fluid.

6.10 The Restoring Forces And Moments

Displacement

An important term which is fundamental to floating bodies is that of displacement, and it is worth defining clearly what this means. Archimedes' principle states that:

> *"a body wholly or partially immersed in a fluid loses weight equal in amount to the weight of the fluid it displaces"*

This applies whether the body is heavier, lighter or equal in weight to an equal volume of fluid in which it is immersed. For a boat to float freely in water, the following statement must be true:

The weight of the boat = The weight of the volume of water it displaces

This can be generally expressed as:

$$\Delta = \nabla \rho$$

where Δ is the displacement of the boat, ∇ is the volume displaced and ρ is the density of the water. In most naval architecture applications where displacement is expressed in tons and volume in cubic feet, the density of seawater is 1/35 tons per cubic feet. However, in this analysis SI units will be used, thus displacement will be in kilogrammes and volume in cubic metres with the density of seawater being 1025 kgm^{-3}.

When a boat enters a harbour or estuary which has a freshwater input from a river, the water density will be reduced. Consequently the volume of water displaced by the boat must increase, since its weight remains the same. In other words, the boat's draft will change causing sinkage to occur. For large ships this can cause problems when entering ports with little under-keel clearance. Sinkage in small boats is given little attention, but this principle is developed later with respect to the heave restoring force.

The water displaced by the boat is also referred to as the buoyant force or simply, buoyancy. When a boat floats at its equilibrium condition the forces of weight and buoyancy must be equal and opposite, otherwise the boat would sit on top of the water surface or continue sinking. Classically these forces can be considered to act at points within the boat. These points will be the centre of gravity and the centre of buoyancy.

Centre Of Gravity (Cg)

This is the point at which the whole weight of the boat is assumed to be concentrated and from where it is considered to act. The weight acts vertically downwards (perpendicular to the gravitational potential) from the centre of gravity. The position of the centre of gravity depends upon the distribution of weight about the boat, additional top weight causes its location to rise whilst ballast lowers the position. The actual position can be determined from an inclining experiment.

Centre Of Buoyancy (Cb)

This is essentially the centre of gravity of the displaced water or underwater volume of the boat. It is the point at which the resultant upthrust of the surrounding water is considered to act. The location of the centre of buoyancy depends upon the geometry of the underwater portion of the boat.

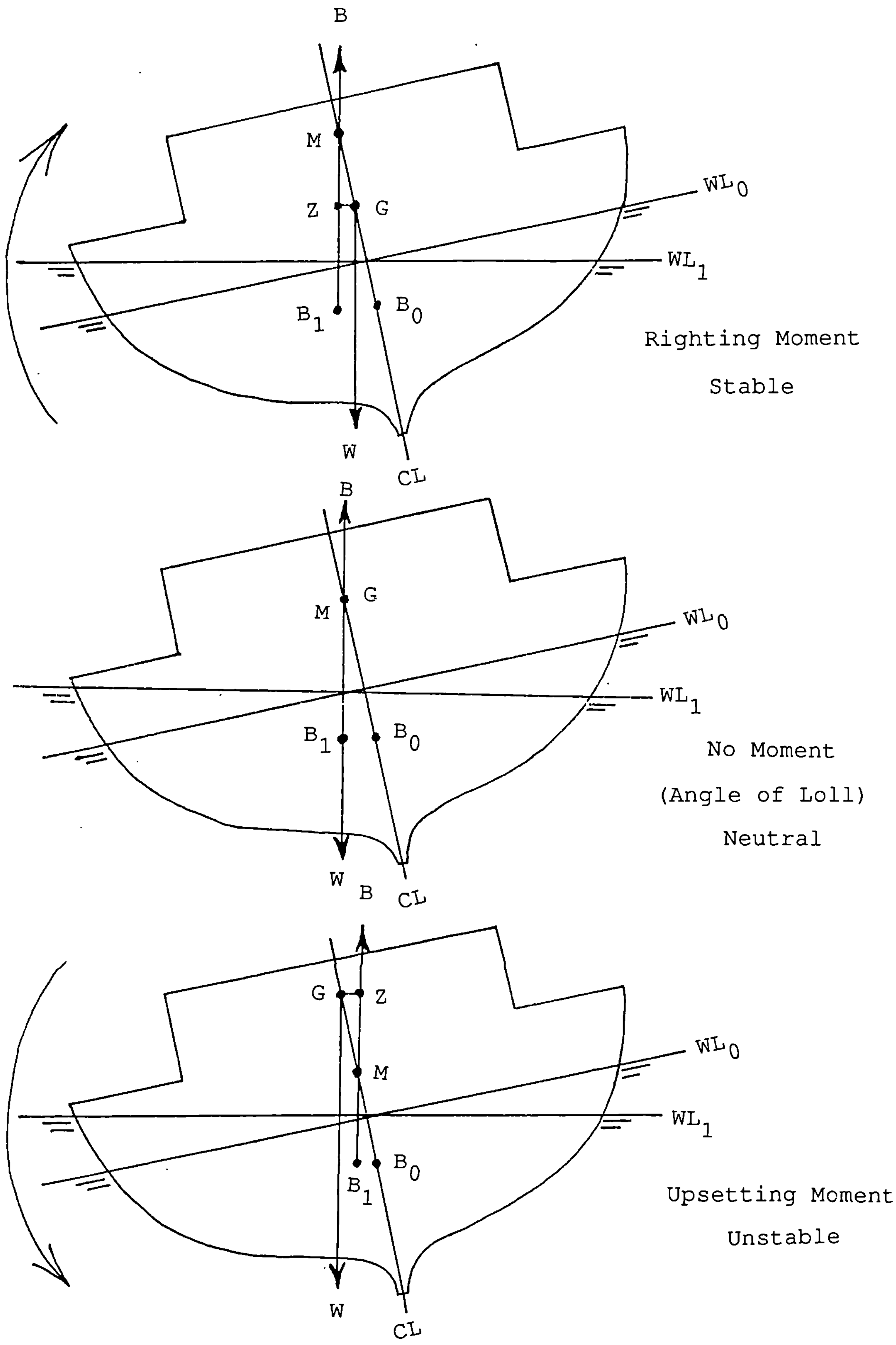
It is a combination of the movement of the centre of buoyancy as a boat rotates from the upright and level conditions and the difference in the weight and buoyancy forces that gives rise to the restoring forces and moments. The primary restoring forces and moments are those in roll, pitch and heave and these will be dealt with first. However, restoring forces and moments also arise in surge, sway and yaw as a result of the boat rolling and pitching.

Stability And Equilibrium

When designing boats, naval architects pay a great deal of attention to their stability. It is the initial stability condition that determines the equilibrium of the boat when it floats freely. While dynamical stability is a measure of the boat's ability to return to the initial equilibrium state when perturbed. Because of its slender nature, a boat is far more susceptible to capsizing from rolling than from pitching, therefore it is normal to only supply information about the righting moment in roll.

Roll Restoring Moment

By designing a boat to have symmetry with respect to the XZ-plane, in other words to be a mirror image along the fore-aft centreline, and by distributing weight evenly either side of the centreline, the initial equilibrium condition of the boat will be such that it is upright with the weight (W) acting vertically

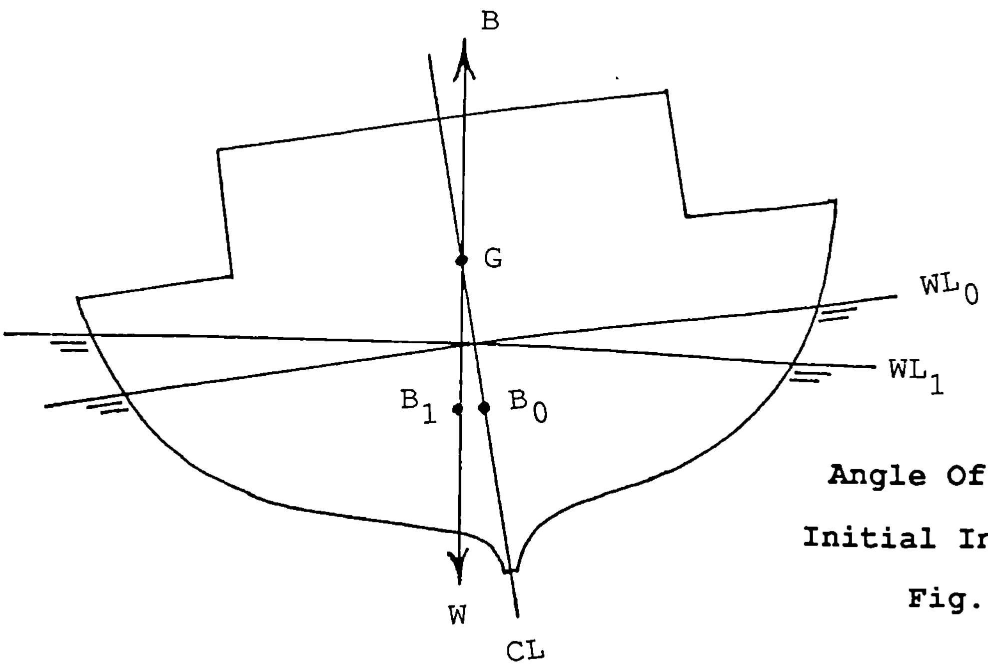


Stable, Neutral and Unstable Equilibrium **Fig.6.2**

downwards along the centreline and buoyancy (B) acting in direct opposition vertically upwards.

When the boat is slightly inclined to some small angle, the centre of buoyancy will move off of the centreline to a location (B_1) dependent upon the change in shape of the underwater section of the body. Assuming all objects on board are immovable, the centre of gravity will remain unaltered (G). The two equal forces of weight and buoyancy will still be acting in opposite directions, but along verticals which are now separated by a horizontal distance GZ known as the righting lever or righting arm. A moment corresponding to the product of weight and the righting lever is thus formed which will rotate the boat either back towards the upright or further away from it depending on the relative separation of the forces (Fig.6.2). Only when the moment disappears with B and G again acting along the same vertical line will equilibrium be regained. Occasionally, a boat may attain a situation where B and G are acting in the same vertical line, but the boat is not actually upright. Under these circumstances the boat will maintain a permanent angle of loll (Fig.6.3a).

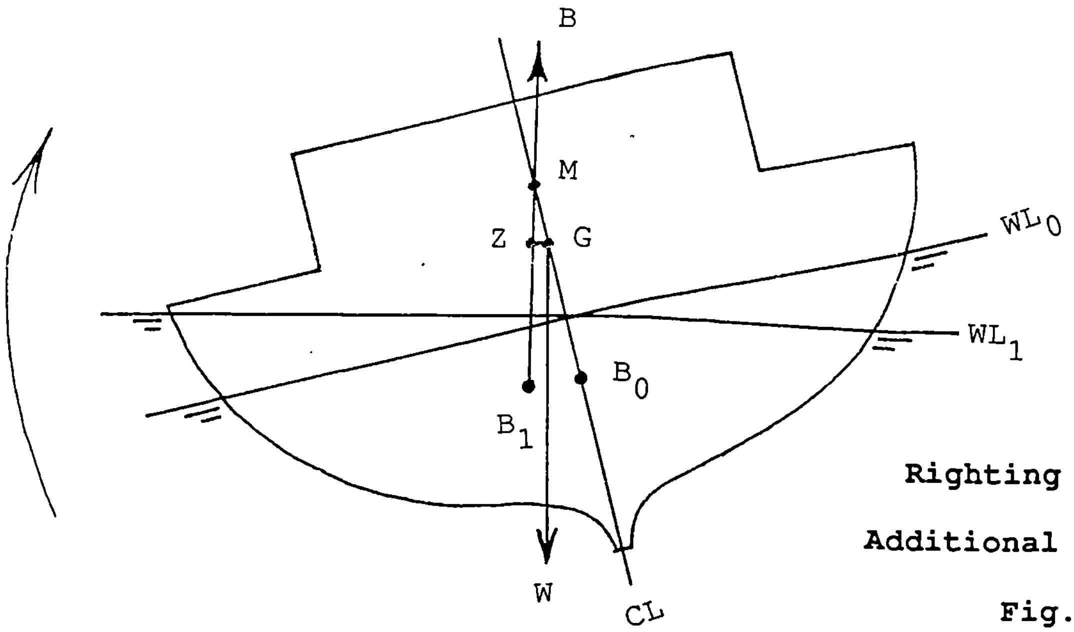
In order to assess a boat's initial stability, naval architects refer to a quantity called the metacentric height (meta being a prefix from the Greek meaning



Angle Of Loll At

Initial Inclination

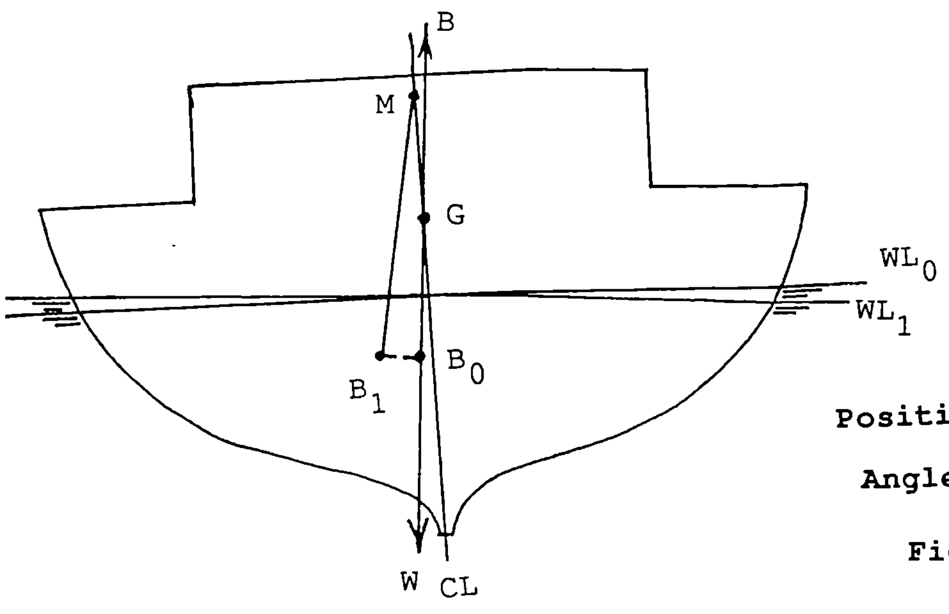
Fig.6.3a



Righting Moment At

Additional Inclinatior

Fig.6.3b



Positive GM At

Angle Of Loll

Fig.6.3c

"change of condition"). The metacentre (M) can be defined as:

> *"the limiting height to which the centre of gravity may be raised without producing initial instability"*

The position of the transverse metacentre will be determined by the intersection of the vertical line through the centre of buoyancy when the boat is upright (which should be the centre line) and the vertical line through the centre of buoyancy at some small inclination from the upright. However, it must be stated that the metacentric height is not the same for all angles of heel and actually tends to rise as the boat is inclined. For large angles of heel, greater than 10°, the position of the metacentre will vary appreciably.

The initial metacentric height (GM) therefore determines the initial stability of the boat. If M is above G then the boat will be stable; if M is at G then the boat is in a neutral condition; and if M is below G then the boat is initially unstable. If the boat acquires an angle of loll, then any further inclination will cause M to rise slightly, which will bring it above G, and the boat will again be stable (Figs.6.3b&c). The magnitude of GM is important as regards the roll acceleration. A small GM will tend to produce a "sluggish" motion, with inadequate GM this can turn into "over-rolling" especially in beam seas. Too great a GM results in a "stiff" boat, giving rise to motions which are capable of causing structural damage.

It is possible to determine the period of roll of the boat knowing the initial metacentric height (GM) and the transverse radius of gyration (k), and is given by the formula:

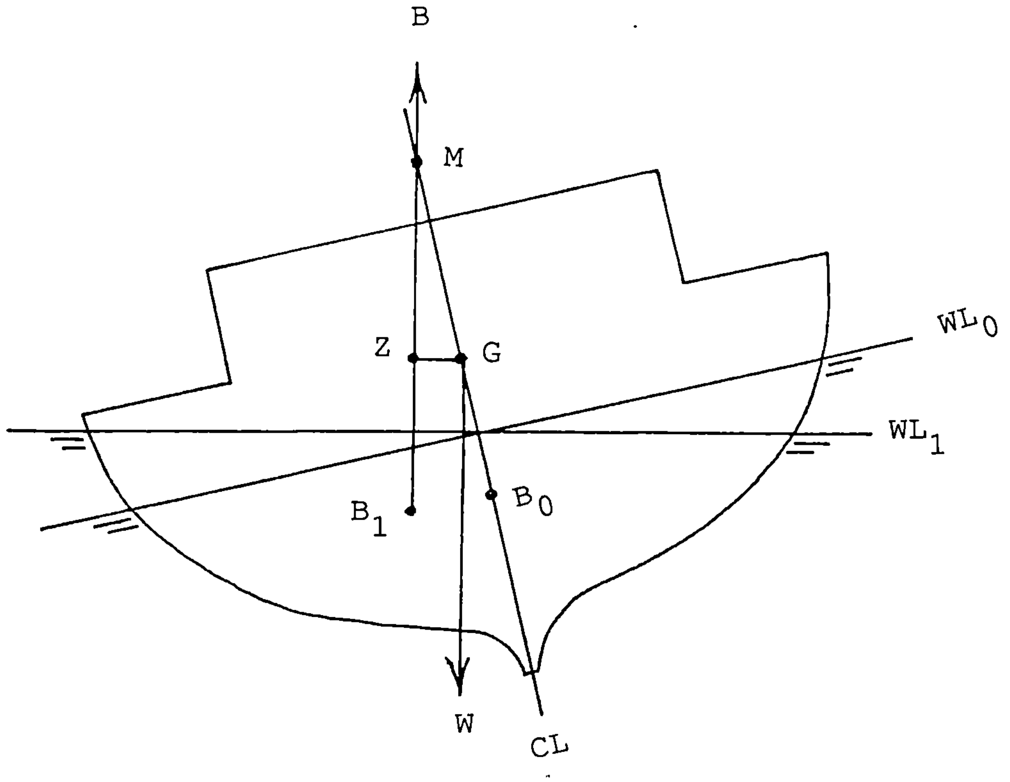
$$2T = 2\pi \frac{k}{\sqrt{g\ GM}}$$

At larger angles of inclination, when the metacentre can no longer be assumed to be fixed, it becomes necessary to obtain the righting moment as a function of the inclination of the boat and the location of the centre of buoyancy at that inclination. By trigonometry (Fig.6.4) it can be shown that for roll, in the absence of pitch, the righting moment is:

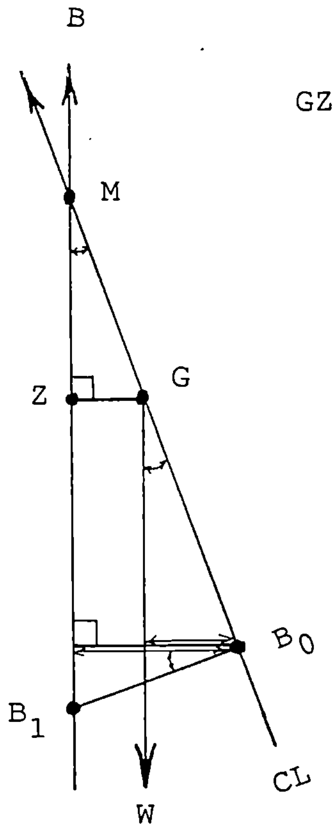
$$(y_g W - y_b B) \cos\phi \quad - \quad (z_g W - z_b B) \sin\phi$$

if, as in the equilibrium condition, B = W then this reduces to:

$$W \; (\; (y_g - y_b) \cos\phi \quad - \quad (z_g - z_b) \sin\phi \;)$$



$$GZ = (y_g - y_b)\cos\phi - (z_g - z_b)\sin\phi$$



Trigonometry Of Roll Righting Lever **Fig.6.4**

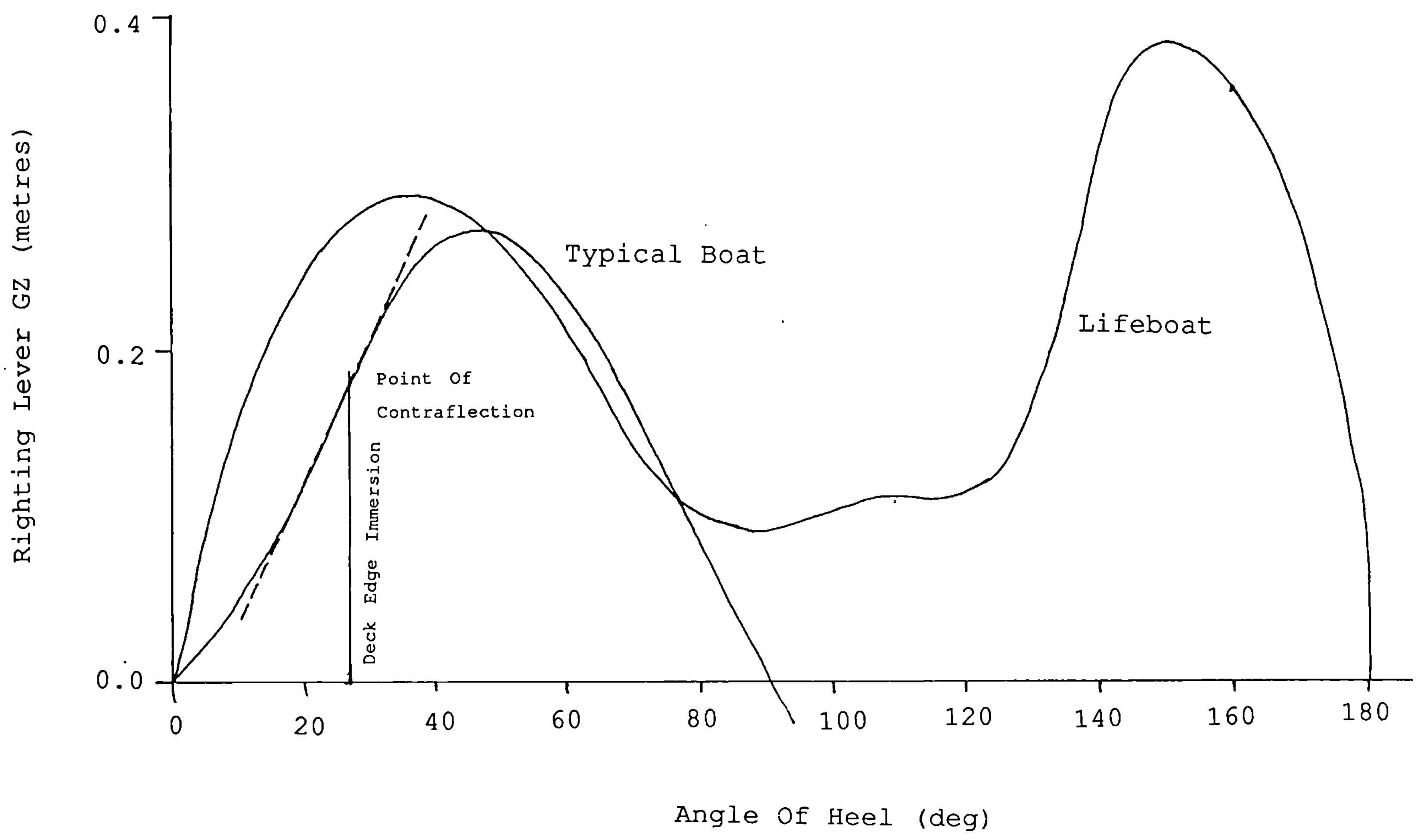
where

$$(y_g - y_b) \cos\phi - (z_g - z_b) \sin\phi = GZ$$

Instead of giving the location of the centre of buoyancy, naval architects produce a statical stability curve or GZ curve (Fig.6.5). This gives the value of GZ against the angle of roll. It is then simply a matter of multiplying the righting lever by weight to obtain the righting moment at any particular angle of roll. Often a curve of dynamical stability is produced which gives a measure of the boat's ability to recover its initial position. This curve is obtained by integrating the curve of statical stability from upright to each angle of heel multiplied by the weight.

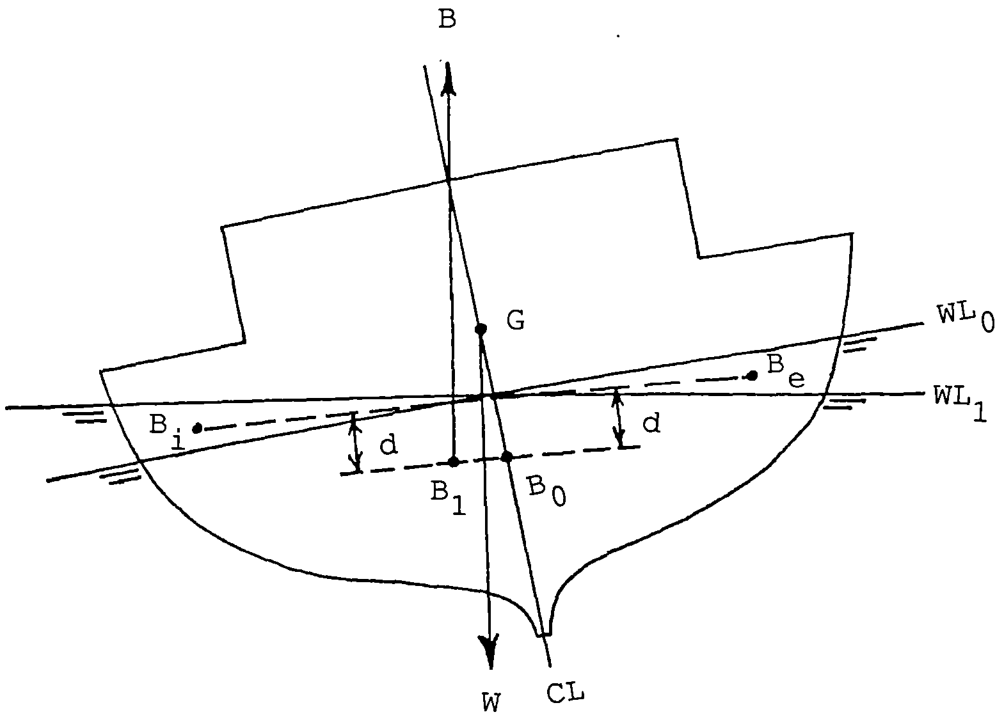
A useful aid in determining the location of the centre of buoyancy at a given angle of heel is provided by Attwood's formula. This shows that the overall centre of buoyancy will move along a line parallel to a line joining the centres of buoyancy of the immersed and emmersed volumes (Fig.6.6).

Several factors influence the shape of the statical stability curve, these include:



Typical Statical Stability Curves

Fig.6.5



After Attwood's Formula - C_b Moves Parallel To Line Joining C_b's Of Immersed And Emmersed Sections

Movement Of Centre Of Buoyancy

Fig.6.6

Beam: this tends to increase GZ, but reduces the range of stability, id est the curve will become negative earlier (Fig.6.7a).

Centre of Gravity: lowering the centre of gravity will improve the stability and increase the range (Fig.6.7b).

Freeboard: this has a bearing on the angle at which the deck edge becomes immersed and is therefore a crucial factor affecting stability. When DEI (Deck Edge Immersion) occurs, the boat will "ship" water and the effects of free-surface can be detrimental to a boat's stability. Up until DEI, the amount of freeboard will have no effect, but beyond this point a boat with increased freeboard will have increased GZ and a greater range of stability (Fig.6.7c).

If the boat is pitching as well as rolling, then the roll righting moment will be affected by the combination of these motions, and becomes:

$$(y_g W - y_b B) \cos\theta \cos\phi \quad - \quad (z_g W - z_b B) \cos\theta \sin\phi$$

which can be expressed in terms of the elements of the direction cosine matrix for transformations from the

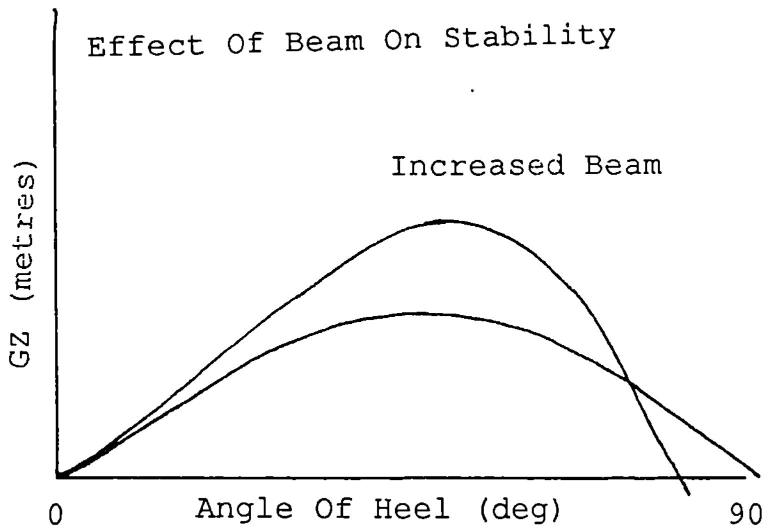


Fig.6.7a

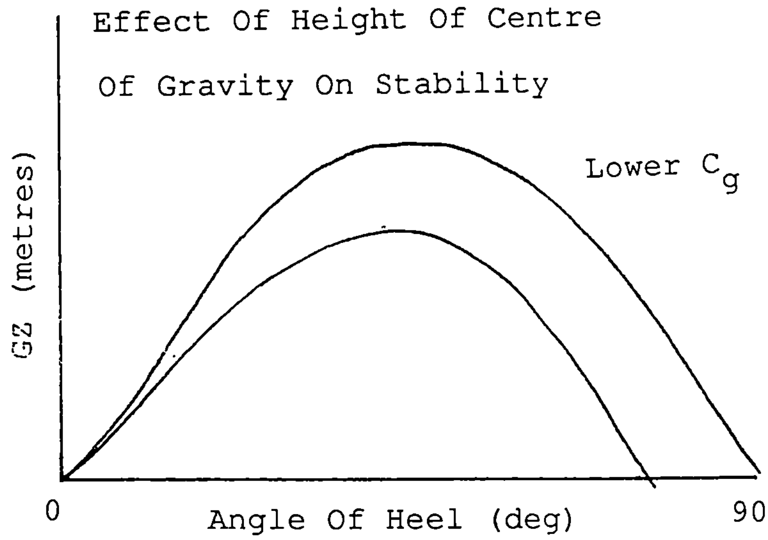


Fig.6.7b

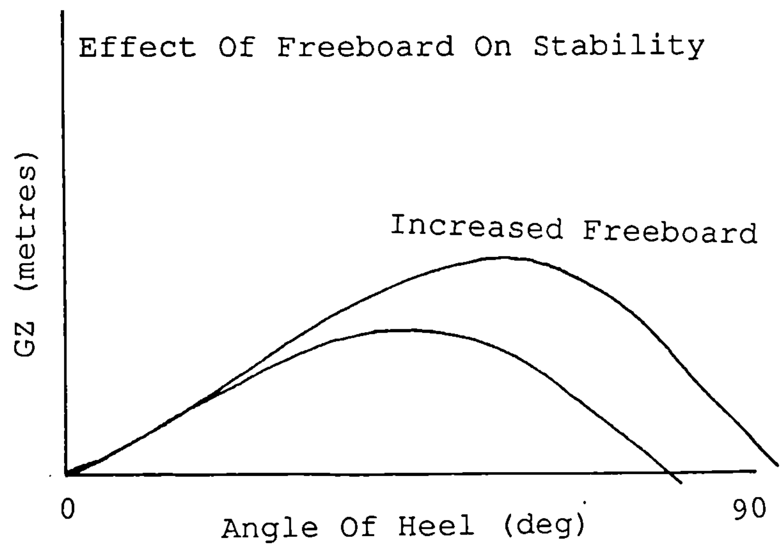


Fig.6.7c

Factors Affecting Roll Stability

moving axes system to the inertial system as:

$$(y_g W - y_b B)\ \lambda_{33} \quad - \quad (z_g W - z_b B)\ \lambda_{32}$$

Pitch Restoring Moment

The principles that determine the roll righting moment also apply to the pitch righting moment. The main difference is that pitch is the longitudinal inclination of the boat about an athwartships axis and does not exhibit symmetry about the YZ-plane when upright. Rotations of pitch again rely on the relative positions of the centres of gravity and buoyancy, but are concerned with their x and z coordinates. The angles of longitudinal inclination are small with respect to the transverse rotations, and consequently naval architects confine themselves to simply expressing changes of trim. Trim is the difference between the draft aft and the draft forward, but for this analysis it will not be distinguished from pitch as no allowance will be made for ballasting or other changes in the weight distribution.

For longitudinal inclinations a boat pivots about a point known as the centre of floatation. This is the centre of the waterplane area and is located somewhere near the amidships position. Normally it will be a little abaft midships and this is due to the asymmetry of the boat's longitudinal shape (Fig.6.8).

The pitch restoring moment, in the absence of roll, can be expressed, from trigonometry, as:

$$- (z_g W - z_b B) \sin\theta \quad - \quad (x_g W - x_b B) \cos\theta$$

if, as in the equilibrium condition, B = W, then this reduces to:

$$W \left(- (z_g - z_b) \sin\theta \quad - \quad (x_g - x_b) \cos\theta \right)$$

where:

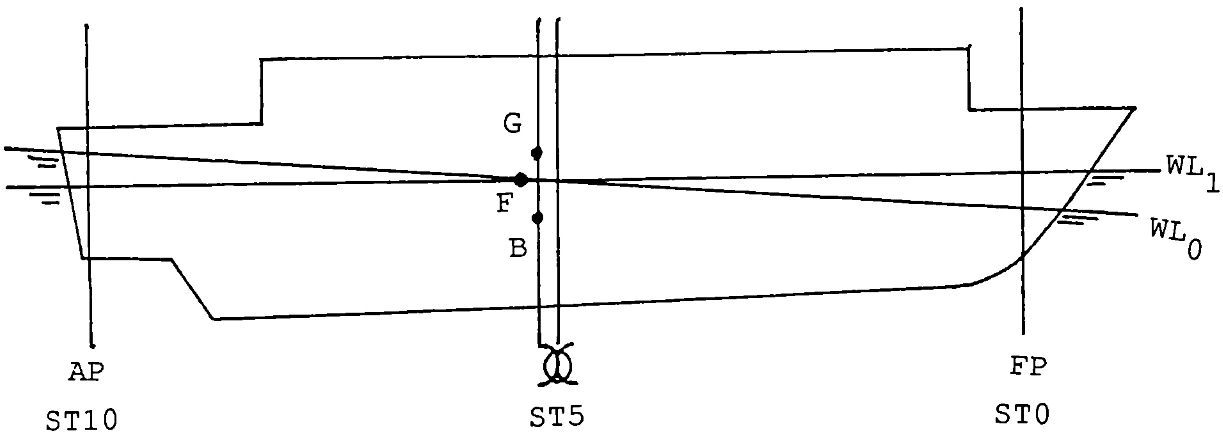
$$- (z_g - z_b) \sin\theta \quad - \quad (x_g - x_b) \cos\theta \quad = \quad GZ_{long}$$

GZ_{long} is the equivalent of GZ, but in the longitudinal sense. It is the horizontal separation of the vertical upwards through B and the vertical downwards through G.

Allowing for the effects of roll, the pitch righting moment becomes:

$$- (z_g W - z_b B) \sin\theta \quad - \quad (x_g W - x_b B) \cos\theta\cos\phi$$

which can be expressed in terms of the elements of the



$$GZ = -(z_g - z_b)\sin\theta - (x_g - x_b)\cos\theta$$

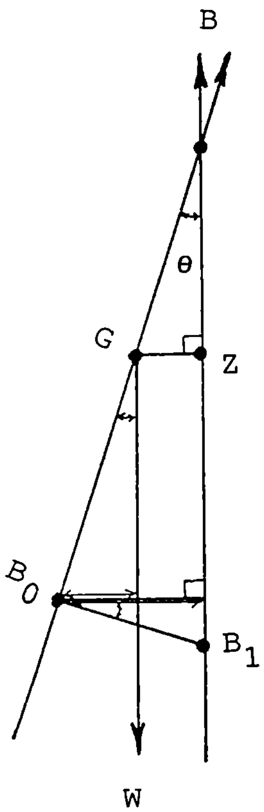


Fig.6.8

Trigonometry Of Pitch Righting Lever

direction cosine matrix for transformations from the moving axes system to the inertial system as:

$$(z_g W - z_b B)\ \lambda_{31} \quad - \quad (x_g W - x_b B)\ \lambda_{33}$$

Heave Restoring Force

When the density of the water alters, the boat will assume a new draft, such that the product of the underwater volume and the water density equate to the displacement. However, if the boat experiences some perturbation which causes a vertical displacement and there is no change in density, then the boat will naturally seek to resume the original underwater volume prior to the disturbance. This is the heave restoring force.

In the equilibrium condition the forces of weight and buoyancy must be equal. When the boat is caused to heave, the underwater volume changes and so, therefore, does the buoyancy force, since:

$$B = \nabla\ \rho\ g$$

Since the weight will remain constant, and is given as:

$$W = m\ g$$

the heave restoring force is dependent on the difference between the weight and buoyancy forces. If the boat is rolling and pitching, then this difference will be reduced and the heave restoring force is given by:

$$(W - B) \cos\theta\cos\phi$$

which can again be represented in terms of the elements of the direction cosine matrix for transformations from the moving axes system to the inertial system as:

$$(W - B) \lambda_{33}$$

The Remaining Restoring Forces

A combination of a change in buoyancy and the boat pitching will yield a surge force. The magnitude of this force will increase with pitch and can be expressed as:

$$- (W - B) \sin\theta$$

Similarly, a combination of a change in buoyancy and the boat rolling and pitching will create a sway force given as:

$$(W - B) \cos\theta\sin\phi$$

The action of rolling and pitching will also generate a yaw moment which is written as:

$$(x_g W - x_b B)\ \cos\theta \sin\phi \quad + \quad (y_g W - y_b B)\ \sin\theta$$

These three equations can, like those for roll, pitch and heave, be expressed in terms of the elements of the direction cosine matrix for transformations from the moving axes system to the inertial system.

The Gravity-Buoyancy Vector

Collecting all the six restoring forces and moments together in vector form and resolving the weight and buoyancy using the direction cosine elements, gives:

$$\begin{bmatrix} \lambda_{31}(W-B) \\ \lambda_{32}(W-B) \\ \lambda_{33}(W-B) \\ \lambda_{33}(y_g W - y_b B) - \lambda_{32}(z_g W - z_b B) \\ \lambda_{31}(z_g W - z_b B) - \lambda_{33}(x_g W - x_b B) \\ \lambda_{32}(x_g W - x_b B) - \lambda_{31}(y_g W - y_b B) \end{bmatrix}$$

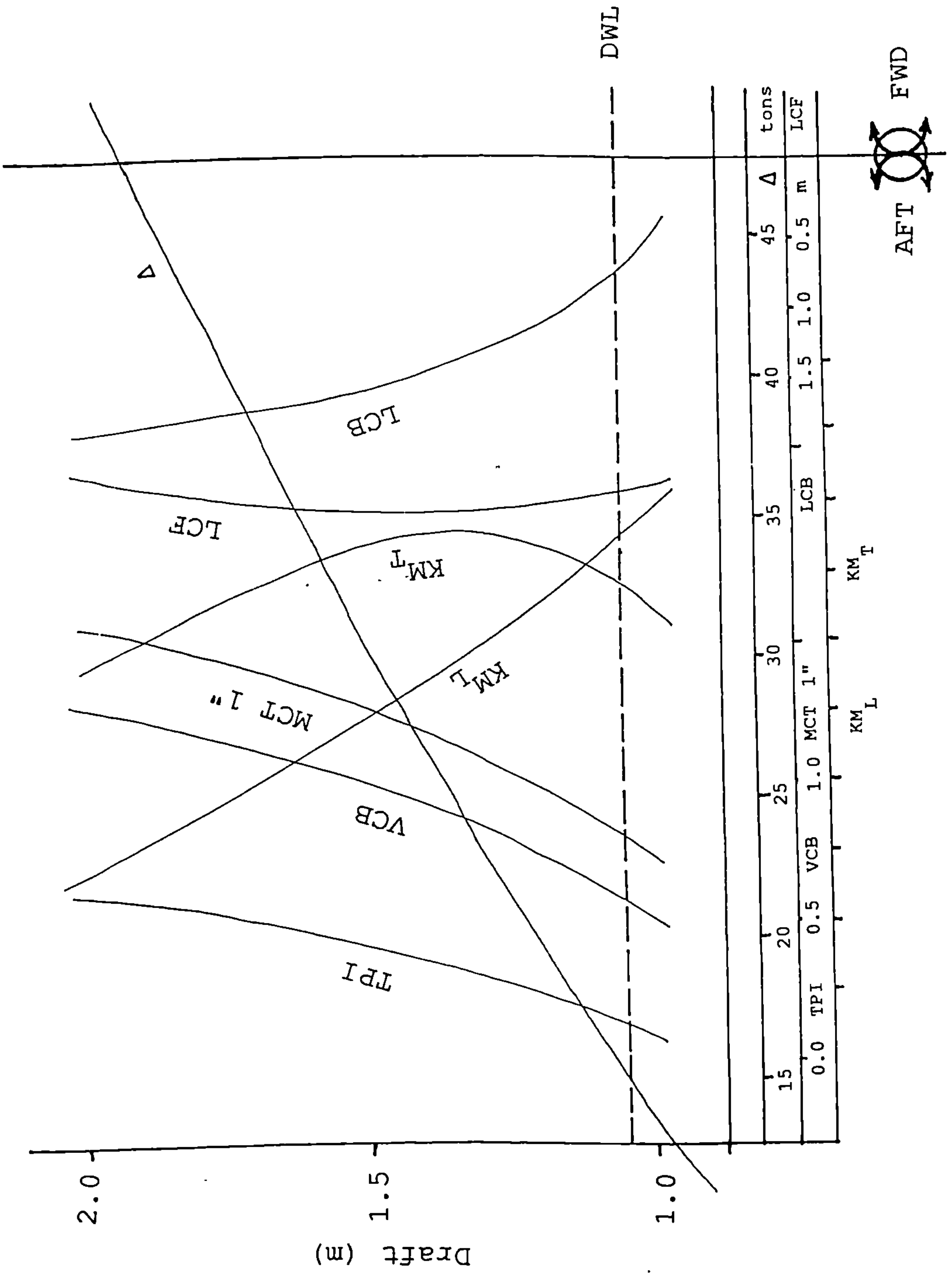
Hydrostatic Curves

Some of the static values which are useful when computing the restoring forces and moments can be gleaned from the hydrostatic curves.

These curves depict a number of terms which vary with draft and are entirely dependent upon the geometrical shape of the underwater part of the boat. The curves are presented with the independent variable of draft on the vertical axis. These figures for draft are often with respect to the underside of the keel (extreme draft) at amidships (station 5). The horizontal axis has a base scale which sets out the varying displacement, but also has a scale of distance from the midships position for some of the remaining terms. The following curves are usually shown (Fig.6.9):

Displacement Δ: This initially curves out from the origin and becomes nearly a straight line. It usually applies to seawater density.

Vertical Centre of Buoyancy (VCB): This also approximates to a steep, straight line and gives the vertical distance of the centre of buoyancy from the underside of the keel.



Typical Hydrostatic Curves

Fig.6.9

Longitudinal Centre of Buoyancy (LCB): This is often a steep, but slightly curving line which shows the longitudinal distance of the centre of buoyancy from amidships.

Longitudinal Centre of Floatation (LCF): This value varies only slightly, often reaching a maximum and then decreasing again. It gives the longitudinal distance of the centre of floatation, about which the boat will pivot, from amidships.

Tons Per inch Immersion (TPI): Gives the number of tons that must be added to cause one inch sinkage. This is a steep, nearly straight line and many curves are still drawn with the imperial system of units.

Moment to Change Trim 1" (MCT 1"): Gives the moment in tons feet required to effect a change in trim of one inch.

Longitudinal Metacentric Height (KM_L): Gives the height above the underside of the keel of the longitudinal metacentres.

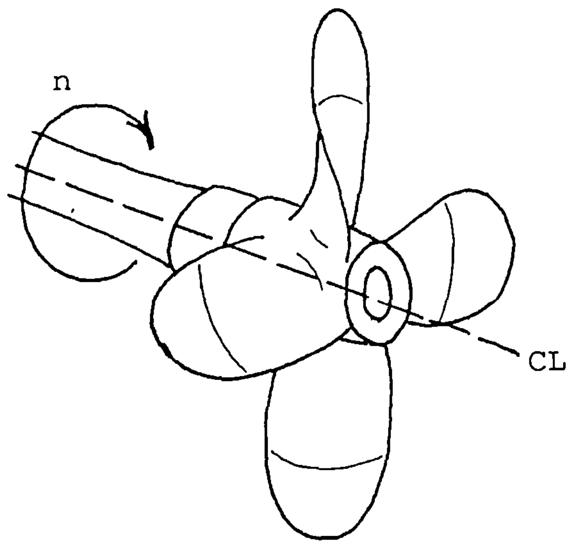
Transverse Metacentric Height (KM_T): Gives the height above the underside of the keel of the transverse metacentres.

For many small boats, the keel line will not necessarily be horizontal, but will be "raked" so that the draft at the aft perpendicular is greater than the draft at the forward perpendicular. The hydrostatic curves are therefore usually calculated for the midships position (station 5) and corrections must be made according to the rake. The rake is expressed as the difference of the aft and forward drafts, and knowing the length between perpendiculars (LBP) the gradient of the keel line can be computed.

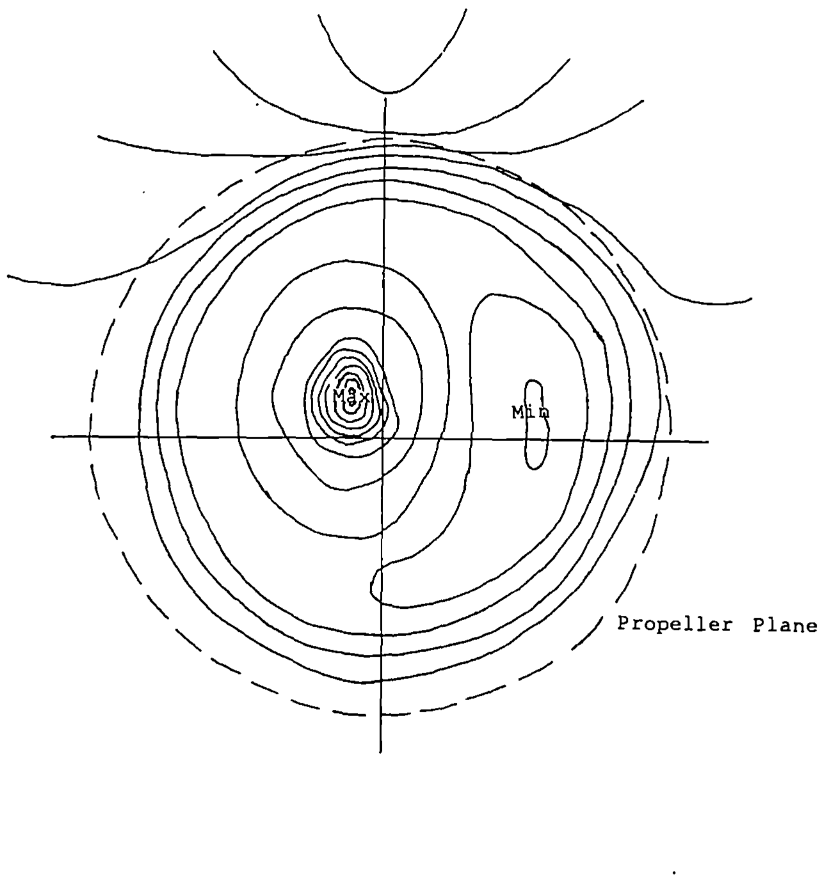
6.11 The Rudder Forces And Moments

Whilst the concept of using a rudder to provide a means of steering a boat seems quite simple, the actual situation is an extremely complex one. Interaction effects between the hull, propeller and rudder cause modifications to the flow past the rudder and the inflow angle (Fig.6.10). Lötveit, 1959 (Ref.95), provides an extremely useful study of rudder action.

The rudder can, in principle, be related to a hydrofoil with a low aspect ratio, and aerodynamic theory can be applied to determine the forces and moments generated by the rudder. The forces acting upon an aerofoil are usually expressed as dimensionless lift and drag coefficients, denoted C_L and C_D respectively. The main



Typical 4-Blade Propeller



Velocity Distribution In Propeller Slip-Stream

Fig.6.10

purpose of the rudder is to generate a side force, by which it is possible to produce yaw and therefore steerability. It is therefore of great importance to know the magnitude of the rudder "lift", or side forces at given speeds and rudder angles. This side force is often termed the rudder normal force.

Rudder Normal Force

The author follows the approach adopted by Japanese researchers such as Ogawa and Kasai, 1978 (Ref.115). The "open" water characteristics of the rudder normal force will be developed first, with additional terms describing the changes of the rudder inflow velocity and angle which occur when a rudder is located behind the hull and in the propeller slip-stream. Open water refers to the condition where the rudder is isolated from the hull and propeller effects and placed deep in the water.

In open water the rudder normal force is usually expressed as:

$$F_N = C_{FN} \frac{\rho}{2} A_R V_R^2$$

where: ρ is the water density, A_R is the rudder area, V_R is the rudder inflow velocity and C_{FN} is the rudder normal coefficient and can be regarded as:

$$C_{FN} = f(\delta, \Gamma, R_e, t/c, \text{Rudder Outline})$$

where: δ is the rudder angle, Γ is the rudder aspect ratio, R_e is the Reynold's number for the particular rudder flow and t/c is the rudder thickness ratio.

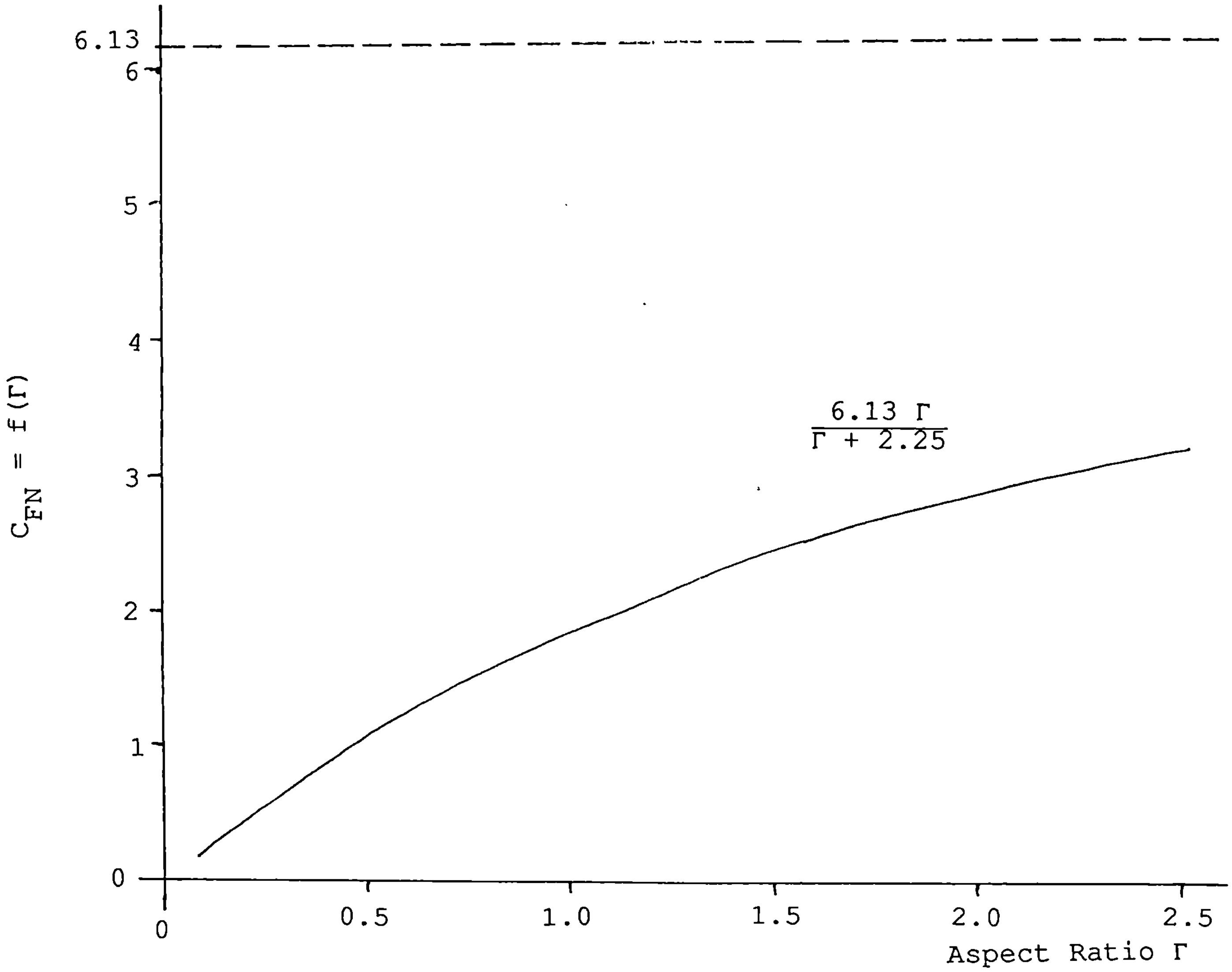
The rudder outline and thickness ratio apparently have minimal effect and especially for rudders of near rectangular shape can be regarded as being constant. Whilst the Reynold's number is important in the open water characteristics, for rudders operating in the propeller slip-stream the "burbling" point will usually occur above the maximum rudder angle. This leaves just the two primary parameters:

$$C_{FN} = f(\Gamma, \delta)$$

Tests monitoring the effect of rudder aspect ratio upon the rudder normal force produce the following empirical relationship (Fig.6.11):

$$C_{FN} \propto \frac{6.13\ \Gamma}{\Gamma + 2.25}$$

The effect of rudder angle is included from a



Rudder Normal Force Coefficient

Fig.6.11

theoretical stand point and gives:

$$C_{FN} = \frac{6.13\ \Gamma}{\Gamma + 2.25} \sin\delta$$

This result is widely used and accepted in practice. The rudder normal force is therefore given as:

$$F_N = \frac{6.13\ \Gamma}{\Gamma + 2.25} \frac{\rho}{2} A_R V_R^2 \sin\delta$$

Hull Wake

The wake from the hull is a complicated feature which entirely depends upon the shape of the aft section of the boat. It is therefore usual to represent the effect of the hull wake as a fraction, w_R. The boat's velocity V is then modified by this wake fraction as:

$$V_P = (1 - w_R)\ V$$

where: V_P will be the flow velocity appearing at the propeller and V is determined from:

$$V^2 = u^2 + v^2$$

where: u is the forward velocity of the boat and v is its lateral velocity.

Propeller Slip-Stream

If V_P is the uniform propeller inflow velocity and ΔV_S is the added velocity due to the propeller at the rudder positioned near the propeller, then the velocity at the rudder, V_R, is:

$$V_R = V_P + \Delta V_S$$

by utilising the axial momentum theory for an actuator disc, ΔV_S can be represented thus:

$$\Delta V_S = V_P K_m \left(\sqrt{1 + \frac{8}{\pi} \frac{K_T}{J_P^2}} - 1 \right)$$

where: K_T is the propeller thrust coefficient and J_P is the corresponding advance constant (see the section on the propeller vector for a more detailed discussion of these two terms). Also K_m is a coefficient which is a function of the axial position of the rudder with respect to the propeller. At the propeller centre K_m=0.5, at a point infinitely downstream of the propeller K_m=1.0, therefore at the normal rudder position K_m will be between these two limits.

However, the non-uniformity of the rudder in flow velocity requires the inclusion of an attenuation of the acceleration effect. For relatively low propeller loadings, Ogawa and Kasai show that the following established formula for V_R is valid:

$$V_R = V_P \sqrt{1 + ks^{3/2}}$$

where: k is an empirical factor and s is the propeller slip-stream ratio, given by:

$$s = 1 - (1 - w_P) \frac{V_P}{nP}$$

where: w_P is the propeller wake fraction, n is the propeller revolutions and P is the propeller pitch.

Effective Rudder Inflow Velocity

The effective rudder inflow velocity can be obtained by comparing the rudder normal forces for the open water condition with the condition when the hull and propeller are in place for the same rudder angles. The inflow velocity, V_R, is then:

$$V_R = V \sqrt{\frac{F_N}{F_{No}}}$$

where: V is the open water inflow velocity, F_N is the rudder normal force with the hull and propeller effects and F_{No} is the rudder normal force in open water.

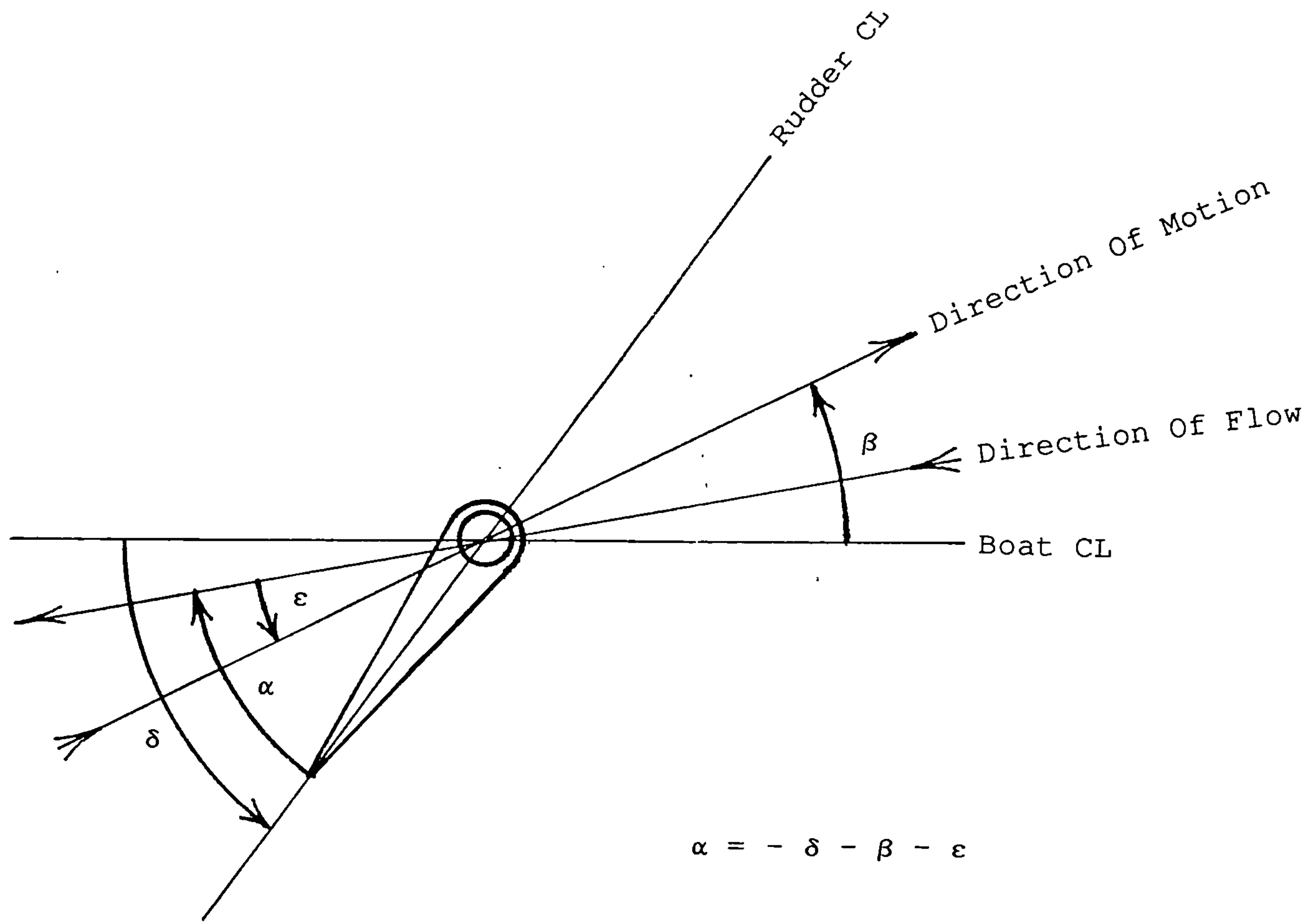
Combining the boat's forward and lateral velocities with the hull wake effect and the propeller slip-stream effect, gives the rudder inflow velocity as:

$$V_R = (1 - w_R)\, V \sqrt{1 + ks^{3/2}}$$

where: k, the empirical constant, equals 1.065 for port turns and 0.935 for starboard turns for single screw, single rudder arrangements. However, the Arun class lifeboat has a twin propeller, twin rudder arrangement with the rudder mounted behind the counter rotating propellers. Therefore it is assumed that any uneven flows due to the rotation of one propeller will be cancelled by the other propeller's opposite rotation, hence k will be equal to 1.0.

Effective Rudder Inflow Angle

As well as the modification of the flow velocity, there is also the need to determine the effective rudder inflow angle (Fig.6.12). This will be a function of the



$$\alpha = -\delta - \beta - \varepsilon$$

Effective Rudder Inflow Angle

Fig.6.12

actual rudder angle δ, the direction of motion of the boat with reference to its centreline β and the additional change in the flow angle due to the hull and propeller effects ε. The effective rudder angle, α, is therefore:

$$\alpha = -\delta - \beta - \varepsilon$$

The angles are measured in specific directions and the signs reflect this. β can be determined from:

$$\beta = \tan^{-1}(v/u)$$

or, as will be used in the simulation:

$$\beta = \sin^{-1}(v/V)$$

ε is sometimes referred to as the flow rectifying angle which after Inoue, Hirano, Kijima and Takashina, 1981 (Ref.76), is given as:

$$\varepsilon = \beta(\gamma-1) + 2\gamma x_R r/V$$

where: x_R is the x-coordinate of the centre of effect of the rudder, r is the yaw rate, V is the boat's resultant velocity and γ is a flow rectification coefficient which is split into two parts:

$$\gamma = C_P \, C_H$$

The propeller flow rectification coefficient, C_P, is expressed by the empirical formula:

$$C_P = \left[1 + 1.2\eta \frac{(1 - 0.7s)s}{(1 - s)^2} \right]^{-1/2}$$

where: s is the propeller slip-stream ratio and η is the propeller diameter, D, to rudder height, H, ratio:

$$\eta = D/H$$

The hull rectification coefficient, C_H, is given in a slightly different form as:

$$C_H = \begin{cases} 0.45\,\nu & \nu <= 0.5/0.45 \\ 0.5 & \nu > 0.5/0.45 \end{cases}$$

where: the numbers are empirically based constants and ν is given as:

$$\nu = (\varepsilon - 2x_R r/V) \; / \; (1 - \gamma)$$

Many models based on Taylor series expansions simply include coefficients for the rudder-exciting forces. When viewed in the light of the preceding discussion, it is obvious that coefficients based on regression techniques alone may not be sufficient for modelling purposes. It is always desirable to relate the coefficients to the physical situation and avoid abstract notations.

The preceding may not be a full representation of the flow around the aft section of a boat, but at least approaches logically the factors involved. As a result of the complex nature of the flow patterns and the difficulty of accurately reproducing them, it may be desirable to make some simplifying assumptions, particularly with reference to the effective rudder inflow angle, to reduce the computations required.

The Rudder Vector

The rudder normal force can now be represented as follows, with α replacing the δ initially suggested:

$$F_N = \frac{6.13\ \Gamma}{\Gamma + 2.25} \frac{\rho}{2} A_R V_R^2 \sin\alpha$$

For surge any deflection of the rudder angle will introduce a retarding or drag force acting so as to reduce the forward velocity. As the deflection increases either to port or starboard, so this force increases negatively. The rudder normal force for surge will be modified by a $-\sin\delta$ factor.

Sway will be similarly affected, but since it is at right angles to surge, the modifying factor will be $-\cos\delta$. Heave will be unaffected by the rudder deflection. Roll will follow the same tendency as sway, but since it is the moment about the fore-aft axis, the sign of the modifying factor will depend upon the vertical position of the centre of effect of the rudder. Pitch, like heave will not be affected by rudder deflections directly.

Yaw, which is the motion that the rudder is designed to create so that course holding and changing can be achieved, will vary as a function of $-\cos\delta$. The moment is affected by the longitudinal position of the centre of effect of the rudder and since in virtually all boats the rudder is placed aft, a minus sign is included.

The rudder vector, R, will become:

$$\begin{bmatrix} -C_{RU}\, F_N \sin\delta \\ -C_{RV}\, F_N \cos\delta \\ 0.0 \\ C_{RP}\, F_N\, z_R \cos\delta \\ 0.0 \\ -C_{RR}\, F_N\, x_R \cos\delta \end{bmatrix}$$

C_{RU}, C_{RV}, C_{RP} and C_{RR} are rudder coefficients. Comparing these to the Japanese researchers' work gives:

$$C_{RU} = 1$$
$$C_{RV} = (1 + a_H)$$
$$C_{RP} = (1 + a_H)$$
$$C_{RR} = (1 + a_H)$$

where: a_H is the ratio of the hydrodynamic force, induced on the boat hull by rudder action, to the rudder force. A typical value for a_H is 0.22, although it is thought to be a function of the block coefficient C_B of the vessel.

Rudder Angle

Two methods of determining the rudder angle are to be used. Firstly, for verifying the model action and for subsequent simulation runs where the helmsman or user will have control of the rudder, the following differential equation will apply:

$$\dot{\delta} = (\delta_d - \delta) / \tau_r$$

where: δ_d is the rudder angle demanded by the helmsman (or simulation user) and τ_r is the rudder time constant.

When the autopilot is attached to drive the boat model, a different method will be used to determine rudder position. The function of the autopilot is to maintain a desired course, which it does by attempting to reduce the course error at every stage. The autopilot computes the course error from heading information and then determines both the direction in which the rudder must be moved to reduce this error, and also the rate which will best accomplish this.

Instead of being sent to the steering gear of a "real" rudder onboard boat, this message will be passed to the boat simulation. The rudder rate code is a number between 0 and 8 inclusive, which represents so many eighths of the maximum rudder rate. The direction, port or starboard, will indicate the sign of the rate. Therefore, the rudder rate can be expressed as:

$$\dot{\delta} = \delta_{dir} \frac{i}{8} \dot{\delta}_{max}$$

where: δ_{dir} is -1 for port rudder or +1 for starboard rudder, i is the rate number and $\dot{\delta}_{max}$ is the magnitude of the maximum rudder rate.

6.12 The Propeller Forces And Moments

A good first text on representing the propeller characteristics is by Baker and Patterson, 1969 (Ref.20). There are in fact four propeller regimes that could be considered, which are:

1) Boat with forward velocity & propeller advancing;
2) Boat with reverse velocity & propeller advancing;
3) Boat with forward velocity & propeller reversing;
4) Boat with reverse velocity & propeller reversing.

Only the first condition will be dealt with, since boats under way seldom enter the other three regimes. The latter three are mainly required when entering a marina or performing some other delicate adjustment manoeuvre.

Fundamental to the analysis of propeller forces and moments are the terms propeller thrust, P_T and propeller torque, P_Q. These can be expressed as:

$$P_T = \rho n^2 D^4 K_T$$

$$P_Q = -\rho n^2 D^5 K_Q - 2\pi J_{pp} \dot{n}$$

where: ρ is the water density, n is the propeller revolutions, $\dot{n}$ is the rate of change of the propeller revolutions, D is the propeller diameter, J_{pp} is the added moment of rotary inertia of the propeller, K_T is the thrust coefficient and K_Q is the torque coefficient.

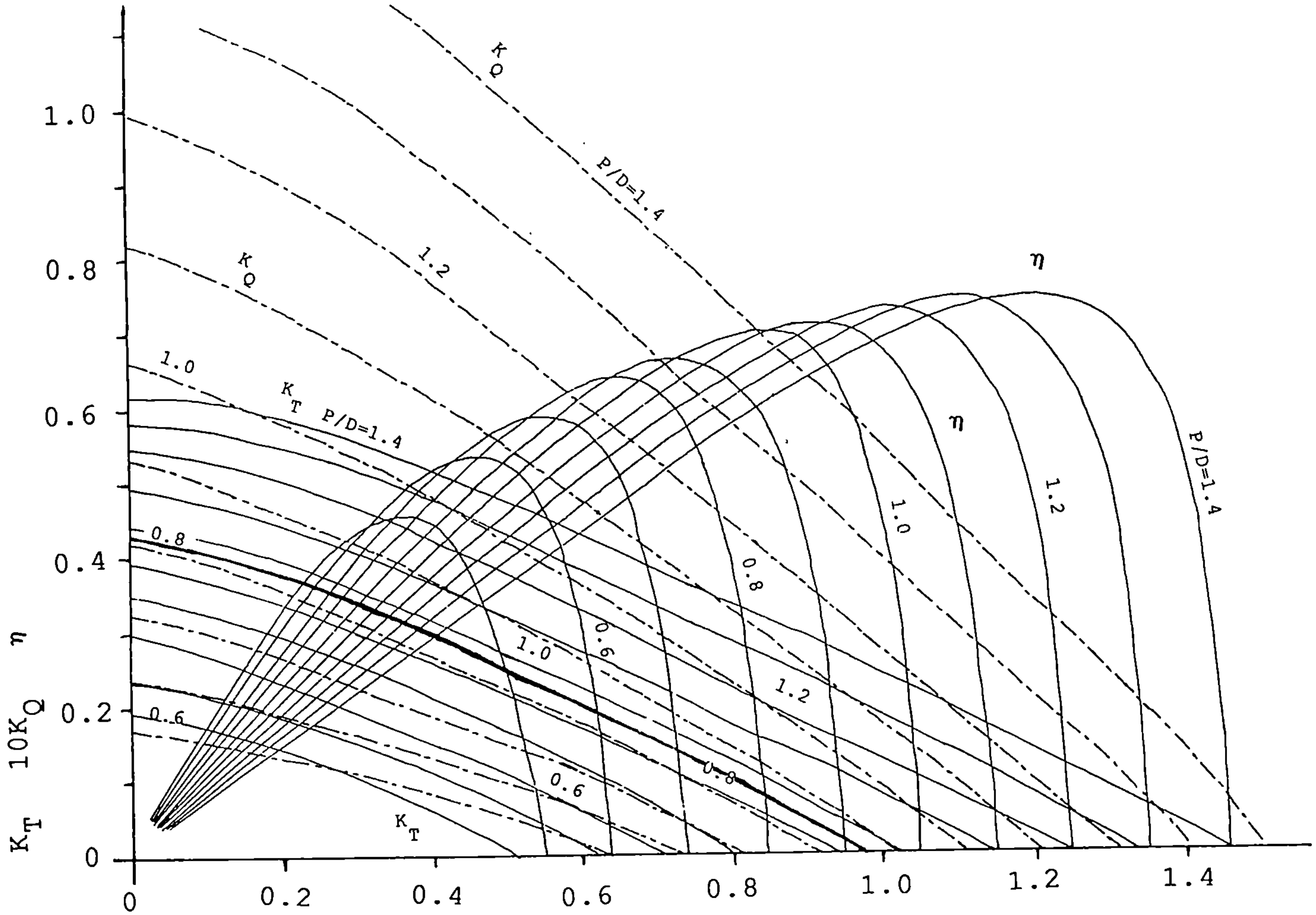
Both K_T and K_Q are functions of the advance constant, J_P (not to be mistaken with J_{pp}) which is expressed as:

$$J_P = \frac{u}{nD}$$

where u is the boat's forward velocity. J_P sometimes includes the propeller wake fraction, thus:

$$J_P = (1 - w_P) \frac{u}{nD}$$

Most propeller forms will have their characteristic documented in the form of curves. These typically show plots of the thrust coefficient, torque coefficient and propeller efficiency against a base of the advance constant (Fig.6.13). The curves are plotted for a number



Propeller Characteristic Curves

Fig.6.13

of pitch to diameter values ranging from about P/D=0.5 to P/D=1.5. Separate sets of curves are generated for different DAR's (Developed Area Ratios).

The DAR effectively gives the ratio of blade area to the circumscribed circle or disc, so that if the developed area (sum of the area of each blade) is F_a and the circumscribed area is F, the DAR will be:

$$F = \pi \left(\frac{D}{2} \right)^2$$

$$DAR = F_a / F$$

For the Arun lifeboat, the diameter of each propeller is 0.826m and developed area is 0.375m^2, therefore:

$$F = \pi \; 0.826^2 / \; 4 = 0.536m^2$$

$$F_a = 0.375m^2$$

$$DAR = 0.375 \; / \; 0.536 = 0.70$$

For the purposes of the simulation, the thrust coefficient curve will be approximated by a cubic polynomial in the advance constant J_p knowing the actual propeller pitch to diameter ratio and DAR for the Arun.

$P/D = 0.772 / 0.826 = 0.935$

$DAR = 0.70$

Therefore, K_T will be expressed as:

$$K_T = 0.154\, J_P^3 - 0.361\, J_P^2 - 0.222\, J_P + 0.430$$

The Propeller Vector

The propeller thrust will almost entirely be directed along the fore-aft line, thus the surge equation will equal the propeller thrust with an additional coefficient. Thus:

$$X_{propeller} = C_{PU}\, \rho n^2 D^4 K_T$$

where: C_{PU} is the thrust reduction coefficient.

The only sway that can be generated from the propeller is if there is some sort of flow anomaly or a secondary effect if the propeller causes any yaw. Heave is also assumed zero, though there is likely to be a small component associated with pitch. There will be no roll moment produced by the propeller.

As the propeller revolutions increase so, for small boats, will the bow rise. The boat will attain a constant angle of pitch, balanced by the pitch restoring moment. The pitch moment will be expressed as:

$$M_{propeller} = C_{PQ}\,\rho n^2 D^5 K_T$$

where: C_{PQ} is a pitch propeller coefficient.

With single screw arrangements, the rotation of the propeller will actually generate a slight yawing motion. If left unchecked therefore, the boat would follow a large arced trajectory. This phenomenon is well-known in naval architecture and twin propeller arrangements are set counter rotating in order to nullify this effect. In large ship models the propeller yaw is usually assumed zero and since the Arun lifeboat, used in the model validation, has a twin arrangement this assumption will be followed here.

The propeller vector, P, will therefore be:

$$\begin{bmatrix} C_{PU}\ \rho n^2 D^4 K_T \\ 0.0 \\ 0.0 \\ 0.0 \\ C_{PQ}\ \rho n^2 D^5 K_T \\ 0.0 \end{bmatrix}$$

Propeller Revolutions

The propeller revolutions will be determined from the following differential equation:

$$\dot{n} = (n_d - n) / \tau_n$$

where: n_d is the revolutions demanded by the helmsman (or simulation user) and τ_n is the propeller time constant.

6.13 Trim Tabs Forces And Moments

The Arun lifeboat has a pair of trim tabs mounted symmetrically either side of the centreline on the aft of the stern and a little below the waterline. The purpose of these is to reduce the angle to which the bows rise when propeller thrust is applied. In their zero position the tabs lie horizontal and no moment is produced. The further the tabs are lowered, the greater the reduction in the trim angle. Typically, the tabs have a maximum angle of about 15° and can make a difference of about 3° to 4° in the trim.

The trim tabs will be treated in a similar manner to that of the rudders, since they represent a flow over a flat surface inclined to the direction of flow. The main difference is because of the method of mounting the tabs, the flow of water is only over the underside. Allowance will be made for this in a reduction coefficient.

In practice, lowering the trim tabs will cause a slight retardation in surge, but this effect is assumed to be small in comparison with the rudder drag and other complex flow phenomena around the after section of the boat. Apart from pitch, all of the forces and moments are assumed to be negligible or zero, especially as it is the trimming effect that is under study.

The pitch moment due to the trim tabs will be expressed as:

$$M_{tabs} = C_{TQ} \frac{\rho}{2} \Gamma_T A_T x_T V_T^2 \sin\alpha_T$$

where: C_{TQ} is the trim tab pitch coefficient, ρ is the water density, Γ_T is the aspect ratio of a tab, A_T is the area of a tab, x_T is the longitudinal position of the centre of effect of the trim tabs, V_T is the velocity of the flow past the tab and α_T is the angle of deflection of the tabs.

The Trim Tabs Vector

The trim tabs vector, T, looks like:

$$
\begin{bmatrix}
0.0 \\
0.0 \\
0.0 \\
0.0 \\
C_{TQ}\ 0.5\ \rho \Gamma_T A_T x_T V_T^2 \sin\alpha_T \\
0.0
\end{bmatrix}
$$

The simulation user, or pseudo helmsman, will have control of the trim tabs setting, therefore the following differential equation for the tabs setting is proposed:

$$\dot{\alpha}_T = (\alpha_{Td} - \alpha_T) / \tau_t$$

where: α_{Td} is the demanded tabs setting in an angular form and τ_t is the tabs time constant.

6.14 The Wind Forces And Moments

While there are fluid flow computer simulators, accurate theoretical prediction of the wind flow around a boat is still not feasible. To include such fluid flow programs within the overall boat model would dwarf the boat manoeuvring mathematics and impose unacceptable time demands on the simulation interval. It is therefore necessary to rely upon experimental tests carried out in wind tunnels and pre-simulation analysis of the wind effect on the boat. Clearly the surface area of the boat above the waterline exposed to the wind will be the prime factor influencing windage. This will depend upon the amount and distribution of the superstructure and the relative wind angle.

Such data is usually expressed in terms of dimensionless coefficients plotted as a function of the relative wind angle. Aage, 1971 (Ref.1), provides typical data for three cargo ships in different conditions, a tanker, a passenger liner, a ferry and a trawler. The wind forces and moments will be expressed thus:

$$X_{wind} = C_x \frac{1}{2}\rho_a V_w^2 A_{TS}$$

$$Y_{wind} = C_y \frac{1}{2}\rho_a V_w^2 A_{LS}$$

$$Z_{wind} = C_z$$

$$L_{wind} = C_l \frac{1}{2}\rho_a V_w^2 A_{LS} H_{WL}$$

$$M_{wind} = C_m$$

$$N_{wind} = C_n \frac{1}{2}\rho_a V_w^2 A_{LS} L_{OA}$$

where: C_x, C_y, C_z, C_l, C_m and C_n are the dimensionless coefficients, ρ_a is the air density, V_w^2 is the wind velocity, A_{TS} is the projected transverse area of the boat above the waterline, A_{LS} is the projected longitudinal area of the boat above the waterline, H_{WL} is the height of the centre of gravity above the waterline and L_{OA} is the overall length of the boat.

By examining the plots of the dimensionless coefficients against the relative wind angle in Ref.1, it can be seen that the general shape and magnitude of the curves is the same for all ship types. Ideally it is desirable to carry out air flow experiments on a model of the boat under consideration, however this is not possible in this instance and it will be sufficient to approximate the coefficients as:

$$C_x = -0.80 \cos(\gamma-\psi)$$
$$C_y = -0.80 \sin(\gamma-\psi)$$
$$C_z = 0.00$$
$$C_l = -1.20 \sin(\gamma-\psi)$$
$$C_m = 0.00$$
$$C_n = -0.05 \sin 2(\gamma-\psi)$$

where: γ is the absolute wind angle. The magnitude of the coefficients are obtained from Ref.1, with the trigonometric multipliers being approximations to the curves. The trigonometric functions are as would be expected for windage variations. The Arun class lifeboat, used to validate the model, is actually thought to align itself with the wind coming from slightly abaft of beam on when allowed to float freely.

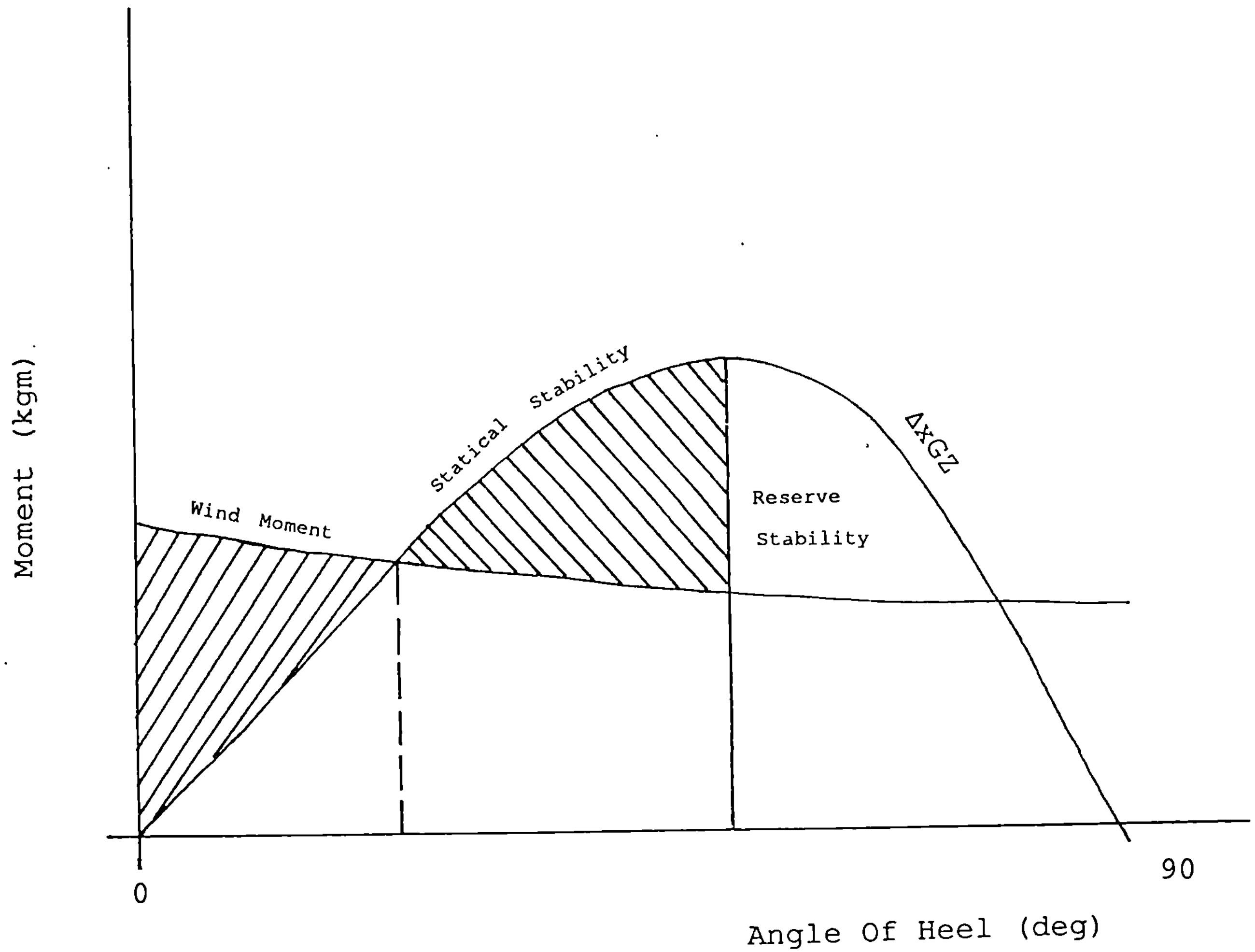
Isherwood, 1972 (Ref.77), presents equations to represent these coefficients based upon an analysis of the wind resistance of several different types of merchant ships. The surge coefficient is given thus:

$$C_x = A_0 + A_1 \frac{2A_L}{L_{OA}^2} + A_2 \frac{2A_T}{B^2} + A_3 \frac{L_{OA}}{B} + A_4 \frac{S}{L_{OA}} + A_5 \frac{C}{L_{OA}} + A_6 M$$

where: A_L is the lateral projected area, A_T is the transverse projected area, B is the breadth of the ship, L_{OA} is the length overall, C is the distance from the bow of the centroid of the lateral projected area, M is the number of distinct groups of masts or kingposts seen in the lateral projection and S is the length of perimeter of lateral projection of model excluding waterline and slender bodies such as masts and ventilators. A_0 to A_6 are coefficients which are tabulated at 10° intervals of the relative wind angle, and will form look-up tables for simulation purposes. Similar expressions exist for the sway and yaw coefficients.

The level of complexity introduced by Isherwood is not deemed pertinent to this analysis of small boats and the equations giving an approximate continuous function will be used.

A point of note is that the angle of steady heel caused by a steady wind can be found by superimposing the wind heeling moment on the curve for the statical stability. The intersection of these curves will determine the angle of steady heel (Fig.6.14).



Wind Heeling Moment

Fig.6.14

The Wind Vector

The wind vector, W, therefore looks like:

$$\begin{bmatrix} -0.80 \cos(\gamma-\psi)\ 0.5\ \rho_a V_w^2 A_{TS} \\ -0.80 \sin(\gamma-\psi)\ 0.5\ \rho_a V_w^2 A_{LS} \\ 0.0 \\ -1.20 \sin(\gamma-\psi)\ 0.5\ \rho_a V_w^2 A_{LS} H_{WL} \\ 0.0 \\ -0.05 \sin 2(\gamma-\psi)\ 0.5\ \rho_a V_w^2 A_{LS} L_{OA} \end{bmatrix}$$

6.15 The Wave Forces And Moments

Although the seaway is far from regular, an idealised water wave having a sinusoidal shape will be assumed. In reality individual wave trains can vary and many will have a trochoidal profile. The addition of a multitude of wave profiles with varying amplitudes and periods leads to the irregular seaway. Predictions of this form of seaway are approached from a statistical point of view. The subject of the motion of a ship in a seaway is covered in a comprehensible manner by Bhattacharyya, 1978 (Ref.26).

The Zeroth Approximation

The wave equation is:

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \phi = 0$$

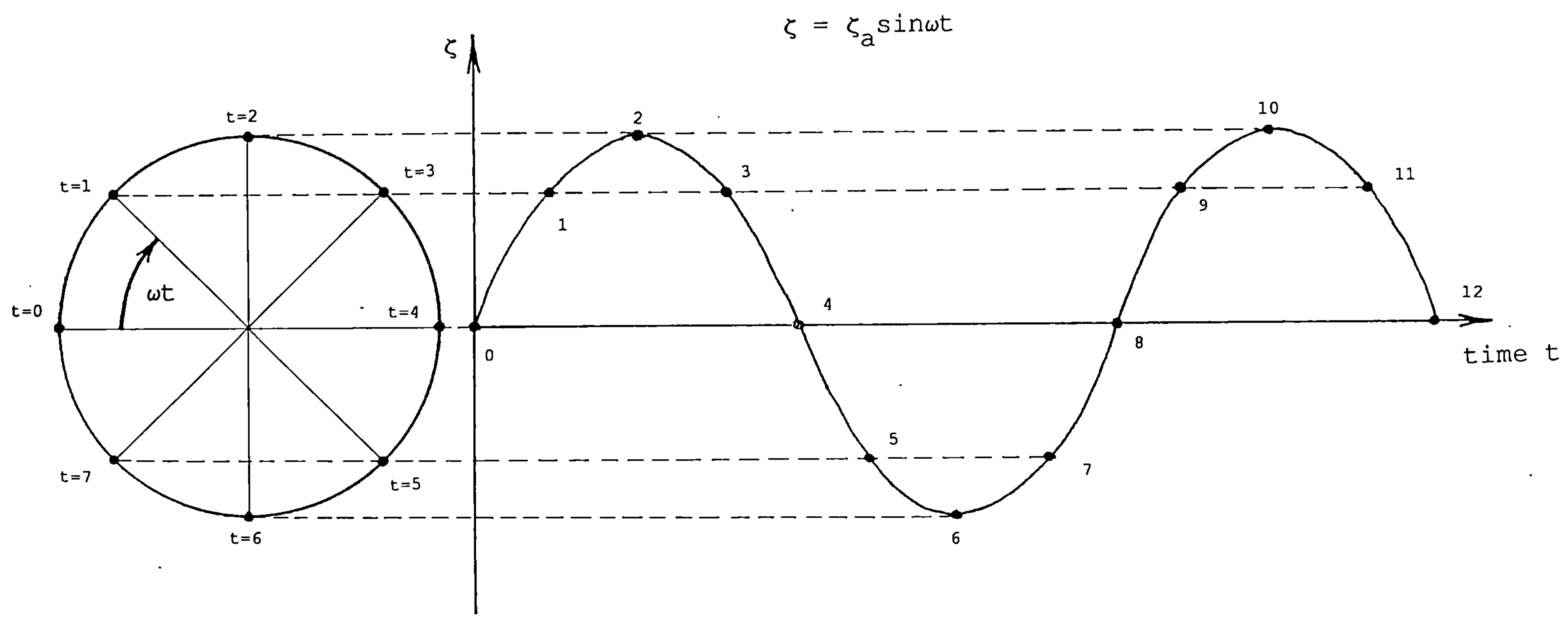
where: ϕ is the gravitational potential and ∇^2 is the del-squared second order partial differential operator which, in cartesian coordinates, is given as:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

The zeroth approximation to the solution of the wave equation assumes two-dimensional irrotational motion satisfying Laplace's equation, for which the influence of the acceleration due to gravity is taken into account. The resultant wave is sinusoidal and each particle is executing simple harmonic motion vertically, but slightly out of phase with its neighbour (Fig.6.15). the wave equation is thus given by:

$$\zeta(x,t) = \zeta_a \sin \left(\frac{2\pi x}{\lambda_v} - \frac{2\pi t}{T} \right)$$

where: ζ is the wave elevation at a given point, ζ_a is the wave amplitude or maximum elevation, t is a measure of time, T is the wave period, x is the horizontal



Sinusoidal Wave Profile

Fig.6.15

distance and λ_v is the wave length.

Two terms are usually introduced at this stage, namely the radian wave number, k and the radian wave frequency, ω. These are expressed as:

$$k = \frac{2\pi}{\lambda_v}$$

$$\omega = \frac{2\pi}{T}$$

and the wave equation reduces to:

$$\zeta = \zeta_a \sin(kx-\omega t)$$

assuming that the incremental distance x is zero this further reduces to the analytical equation:

$$\zeta = - \zeta_a \sin\omega t$$

The rate of change of the wave profile can be obtained by differentiating the wave profile equation with respect to time, and is:

$$\dot{\zeta} = - \zeta_a\omega \cos\omega t$$

The wave slope α can be obtained by differentiating the wave profile equation with respect to distance, and is:

$$\alpha = -\zeta_a k \cos\omega t$$

or

$$\alpha = -\frac{2\pi\zeta_a}{\lambda_v} \cos\omega t$$

Wave Velocity

The wave celerity (or phase velocity) for sinusoidal waves can be drawn from oceanography as:

$$c^2 = \left(\frac{g\lambda_v}{2\pi} + \frac{2\pi\gamma}{\rho\lambda_v} \right) \tanh\left(\frac{2\pi h}{\lambda_v} \right)$$

where:

	c	is the wave celerity
	$\frac{g\lambda_v}{2\pi}$	is the gravitational term
	$\frac{2\pi\gamma}{\rho\lambda_v}$	is the surface tension term
	$\tanh\left(\frac{2\pi h}{\lambda_v}\right)$	is the depth factor

and:

g = acceleration due to gravity

λ_v = wavelength

γ = surface tension ($\cong 7.0 \times 10^{-2}$ kgs^{-1})

ρ = water density

h = water depth

The surface tension term is usually only included if capillary waves are being analysed; however these tiny waves will have a negligible effect on boat motion and the term will be omitted. If the water depth is large in comparison to the wavelength, then the depth factor will asymptotically tend towards unity and the celerity will reduce to:

$$c = \sqrt{\frac{g\lambda_v}{2\pi}}$$

The time between successive wave crests is the period of the wave and is given by:

$$T = \lambda_v / c$$

or:

$$T = \sqrt{\frac{2\pi\lambda_v}{g}}$$

Relative Wave Motion

So far, the wave has been assumed to be passing a stationary point, however since a boat can itself be moving, it is now necessary to consider relative motion. The period of the waves encountered by the boat may not be the same as the absolute wave period. For example, a boat heading directly into waves (head sea) will encounter each successive wave crest more quickly, thus giving a relatively shorter period. The reverse will be the case for a following sea where the period will be longer, but in a beam sea the period will not be altered by the boat's speed.

The period of encounter, or time for the boat to travel from one crest to the next is:

$$T_e = \frac{\lambda_v}{V_r}$$

where: V_r is the relative velocity and is given by:

$$V_r = V_v - V_b \cos(\beta_v - \psi)$$

with V_v being the speed of the wave, V_b the speed of the boat, β_v the absolute wave direction and ψ the boat's heading. Therefore:

$$T_e = \frac{\lambda_v}{V_v - V_b \cos(\beta_v - \psi)}$$

recalling:

$$\lambda_v = V_v T$$

$$T_e = \frac{V_v T}{V_v - V_b \cos(\beta_v - \psi)}$$

$$T_e = \frac{T}{1 - (V_b/V_v)\cos(\beta_v - \psi)}$$

The relative velocity V_r can also be represented in terms of easterly and northerly components thus:

$$V_N = V_v \cos\beta_v - u\cos\psi + v\sin\psi$$

$$V_E = V_v \sin\beta_v - u\sin\psi - v\cos\psi$$

and:

$$V_r^2 = V_N^2 + V_E^2$$

The wave frequency of encounter is given by:

$$\omega_e = \frac{2\pi}{T_e} = \frac{2\pi V_r}{\lambda_v}$$

hence:

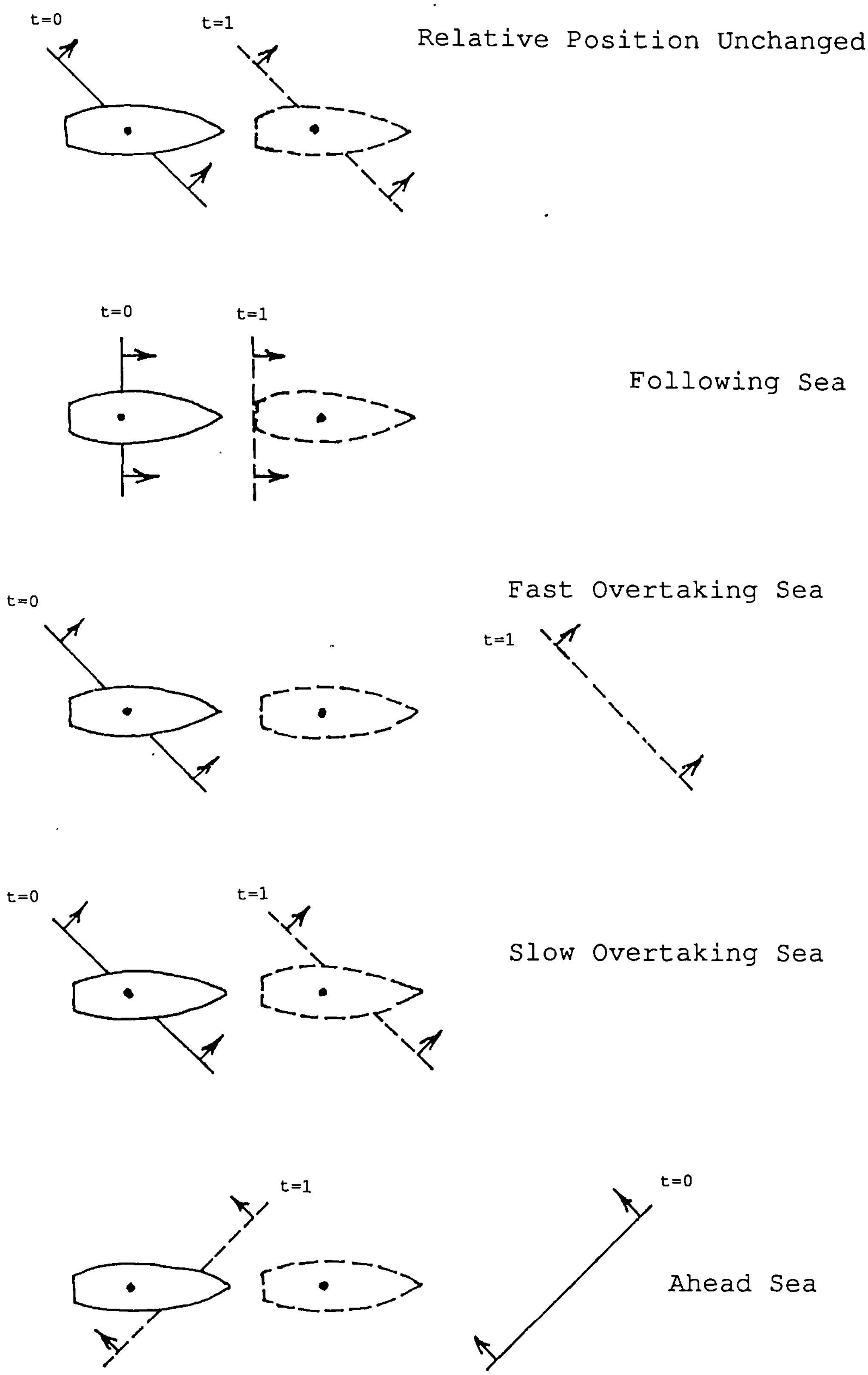
$$\omega_e = 2\pi \, \frac{1 - (V_b/V_v)\cos(\beta_v - \psi)}{T}$$

$$\omega = \frac{2\pi}{T}$$

The wave encounter frequency can therefore be related to the actual wave frequency, the boat to wave velocity ratio and the relative wave angle thus:

$$\omega_e = \omega \left(1 - \frac{V_b}{V_v} \cos(\beta_v - \psi) \right)$$

The significance of the wave encounter frequency can be demonstrated in Fig.6.16. When ω_e is negative the boat will be moving faster than the waves so that they appear to be coming off the bow although the opposite is the case. If ω_e is zero, the boat will be moving at the same speed as the wave and subsequently remains in the same relative position to the wave. When ω_e is positive, the waves will overtake the boat. The rate at which they pass the boat depends on the relative sizes of V_v and $V_b \cos(\beta_v - \psi)$. If these terms are of similar magnitude then a slow-overtaking sea results, whereas if V_v is much greater than $V_b \cos(\beta_v - \psi)$ then a fast-overtaking sea occurs. A fourth condition exists where V_v and $V_b \cos(\beta_v - \psi)$ are of opposite direction and sign, the relative velocity is therefore positive and large. Under these circumstances, the waves approach the boat from the bow and ω_e is everywhere greater than ω. This is the ahead sea condition.



Different Relative Wave Motions

Fig.6.16

The encountered wave slope will therefore become:

$$\alpha_e = -\zeta_a k_e \cos\omega_e t$$

Wave Energy

A wave system possesses both kinetic and potential energy due to the periodic motion. The kinetic energy is due to the orbital motion of the wave particles, and the potential energy is due to the elevation of the water surface.

For a sinusoidal wave the potential energy per unit wave surface area is:

$$E_P = \frac{1}{4}\rho g \zeta_a^2$$

Likewise the kinetic energy per unit wave surface area will also be:

$$E_K = \frac{1}{4}\rho g \zeta_a^2$$

The total energy per unit wave surface area for a sinusoidal wave will be:

$$E = E_P + E_K = \frac{1}{2}\rho g \zeta_a^2$$

Motion Of A Boat In Waves

The exciting or disturbing forces and moments generated by a passing wave are basically due to the additional buoyancy created by the wave along the boat. It is fairly easy to visualise the heaving, pitching and rolling motions resulting from waves, and equations will be developed to model these primary effects. However, the wave does not necessarily directly generate yaw, but when the boat encounters waves form oblique angles, that is on the bow or quarter rather than ahead, abeam or astern, broaching and other yaw "slipping" motions can occur. This can be explained by referring to the gravity vector, where the yaw term is expressed as:

$$(x_g W - x_b B)\cos\theta\sin\phi + (y_g W - y_g B)\sin\theta$$

which indicates that if the boat rolls and pitches, including any subsequent movement of the centre of buoyancy, the boat will yaw as a result. As an example, if a wave approaches the boat on the starboard bow, then the boat's bows will rise (positive θ), the boat will roll to port (negative ϕ), the centre of gravity will remain unchanged and the centre of buoyancy will move out to port and aft which all adds up to a yaw to port which appears as if the boat "falls" off of the wave.

Similar reasoning is assumed to apply to the other non-oscillatory motions of surge and sway. The wave vector will therefore have no components in surge, sway and yaw, but will rely on the secondary effects from the gravity vector.

The heave wave disturbing force is obtained by integrating the additional buoyancy due to the wave longitudinally along the waterplane area of the boat. This can be represented as:

$$Z_{wave} = -C_{VW}\zeta A_{WL}\rho g$$

but, for a wave with the profile of:

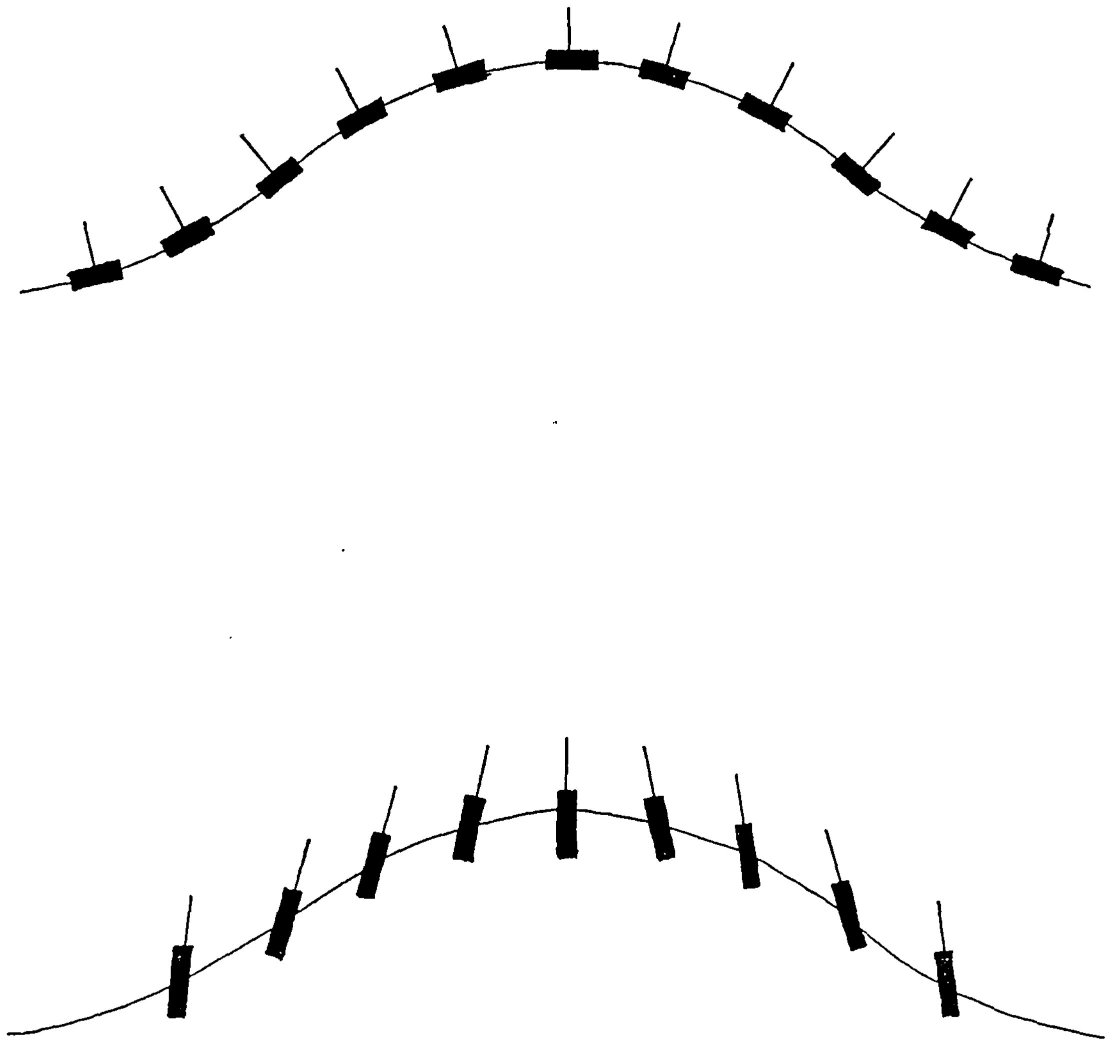
$$\zeta = \zeta_a \sin\omega_e t$$

gives:

$$Z_{wave} = -C_{VW}\,\rho g A_{WL}\,\zeta_a \sin\omega_e t$$

where: C_{VW} is a dimensionless wave coefficient and A_{WL} is the waterplane area.

The beam to draft ratio and length to draft ratio can affect the roll and pitch motions in waves respectively (Fig.6.17). Whilst all boats will have a considerably



Effect Of Beam And Draft On Rolling In Waves

Fig.6.17

greater length than draft, it is assumed that most small boats will also be beamy with respect to their draft.

The roll and pitch moments will be proportional to the wave slope and can be represented as:

$$L_{wave} = C_{VP} \sin(\beta_v - \psi) \; \rho g A_{WL} B \; \zeta_a^2 k_e \cos\omega_e t$$

$$M_{wave} = -C_{VQ} \cos(\beta_v - \psi) \; \rho g A_{WL} L_{BP} \; \zeta_a^2 k_e \sin\omega_e t$$

where: C_{VP} and C_{VQ} are dimensionless wave coefficients, B is the boat's breadth and L_{BP} is the boat's length between perpendiculars.

Some large ship models incorporate wave drifting forces, Hirano, Takashina, Takaishi & Saruta, 1980 (Ref.67). The large ship problem is different from the small boat one, due principally to effects of scale. The wavelength to ship length ratio will typically be less than unity for large ships so that several wave crests may occur along its length. Whereas small boats tend to sit within a single wave and the wavelength to boat length ratio will be greater than unity.

Disturbances of pitch and heave will be far smaller for large vessels due to the fact that the effect of a wave crest towards the bow of the ship can be opposed by the effect from a second wave crest near the stern. The typical equations used in ship models for the horizontal wave drifting forces are written:

$$X_{wave} = X'_{wave} \frac{1}{2}\rho g L_{BP} \zeta_a^2$$

$$Y_{wave} = Y'_{wave} \frac{1}{2}\rho g L_{BP} \zeta_a^2$$

$$N_{wave} = N'_{wave} \frac{1}{2}\rho g L_{BP}^2 \zeta_a^2$$

where the non-dimensionalised wave drifting coefficients X'_{wave}, Y'_{wave} and N'_{wave} are considered to be functions of wave length, wave encounter angle, ship speed and wave encounter frequency. Theoretical determination of these coefficients still leaves much to be desired and emphasis is placed upon experimental data.

From graphs provided in Ref.67 it seems that, for large ships, once the wavelength becomes greater than the ship length, the wave drifting coefficients tail off toward zero. This indicates that as wavelength increases, the wave tends to appear to the ship like a flat surface. The coefficients seem to peak at a wavelength to ship length ratio λ_v/L_{BP}=0.4. While no data is given for ratios less than 0.3, it would seem that as the number of wave crests occurring along the ship length increases, the wave effect from any particular wave will tend to be cancelled by components from other waves and the coefficients will again tend to zero.

The Wave Vector

Entering the above equations into vector format gives the wave vector, V, as:

$$\begin{bmatrix} 0.0 \\ 0.0 \\ -C_{VW}\ \rho g A_{WL}\ \zeta_a\ \sin\omega_e t \\ -C_{VP}\ \sin(\beta_V-\psi)\ \rho g A_{WL} B \cdot \zeta_a^2 k_e \cos\omega_e t \\ C_{VQ}\ \cos(\beta_V-\psi)\ \rho g A_{WL} L_{BP}\ \zeta_a^2 k_e \cos\omega_e t \\ 0.0 \end{bmatrix}$$

6.16 Sea Inertia Forces And Moments

Tied in with the wave vector and any other motions of the water are the equations describing the inertia of the sea. This vector contains the forcing terms associated with the motion of the water $\dot{u}_s$, $\dot{v}_s$, $\dot{w}_s$, u_s, v_s, w_s, p, q, and r which cannot be paired with the inertial motion.

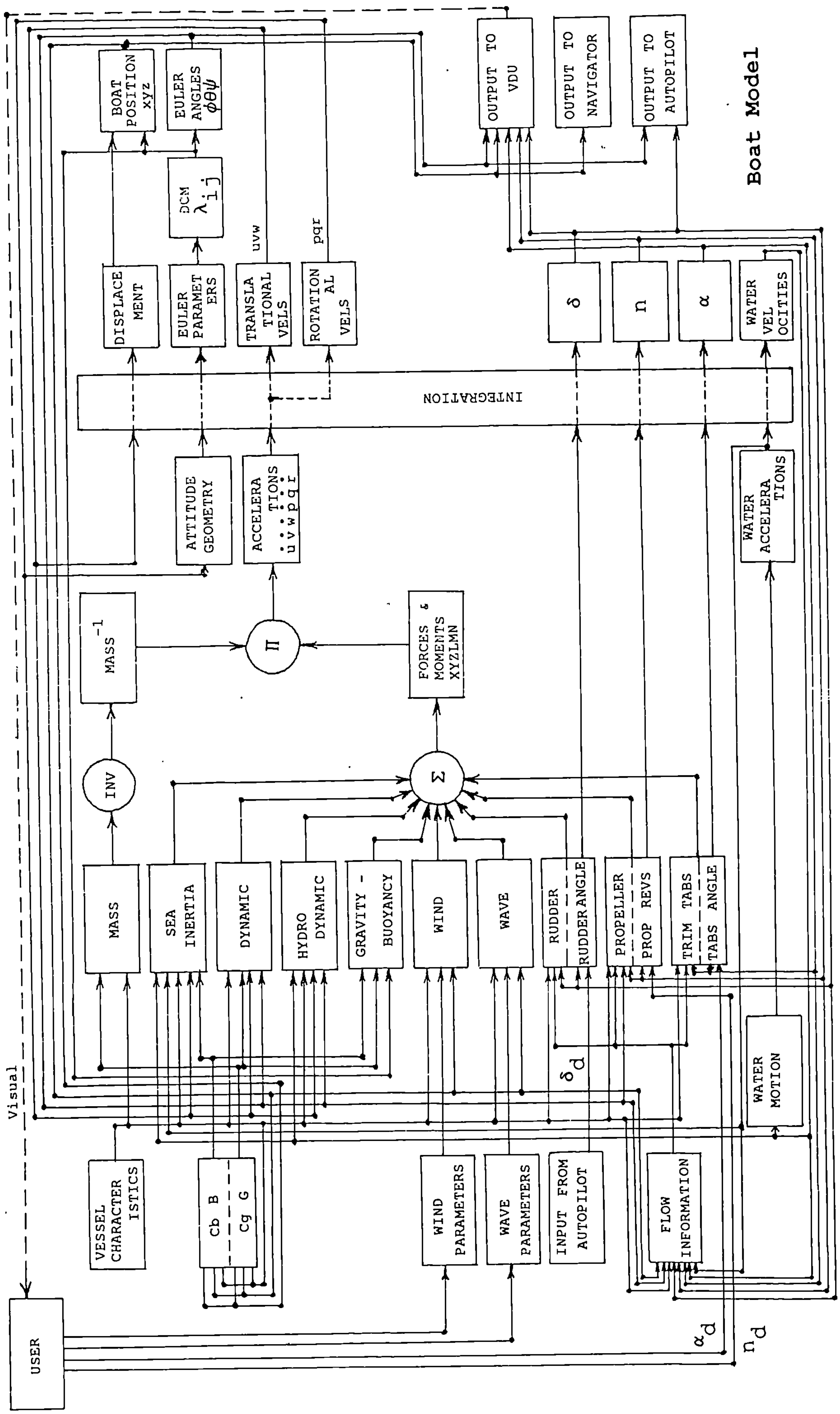
The sea inertia vector, S, is written:

$$\begin{bmatrix} (\bar{m}-X_{\dot{u}})\dot{u}_s-(\bar{m}-Y_{\dot{v}})v_s r+(\bar{m}-Z_{\dot{w}})w_s q \\ (\bar{m}-Y_{\dot{v}})\dot{v}_s-(\bar{m}-Z_{\dot{w}})w_s p+(\bar{m}-X_{\dot{u}})u_s r \\ (\bar{m}-Z_{\dot{w}})\dot{w}_s-(\bar{m}-X_{\dot{u}})u_s q+(\bar{m}-Y_{\dot{v}})v_s p \\ -L_{\dot{u}}\dot{u}_s-(z_b\bar{m}+L_{\dot{v}})\dot{v}_s+(y_b\bar{m}-L_{\dot{w}})\dot{w}_s+\bar{m}[y_b(v_s p-u_s q)-z_b(u_s r-w_s p)] \\ -M_{\dot{v}}\dot{v}_s-(x_b\bar{m}+M_{\dot{w}})\dot{w}_s+(z_b\bar{m}-M_{\dot{u}})\dot{u}_s+\bar{m}[z_b(w_s q-v_s r)-x_b(v_s p-u_s q)] \\ -N_{\dot{w}}\dot{w}_s-(y_b\bar{m}+N_{\dot{u}})\dot{u}_s+(x_b\bar{m}-N_{\dot{v}})\dot{v}_s+\bar{m}[x_b(u_s r-w_s p)-y_b(w_s q-v_s r)] \end{bmatrix}$$

Whilst it is possible to describe the vertical motion of a wave with a sinusoidal profile by analytically integrating the equation of the profile twice, predicting other water motions and thus the sea inertias is far from easy.

6.17 The Overall System

The forces and moments acting on a small boat have been presented in the form of nine vectors which are separate, well-defined entities of the overall system.



Boat Model

Fig.6.18

Should any particular aspect of the model require alteration or upgrading to suit a new application, it will be possible to replace any individual module without affecting the remainder. Other effects, for example the tunnel or axial thrusters on a tug or the sail on a yacht, not so far included can quite easily be tagged on as additional modules once the equations have been formulated.

The forces and moments generated by each module are summed, using vector addition, to obtain the total effect in six degrees-of-freedom. The inverted mass matrix is then multiplied by this vector sum of the forces to produce the six accelerations along and about the boat coordinate axes (Fig.6.18). Integration of the six accelerations yields the six velocities and, if required, the integration of the three translational velocities yields the boat's change of position. The angular rotations can be determined from Euler parameters which are obtained from the angular velocities (see earlier chapter for development on quaternions).

6.18 The Method Of Integration

The differential equations produced have been purposely arranged in terms of the highest state derivatives, namely the accelerations. However, the complexity of the expressions means that analytical integration methods cannot normally be applied to provide a solution. It is, therefore, necessary to rely upon approximate integration techniques such as the Runge-Kutta methods.

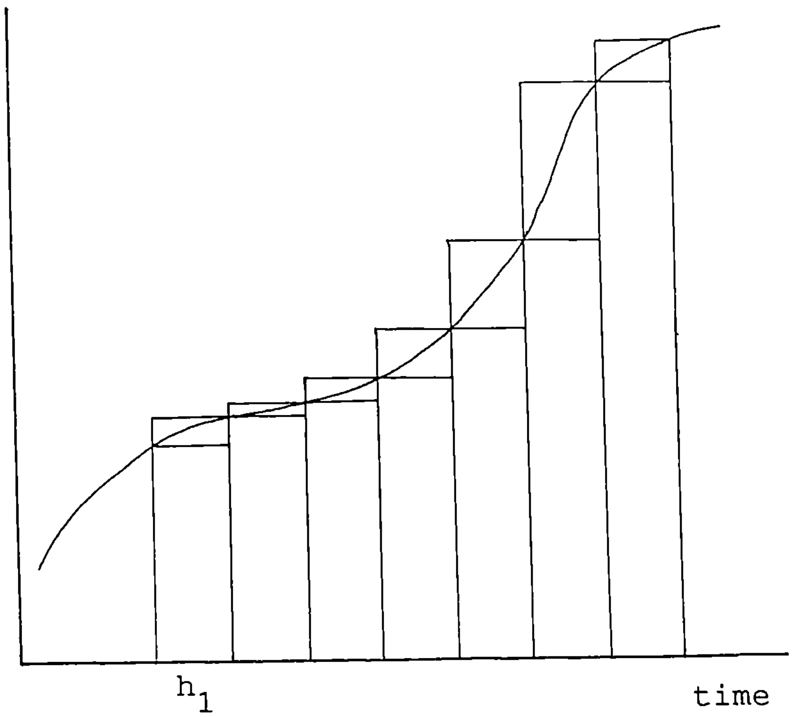
In its simplest form, approximate integration can be achieved by evaluating the expressions for a small time interval from the present, given the current conditions of the states and assuming the values of the state derivatives remain unchanged during the integration step. The result can be depicted graphically (Fig.6.19) with the continuous function being split into a series of discrete values at regular intervals.

The classical RK4 (Runge-Kutta fourth order) approximate integration method is perhaps the most widely used technique in simulation of dynamic systems. For a dynamic system described by a set of equations of the form :

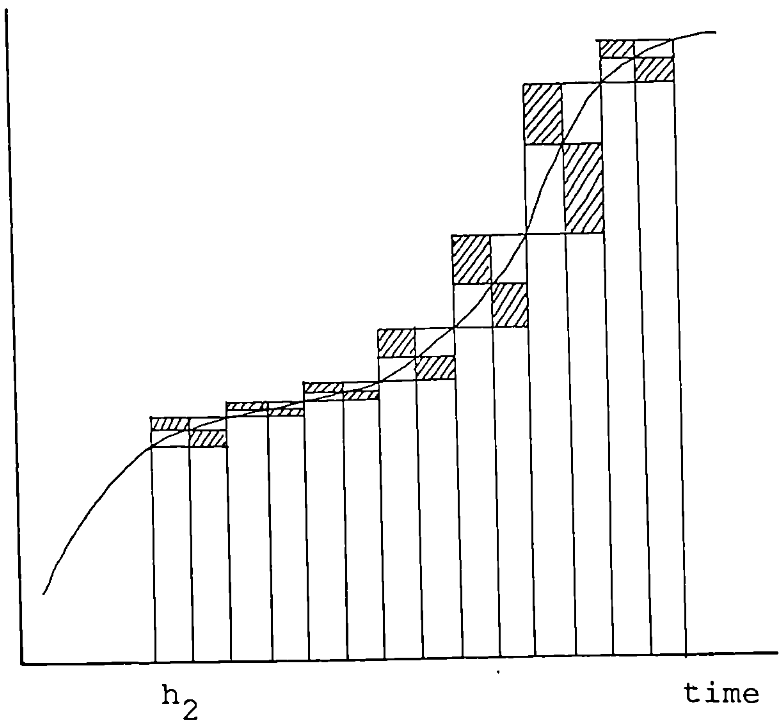
$$\dot{y}(t) = f(\ t,\ y(t),\ x(t)\ )$$

The RK4 method uses the following algorithm to compute the set of state variables y_{n+1}

$$y_{n+1} = y_n + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$



$h_2 = h_1/2$



Integration Step Size

Fig.6.19

where:

$$K_1 = h\, f(\, t_n,\ y_n,\ x(t_n)\ )$$

$$K_2 = h\, f(\, t_n + \tfrac{1}{2}h,\ y_n + \tfrac{1}{2}K_1,\ x(t_n + \tfrac{1}{2}h)\ )$$

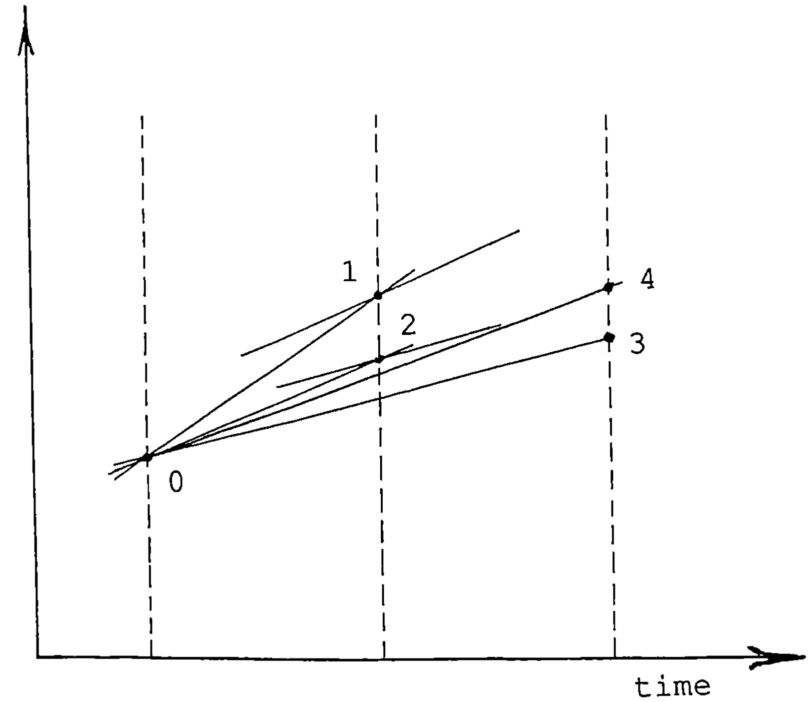
$$K_3 = h\, f(\, t_n + \tfrac{1}{2}h,\ y_n + \tfrac{1}{2}K_2,\ x(t_n + \tfrac{1}{2}h)\ )$$

$$K_4 = h\, f(\, t_n + h,\ y_n + K_3,\ x(t_n + h)\ )$$

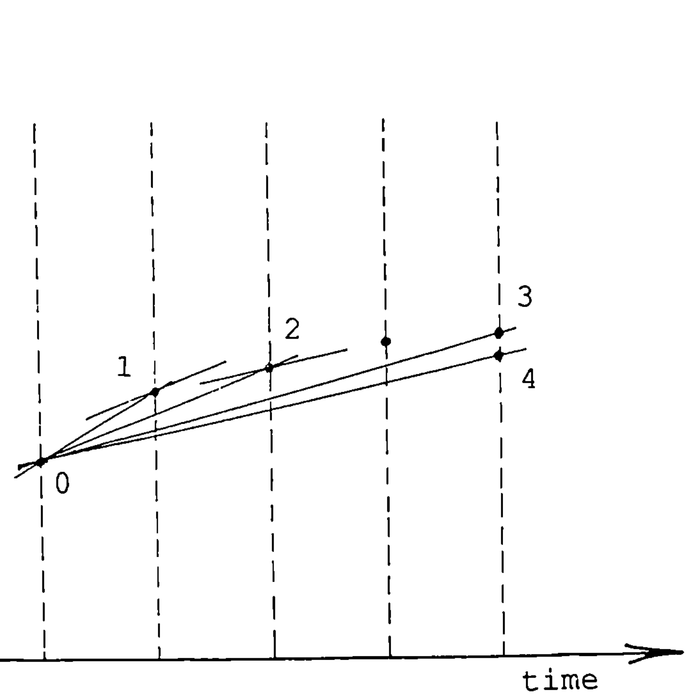
with h being the time interval.

K_1 is essentially the known slope of the function at time n. Projecting this slope K_1 half way across the sample interval and re-computing the new slope at that point produces K_2. The slope K_2 is brought back to the point at time n and then projected half way across again giving K_3. The slope K_4 is then determined at the point where K_3 is projected right the way across the sample interval to time n+1. The weighted average of these four slopes is combined to give the new value at time n+1 (Fig.6.20).

A useful variation to the classic RK4 method provides intermediate output by computing four slopes as before, but at intervals of h/4. This algorithm can be



Fourth Order Method



Modified Fourth Order Method

Runge-Kutta Integration Method

Fig.6.20

summarised as:

$$K_1 = h\, f(\, t_n,\ y_n,\ x(t_n)\ )$$

$$K_2 = h\, f(\, t_n + \tfrac{1}{4}h,\ y_n + \tfrac{1}{4}K_1,\ x(t_n + \tfrac{1}{4}h)\ )$$

$$K_3 = h\, f(\, t_n + \tfrac{1}{2}h,\ y_n + \tfrac{1}{2}K_2,\ x(t_n + \tfrac{1}{2}h)\ )$$

$$K_4 = h\, f(\, t_n + h,\ y_n + K_1 - 2K_2 + 2K_3,\ x(t_n + h)\ )$$

and the value of y can, if desired, be determined at the same h/4 intervals by:

$$y_{(n+1/4)} = y_n + \tfrac{1}{4}K_1$$

$$y_{(n+1/2)} = y_n + \tfrac{1}{2}K_2$$

$$y_{(n+3/4)} = y_n + \tfrac{3}{16}(K_1 + 3K_3)$$

$$y_{(n+1)} = y_n + \tfrac{1}{6}(K_1 + 4K_3 + K_4)$$

Since four slopes have to be computed using the RK4 method, it seems highly useful to have a method of determining three intermediate values and giving a resolution four times greater than otherwise.

The RK4 methods of integration were used with ACSL (Advanced Continuous Simulation Language) as a tool during the development of the model. However, the overhead of the additional computations makes these methods redundant in the real-time simulation.

The small boat simulation will be required to run at either real time or fairly close to real time. It is also desirable for the sample interval to be quite short, in the order of 50ms to 100ms, since the Cetrek autopilot requires heading and rudder information at about this rate. It was therefore decided to initially use the most simplistic of integration methods and effectively multiply the derivative by the sample interval, and use the result to update the state variable for the next time interval.

By using a sufficiently small sample interval, the function can be assumed monotonic between successive steps. Theoretically, as the interval is reduced, the estimate of the integral becomes progressively better, and for an infinitely small step any error tends towards zero. Therefore, if the chosen interval proves inadequate, it can be reduced to improve the simulation.

Limitations exist in numerical integration, particularly when implemented on digital computers. The main source of error is from "rounding-off", a computer can only work to a finite number of significant figures. Consequently there will inevitably be a round-off error at each iteration of the computation. There is a danger of a greater cumulative error being generated when more steps are taken. Clearly a balance between poor approximations and the number of integration steps is required. Taken to the extreme, any cumulative error will suggest that if a model is run for long enough the errors will reach a stage where the simulation becomes invalid!

Two other sources of error in all approximate integration methods are "formula error" and "inherited error". Formula error results from the approximation that the slope of the function remains constant which can lead to an incorrect estimate of the new value. There will, of course, be an exact answer for the state trajectory guaranteed by the mean value theorem if there are suitable conditions of continuity and differentiability, however, finding it is another matter! Inherited error recognises that the starting point from which the projection of the state is to take place will, unless it is the initial condition, be an estimate from the previous iteration.

These three types of errors can work to accumulate errors, but in most instances they will balance out over a number of integration steps. The answer for establishing the correct integration interval would seem to depend on the application, but the step size does not want to be so large that errors are capable of getting out of hand, or too small where the changes in the states are only of the order of the rounding-off errors. The method of approximate numerical integration to choose will undoubtedly depend upon the application, but more sophisticated routines will demand much greater computing power than simpler ones. However, the sampling interval can often be increased when using higher order techniques, since these methods often compute intermediate slopes as part of the overall integration step.

CHAPTER 7

PARAMETER DETERMINATION AND MEASUREMENT

7.01 Model Verification And Validation

Initial verification of the model was achieved by taking each module in isolation and supplying it with inputs which have a known result. Checks on fundamental principles, such as a starboard rudder angle generating a starboard turn, and other intuitively obvious concepts were performed, the exact magnitude of the forces and moments not yet being required.

Combining the modules, simple manoeuvres were reproduced and the correct performance verified. For example, a straight ahead run with rudder amidships without disturbing effects for waves or wind should continue in a straight line (assuming the real boat is course stable). Steady state turning simulations were used to exercise additional areas of the model.

Once the verification procedure had been satisfied, it became necessary to validate the model against the performance of a real boat. This entailed two separate parts, first obtaining measurements of manoeuvres carried out during boat trials and secondly establishing the values of the various coefficients used in the model for the boat under consideration.

7.02 Boat Trials

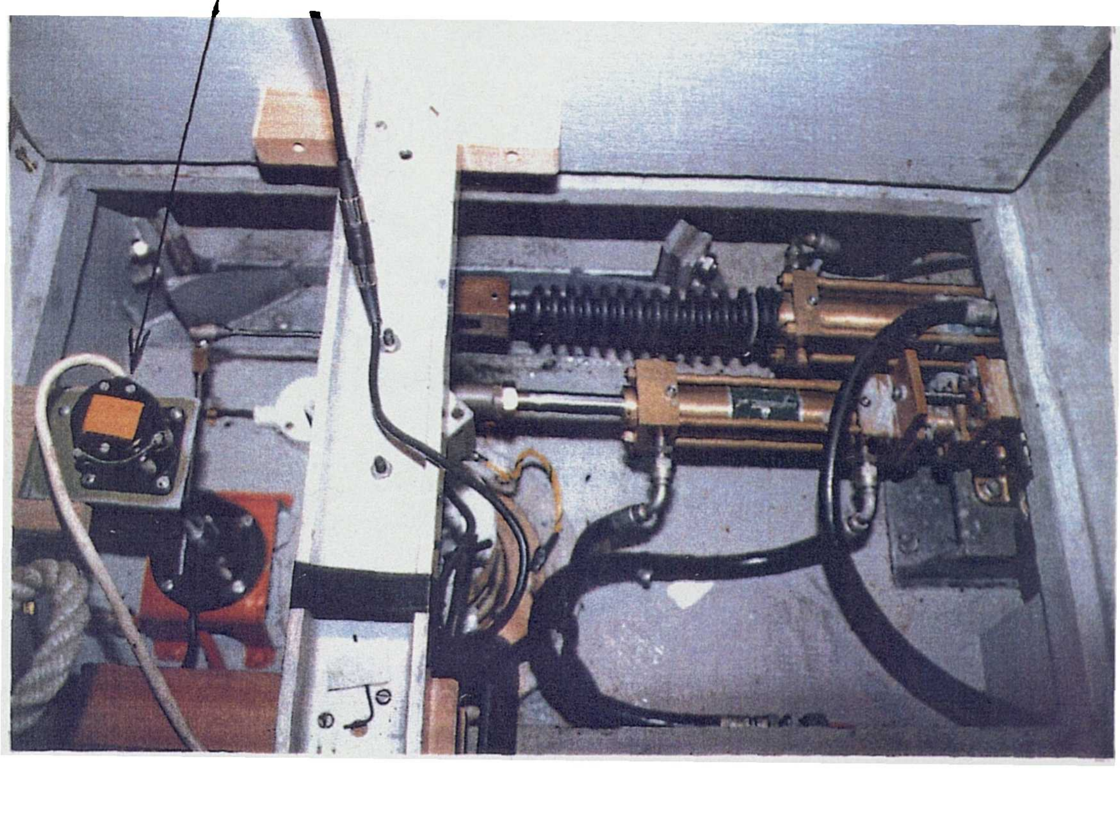
A series of trials were performed on board a RNLI 52ft Arun class lifeboat (see plates 1 to 4) in the spring of 1989. On the whole the weather and sea conditions were relatively mild and calm, a far cry from what this type of boat can be expected to endure!

An Amstrad (IBM compatible) Personal Computer equipped with a PC-30 Analogue Multifunction Board was used as a data-logging device (see plates 5 and 6). The PC-30 board has a maximum of 16 channels which can be used to monitor as many instruments. An interface box, designed by Mr J.Reynolds of Bournemouth Polytechnic and built by the author, provided potential division instruments with voltage and then buffered the resulting measurement through an anti-aliasing arrangement. Where necessary, voltage scaling was performed to optimise the voltage range of the PC-30 board.

The various instruments to be recorded were, therefore, connected to the computer via the interface box. Power for the instruments was drawn from the boat's DC supply and regulated within the interface box. Useful technical information on the interface box and the PC-30 multifunction board can be found in an appendix at the end of this thesis.

Plate 1
Arun Lifeboat

Plate 2
Rudder System



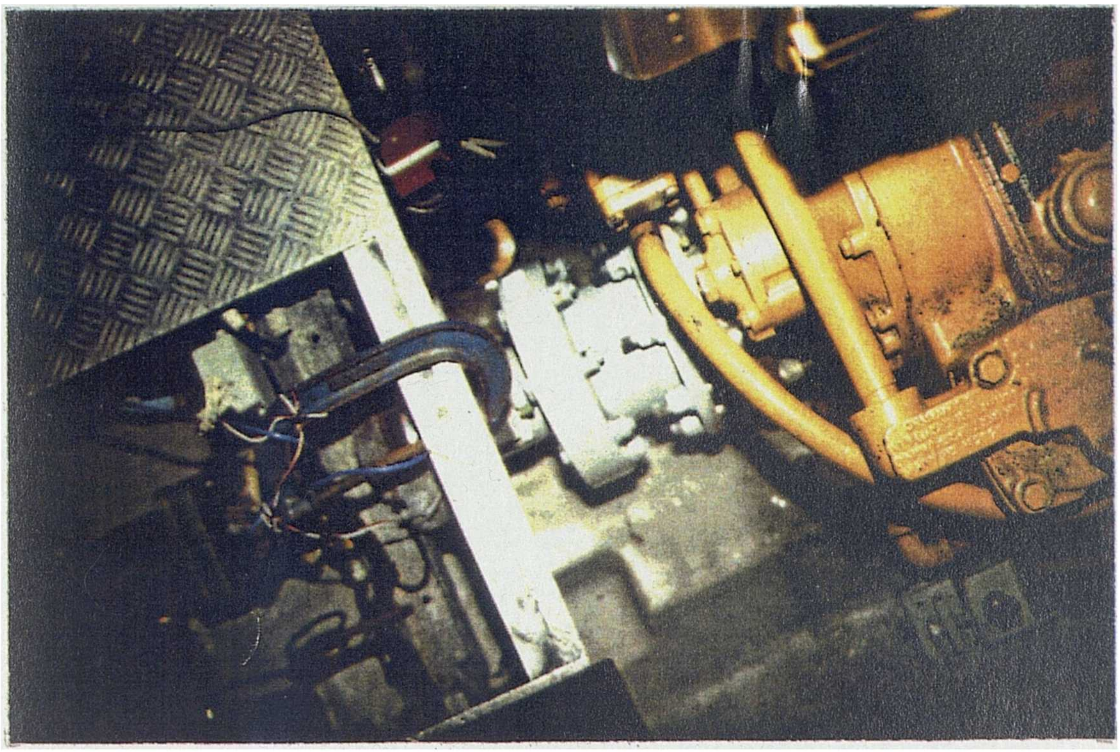


Plate 3

Propeller Shaft

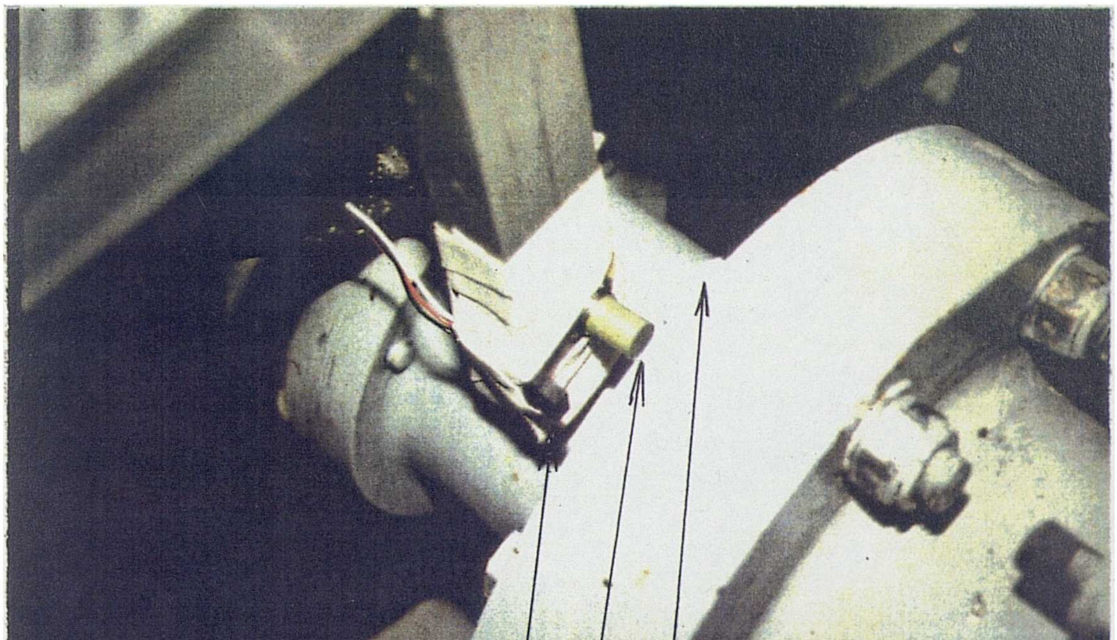
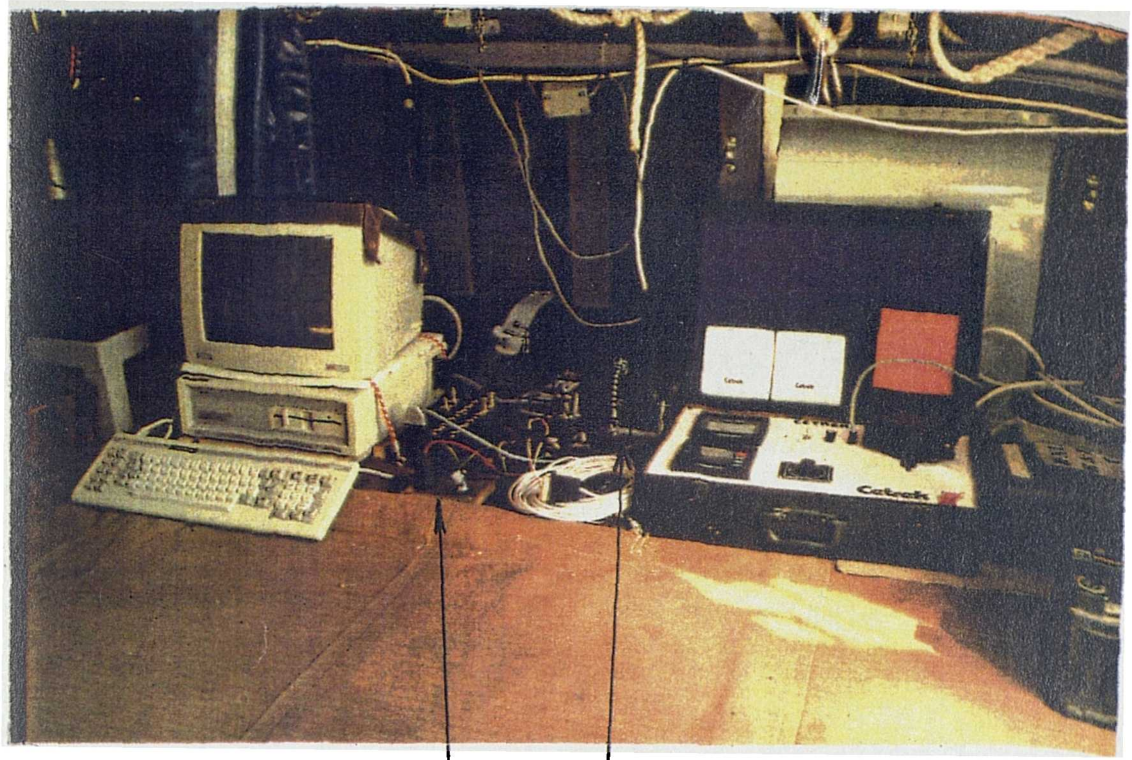


Plate 4

Phototransistor

Magnetic Sensor

Both Use The 8 Bolt Heads



Roll/Pitch Gyro

Interface Box

Plate 5

Data-Logging Setup

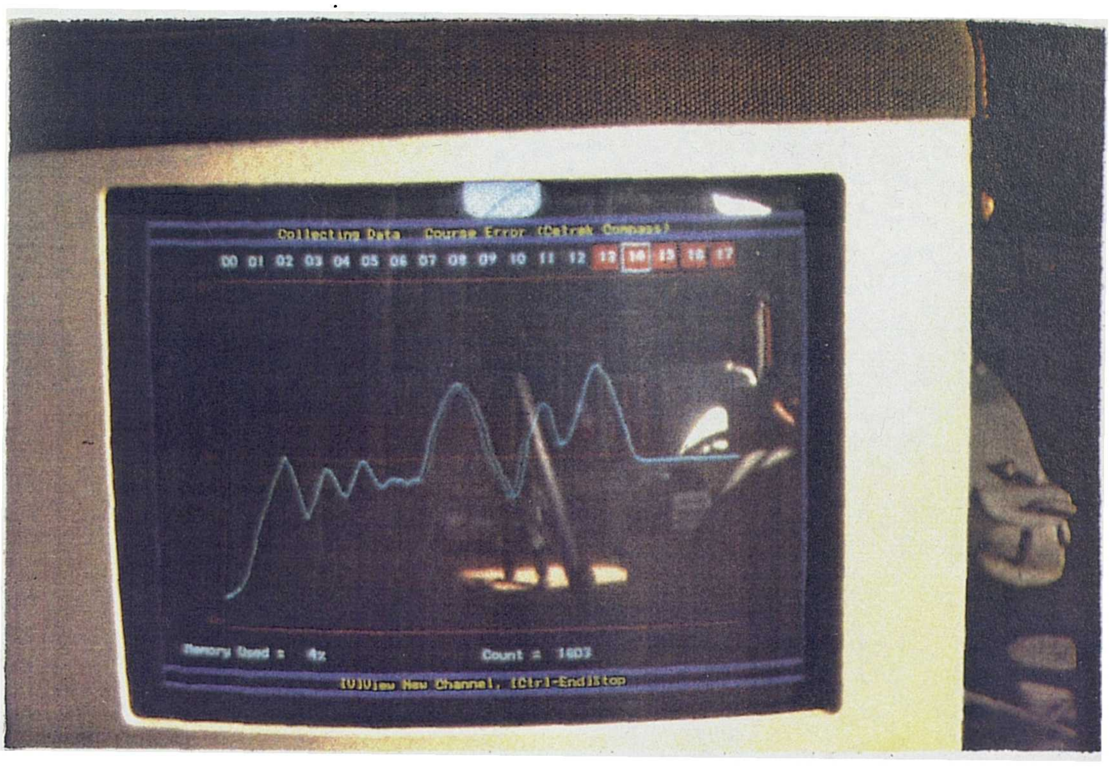


Plate 6

Software, designed by the author and written in 'C', was used to select channels to be read, record information during trials, save the data to disk between individual test runs and plot the results back in the office. A sampling rate of between 10Hz and 20Hz was deemed sufficient for this analysis, and since the computer's BIOS (Basic Input Output System) timer functions at 18.2Hz this was used to provide an interrupt routine to read the multifunction board. As data *during a run is stored in memory (to avoid the* unreliable and slow process of writing to disk during the run) the maximum duration of recording depends on the number of channels in use.

For the purposes of the small boat simulation, capable of modelling motions in all six degrees-of-freedom, the primary measurements required are the three Euler angles and the rudder position. The angles of roll and pitch were obtained from a roll and pitch gyro with the free-floating condition giving the zero position. Yaw was extracted from a Cetrek fluxgate compass with the digital reading being fed into the serial port of the computer. The rudder angle was measured from a potentiometer mounted above one of the rudder stocks.

Additionally the propeller shaft revolutions were recorded using a photoelectric sensor placed near a propeller coupling which had eight equally spaced bolts. Reflective strips were attached to the bolt heads and the pulses generated from the sensor were fed into a counter. The propeller shaft revolutions can vary between about 300rpm and 1100rpm, therefore by determining the number of revolutions from the number of counts per second means that the arrangement of eight pulses per revolution gives a resolution of 7.5rpm. Clearly the resolution can be increased by taking the count over a longer time interval, but this will be limited by how often the rpm is to be updated and how rapidly it is changing. The count is read in on the digital port of the multifunction board at the same rate as the other measurements, and can be processed at a later stage.

Provision was made to record propeller torque, but due to flooding in the bilges the instrument was not mounted during the trials. Similarly, a load cell was to have been used to determine the force on the rudders, but it proved difficult to actually incorporate it within the rudder system.

Heave accelerations were recorded using an accelerometer, but these measurements consistently registered less than ±1g since the sea surface was a low amplitude swell. The only onboard measurement of the horizontal translational velocity was from a navigator system, however, since it determined velocity from the differentiation of the change of distance with respect to time, it could only provide an indication of the steady state velocity after holding a particular course and speed for at least 2 or 3 minutes.

As well as the five principal measurements of roll, pitch, yaw, rudder and propeller revolutions, the regulated voltage level was monitored and found to remain very stable whilst recording.

Briefly, the designation of the channels is as follows:

Channels 0 to 10 inclusive: had three pin DIN connectors, allowing voltage to be supplied to, and measured from, a number of potential division devices, such as accelerometers, rudder reference, roll and pitch gyro, et cetera (not all these channels ended up being used).

Channel 11: had a three pin DIN connector and a voltage amplifier. This was dedicated to the rudder load cell.

Channels 12 and 13: had BNC connectors and were dedicated to the propeller torque meters, which have their own power supply.

Channel 14: had a unique connector which allowed the analogue course error (±40°) signal to be recorded from the Cetrek fluxgate compass.

Channel 15: recorded the regulated voltage directly from within the interface box.

Channel 16: (in reality, the digital port on the PC-30 board) was used for the propeller revolutions counter.

Channel 17: (in reality, the computer's serial communications port and nothing to do with the PC-30 board) was used to read the 10bit heading from the Cetrek compass (10bits gives a resolution of about $\frac{1}{3}°$).

The plots of several different manoeuvres performed at various rudder angles and approach speeds are presented in an appendix towards the end of the thesis. Most are plotted over a common time base to ease comparison.

7.03 Parameter Estimation

It is a fairly straight forward matter to determine the principal dimensions of a boat, since this information is readily available for all craft. It is standard to quote figures for the length between perpendiculars, length overall, moulded breadth or beam and moulded draft. Other quantities such as the projected transverse and longitudinal areas above the waterline and the projected areas below the waterline can be estimated from the general arrangement or lines plans for the boat. Rudder span and area, plus propeller pitch and diameter and other such dimensions can also be ascertained from draughtsman's drawings.

The hydrostatic curves (see section on the gravity-buoyancy vector), drawn for most boats, provide the static values for displacement and the positions of the centres of buoyancy and floatation as a function of the draft. The static underwater volume can be obtained from displacement in a given density of seawater. A combination of the static conditions and the lines plans allows estimates for the dynamic values of the underwater volume.

The rudder acts like a hydrofoil and, therefore, can be related to aerodynamic theory. The rudder normal force equation contains an aspect ratio term which has been based on Japanese research. Since the general shape of the rudder is not dissimilar to those of large ships, it is assumed that the theory is valid for all sizes of rudder. The rudder normal force was subsequently found to be of the correct order for this analysis. The rudder coefficients were slightly altered once comparisons of actual and simulated steady state rates of turn for a range of rudder angles were made.

The propeller vector relies on propeller thrust which can be deduced from propeller characteristic curves given the advance constant. The surge propeller coefficient was determined by balancing the forces producing a steady state forward speed attained at set propeller revolutions. The pitch propeller coefficient basically came down to a balance between the pitch restoring moment and the moment applied by the propeller. As propeller revolutions increase, so the bows rise; at constant propeller revolutions, the resulting angle depends on the restoring moment. The actual measurements had to be with trim tabs horizontal since these effects are modelled in the trim tabs vector.

The trim tabs coefficients were determined by observing the changes in trim or pitch as the trim tabs are lowered. Measurements of pitch were taken in the trim tabs up and down positions at the same forward speeds.

The centre of gravity is assumed fixed, though in practice this can move if weights are added or removed from the boat or as fuel is used. Although the equations allow for differences in the buoyancy and weight, these are assumed to remain about the same for the majority of the time.

The GZ curve gives the horizontal separation of the centre of buoyancy from the centre of gravity. This helps to indicate the motion of the centre of buoyancy at varying angles of heel. In the absence of a method of accurately determining the true centre of buoyancy at any angle of pitch and roll, two elliptical equations will be generated to approximate its locus. For the Arun lifeboat the GZ curve is always positive; that is there is always a righting moment, and the locus of the centre of buoyancy is more likely to be closer to a cardioid than an ellipsoid. However, from the point of view of testing the autopilot, it is sufficient to cater for angles less than 40°. Measurements of the amplitude and duration of roll motions when rudder is initially applied allow the approximations made above to be matched with the real situation.

Large ship modellers usually approximate the GZ curve with a fifth order polynomial and determine the roll righting moment based on the GZ, whereas the gravity-buoyancy vector is capable of accurately reproducing the restoring forces and moments for all six degrees-of-freedom if the location of the centre of buoyancy is known.

The various coefficients that make up the Mass, Dynamics and Hydrodynamic vectors are perhaps the most difficult to assign values to without the use of complex computational programs. Virtually the only method is to simplify the equations and isolate individual terms.

The moments of inertia about the three axes can be determined from the boat's radius of gyration. This is usually obtained by swinging the boat when it is suspended from a pivot point. Empirical formulae give close approximations to the moments of inertia given the mass and length of the boat. The products of inertia, those appearing in the off-diagonal positions in the inertia tensor, will be small and are initially assumed negligible. However, I_{xy} often plays a significant role and has a non-zero value.

The added mass coefficients are extremely difficult to determine precisely without the aid of complex computational programs. As Lamb indicates, they are determined by the form and position of the surface relative to the coordinate system. Theoretical expressions involve double integrals of the velocity potential with respect to the entire surface. However, since the boat exhibits symmetry about the XZ-plane, a number of the added mass terms are zero. Furthermore, it is possible to determine the coefficients for regular mathematical solids such as the ellipsoid (the equations for which have been presented earlier). Using the length and either beam or draft as the major and minor axes of a prolate ellipsoid provides a reasonable first approximation of the important added mass terms. It should be noted that, the added mass coefficients can vary greatly with changes in the depth of water, with increases reaching as much as twice the actual mass in shallow water. It is assumed that sufficiently deep water exists so that these effects can at present be ignored.

Clarke, Gedling and Hine, 1982 (Ref.40), produce empirical formulae based upon regressional analysis of large ship data in an attempt to quantify some of the fundamental hydrodynamic coefficients. The sort of equations which result are:

$$-Y'_{\dot{v}} = \pi(T/L)^2[1 + 0.16C_BB/T - 5.1(B/L)^2]$$

$$-Y'_{\dot{r}} = \pi(T/L)^2[0.67B/L - 0.0033(B/T)^2$$

$$-Y'_{v} = \pi(T/L)^2[1 + 0.40C_BB/T]$$

$$-Y'_{r} = \pi(T/L)^2[-0.5 + 2.2B/L - 0.080B/T]$$

$$-N'_{\dot{v}} = \pi(T/L)^2[1.1B/L - 0.041B/T]$$

$$-N'_{\dot{r}} = \pi(T/L)^2[0.0833 + 0.017C_B B/T - 0.33B/L]$$

$$-N'_{v} = \pi(T/L)^2[0.5 + 2.4T/L]$$

$$-N'_{r} = \pi(T/L)^2[0.25 + 0.039B/T - 0.56B/L]$$

where: T is the draft, B is the breadth, L is the length and C_B is the block coefficient.

These are dimensionless coefficients, as indicated by the prime, and can be dimensionalised by:

$$Y_{\dot{v}} = Y'_{\dot{v}} \frac{\rho}{2}L^3$$

$$Y_{\dot{r}} = Y'_{\dot{r}} \frac{\rho}{2}L^4$$

$$Y_{v} = Y'_{v} \frac{\rho}{2}L^2u$$

$$Y_{r} = Y'_{r} \frac{\rho}{2}L^3u$$

$$N_{\dot{v}} = N'_{\dot{v}} \frac{\rho}{2}L^4$$

$$N_{\dot{r}} = N'_{\dot{r}} \frac{\rho}{2}L^5$$

$$N_v = N_v' \frac{\rho L^3 u}{2}$$

$$N_r = N_r' \frac{\rho L^4 u}{2}$$

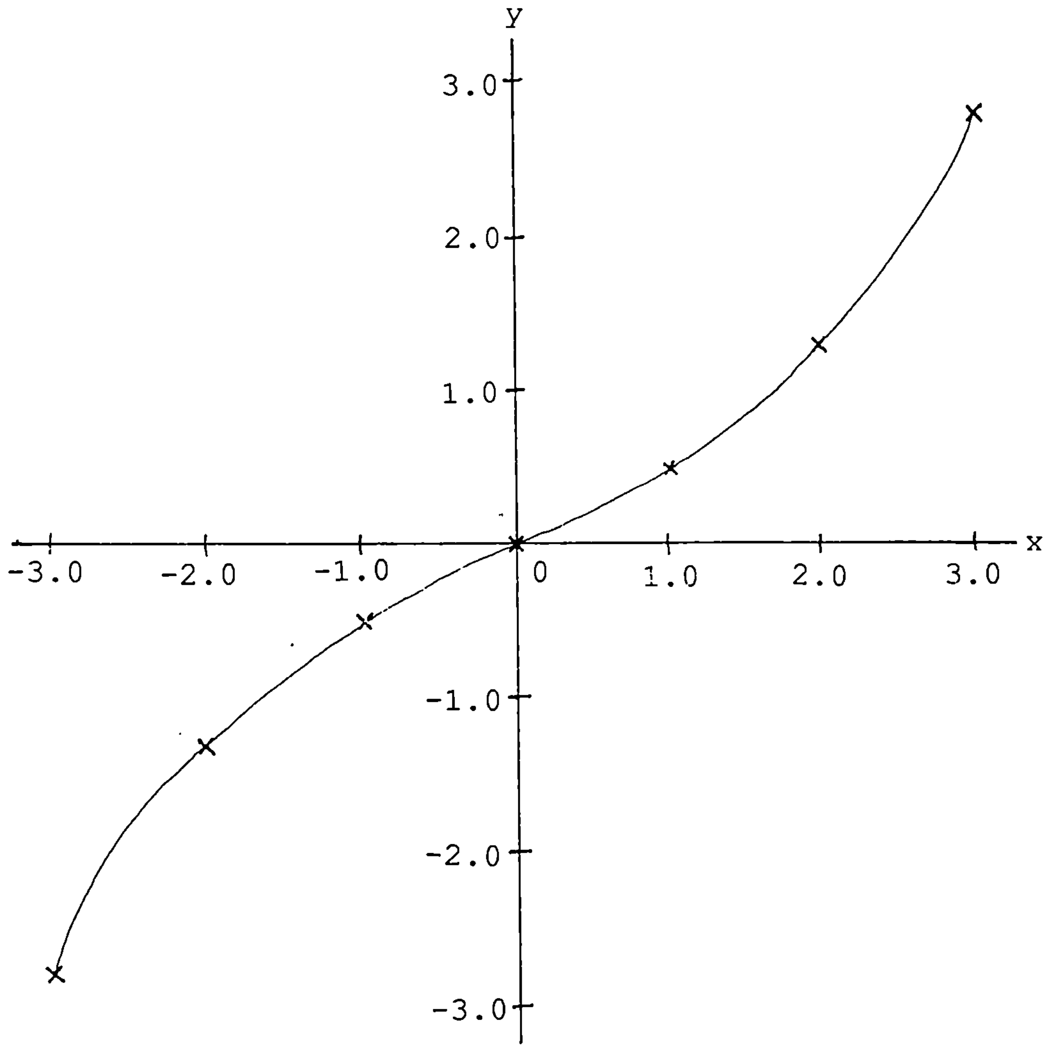
These equations help to provide an idea of the relative importance of each coefficient, but since they are based upon large ship data, they are of little aid to determining the magnitude of the small boat coefficients.

The most practical method of actually assigning values to the hydrodynamic coefficients is from real data obtained during boat trials. The idea here is to compare results of simple manoeuvres against the actual equations. Choosing appropriate manoeuvres, many of the terms can be omitted since they play no part in that particular manoeuvre. By examining the equations and selecting a set of manoeuvres, many of the parameters can be established. Beginning with the simplest of manoeuvres, such as straight ahead movement with undeflected rudder, no disturbances and running in a steady state condition, the majority of terms can be excluded. The remainder can then be matched with the recorded data. During turning circle manoeuvres, the steady state condition can yield further hydrodynamic coefficients in yaw and sway. If the motion is steady state, then there is no acceleration and all those terms can be omitted. Further comparisons of data, such as accelerations or decelerations occurring when there are changes in throttle position and the initial roll response to rudder deflections, can provide estimates of damping and other terms.

The hydrodynamic coefficients can be determined by utilising a number of techniques, which include: full scale sea trials, tank testing with scale models (here Froude and Reynold's numbers need to be introduced when comparing results to the full size ship), and parameter estimation using empirical formulae or regressional techniques.

A weakness of the concept of polynomial fitting is now demonstrated. Increasing or decreasing the number of terms included in the polynomial approximation can affect the values of the coefficients of the variable under consideration. For example, if the points (Fig.7.1):

x	3.0	2.0	1.0	-1.0	-2.0	-3.0
y	2.8	1.3	0.5	-0.5	-1.3	-2.8



Polynomial Approximation Fig.7.1

are approximated by a cubic, the equation would look like:

$$y = 0.44583\,x + 0.05417\,x^3$$

whereas if the quintic term is included the equation becomes:

$$y = 0.45333\,x + 0.04583\,x^3 + 0.00083\,x^5$$

The values of the coefficients change according to the number of terms used in the approximation. Note also that neither approximation gives the correct slope of the curve at the origin, 0.5 being the true value.

This also applies to models based upon a Taylor series expansion where the coefficients are subsequently obtained from regressional analysis of full scale sea trials data. If more or fewer terms are included in the expansion then the value of all of the coefficients will have to change in order for the same result to be achieved. Polynomial fitting and regressional analysis can therefore produce models which fit the data extremely well for the manoeuvres considered, but the values of the coefficients can be meaningless and not actually the correct ones.

So far the technique of parameter estimation has assumed that the boat is floating in an ideal fluid and that there are no external disturbances of wind or waves. This analysis has developed mathematical equations which can be used to exert external forcing functions on the boat. This is extremely important as regards the autopilot control, since without these disturbing influences the autopilot would always maintain a precise heading.

The wind coefficients that determine the magnitude of the wind effect are based on work by Aage, since there was no real data available for the Arun lifeboat. The shape of the curves of the dimensionless wind coefficient against relative wind angle are very similar for the nine ships presented by Aage. These curves can be approximated by trigonometric functions of the relative wind angle and agree with intuitive logic. For example, the maximum yaw moment is likely to be produced at a relative wind angle equal to the inter-cardinal points, that is on the starboard and port bow and quarter. The values of the maximum dimensionless wind coefficients are used to scale these trigonometric functions. The dimensionless coefficients can be related to an individual boat by appropriate cross-sectional area or the projected area that the wind can act upon. The projected cross-sectional areas, both longitudinal and transverse, can be ascertained from lines plans. Allowances will have to be made to the equations for uneven distribution of superstructure.

The wave vector presents a sinusoidal forcing function which again has coefficients which determine the magnitude of this disturbance. The boat trials were carried out in slight seas, but no wave data was available to assess the perturbations in the recorded boat runs. By observing the deviations from a mean value during steady state sections of the recorded data, and by noting the sea conditions on the days of the trials, gestimates were made for the wave coefficients. The model was used to perform simulation runs in order to ensure that the values chosen were reasonable.

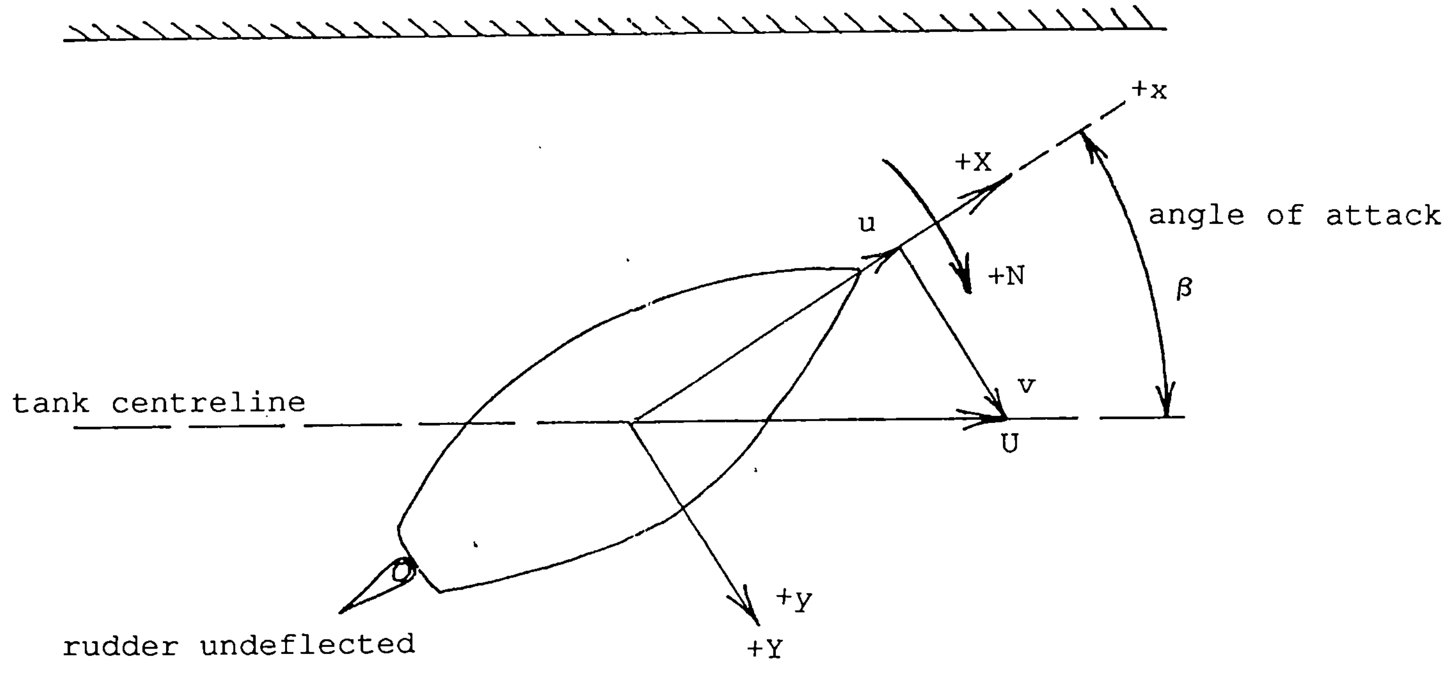
The main purpose of the thesis is to assess the performance of small boat autopilots whilst exercising the mathematical model. Accurate prediction of an Arun lifeboat is, therefore, not desperately required. It is sufficient to have a model which behaves in a similar manner to a 52feet long semi-displacement vessel. So long as the disturbance forces and moments are not unrealistic of the actual situation, simulations can be performed without cause for concern. Once the parameter estimation stage was complete, the Kempf manoeuvres were used as the acid test of the model's ability to simulate small boat behaviour.

7.04 Tank Testing Techniques

To obtain more accurate values for the hydrodynamic coefficients, and therefore get the model closer to the real world situation, the modeller can make use of other techniques like tank testing.

As discussed in the previous sub-section, the hydrodynamic coefficients depend upon the geometry of the boat; however no adequate theory or calculation procedure exists to accurately determine these values. Model tests of a special nature are often undertaken in order to obtain the hydrodynamic coefficients of a particular vessel shape. Even this method of parameter determination is limited to those coefficients which can be isolated for analysis.

The simplest test is for the Y_v and N_v derivatives. These are obtained by towing a model vessel, at the "proper" speed given by the Froude number, at various angles of attack to the path of the model (Fig.7.2). A dynamometer is used to record the Y (sway) force and N (yaw) moment experienced by the model. The slope at the origin of a graph of Y or N versus sway velocity v gives the numerical values of Y_v or N_v.



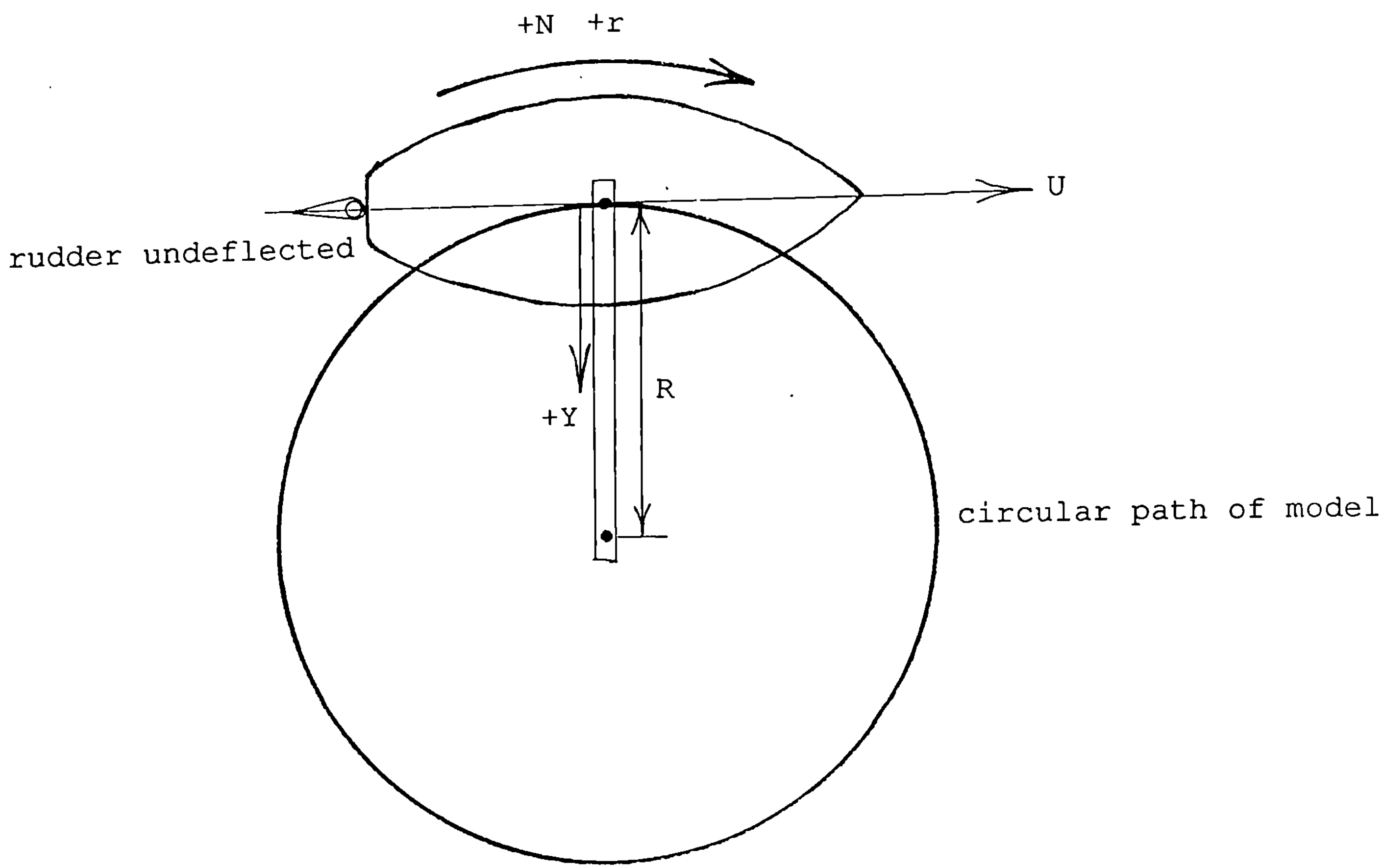
Tank Testing

Fig.7.2

The test for Y_r and N_r involves towing the model at a constant forward speed whilst imposing various values of angular velocity r on the model. A "rotating arm" apparatus is used to impose the angular velocity on the model by rotating it in a circle at the end of the arm (Fig.7.3). The model is towed with a forward velocity u_o (Froude number $u_o(gL)^{-1/2}$), tangential to the circular path, for various radii R. The only way of varying the angular velocity r for a fixed forward velocity u_o is to alter the radius, since $r=u_o/R$. A dynamometer is used to record the Y force and N moment during each test and the slope at the origin of the graph of Y or N versus r will fix the numerical values of Y_r or N_r.

In order to avoid the expense of an additional tank facility such as the rotating arm apparatus, a device known as the planar motion mechanism was devised for use in a long narrow tank. This allows Y_r and N_r as well as other derivatives to be determined.

The mechanism consists of two transverse oscillators, one positioned at the bow and the other at the stern of the model. These are set oscillating whilst the model is towed down the tank. The two oscillators are given the same amplitude a_o and frequency ω of oscillation, but the phase of the oscillators can be adjusted. Dynamometers at the bow and stern measure the



Rotating Arm Facility

Fig.7.3

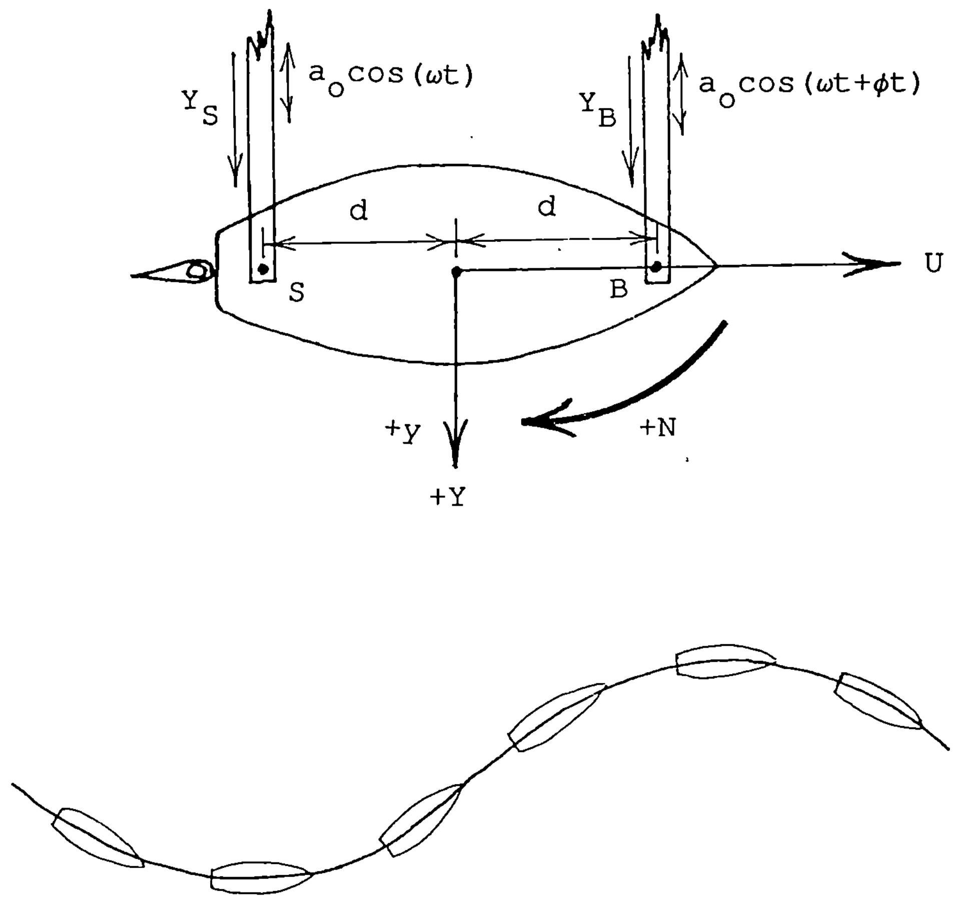
oscillatory Y forces experienced by the model (Fig.7.4).

Eight principal hydrodynamic coefficients can be obtained according to the following. Since the sway velocity v (sine function) is out of phase with the displacement y (cosine function) then the out of phase measurements of Y_B and Y_S are the forces arising from the effects of v. Where Y_B is the sway force at the bow and Y_S is that at the stern. Therefore:

$$Y_v = \pm \frac{\text{Out Of Phase Amplitude Of } (Y_B + Y_S)}{-a_o \omega}$$

$$N_v = \pm \frac{\text{Out Of Phase Amplitude Of } (Y_B - Y_S)d}{-a_o \omega}$$

where: d is half the distance between the two oscillators. Similarly, since the sway acceleration $\dot{v}$ (cosine function) is in phase with displacement y (cosine function) then the in phase measurements of Y_B and Y_S are forces arising from the effects of $\dot{v}$, therefore:



Planar Motion Mechanism

Fig.7.4

$$Y_{\dot{v}} = \pm \frac{\text{In Phase Amplitude Of } (Y_B + Y_S)}{-a_o\omega^2}$$

$$N_{\dot{v}} = \pm \frac{\text{In Phase Amplitude Of } (Y_B - Y_S)d}{-a_o\omega^2}$$

In order to obtain the derivatives Y_r and N_r, the measurements must be made at the time or phasing when $v = \dot{v} = \dot{r} = 0$. Whereas for $Y_{\dot{r}}$ and $N_{\dot{r}}$ the measurements must be taken when $v = \dot{v} = r = 0$. In order to impose an angular velocity and angular acceleration in the body with $v = \dot{v} = 0$, the model must travel down the tank at a speed of u with its centreline always tangential to its path. This oscillatory path of the model will be followed if the phase angle φ between bow and stern oscillators satisfies:

$$\varphi = 2 \tan^{-1} \frac{\omega d}{u}$$

With the phase angle set at this value, the out of phase components of Y_B and Y_S will provide the force and moment due to r. If ψ_o is the orientation angle, then:

$$\psi = \psi_o \cos\omega t$$

$$r = -\psi_o \omega \sin\omega t$$

$$\dot{r} = -\psi_o \omega^2 \cos\omega t$$

hence r is out of phase with ψ, and $\dot{r}$ is in phase with ψ. ψ_o is determined from the amplitude a_o, the distance d and the phase φ. The derivative values which are functions of r and $\dot{r}$ are thus:

$$Y_r = \pm \frac{\text{Out Of Phase Amplitude Of } (Y_B + Y_S)}{-\psi_o \omega^2}$$

$$N_r = \pm \frac{\text{Out Of Phase Amplitude Of } (Y_B - Y_S)d}{-\psi_o \omega^2}$$

$$Y_{\dot{r}} = \pm \frac{\text{In Phase Amplitude Of } (Y_B + Y_S)}{-\psi_o \omega^2}$$

$$N_{\dot{r}} = \pm \frac{\text{In Phase Amplitude Of } (Y_B - Y_S)d}{-\psi_o \omega^2}$$

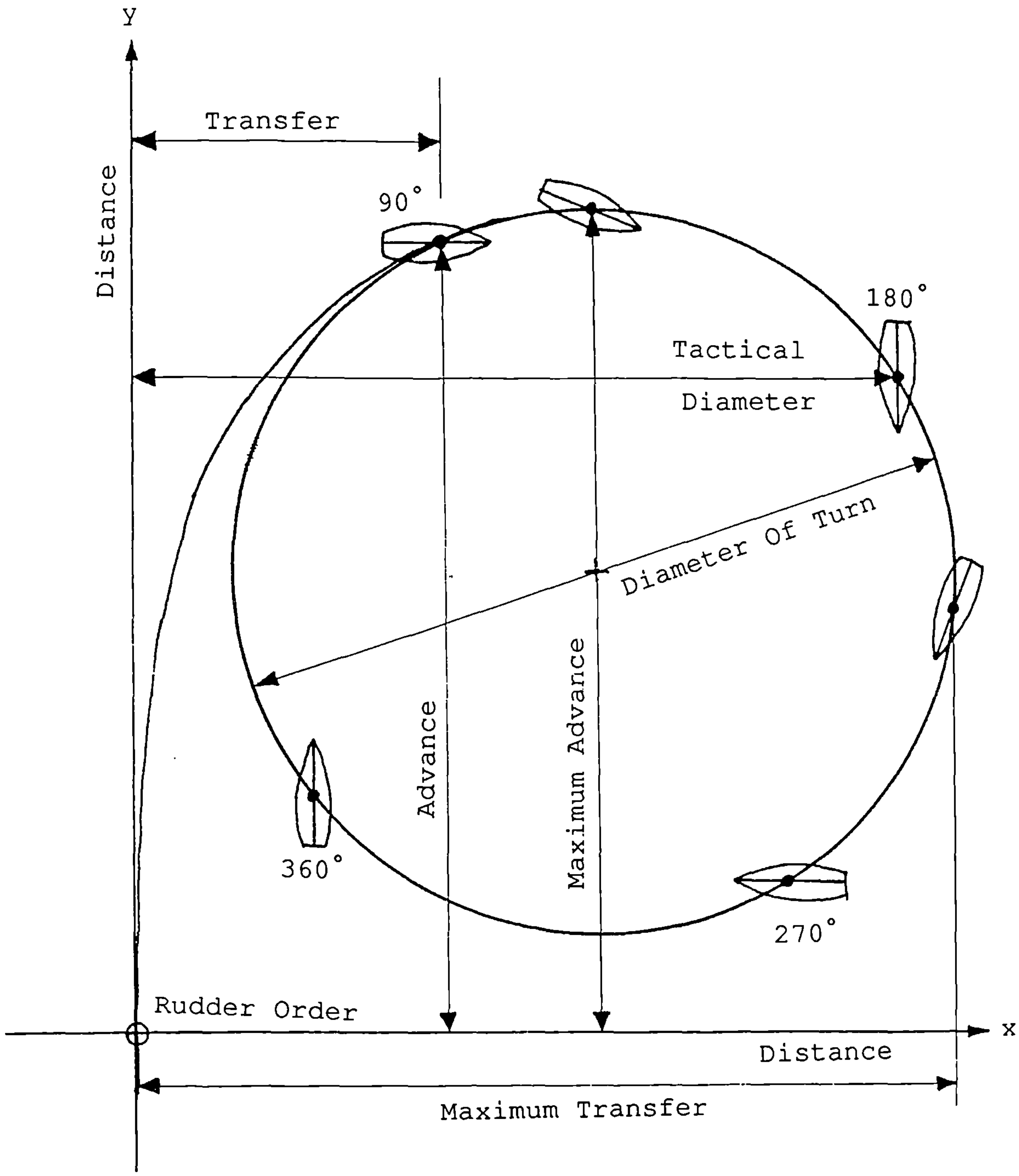
Note that since a rotating propeller and the undeflected rudder both act as lifting surfaces, the various model tests must be performed with propellers operating and the rudder included in the undeflected condition.

The drawback with model testing is that it is both time consuming and expensive. For small boats it is as easy to conduct full scale tests onboard the real boat, which also dispenses with troublesome scaling problems and linking the experiments at the Froude number. The funds and time available to the author are not sufficient to cover tank testing experiments and measurements will be made onboard a real boat.

7.05 Standard Manoeuvres

In order to assess how closely the mathematical model describes the real boat, it is usual to compare sets of standard manoeuvres performed on the boat simulator with real boat data. Typical manoeuvres used include (Ref.33):

Turning Circles: these are used to determine the effectiveness of the rudder to produce steady state turning motion. These are usually performed at a number of rudder angles, both port and starboard. All turns should be performed with approach runs at the same relative wind angle. A short approach run is used to ensure the boat is at speed before the rudder is put

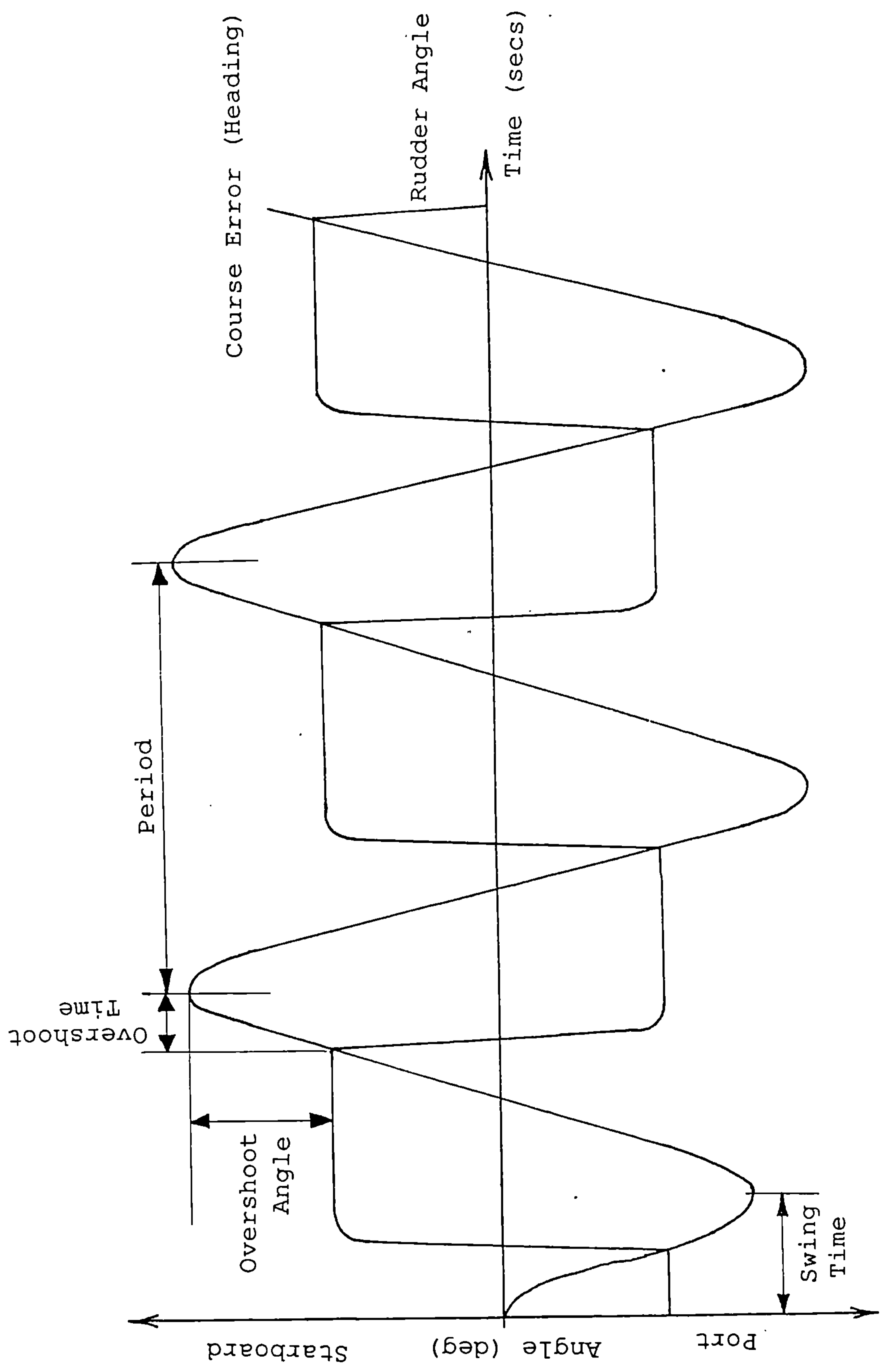


Turning Circle

Fig.7.5

over. Ideally, it is desirable to continue for a turn and a half, 540°, in order to ensure that the manoeuvre is properly terminated. Plots of turning circles are normally presented in terms of horizontal translation from the start point (Fig.7.5), however, since the Arun lifeboat is neither equipped with an accurate means of determining the horizontal translational velocities nor an accurate position fixing system, the plots will be in terms of heading against time. This at least allows the steady state turn rate to be determined easily. It should be noted that since small boats exhibit tight turning ability, it is often possible for the boat to encounter its own wake before completing the turn, obviously this condition will not occur in the simulation.

Kempf (Zig-Zag) Manoeuvre: this provides a means of investigating the steering ability of a boat. It shows the effectiveness of the rudder to initiate and correct changes in heading. The boat is initially lined up on a chosen course at a constant speed. The rudder is then quickly, but smoothly, deflected to a certain angle, say 20°. The boat's heading is then allowed to change a prescribed amount from the initial course, say 30°, at which point the rudder is applied to the same chosen angle of deflection, 20°, but in the opposite direction. This angle of rudder is held until the boat's heading

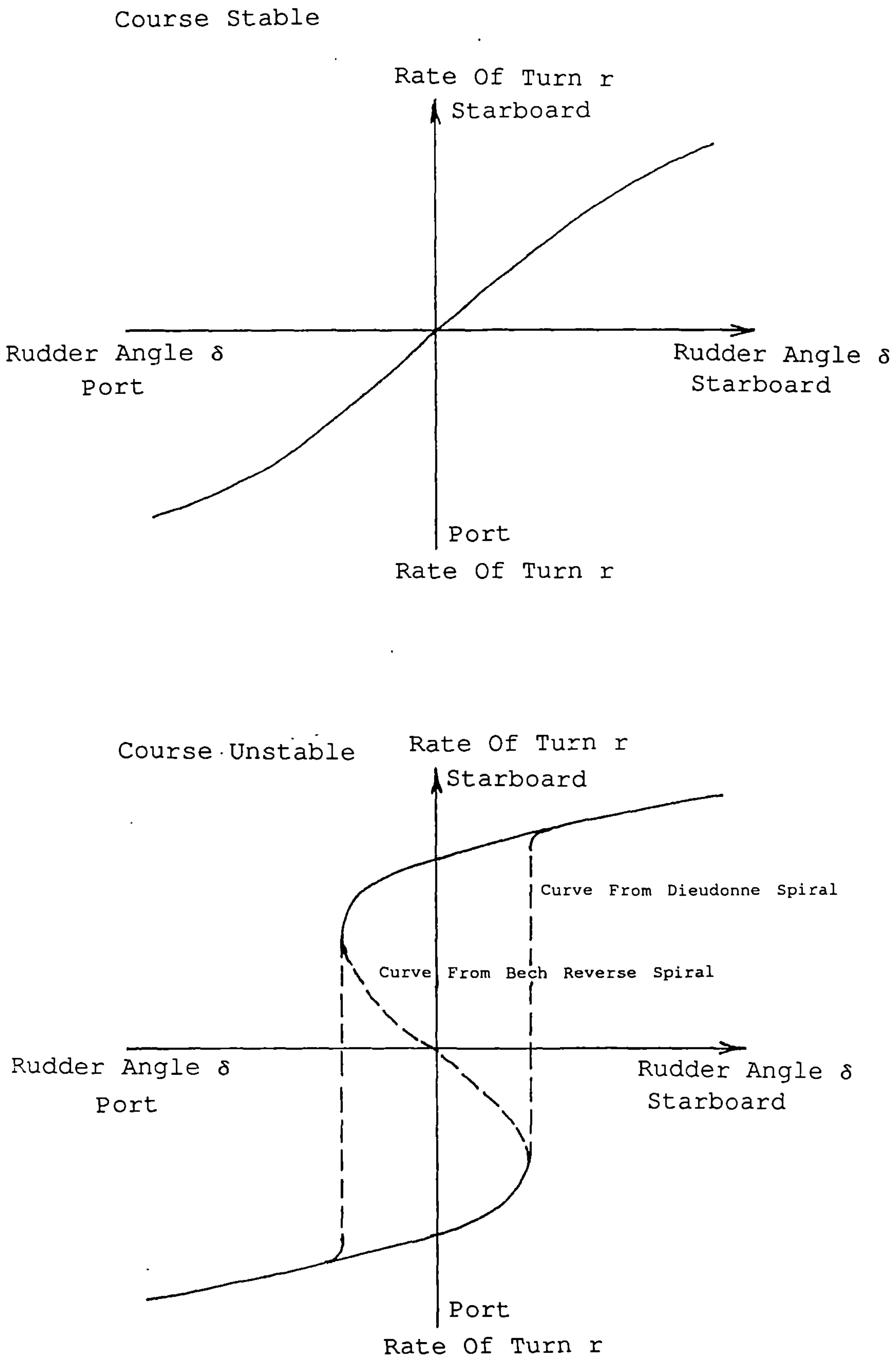


Kempf Manoeuvre

Fig.7.6

has crossed the initial course and is the prescribed change, 30°, in the opposite direction. This process is repeated for a minimum of four or five complete alterations (Fig.7.6). Since a small boat has a much greater turning capability than a large ship, the values used for the rudder angle deflection tend to be smaller, whilst those for the course change are greater. The whole process is usually repeated for a number of different rudder angle/course change values, for example, $10^\circ/30^\circ$, $15^\circ/20^\circ$ et cetera.

Dieudonné Spiral Manoeuvre: This manoeuvre is used to provide a qualitative measure of the course stability of a boat. The procedure involves initially holding the boat on a set course and speed. Then maximum starboard rudder is applied and held while the boat turns through 360° to allow the turn rate to reach a steady value. The rudder is then reduced by a prescribed amount and again held until a steady rate of turn is achieved. The process of reducing the rudder and noting the turn rates is continued until all starboard rudder has been removed, then port rudder is applied by the same increments until maximum is reached. The procedure is then reversed until the rudder is again at maximum starboard deflection. A graph of turn rate against rudder angle is drawn up from the steady state values (Fig.7.7) which will give an indication of the course



Dieudonne Spiral Or Bech Reverse Spiral

Fig.7.7

stability of the boat.

Bech Reverse Spiral: This is the inverse of the Dieudonné spiral and can only be successfully used if there is a rate of turn indicator or an autopilot which responds to rate of turn error instead of heading error. The boat is steered at different constant turn rates, and the mean rudder angle required to produce this yaw rate is measured. This method, although harder to instigate for small boats, allows the rate of turn against rudder angle curve to be plotted as a continuous function (Fig.7.7).

The manoeuvres performed during trials with the RNLI consisted of a set of turning circles, a set of kempf manoeuvres and some straight runs conducted with the relative direction of the sea at the cardinal and inter-cardinal points. Plots of these manoeuvres can be found in an appendix. It was thought that the Dieudonné or Bech spiral manoeuvres would not yield much further information, especially as the turn rates can be determined from the turning circles.

CHAPTER 8
RESULTS

"It is a capital mistake to theorise before one has data"

Sir Arthur Conan Doyle

8.01 Real And Simulated Data

During boat trials with the RNLI, data was obtained for, amongst other manoeuvres, a whole series of turning circles, carried out at various rudder angles and forward speeds, as well as a set of Kempf manoeuvres. Plots of this data are presented in an appendix of this thesis. The plots are arranged in terms of a set of twelve turning circles and four Kempf manoeuvres. For each manoeuvre the real data is given first, and has the time and date when it was recorded at the head of the plot; this is followed by the comparable simulation which has the word 'Simulation' written at the head of the plot.

The turning circles are fairly rudimentary, but nonetheless extremely useful, manoeuvres which yield the steady turn rates for given rudder deflections, and they provide for an initial impression of the performance of the mathematical model. Kempf manoeuvres, although again used to determine how the rudder affects the boat's turning ability, tend to provide continual reversals in the rudder commands, and consequently the boat is set into oscillation. If the model proves able to adequately follow such fluctuations, then this will provide good reason to believe that it will closely match any required manoeuvre of the real boat.

The computer simulation was run with a control routine in order to perform the same manoeuvres carried out in the boat trials. The object was to allow accurate automatic reproductions of the real data. The control routine used here allows the model to run up to speed on the initial given course, prior to applying rudder commands. For the turning circle manoeuvre, the particular rudder angle is applied and left until a full turn and a half has been completed.

The Kempf manoeuvre requires slightly more control. A given rudder angle must be applied until a certain course change has occurred, then the desired rudder angle must be reversed and so on until a prescribed number of iterations have been performed. One drawback with the simulation of the Kempf manoeuvre is that, if the frequency of the oscillations is not quite the same as in the real situation (either slightly longer or slightly shorter), then the result will progressively get further out of synchronisation with the real data.

8.02 Turning Circle Plots

Analysing the turning circles in terms of the five variables plotted shows the following:

Roll: The real data shows that the average value during a turn tends to hang off to one side id est there is a constant angle of heel in a turn (plus the wave effects). A starboard turn causes a starboard angle of heel, whilst a port turn yields a port roll. This becomes more pronounced at higher speeds and at greater rudder angles (that is in tighter turns). Clearly visible on virtually all the plots of the real data is the initial peak at the instant when rudder is suddenly applied. The comparable simulations show all these attributes, except that the roll damps down to constant angle of heel without the wave effect. About two-thirds of the way through each turn the boat encountered its own wake, this is visible on the real trace, but the simulations are not capable of taking this into account.

Pitch: At speed the bow of the semi-displacement boat rises, the greater the speed the higher the constant angle of trim achieved. Due to speed reduction in turns the pitch angle will be reduced, though with the wave effect tends to dwarf this effect in the real data. The effect of crossing the wake is more pronounced on the pitch trace. The mean pitch angle on the real trace seems to vary little for different turn rates or forward speeds. The simulated pitch trace is a little less than the real situation, due to different settings of the trim tabs and the need for a slight adjustment to the propeller effect term.

Yaw: The fluxgate compass provides a very stable output of the change of heading, and a near straight line is produced. Some slight deviations from a constant rate are visible, due to "hanging off" in the swell. Naturally the simulation produces a perfectly straight line with the gradient matching the real data.

Rudder: Some differences exist between the simulated and real rudder angle. Clearly it is a simple matter to set an exact rudder angle during a simulation run, but it is not so easy in a real situation with swell and waves. The rudder increments are $\pm5^\circ$, $\pm15^\circ$ and $\pm25^\circ$.

RPM/Surge: The real boat data consists of the propeller rpm since there was no means of determining forward velocity. The trace shows a virtually constant value throughout the turns. The simulation plot is of the surge velocity, and shows the speed reduction upon commencing a turn. Initially the simulation model came up to speed, then dipped as rudder was applied.

8.03 Kempf Manoeuvre Plots

Similarly, analysing the Kempf manoeuvres in terms of the five variables plotted shows the following:

Roll: The simulated roll is not so sharply spiked as the real data, but it does exhibit the same damping characteristics. This manoeuvre, with the rapid changes in rudder angle from one side to the other, tends to belittle the effect of the waves on the roll, especially as the manoeuvres were performed into the direction of the swell.

Pitch: Little more can be said of the pitch angle trace than was said for the turning circles, except that the wake does not interfere with these manoeuvres.

Yaw: The simulated yaw trace shows a greater overshoot, and hence longer time to return to the base course, than the real data. This is due to the differences between the compass systems used to perform and record the manoeuvre and to perhaps the yaw moment of inertia being set a little high. It was noted in hindsight that there was a deficiency in the yaw assessment in that the coxswain was using a Cestral compass damped by oil in order to determine the deviation from the base course. The accuracy of reading this instrument under such oscillatory motions is not too good, especially as it tends to swing easily. This is reflected in the yaw trace from the fluxgate with the peaks reaching varying heights, however the underlying characteristics are plain enough. The fluxgate compass also suffers from variations due to roll and pitch accelerations. The roll motions can be clearly seen imposed upon the yaw trace and this detracts from the smooth sinusoidal curve.

Rudder: The real rudder traces show that these manoeuvres were a little tricky for the coxswain to perform with absolute accuracy. Since the simulated yaw took slightly longer to return to the base course than the real data, the rudder commands are correspondingly spaced at slightly longer intervals. However, the basic shape of a square wave command format is clearly visible.

RPM/Surge: The surge trace exhibits fluctuations, particularly at higher speeds and more rapid Kempfs, due to the changes in drag from the rudder and the changing yaw rate which gives rise to the sinusoidal style yaw trace. The rpm remains fairly constant, as it did for the turning circles.

Differences in the real and simulated results are principally due to the effects of wave and wake disturbances and the inaccuracy of the helmsman's positioning of the rudder, especially in Kempfs.

Altogether the small boat model is able to mimic the Arun lifeboat exceedingly well. A few minor adjustments could be made to obtain a closer fit, but without supporting reasons for such changes this is hardly worthwhile. From the point of view of the autopilot, the model behaves close enough to the real world situation for simulation purposes in the development environment. The model is capable, with sufficient data, of adequately predicting the behaviour of any small boat, particularly of the displacement or semi-displacement type. It has the flexibility over ship models that it will be capable of reproducing any small boat, and not being necessarily dedicated to one ship type. This is extremely important where the variety and style of small boats is large and diverse.

CHAPTER 9

STRATEGY FOR TESTING AUTOPILOTS

9.01 Communicating With The Autopilot

In order to assess the performance of the autopilot system within the development environment, it became necessary to allow the autopilot to exercise control over the computer simulation of the small boat. This in effect replaces the real boat with the mathematical model, and varying conditions of sea state, tide and wind can be imposed on the simulation at will. The main advantage of this is that tests can be repeated, under exactly the same conditions, for different or modified autopilot designs. Results will then be directly comparable.

The autopilot requires information about the boat's yaw angle (magnetic heading) and rudder position in order to provide automatic control. In a real boat situation these will be provided by the electronic fluxgate compass and rudder reference units. The autopilot then effects control of the boat by sending a command signal to the rudder. The model generates the yaw angle and rudder position data at each discrete interval so, if these are input to the autopilot controller in place of the compass and rudder reference, the model can simulate the boat.

There already exists a marine standard for data transmission between electronic navigational devices known as the NMEA (National Marine Electronics Association) format. This consists of a large number of ASCII (American Standard Code for Information Interchange) messages which start with a two character system identifier, followed by a three character message identifier, the message itself and finally the CR (Carriage Return) and LF (Line Feed) message terminators. The Cetrek autopilot has a number of ports to which Decca Navigators, and other such equipment which produces NMEA messages, can be connected.

The small boat computer simulation has, therefore, been designed to output magnetic heading and rudder angle, in NMEA format (but not a true NMEA message), to the autopilot. In return the autopilot sends back a byte containing the rudder command information. This link provides a closed loop between the autopilot and boat model which can be used to assess autopilot performance.

With the addition of the chart pilot (a combined electronic chart and autopilot unit) to the Cetrek range of products, it became possible to display the boat's position and navigate along a prescribed track defined by a set of waypoints. A second NMEA message (this time a true message) giving the boat's position in latitude and longitude is sent by the model to the autopilot. The computer simulation, therefore, also simulates an electronic position fixing system. There is thus a visual representation of the autopilot's performance on the chart pilot as the boat's track is plotted against the desired track.

The model outputs a further piece of information, that of yaw rate. The reason is two-fold; first it can be used to assess the rate term used by the derivative section of the PID controller, and secondly it is envisaged that it will provide a means of assessing the performance of the rate-gyro presently being developed at Cetrek.

For purposes of analysis, the autopilot returns a second byte of information containing the course error. That is the difference between the desired course to be steered, obtained from either the chart pilot's waypoint information or from the user, and the actual heading. The relevance of this item of data will become apparent in the next section.

The pseudo NMEA message passed by the simulation to the autopilot can be represented thus:

$MAMOD,xxx.x,M,yy.y,L,zz.z,LC_RL_F$

where:

$	denotes the start of the message
MA	is a two character system identifier
MOD	is the message identifier (id est it indicates what information is to follow) and is short for model
xxx.x	is the boat's heading in degrees
M	either M for Magnetic heading or T for True heading
yy.y	is the absolute rudder angle in degrees
L	either L for Left or Port rudder or R for Right or Starboard rudder
C_RL_F	are the two characters (Carriage Return, Line Feed) which terminate the message
,	the commas are field markers

The NMEA message passed by the simulation to the chartpilot can be represented thus:

$MAGLL,xxxx.xx,N,yyyyy.yy,EC_RL_F$

where:

$	denotes the start of the message
MA	is a two character system identifier
GLL	is the message identifier and is short for geographical latitude and longitude
xxxx.xx	the first two x's are the degrees of latitude; the next two x's are the minutes of latitude; and the last two x's are the hundredths of minutes (not seconds)
N	either N for North or S for South
yyyyy.yy	the first three y's are the degrees of longitude; the next two y's are the minutes of longitude; and the last two y's are the hundredths of minutes (not seconds)
E	either E for East or W for West
C_RL_F	are the two characters (Carriage Return, Line Feed) which terminate the message
,	the commas are field markers

In reply the autopilot sends two bytes. Firstly the rudder command message, the eight bits of which can be represented thus:

UDRRVVVV

where:

U	is the highest order bit and is unset=0
D	is the drive condition: 0=standby; 1=drive (drive is when the autopilot has control over the boat)
RR	is the rudder direction command: 0=Freewheel; 1=Port; 2=Starboard; 3=Brake
VVVV	is the speed of rudder movement and is a value between 0 and 8 and represents so many eighths of the maximum possible rudder speed

The other byte contains the course error determined by the autopilot, the eight bits of which can be represented thus:

SDEEEEEE

where:

S	is the highest order bit and is set=1
D	is used to determine the direction of the course error
EEEEEE	is the course error

The simulation of the small boat is run on a Personal Computer. The model code is executed on every BIOS timer interrupt (65536/1193180 = 0.0549s) id est the states are updated approximately once every 55ms. The MAMOD NMEA message is then output to the autopilot, via the Tx (transmit) line on the computer's serial communications port, as close to this rate as is possible.

The MAGLL NMEA message is output on every 50th transmission, id est about every 2.5s, to the chartpilot. This simulates a navigator input.

A second interrupt routine awaits the two incoming bytes on the Rx (receive) line from the autopilot. These are processed as soon as they arrive. As far as the autopilot is concerned the model messages appear on one of its standard navigator ports.

9.02 Graphical Presentation

The simulation communicates with the user by displaying pertinent information on a VDU (visual display unit). The settings of the control surfaces of the rudder, propeller and trim tabs are displayed as bar graphs, whilst the yaw angle is expressed as a plan view outline drawing of a boat within a compass rose. The other Euler angles, latitude, longitude, forward velocity, wind parameters, wave parameters and tidal information are displayed in a numerical format.

Two additional sets of information are presented on the VDU which directly aid the assessment of autopilot performance. The first is a trace recording the course error over the past one minute of the simulation. This graphically shows the amplitude of any overshoots due to insufficient damping in the system or the length of time taken to come up onto course. By altering the autopilot gains, it is possible to watch the differences. This is extremely difficult to produce on board a real boat because the differences are not easily differentiated quantitatively by the helmsman and one can never be sure that the differences are not due to disturbing effects.

The second set of information is a numerical "cost function" value produced for the rudder usage and for the course error. An ideal autopilot will correct all heading errors immediately, but with extremely small use of the rudder. In other words, the autopilot controller should minimise both course errors and rudder usage. Large deviations from the desired course will add extra time and distance to a journey, but close adherence to track requires a greater, unwanted, use of the rudder. A compromise must, therefore, be made between minimising both of these quantities. Cost function values are achieved for both course error and rudder angle by using a low pass filter algorithm.

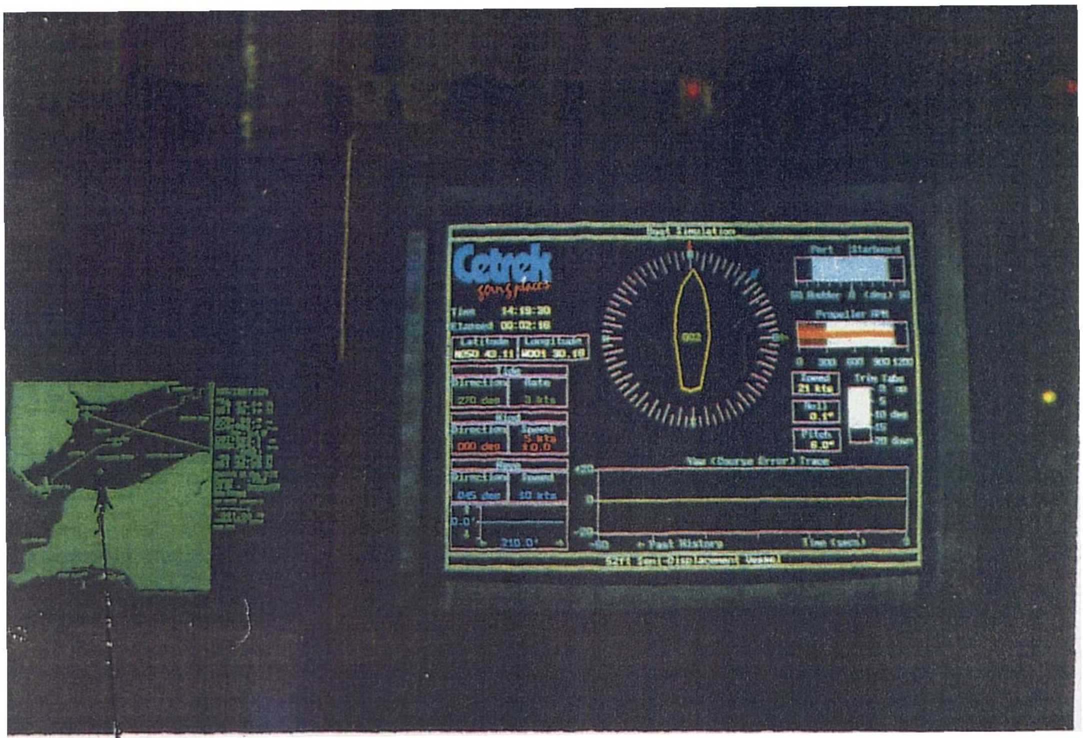
With such information, developers will be able to quantitatively assess the performance of an autopilot prior to conducting extensive and time-consuming sea trials.

9.03 Autopilot Control Of The Model

Preliminary use of the computer simulation with the Cetrek chart pilot has already produced exciting results. Plates 7 to 12 are a set of "snapshots" from a typical simulation run with the autopilot exercising control over the model in order to achieve track keeping.

A number of areas in the autopilot control algorithms have been identified as requiring attention and some modifications to the autopilot code have been made, these include:

1) **Course Deadband:** This defines the limit to which the course may deviate from the desired heading before any rudder is applied. The idea of such a region is to prevent excess rudder movement when the boat is nearly on course. However, it was noticed that if the deadband is set quite wide, then if the course falls outside of the deadband, there is a sudden demand for a large amount of rudder in order to turn the boat back towards the base course. The boat then turns back too quickly and overshoots the other side and a large, unwanted oscillation is induced.



Time: 2mins 16secs

Plate 7

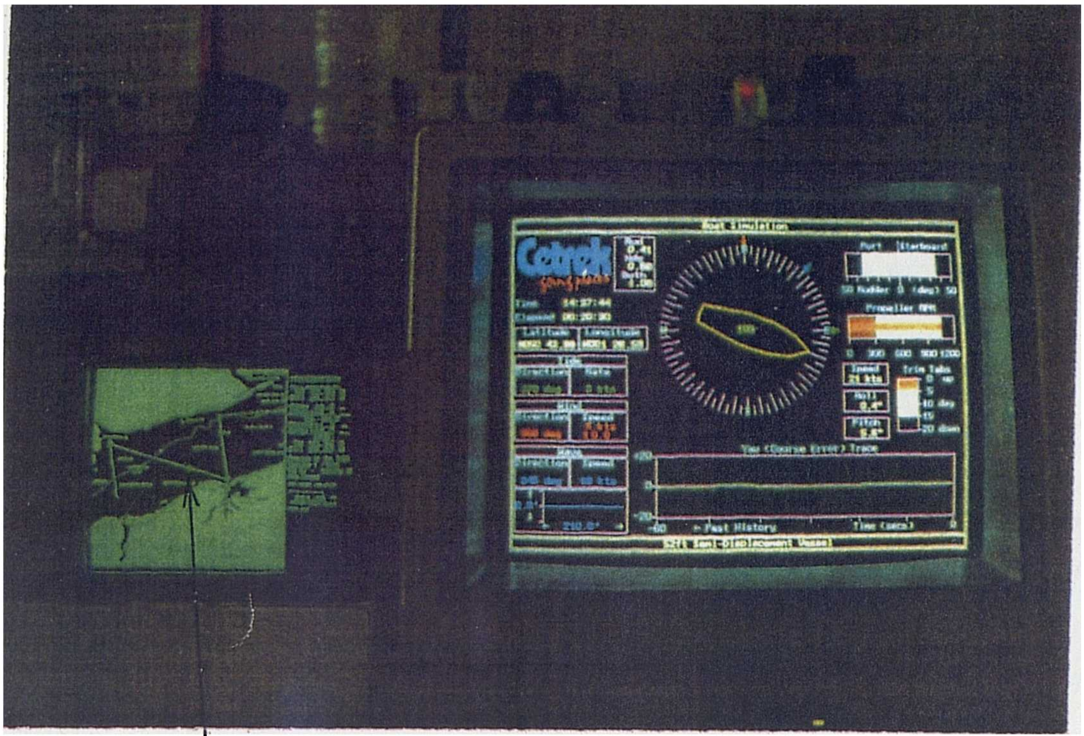
Boat Position (Steering To Waypoint 1)

Time: 14mins 01secs

Plate 8

Boat Position (Steering To Waypoint 2)

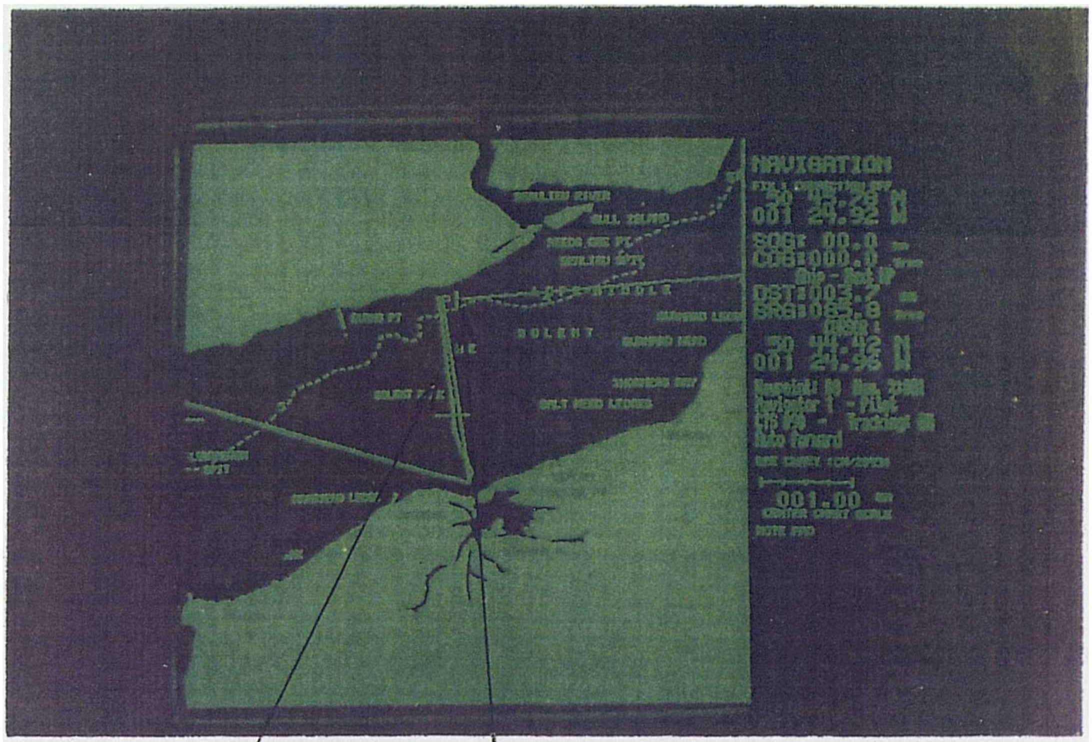
Waypoint Overshoot



Time: 20mins 30secs

Plate 9

Boat Position (Steering To Waypoint 2)
(Good Track Keeping)

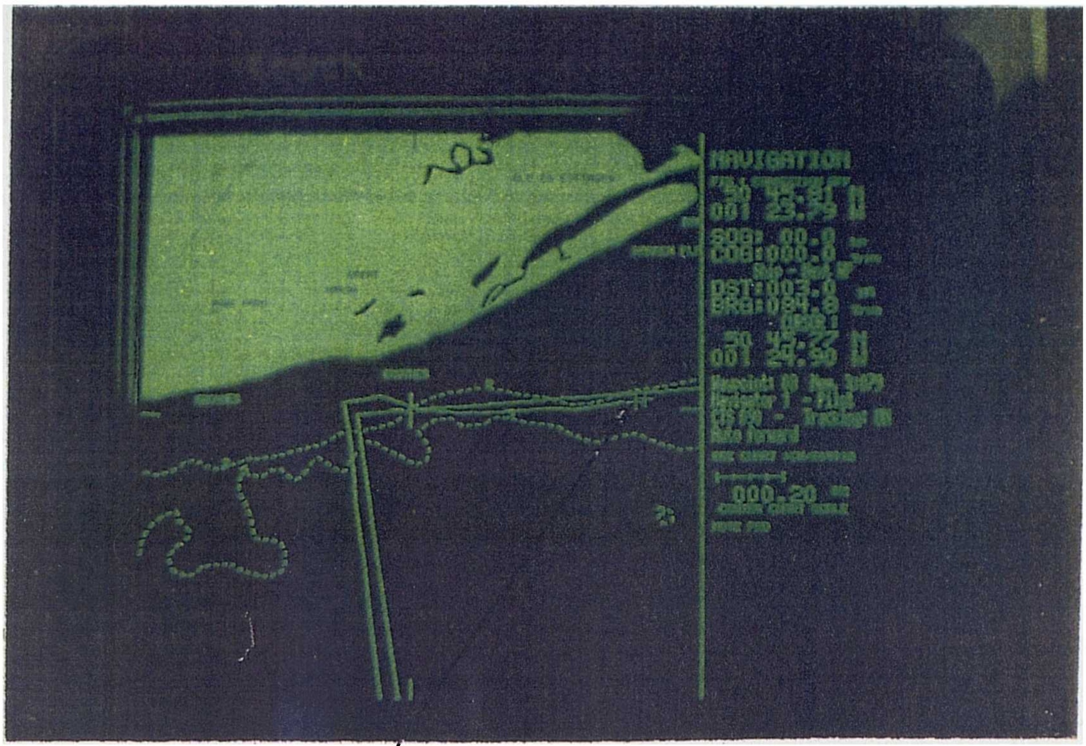


Time: About 32mins

Plate 10

Boat Position (Steering To Waypoint 4)

3Knt Tide Running From East To West Causes Offset
Short Leg Does Not Give Autopilot Time To Correct For Tide



Time: 36mins 28secs

Boat Position (Steering To Waypoint 4)

Plate 11

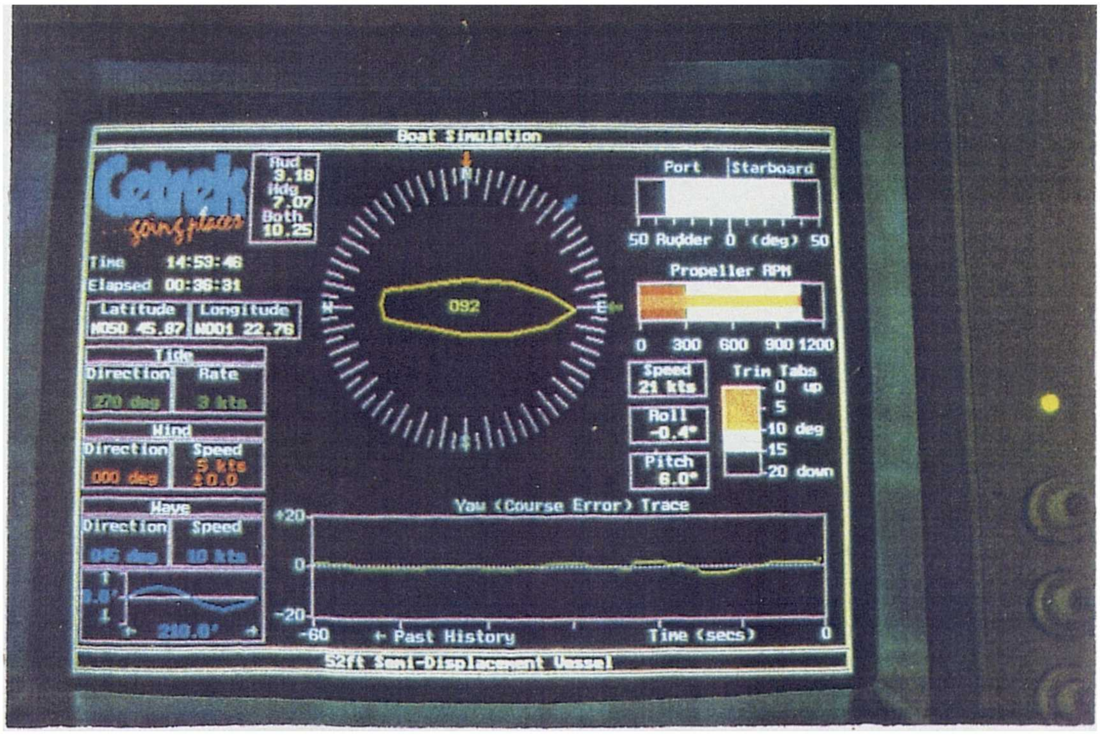


Plate 12

2) **Track Keeping By Cross-Track Error:** Given a navigator input, the chart pilot is capable of steering between waypoints by assessing the cross-track error. However, when the boat arrives at one waypoint and is directed towards the next, the cross-track error can become very large. Consequently the cross-track error integral term increases exceedingly quickly, and too much offset helm is built up, resulting in a severe turn round the waypoint. This problem was solved by clearing the integral term upon waypoint arrival and during the course change.

3) **Assessing Overshoots At Waypoints:** The criterion for waypoint arrival is that the boat must pass abeam of the waypoint mark. The boat will then be navigated along the next leg of the track. The model allows the autopilot developers to assess the overshoot of a boat past a waypoint and how quickly it comes round to the new track.

4) **Other Alterations:** A few additional minor alterations have been put into effect in the autopilot code concerning, for example, the NMEA message handling.

Visual assessment of what is actually occurring is extremely useful and is provided by both the chart pilot and the "front end" graphics of the computer simulation.

CHAPTER 10
CONCLUSIONS

"In research the horizon recedes as we advance, and is no nearer at sixty than it was at twenty. As the power of endurance weakens with age, the urgency of the pursuit grows more intense .... And research is always incomplete."

Mark Pattison

The work contained in this thesis describes in full the investigation and development of a small boat mathematical model. The model is of a highly flexible, modular nature and is not confined to a given boat type, but can, if desired, represent any small boat. The model extends the boundaries of marine simulation and can be easily modified to describe other marine vessels, including large ships.

The initial literature search showed a lack of published material and data about modelling small boats, whereas their large ship counterparts have received much attention. Much of the standard naval architectural theory has held valid for small boats and some of the equations derived in this thesis have been spawned from large ship research.

A number of tools essential to this kind of simulation, namely rigid body dynamics and transformations between axes systems, were discussed and developed. Their particular relevance to the small boat model has been outlined and the mathematics has been in keeping with standard nomenclature.

The approach to the model has been one of modularity, thus easing its construction and maintenance. The equations have been developed around Newton's second law of motion, but split into a number of distinct vectors. Each vector can be treated as an individual entity and can be added to, modified or replaced to suit new applications. The framework of the thesis also allows new modules to be bolted on as further research dictates.

A computer program was developed by the author to provide a simulation of small boat manoeuvres. Additional programs, also written by the author, were used in the real boat trials to record information needed to validate the model. Parameter determination proved to be a process of simplification and careful analysis of the equations that had been developed. Comparisons with the boat trials, that were conducted using an Arun class lifeboat, showed the model to be capable of simulating small boat behaviour. The coefficients were not fine tuned to produce accurate reproductions of the manoeuvres, since the simulator is to be used to assess the performance of small boat autopilots.

Communication pathways between the autopilot and model were set up to allow the autopilot to exercise control over the model. The visual assessment of what is actually occurring, in real-time, is extremely useful and is provided by both the chartpilot and the "front-end" graphics of the computer simulation. Several controlled tests were carried out and a number of weaknesses in the autopilot's algorithms have been detected.

The closed loop system of the Cetrek chart pilot and small boat simulator provides an indispensable tool for assessing autopilot design. This system set-up has also been on display at the Southampton boat show in September 1989, and will be used at other such venues, to demonstrate the action and capabilities of the autopilot.

CHAPTER 11

SUGGESTED FURTHER RESEARCH

"*So little done, so much to do*"

Cecil Rhodes

With such work as is herein described, it is customary to indicate further areas of research, and this document will prove no exception. The following suggestions are a few general observations of possible areas of further research in the realm of small boat modelling.

With respect to the autopilot system, the small boat simulation could be incorporated into a model reference system capable of determining the optimum gains in the autopilot control routine for a particular boat in a given sea condition. The amount of variation of the compass course when attempting to hold a set course needs to be reduced to a minimum and the system algorithms could be designed to learn from predicted boat motion and course deviation the best settings of the autopilot gains. Kalman filter theory can be included if there are measurements of some of the states available.

With the prestige and interest in racing sailing yachts in competitions such as the America's cup and Admiral's cup, the model could be extended to incorporate a module representing the sail forces and moments. This could prove extremely difficult to match to a real situation, due to the unpredictability of a gusty wind. Other additions could be in the form of forces and moments due to tunnel or axial thrusters as used on tugs, various appendages attached to boats and the effects of dragging nets from fishing boats.

Although the effect of added mass is well known, there is still much work required to incorporate the computation of accurate values for the coefficients into the boat model. The change of these parameters with water depth and forward speed requires special attention. Ideally, to be able to extract the added mass terms and other hydrodynamic coefficients from the lines plans and general arrangements would provide an added facility for prediction at the initial design stage.

Similarly, theoretical wind computations based upon a knowledge of the quantity and distribution of the superstructure and the projected transverse and longitudinal areas require further investigation.

In order to formulate progressively more accurate mathematical models and increase the detail and fidelity of simulations, especially as new advances in digital computer technology become available, it is paramount that more experiments are carried out to produce data for all aspects and types of vessels which covers a wider range of operating conditions, so that the mathematics can provide increased continuity and accuracy.

In addition to research, there are other aspects that warrant attention; these include the standardisation of symbology, data, codes of practice and so forth in order to enable advances in theories and research to be easily transposed to other applications within the marine field. A greater pooling of experimental data and trials results would aid comparison of different vessel types and the discovery of commonalities.

In concluding this document, the author wishes to echo the statement made by Dand, (Ref.43) 1987, in his concluding remarks:

> *"The greater the fidelity and detail of the model and the less it relies on 'tuning', the greater demands it places on a clear understanding of the physics of ship hydrodynamics. We can only model accurately that which we understand, so the demands of modular simulation must ultimately lead to deeper understanding."*

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APPENDIX A

PRINCIPAL DIMENSIONS OF AN ARUN

Principal Dimensions

Length Overall (Moulded, Exc. Fenders)	51' 8"$\frac{1}{2}$	15.76m
Length Between Perpendiculars (DWL)	46' 0"	14.02m
Beam (Moulded)	17' 0"	5.18m
Draft (Moulded At ST5, DWL)	3' 7"$\frac{5}{8}$	1.11m
Displacement (Working Draft)	32tons	32514kg

General Notes

Deck Camber (Half Breadth)	7" : 8' 6"
Station Spacing	4'.60
Rake Of Keel	13".92 : 46' 0"

At Draft = 1.35m (Working Draft)

$A_{UT} = 2.93m^2$

$A_{UL} = 18.65m^2$

$A_{WL} = 28.00m^2$

Rudder Characteristics

Number Of Rudders	2	
Rudder Height (Span)	30"	0.762m
Rudder Area	450"^{2}	$0.290m^2$
Rudder Aspect Ratio	2.0 ($30^2/450$)	
x_R		-5.42m
y_R		±0.91m
z_R		+0.96m

Propeller Characteristics

Number Of Propellers	2	
Number Of Blades	4	
Diameter	$32''\frac{1}{2}$	0.826m
Pitch (r/R > 0.6)	$30''\frac{3}{8}$	0.772m
Developed Area	$581''^2$	$0.375m^2$
Projected Area	$500''^2$	$0.323m^2$
Circumscribed Area ($\pi D^2/4$)	$829''^2$	$0.535m^2$
Area Of Each Blade		$0.094m^2$
DAR	0.7	(0.375 / 0.535)
x_P		-4.90m
y_P		±0.83m
z_P		+1.07m

APPENDIX B

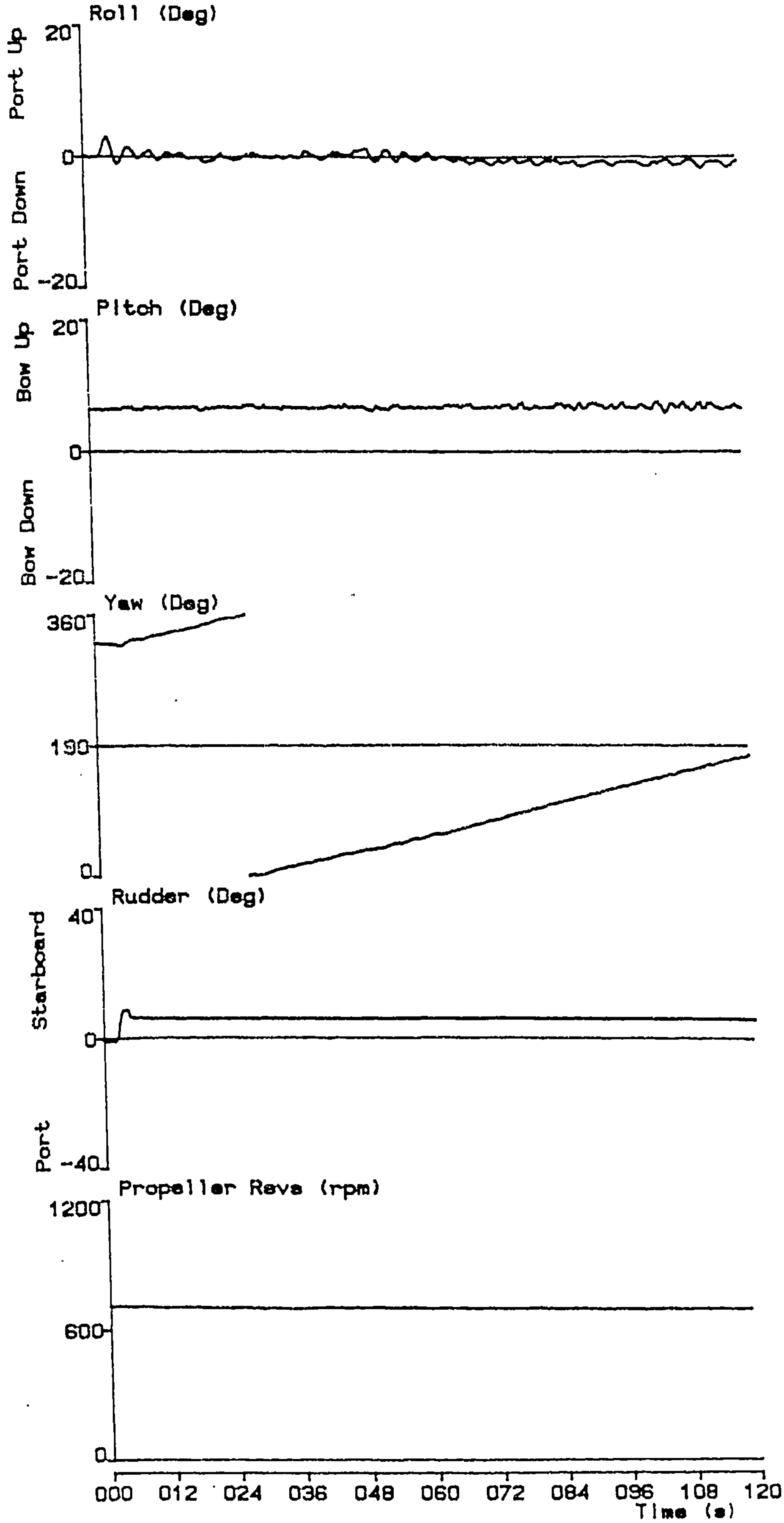
PLOTS

The plots consist of 12 Turning Circles, 4 Kempf Manoeuvres and 1 Trim Tabs Illustration. For each plot the real data is given first, and is indicated by the time and date when it was recorded, followed by the corresponding simulation run, indicated by the word "simulation".

A distinctive feature of interest, which is clearly visible on the pitch plots of the tighter and quicker turning circles, is the point when the boat crosses its own wake. This point is marked on the pitch graph (where distinguishable from the background waves) by a "w".

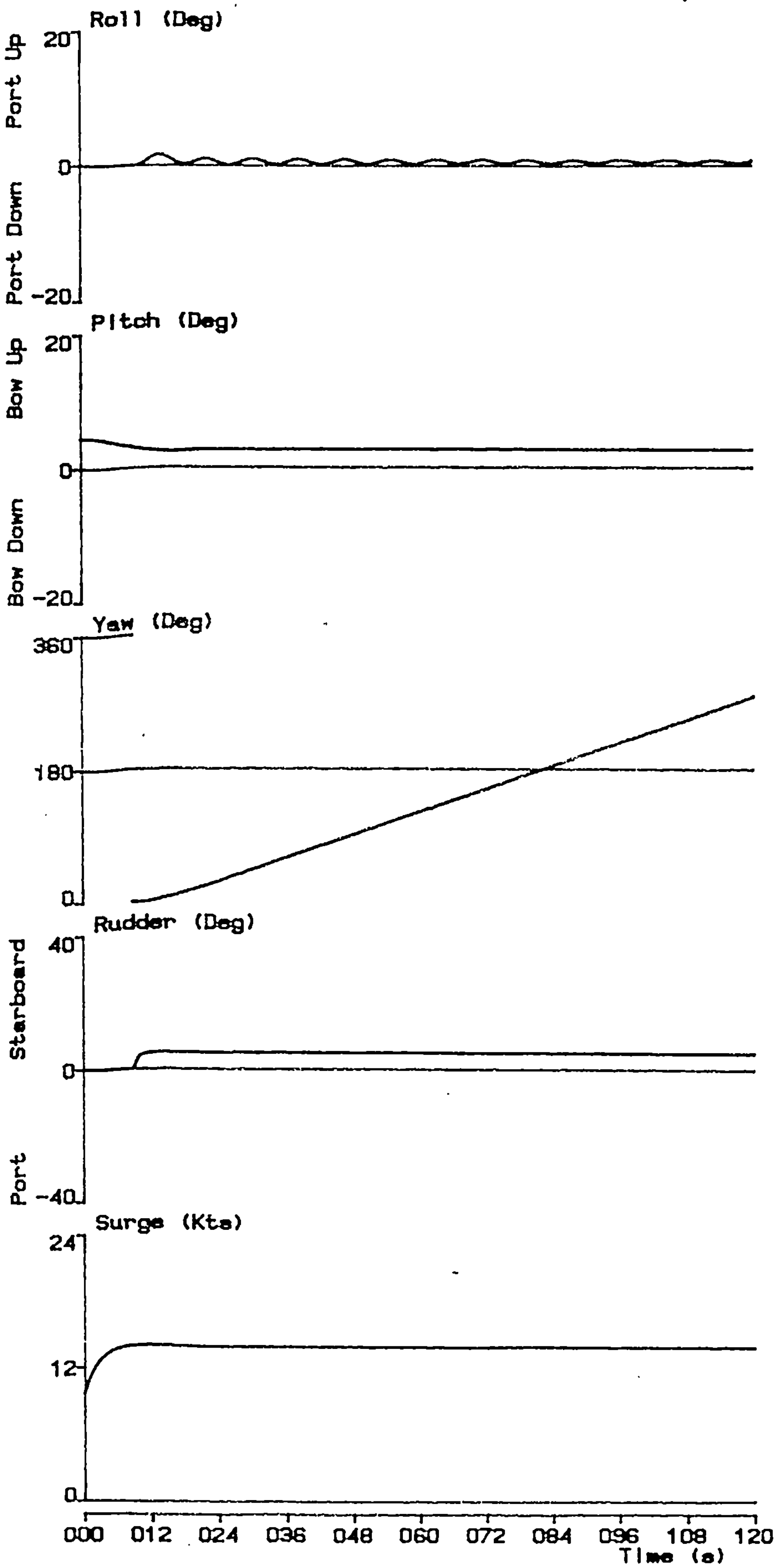
13:54:29 20/04/1989

TURNING CIRCLE 01 (700rpm)



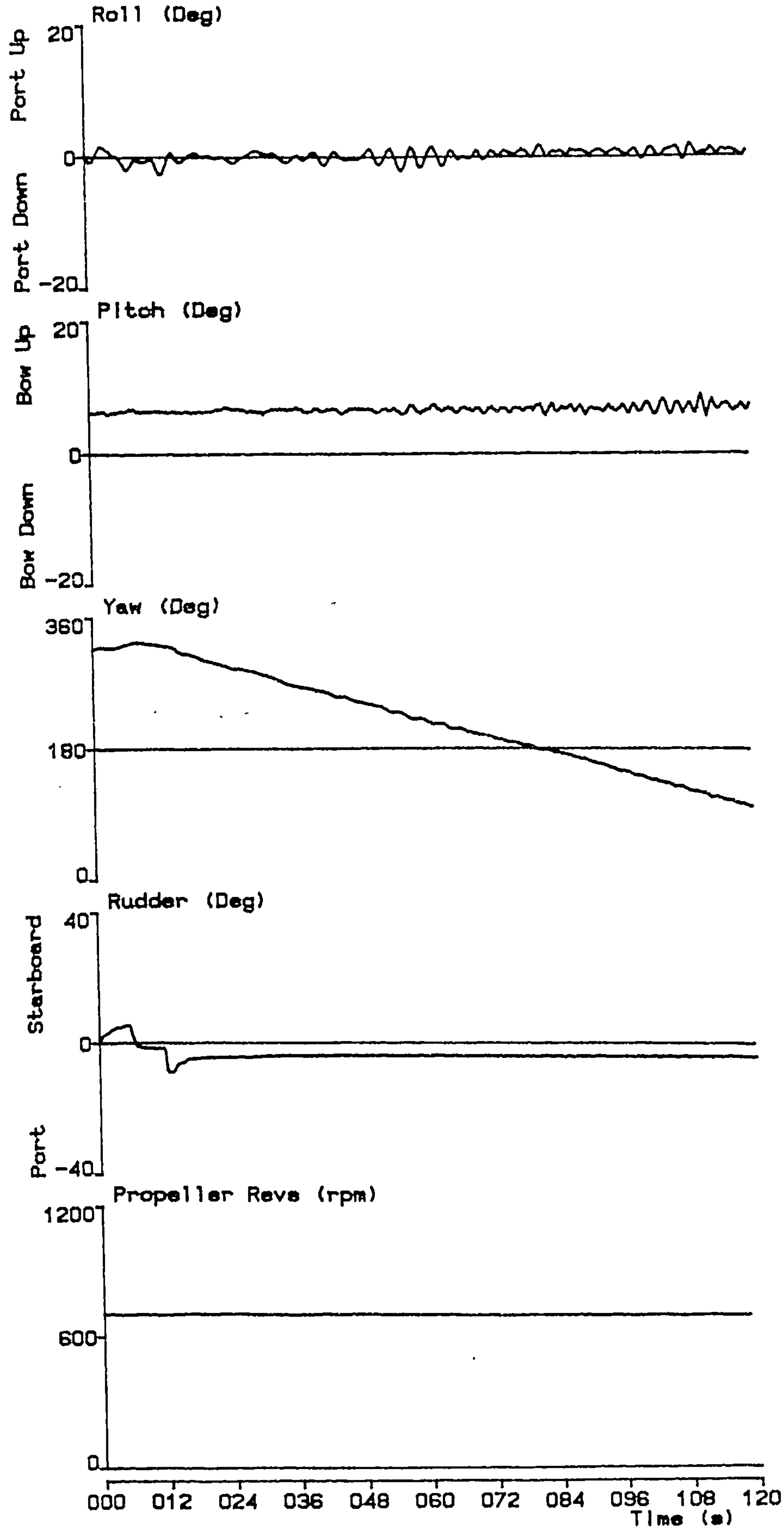
Simulation

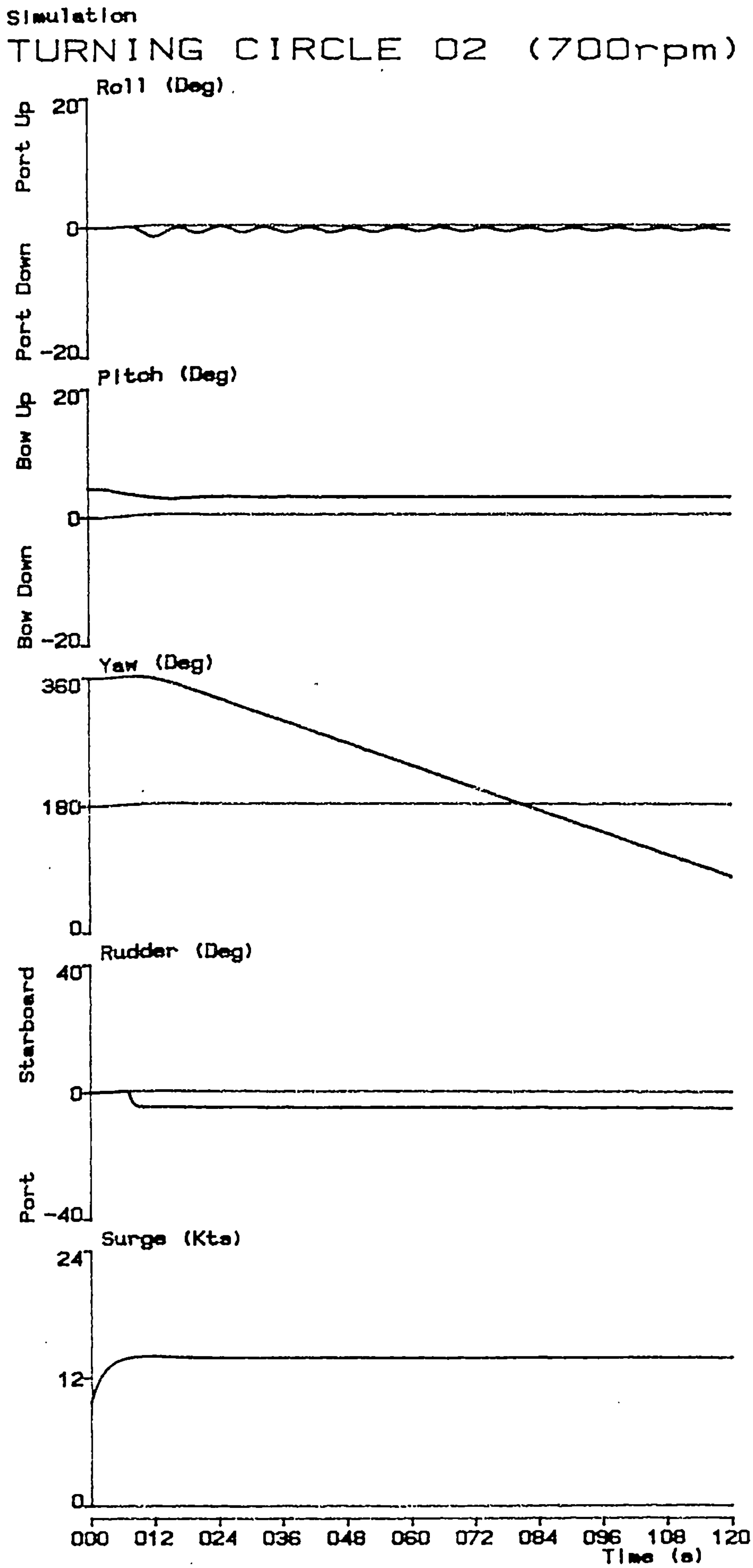
TURNING CIRCLE 01 (700rpm)



13:42:54 20/04/1989

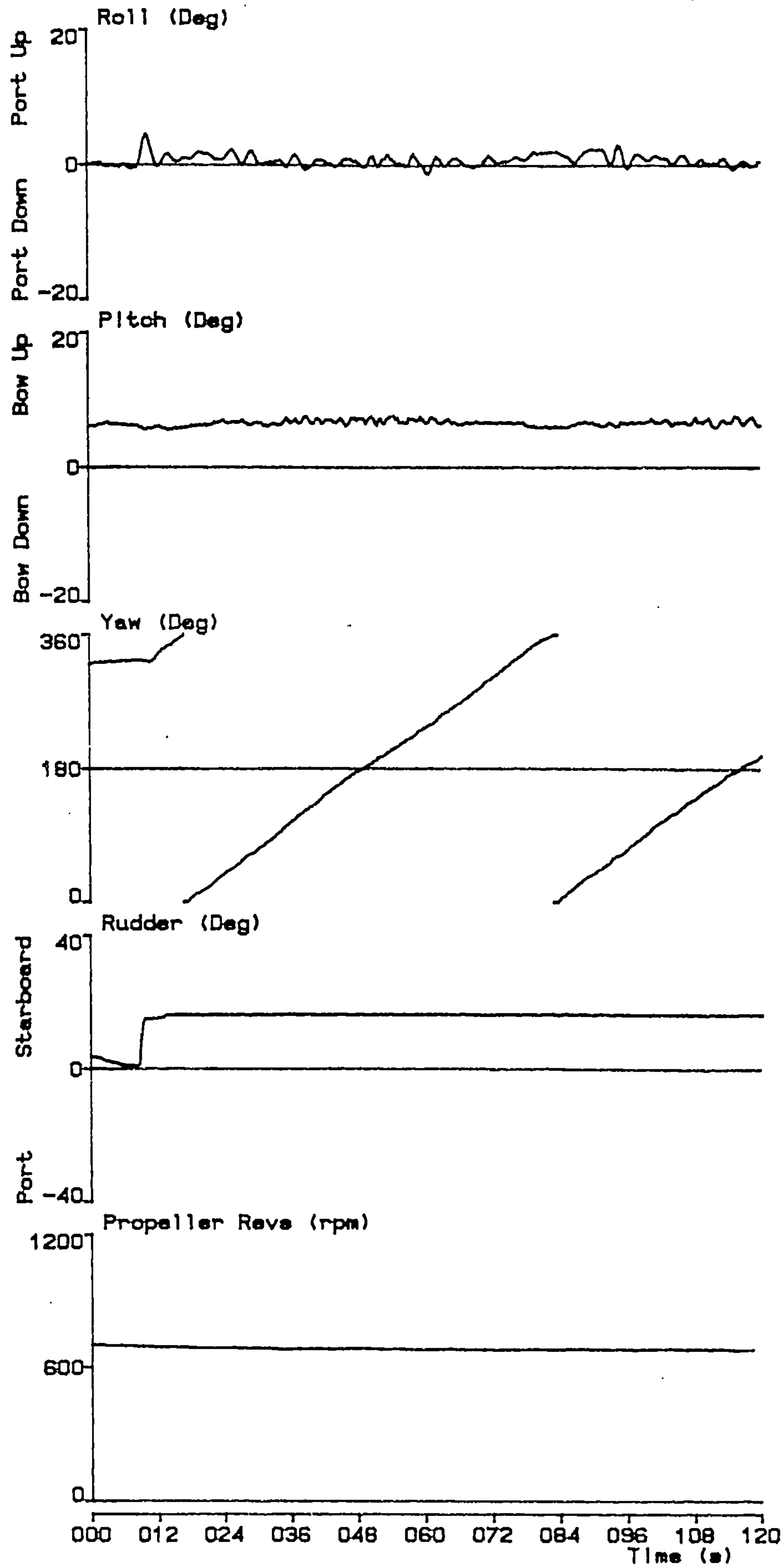
TURNING CIRCLE 02 (700rpm)




Simulation
TURNING CIRCLE 02 (700rpm)
Roll (Deg)
Port Up
Port Down
20
0
-20
Pitch (Deg)
Bow Up
Bow Down
20
0
-20
Yaw (Deg)
360
180
0
Rudder (Deg)
Starboard
Port
40
0
-40
Surge (Kts)
24
12
0
000 012 024 036 048 060 072 084 096 108 120
Time (s)

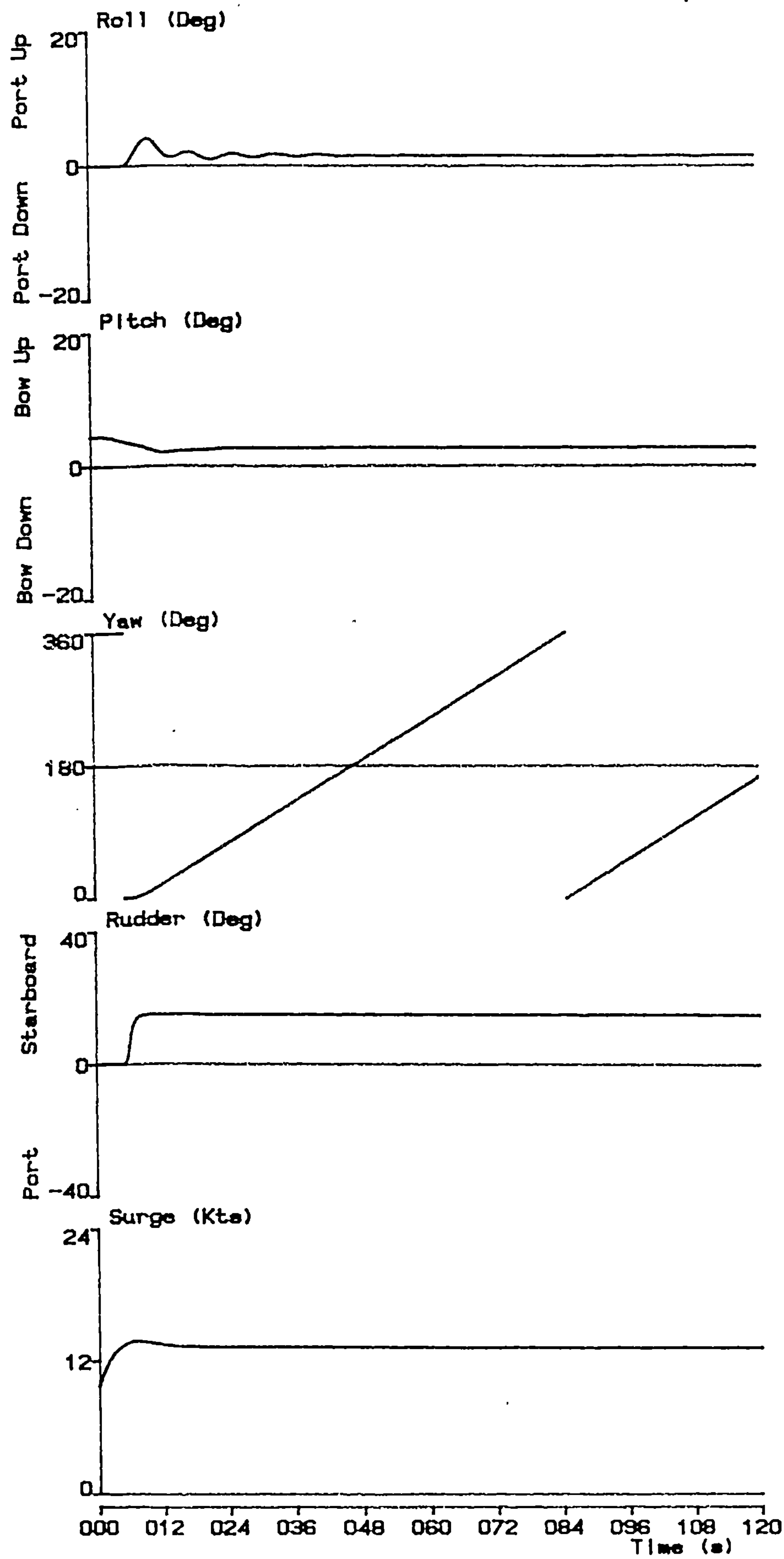
14:14:59 20/04/1989

TURNING CIRCLE 03 (700rpm)



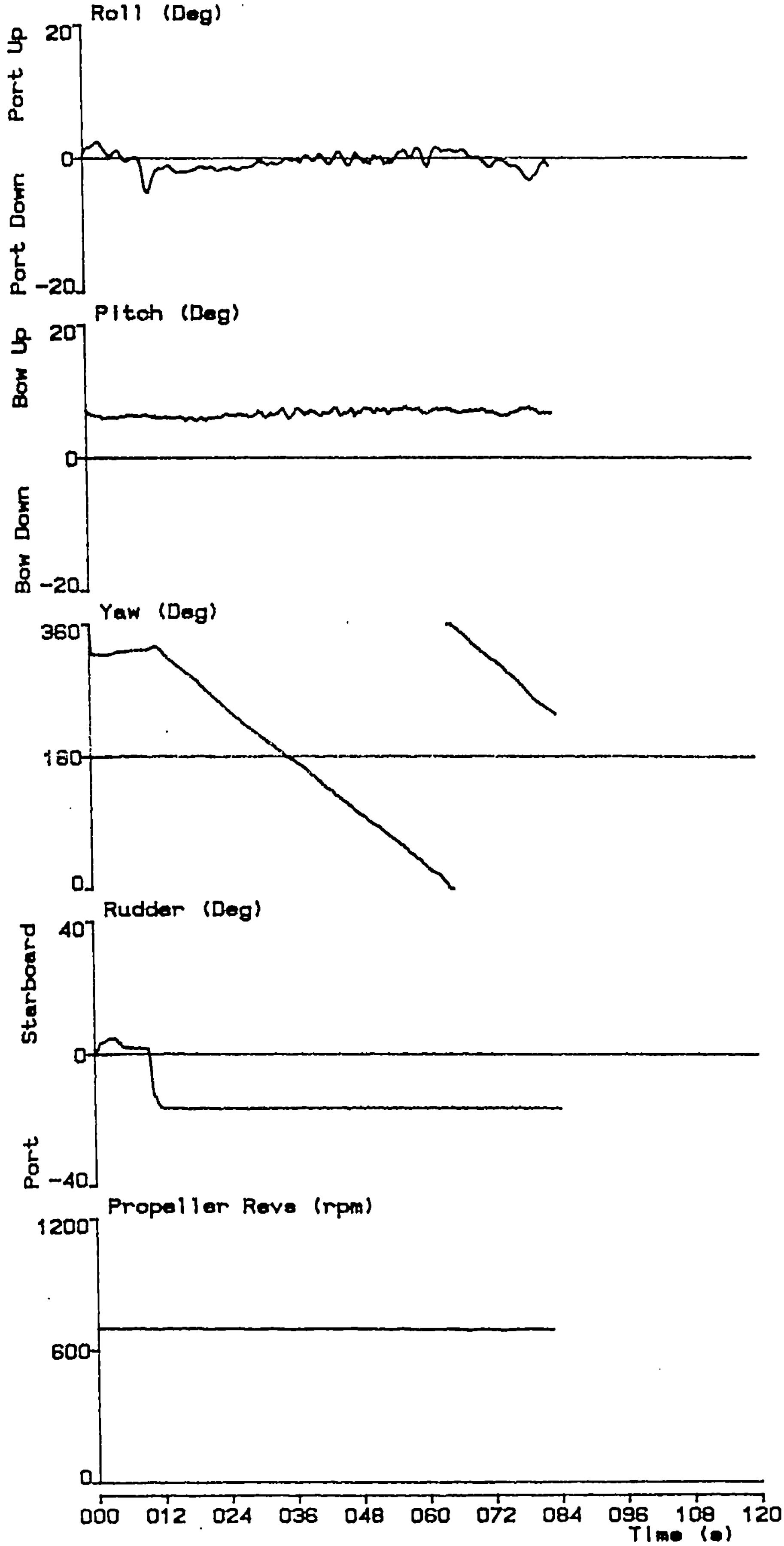
Simulation

TURNING CIRCLE 03 (700rpm)



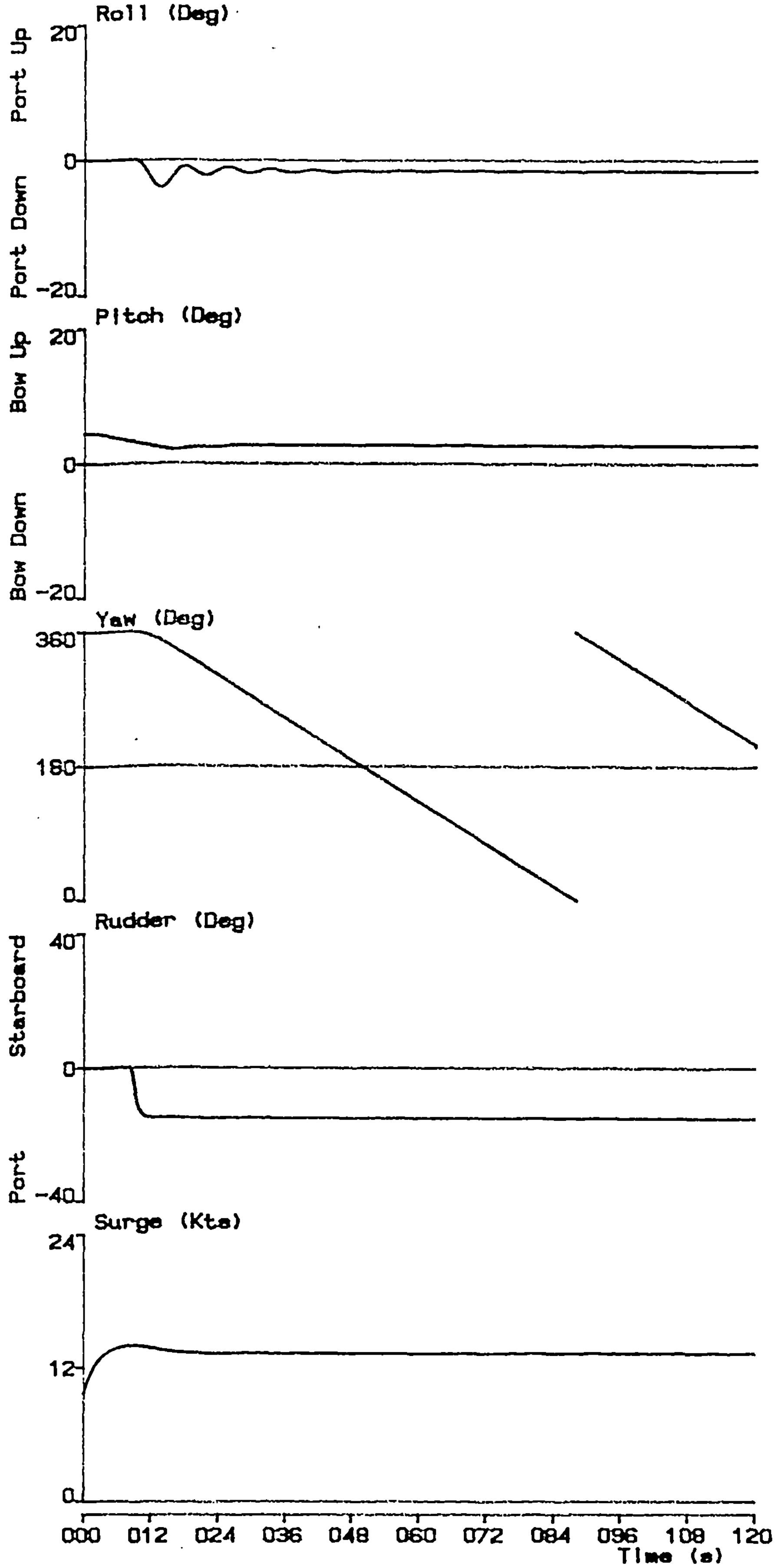
14:05:41 20/04/1989

TURNING CIRCLE 04 (700rpm)



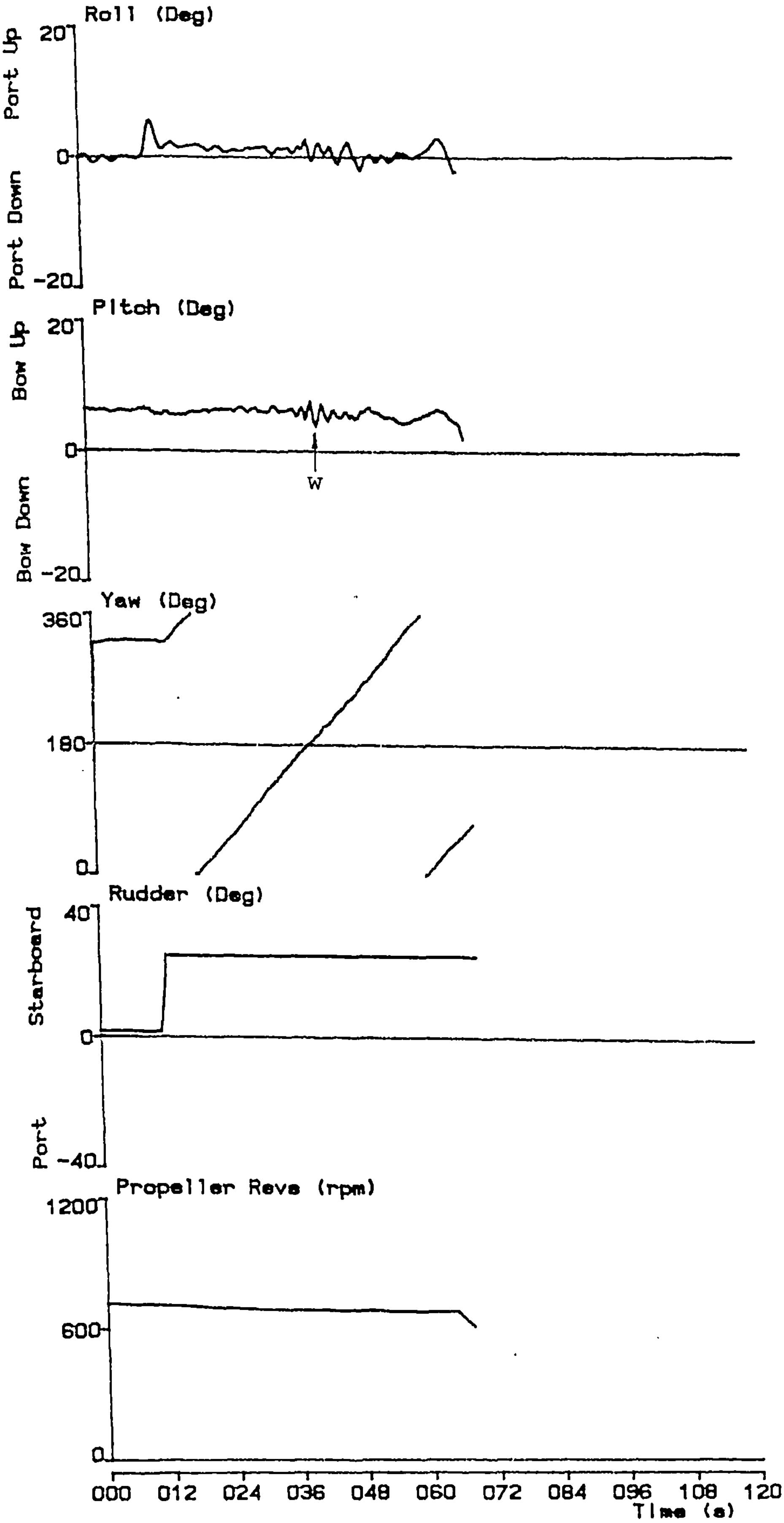
Simulation

TURNING CIRCLE 04 (700rpm)



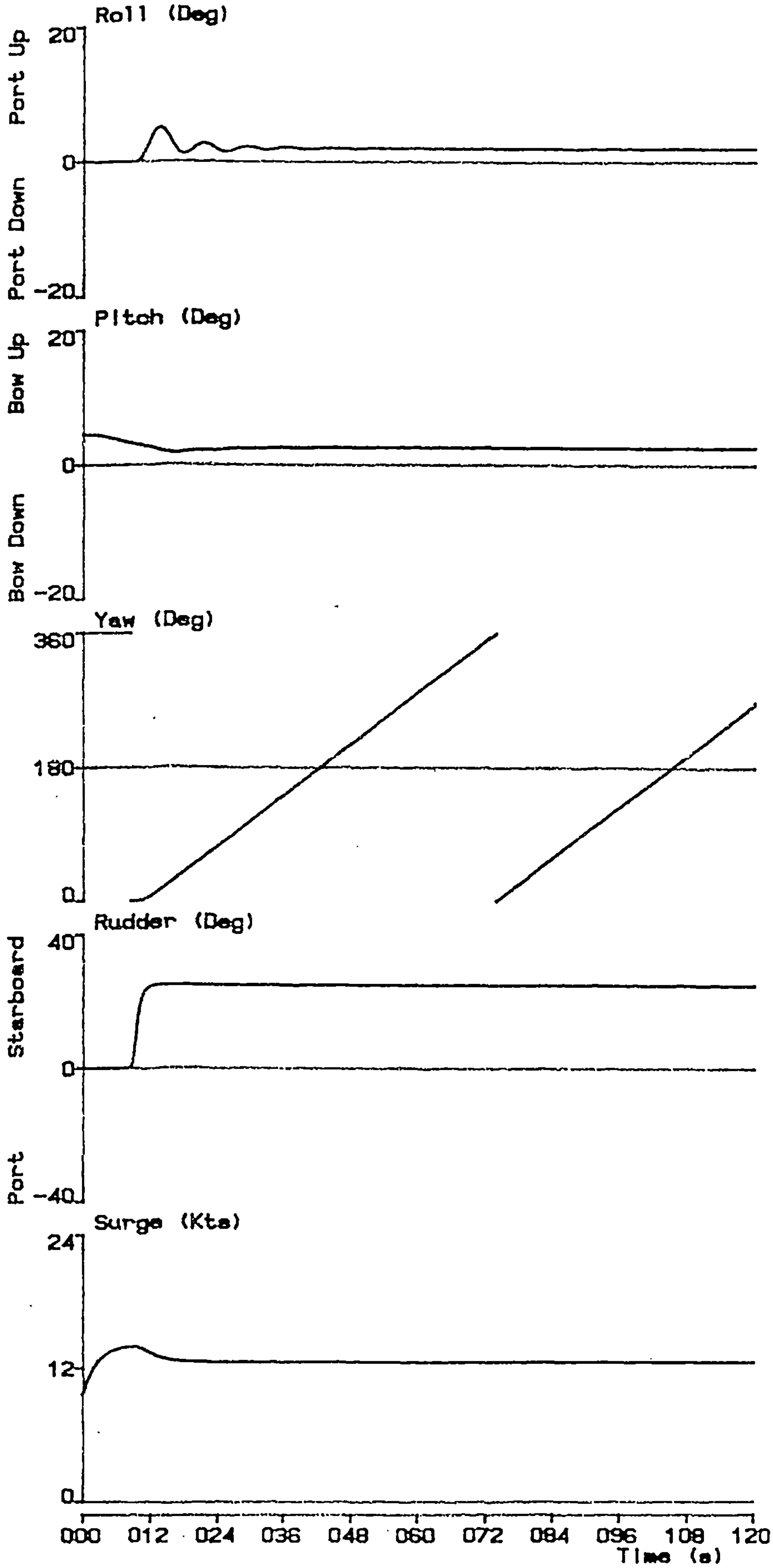
14:30:11 20/04/1989

TURNING CIRCLE 05 (700rpm)



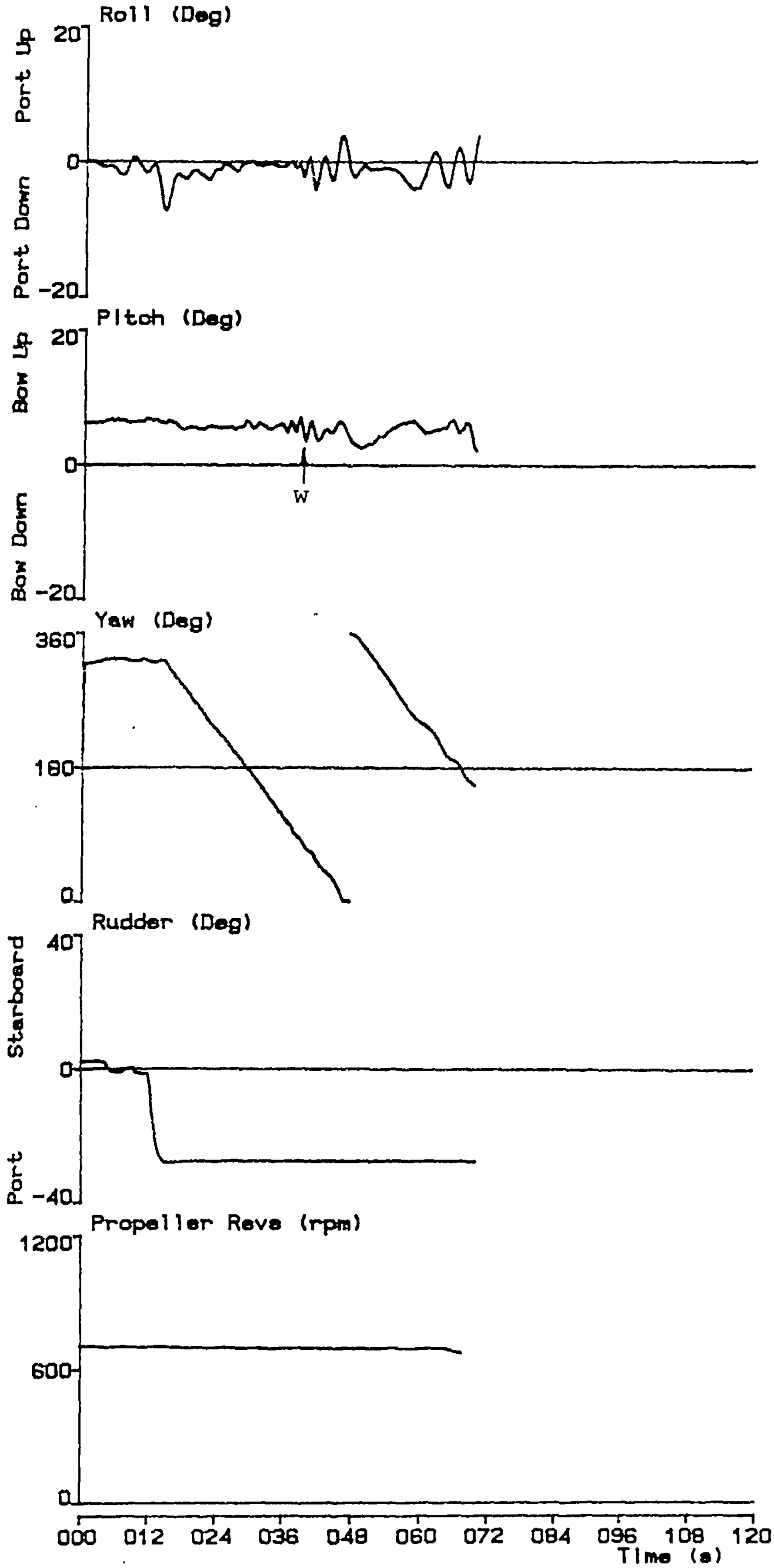
Simulation

TURNING CIRCLE 05 (700rpm)



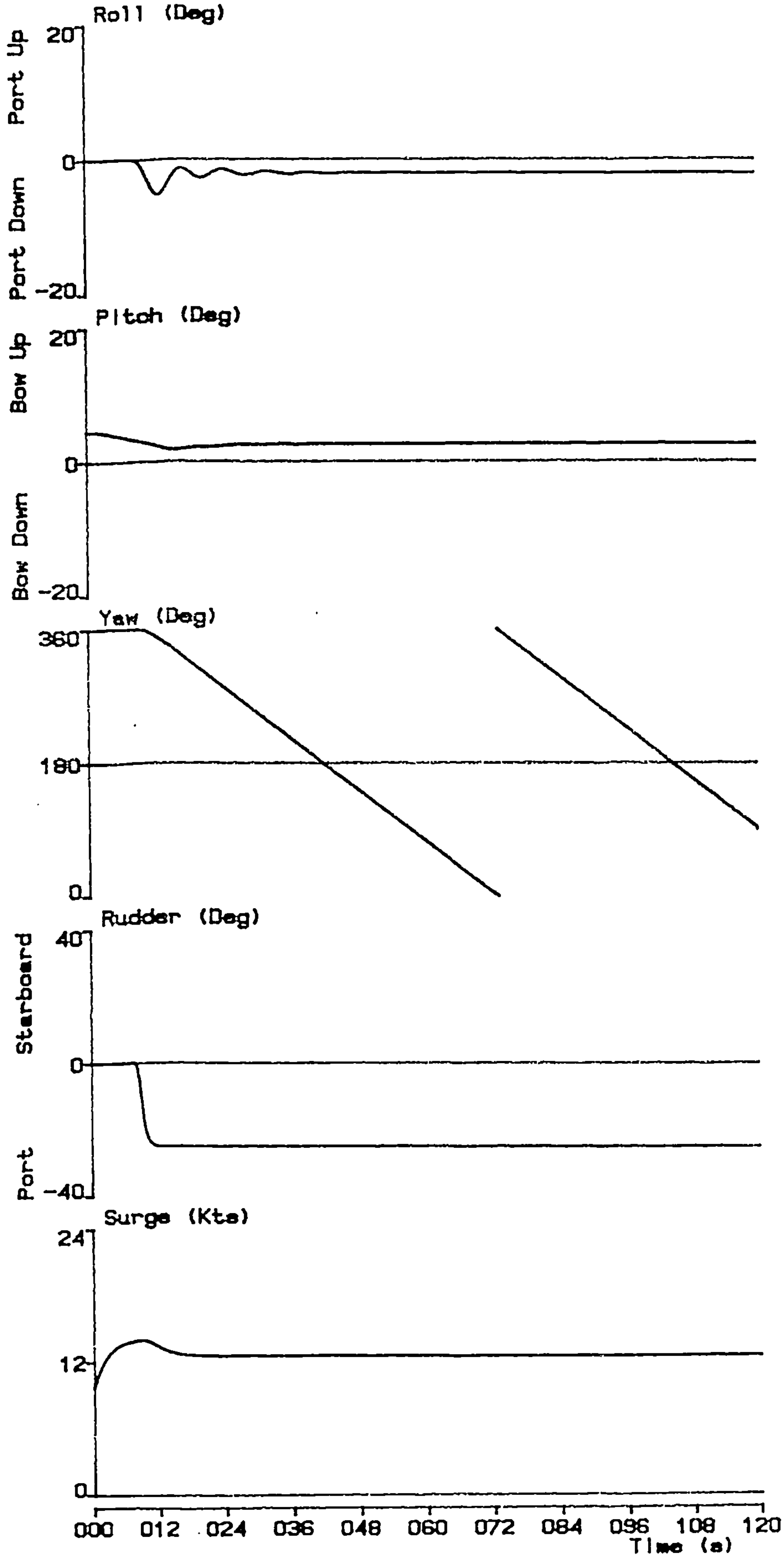
14:24:31 20/04/1989

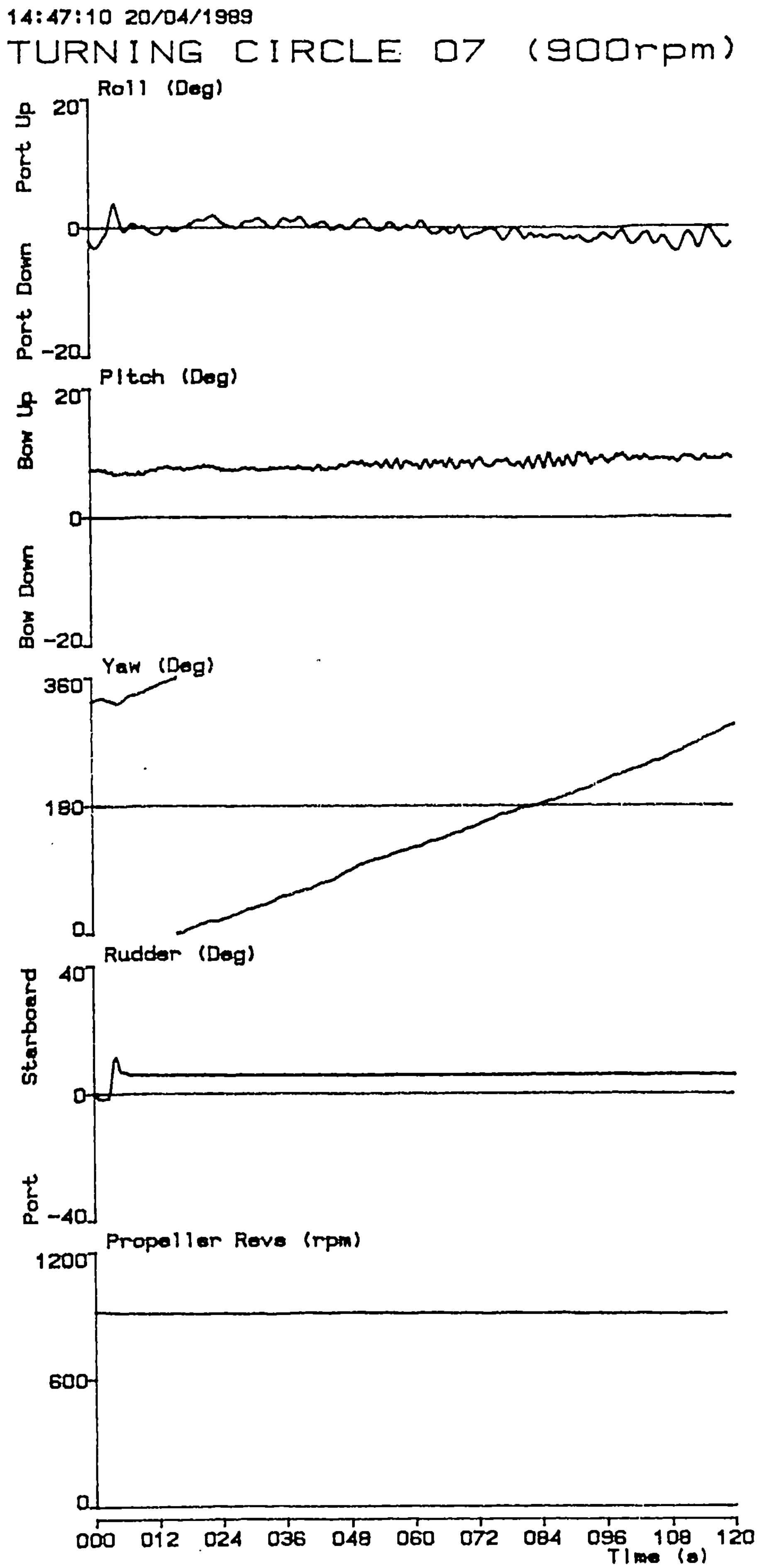
TURNING CIRCLE 06 (700rpm)



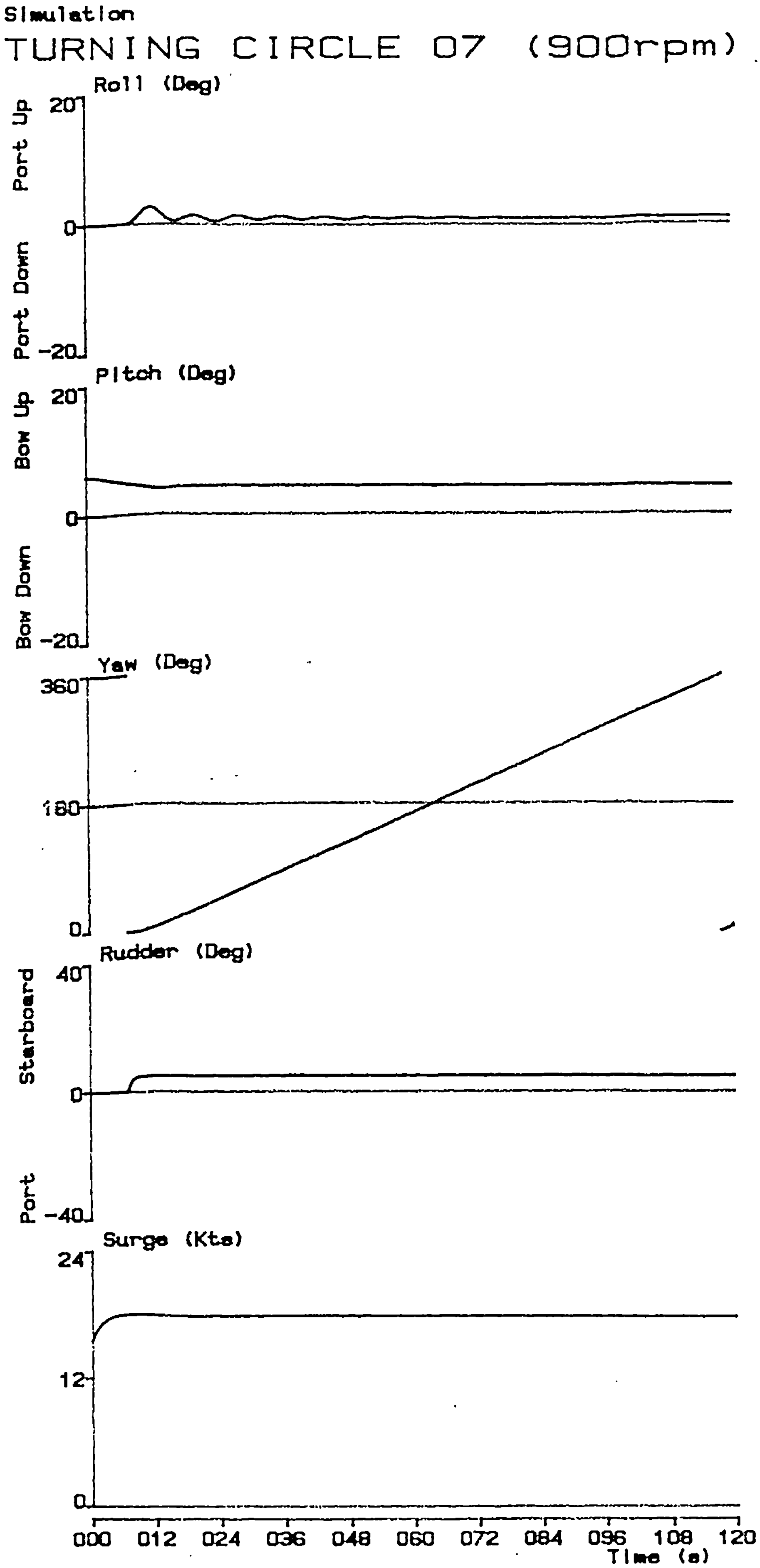
Simulation

TURNING CIRCLE 06 (700rpm)





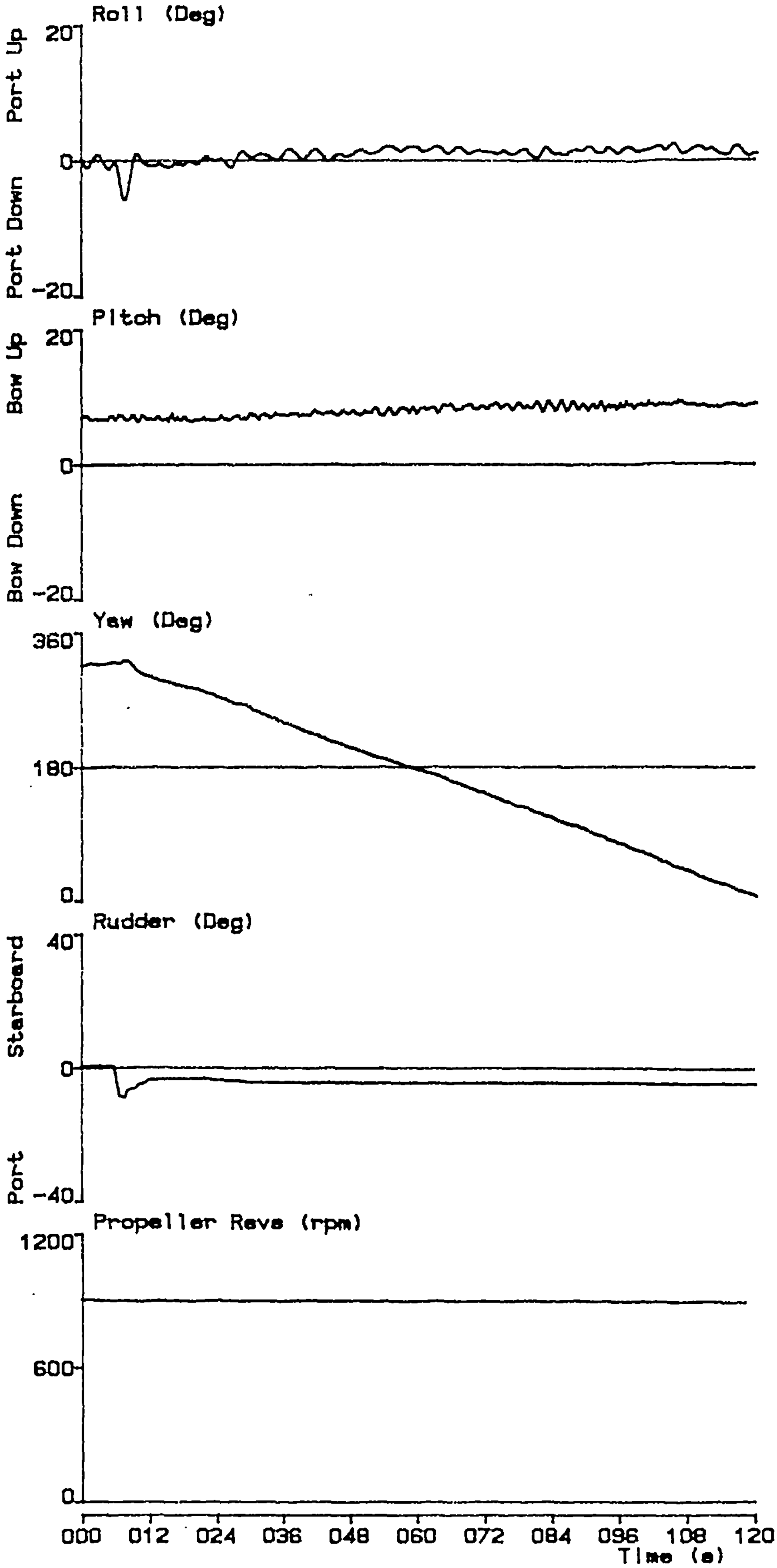
14:47:10 20/04/1989
TURNING CIRCLE 07 (900rpm)
Roll (Deg)
Port Up
Port Down
20
0
-20
Pitch (Deg)
Bow Up
Bow Down
20
0
-20
Yaw (Deg)
360
180
0
Rudder (Deg)
Starboard
Port
40
0
-40
Propeller Revs (rpm)
1200
600
0
000 012 024 036 048 060 072 084 096 108 120
Time (s)



Simulation
TURNING CIRCLE 07 (900rpm)
Roll (Deg)
Port Up
20
0
Port Down
-20
Pitch (Deg)
Bow Up
20
0
Bow Down
-20
Yaw (Deg)
360
180
0
Rudder (Deg)
Starboard
40
0
Port
-40
Surge (Kts)
24
12
0
000 012 024 036 048 060 072 084 096 108 120
Time (s)

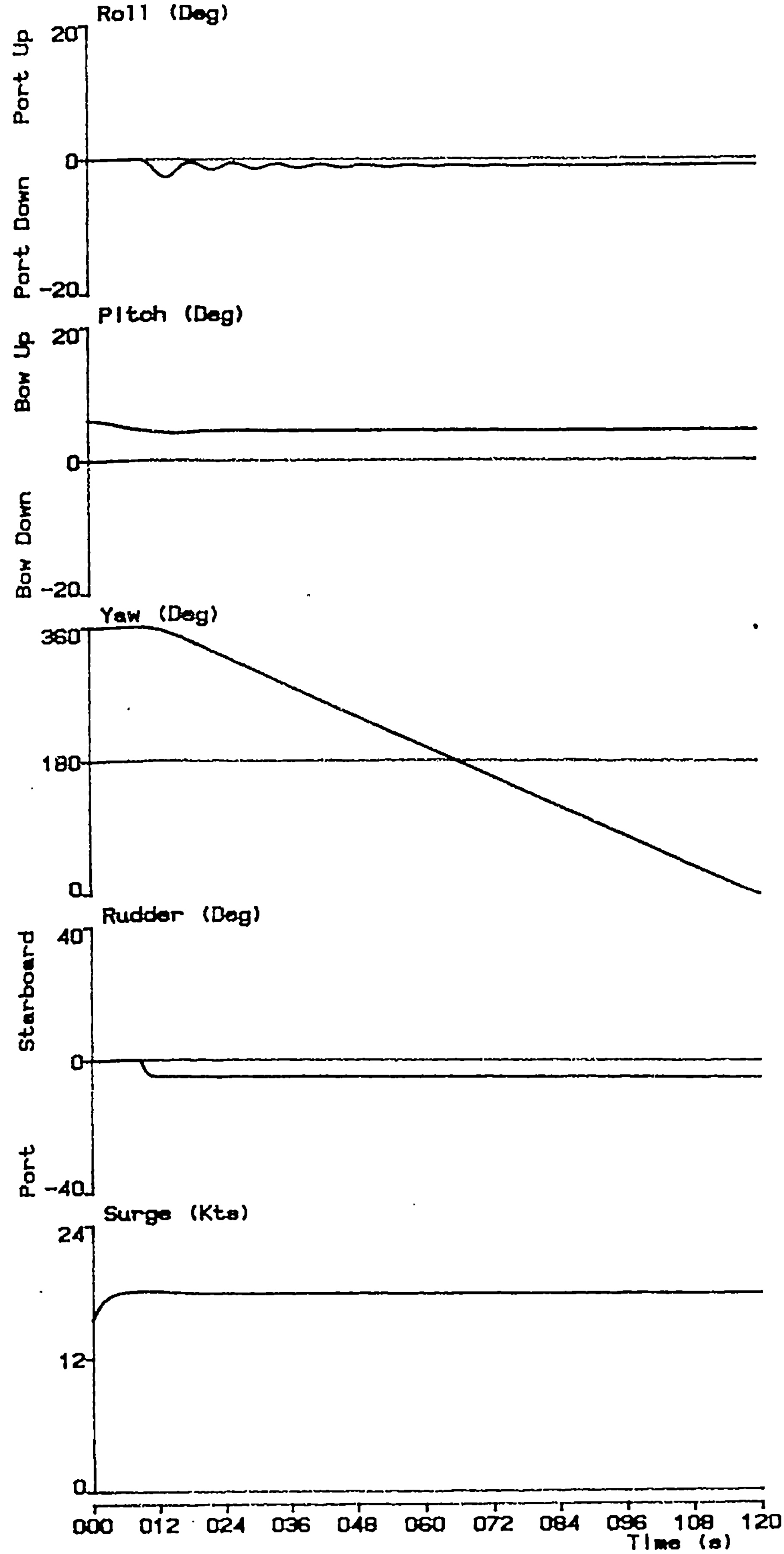
14:36:49 20/04/1989

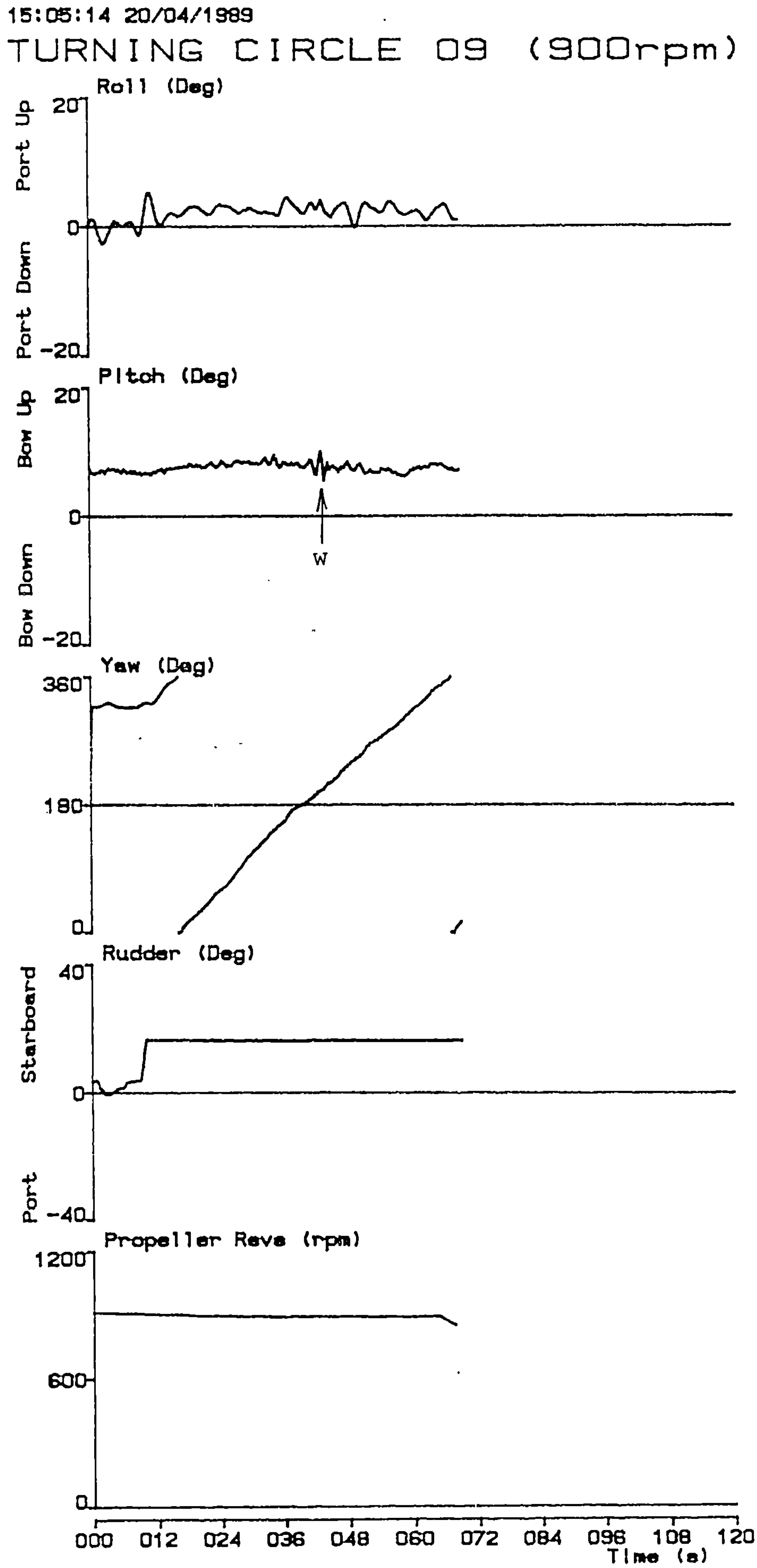
TURNING CIRCLE 08 (900rpm)

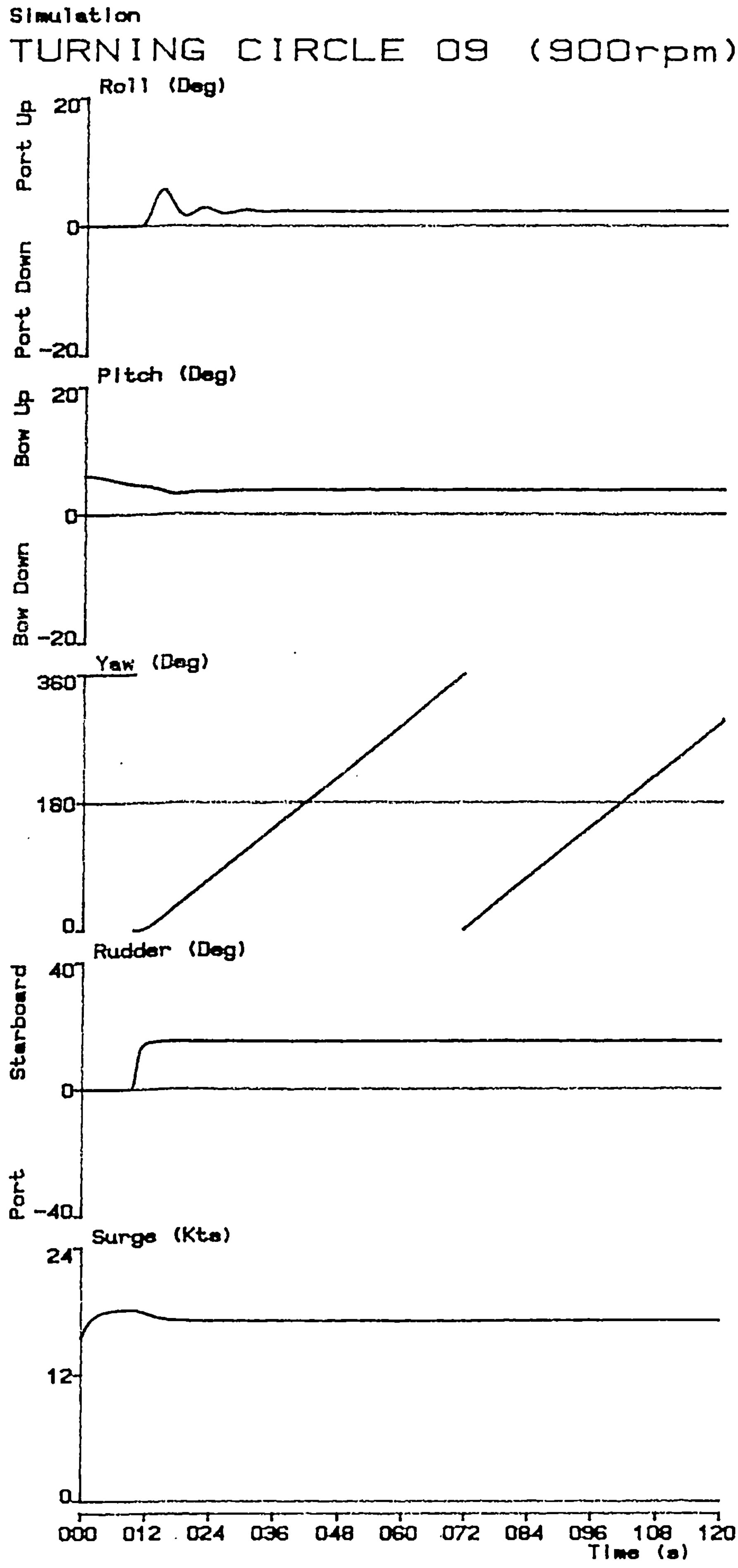


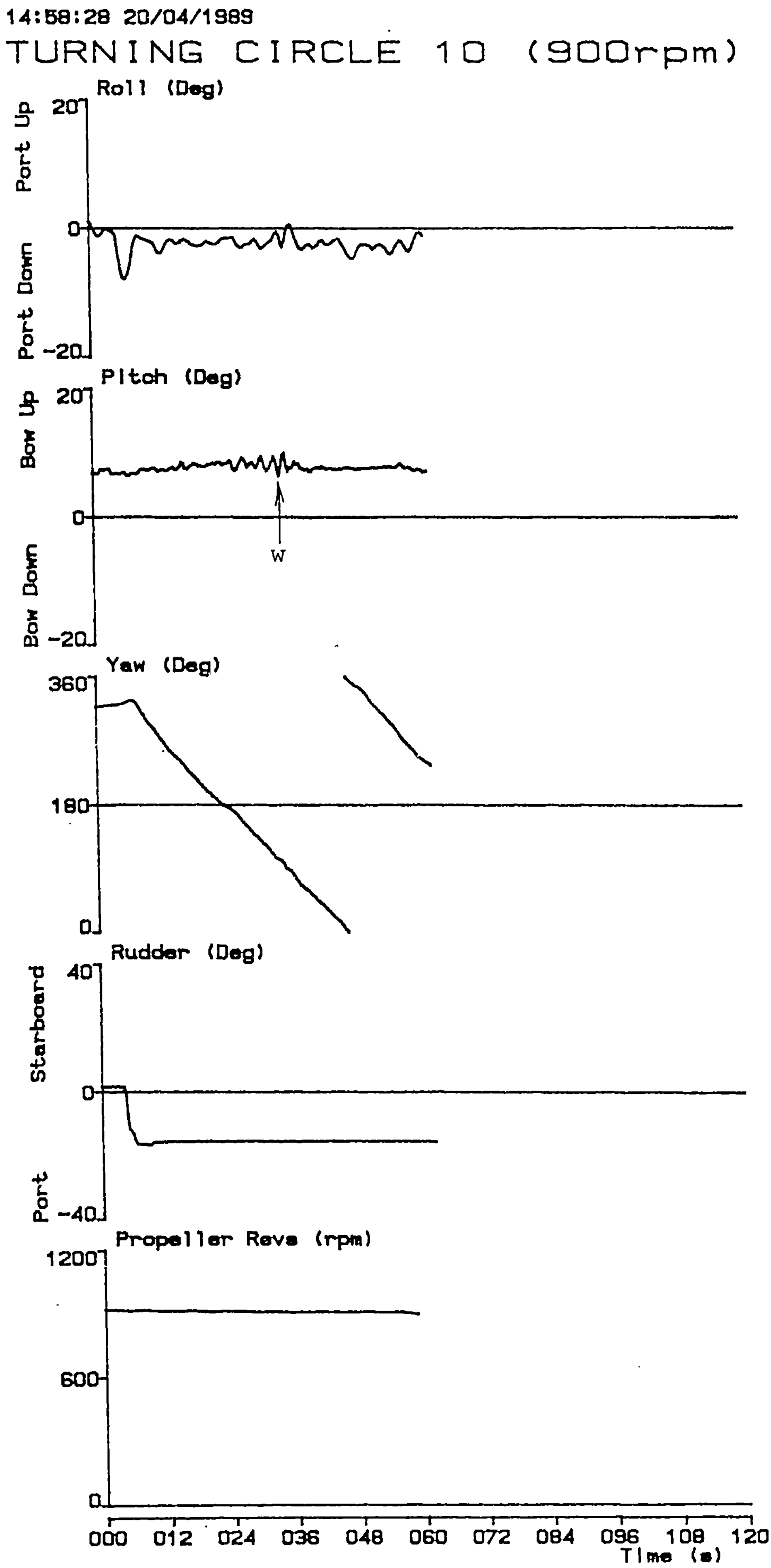
Simulation

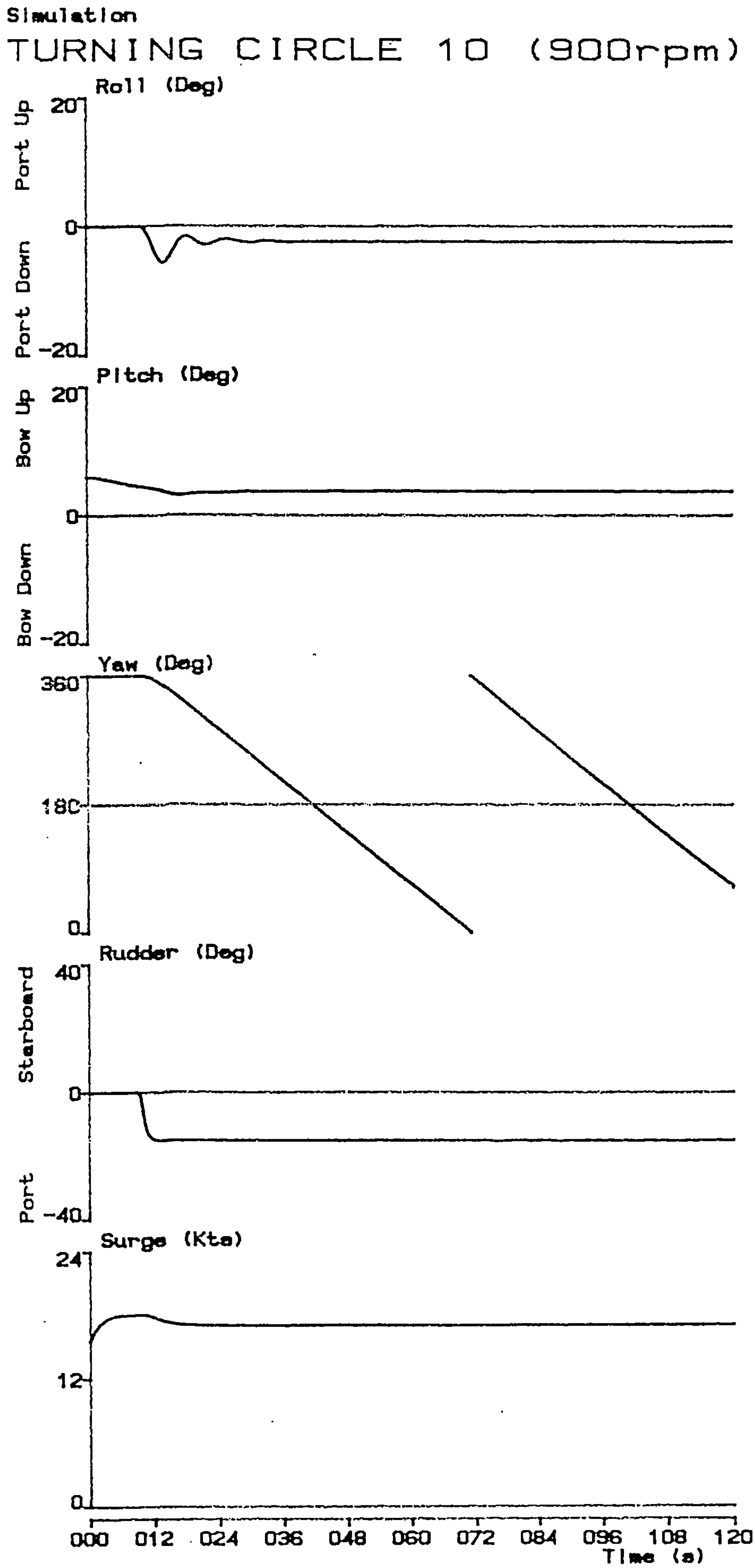
TURNING CIRCLE 08 (900rpm)



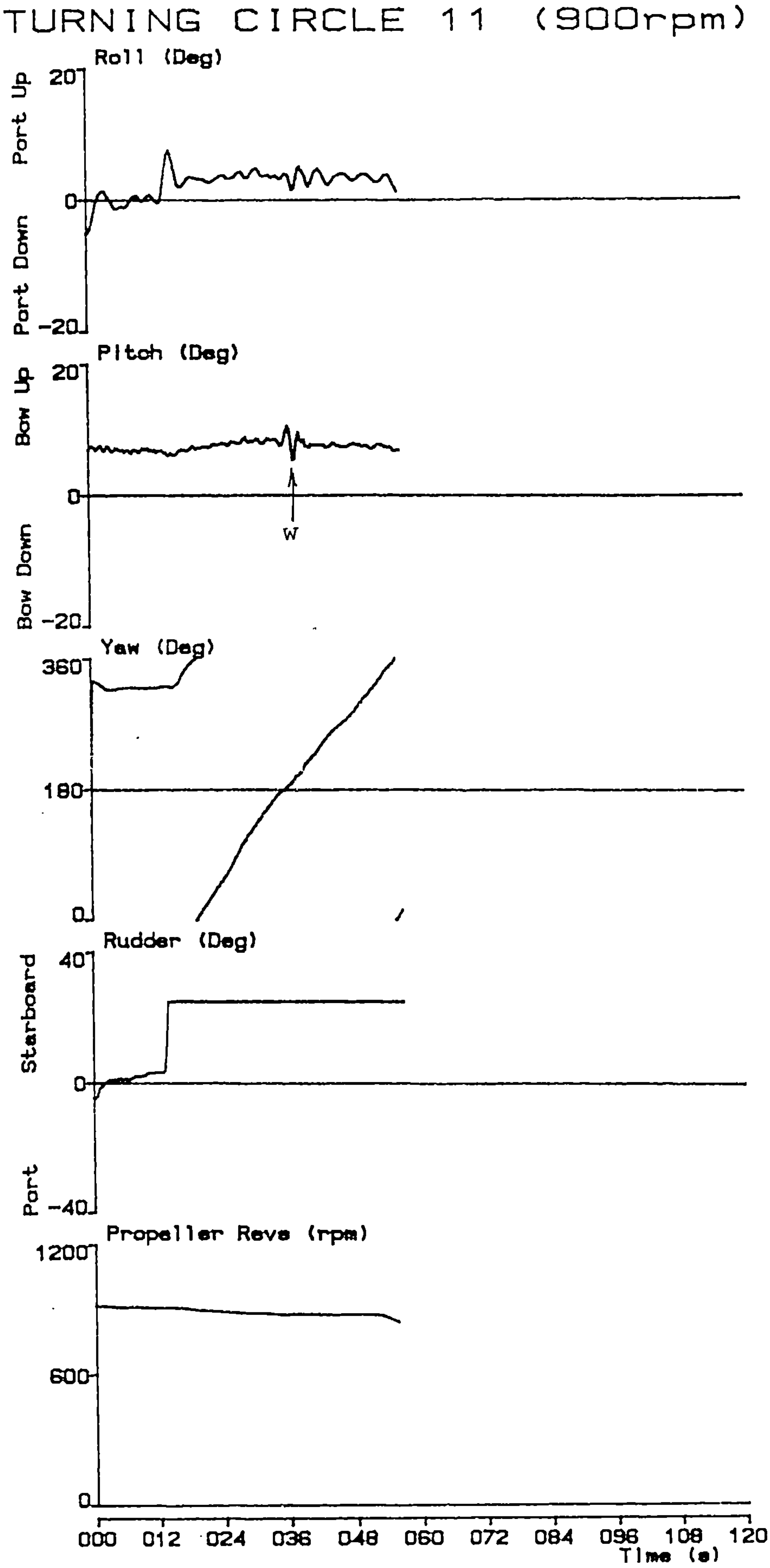

15:05:14 20/04/1989
TURNING CIRCLE 09 (900rpm)
Roll (Deg)
Port Up
Port Down
20
0
-20
Pitch (Deg)
Bow Up
Bow Down
20
0
-20
W
Yaw (Deg)
360
180
0
Rudder (Deg)
Starboard
Port
40
0
-40
Propeller Revs (rpm)
1200
600
0
000
012
024
036
048
060
072
084
096
108
120
Time (s)


Simulation
TURNING CIRCLE 09 (900rpm)
Roll (Deg)
Port Up
Port Down
20
0
-20
Pitch (Deg)
Bow Up
Bow Down
20
0
-20
Yaw (Deg)
360
180
0
Rudder (Deg)
Starboard
Port
40
0
-40
Surge (Kts)
24
12
0
000 012 024 036 048 060 072 084 096 108 120
Time (s)


14:58:28 20/04/1989
TURNING CIRCLE 10 (900rpm)
Roll (Deg)
Port Up
Port Down
20
0
-20
Pitch (Deg)
Bow Up
Bow Down
20
0
-20
W
Yaw (Deg)
360
180
0
Rudder (Deg)
Starboard
Port
40
0
-40
Propeller Revs (rpm)
1200
600
0
000 012 024 036 048 060 072 084 096 108 120
Time (s)

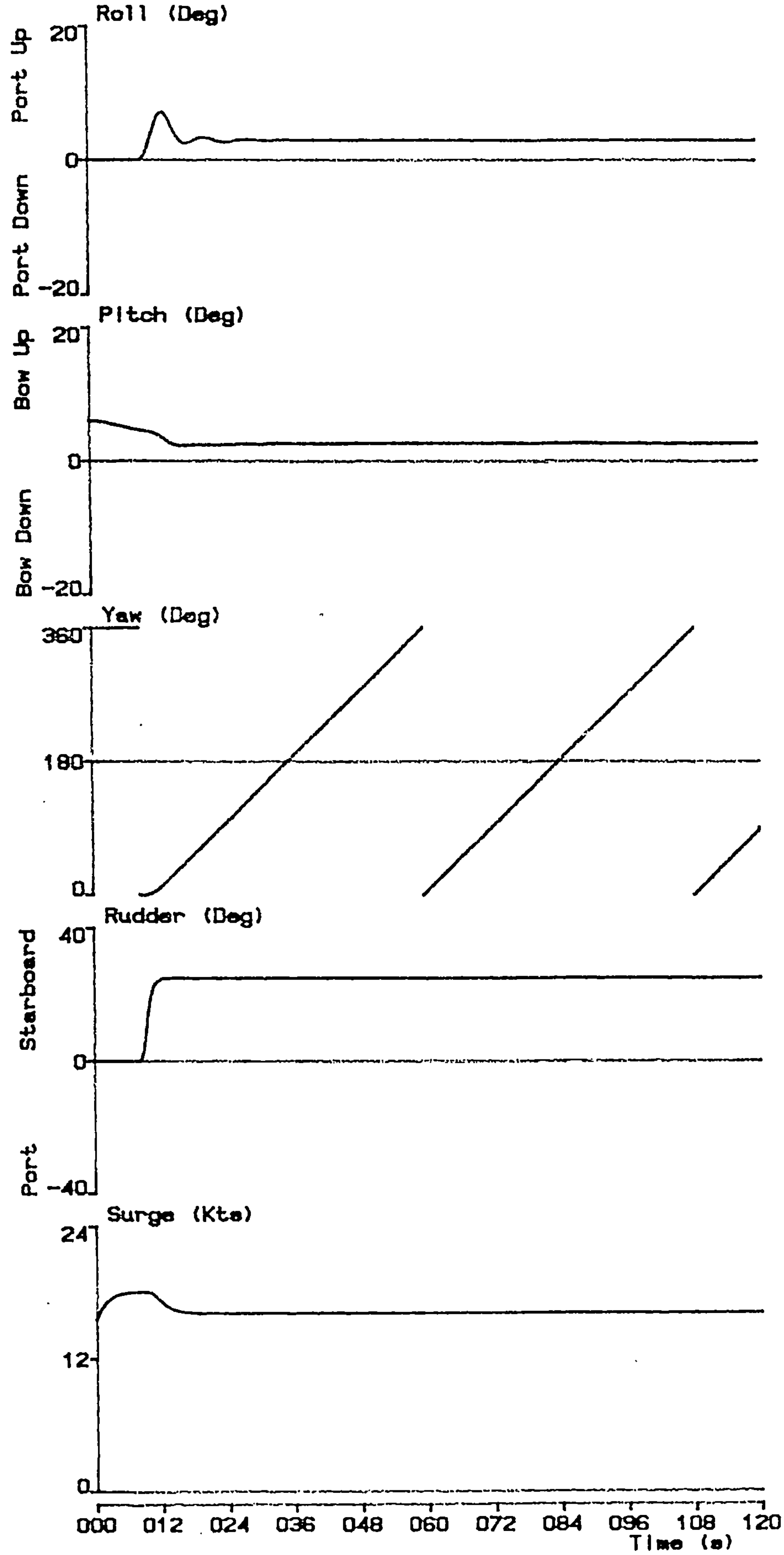

Simulation
TURNING CIRCLE 10 (900rpm)
Roll (Deg)
Port Up
Port Down
20
0
-20
Pitch (Deg)
Bow Up
Bow Down
20
0
-20
Yaw (Deg)
360
180
0
Rudder (Deg)
Starboard
Port
40
0
-40
Surge (Kts)
24
12
0
000 012 024 036 048 060 072 084 096 108 120
Time (s)

15:19:21 20/04/1989



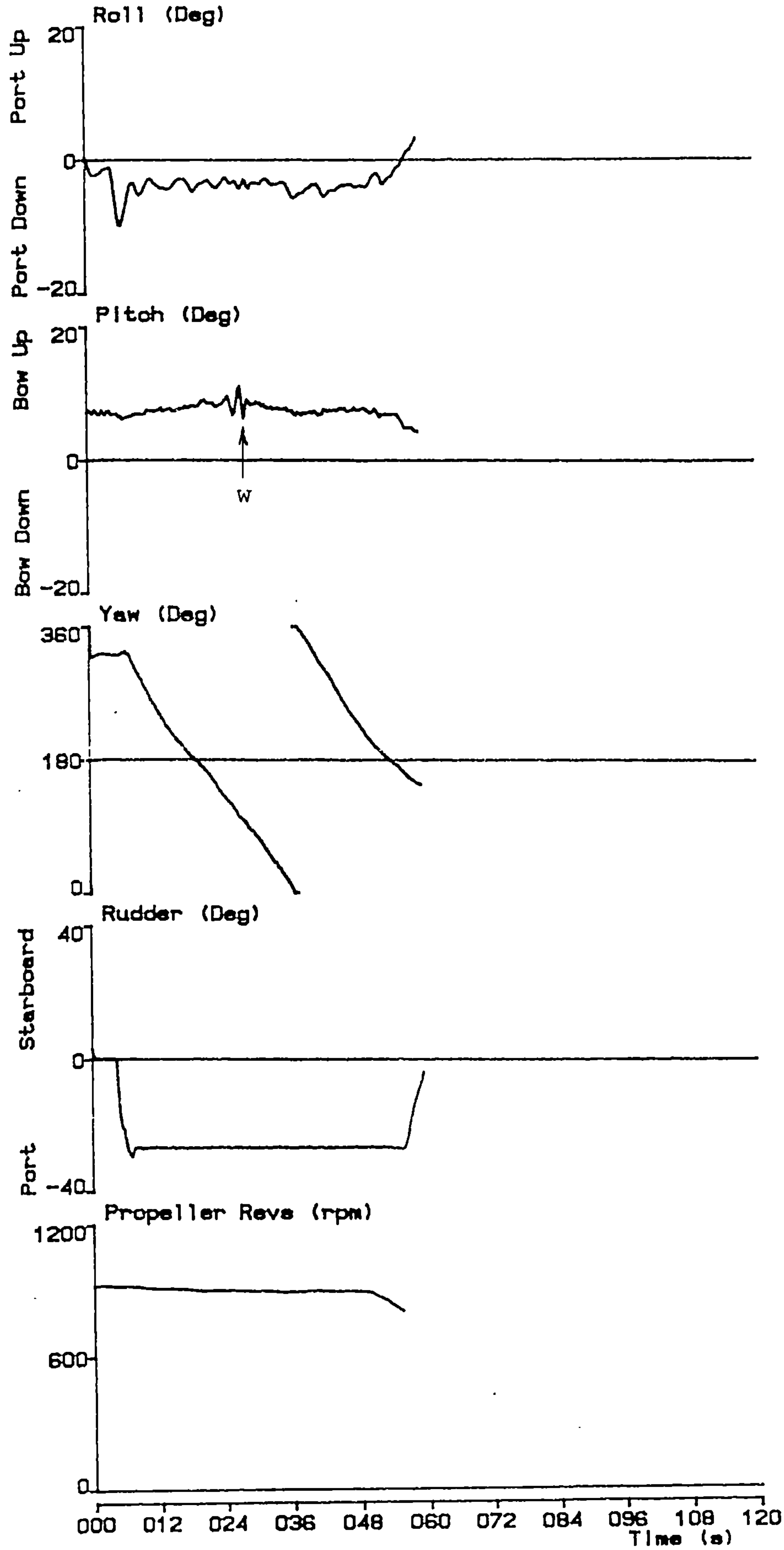
Simulation

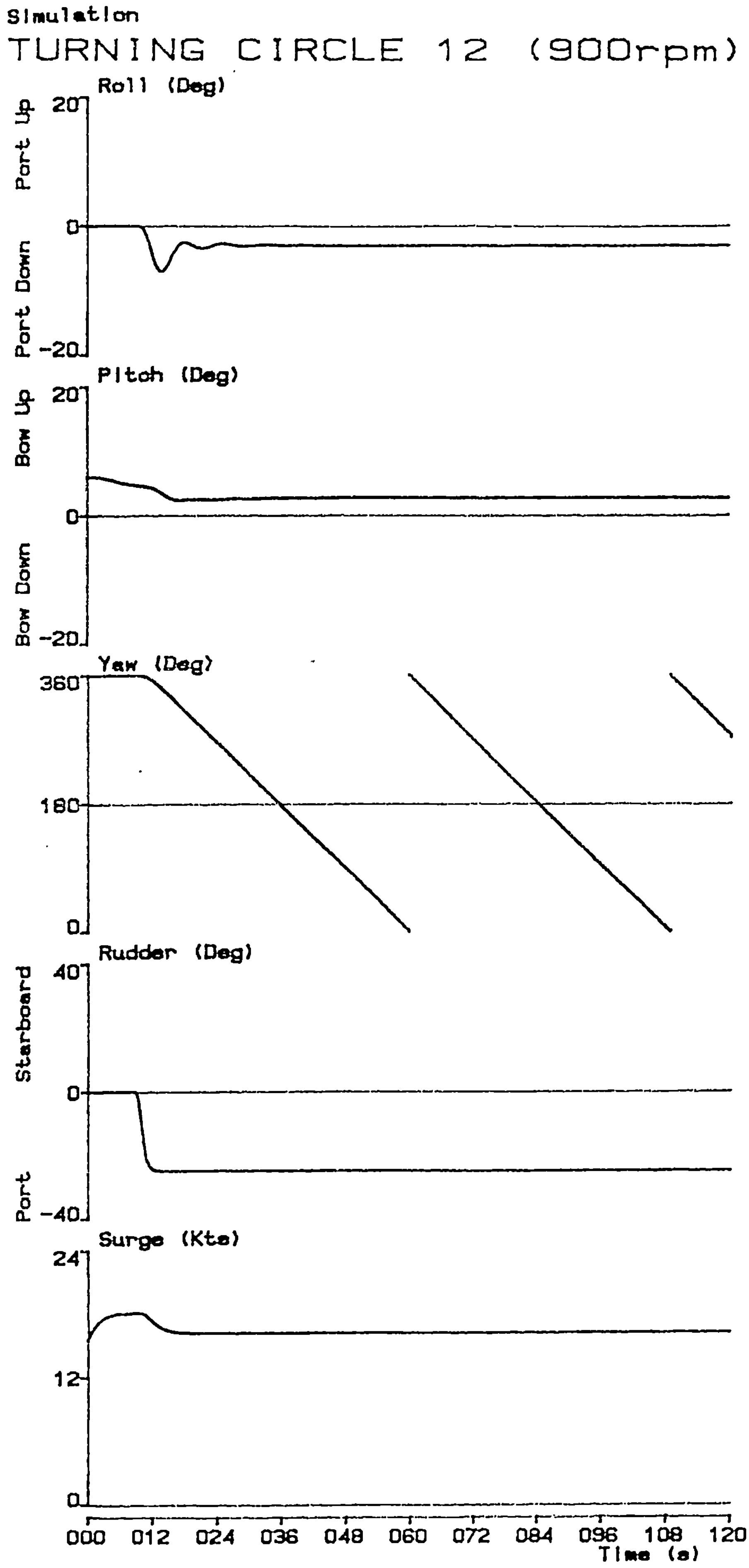
TURNING CIRCLE 11 (900rpm)



15:12:22 20/04/1989

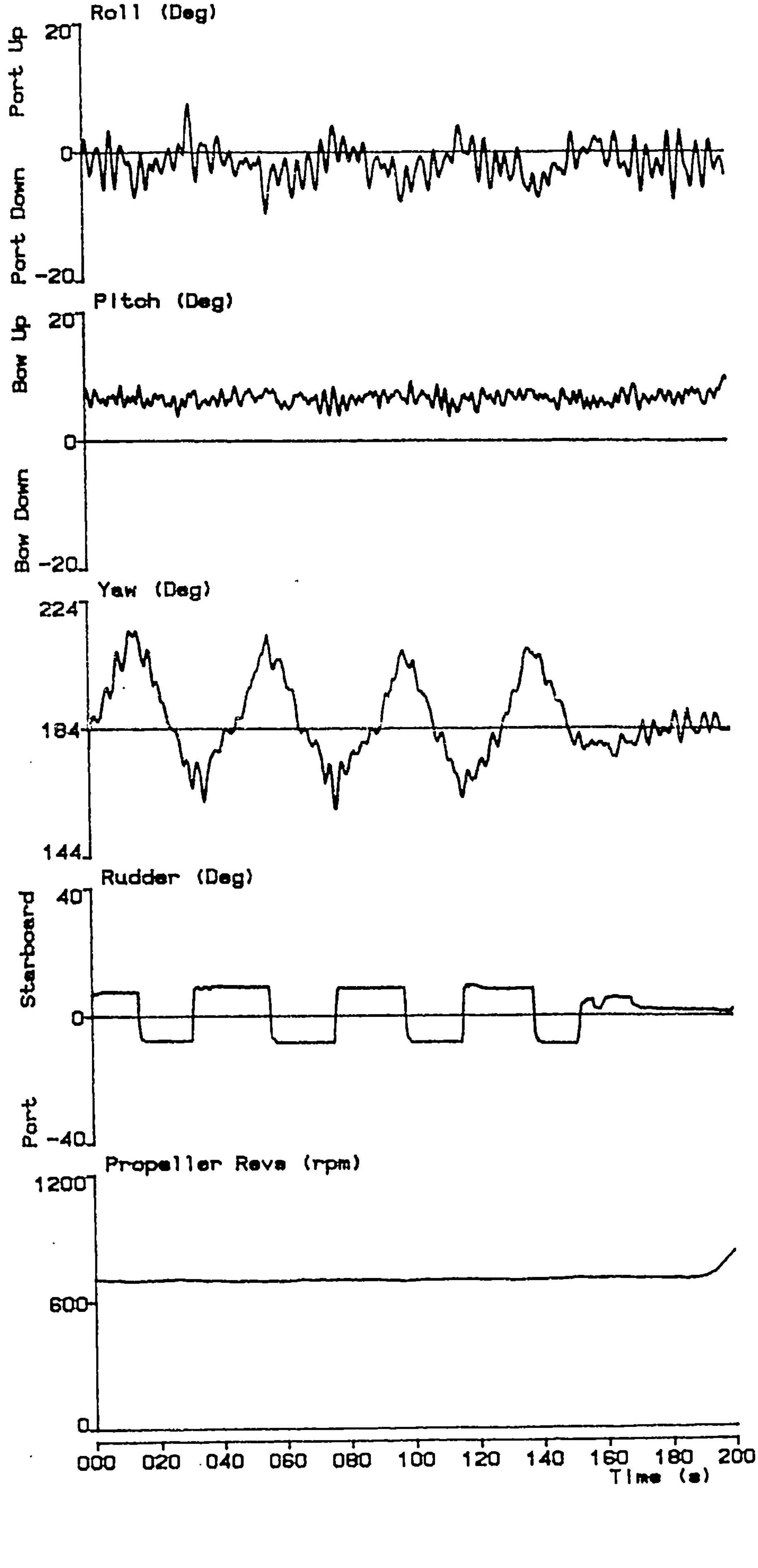
TURNING CIRCLE 12 (900rpm)

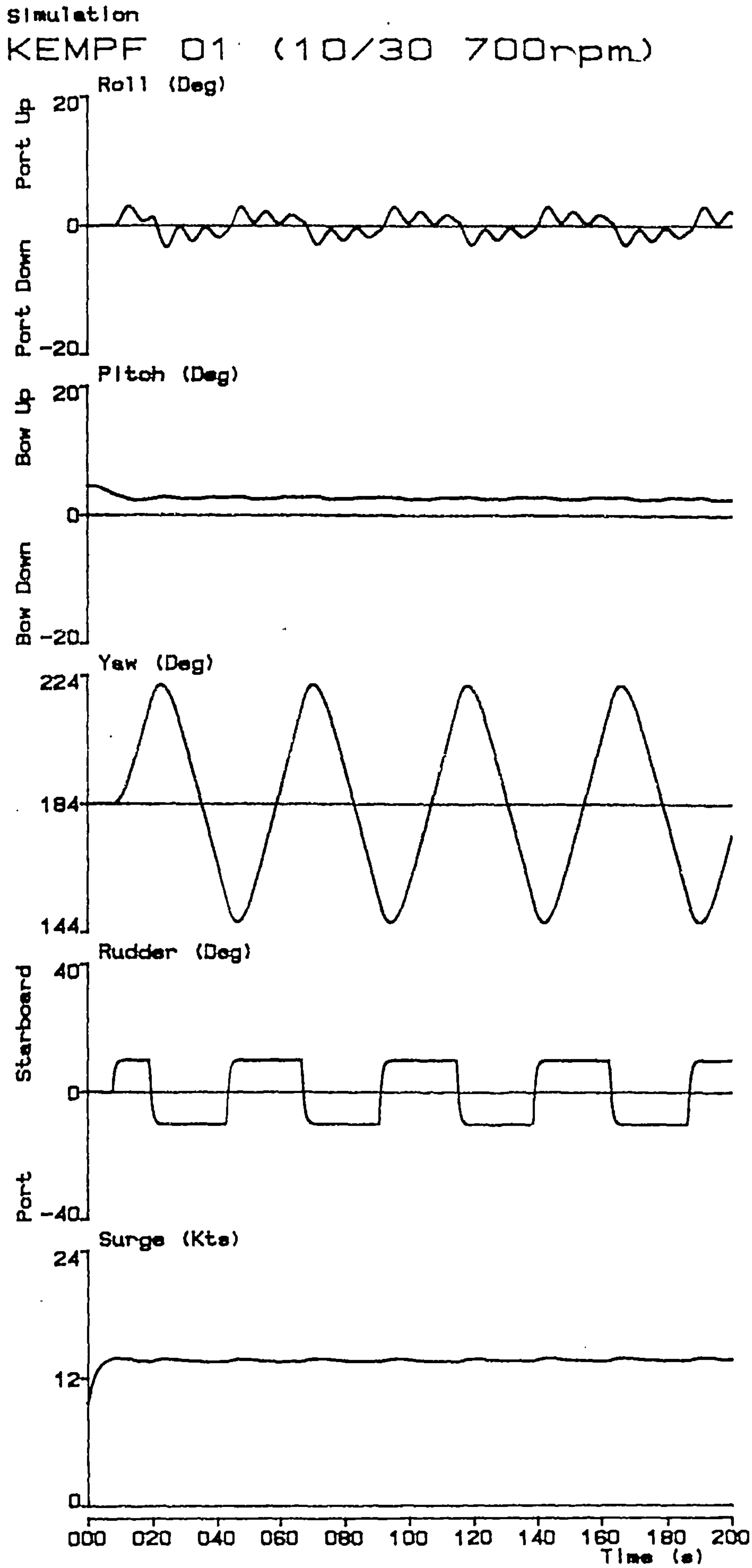



Simulation
TURNING CIRCLE 12 (900rpm)
Roll (Deg)
Port Up
Port Down
20
0
-20
Pitch (Deg)
Bow Up
Bow Down
20
0
-20
Yaw (Deg)
360
180
0
Rudder (Deg)
Starboard
Port
40
0
-40
Surge (Kts)
24
12
0
000 012 024 036 048 060 072 084 096 108 120
Time (s)

11:12:06 17/04/1989

KEMPF 01 (10/30 700rpm)

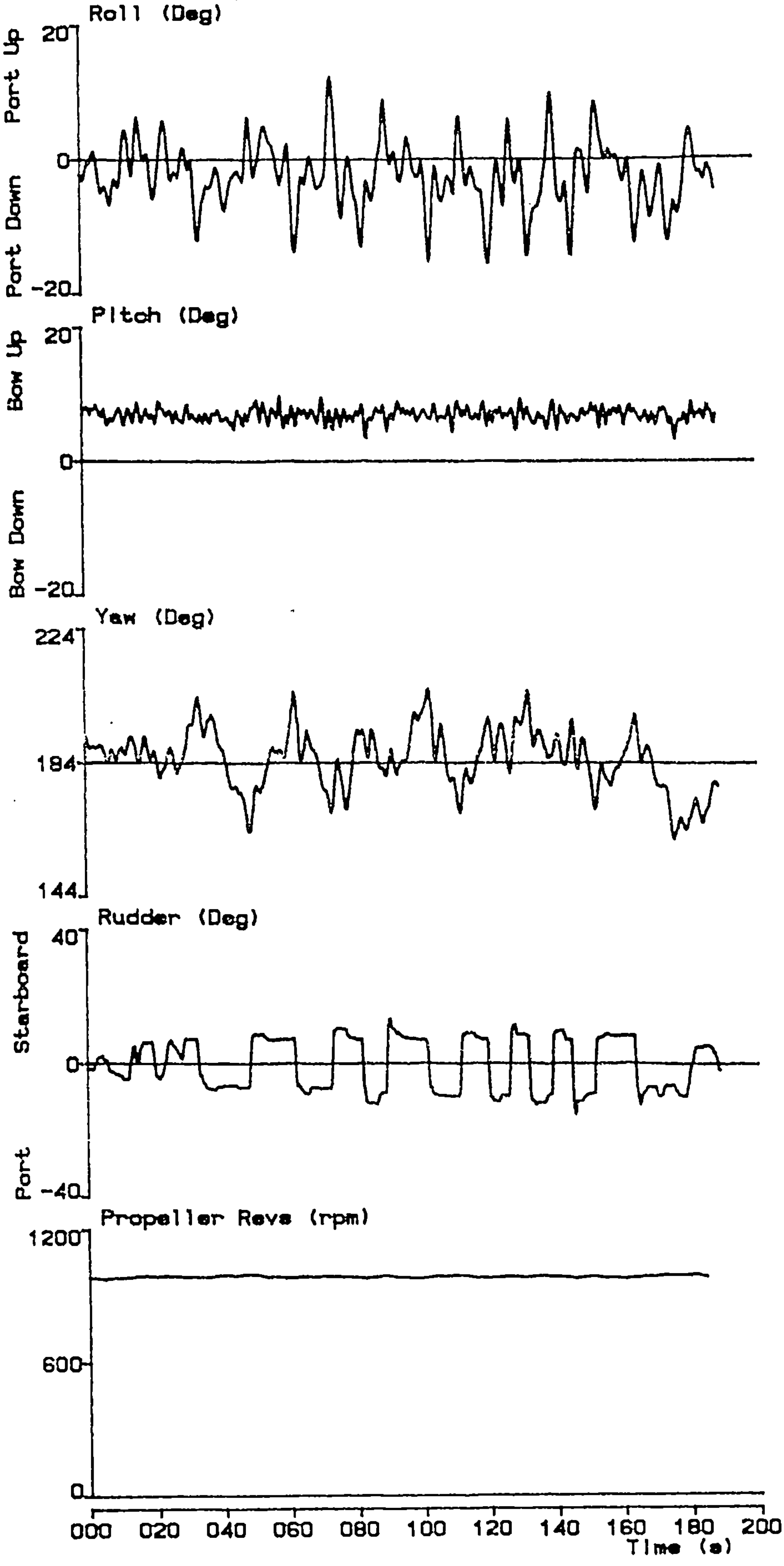


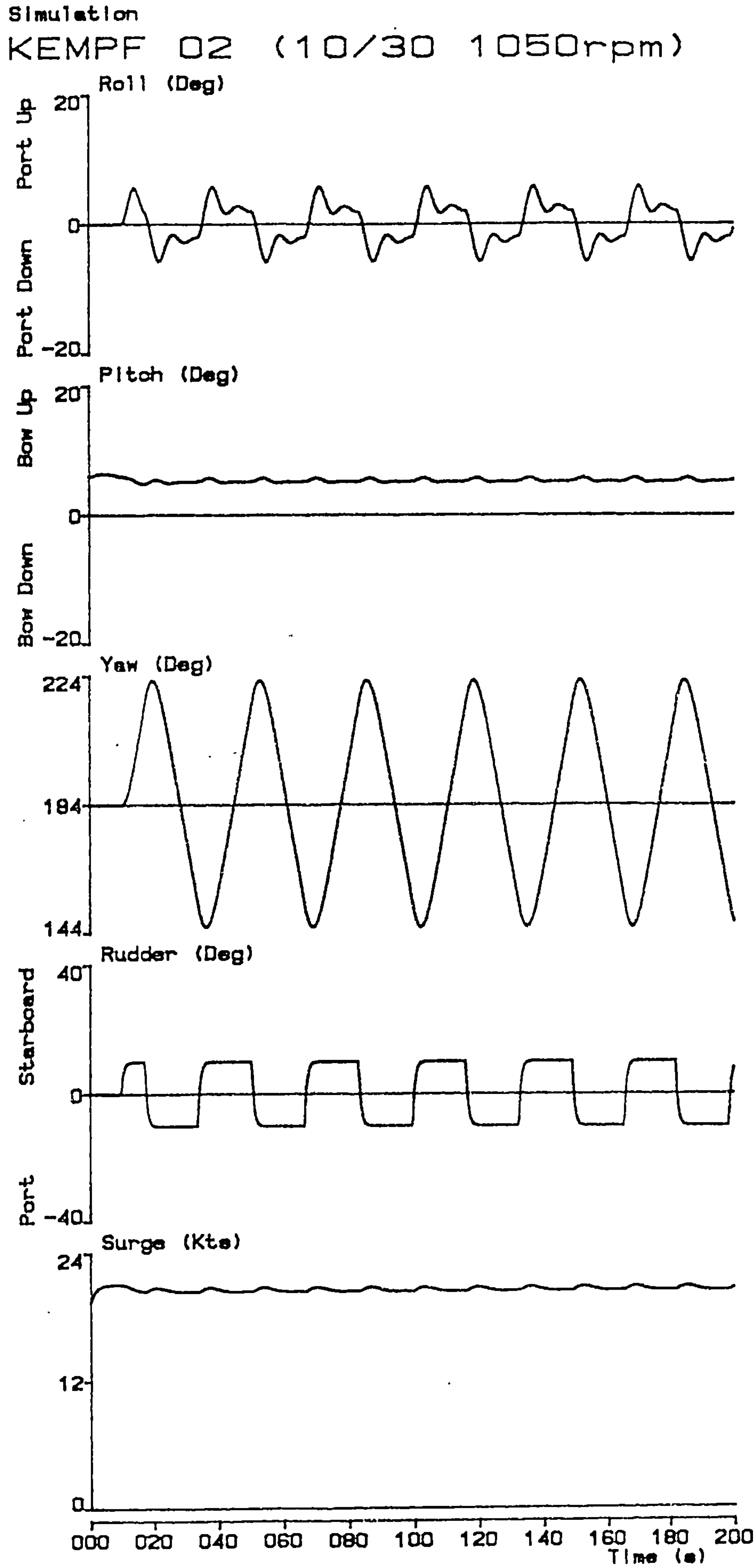


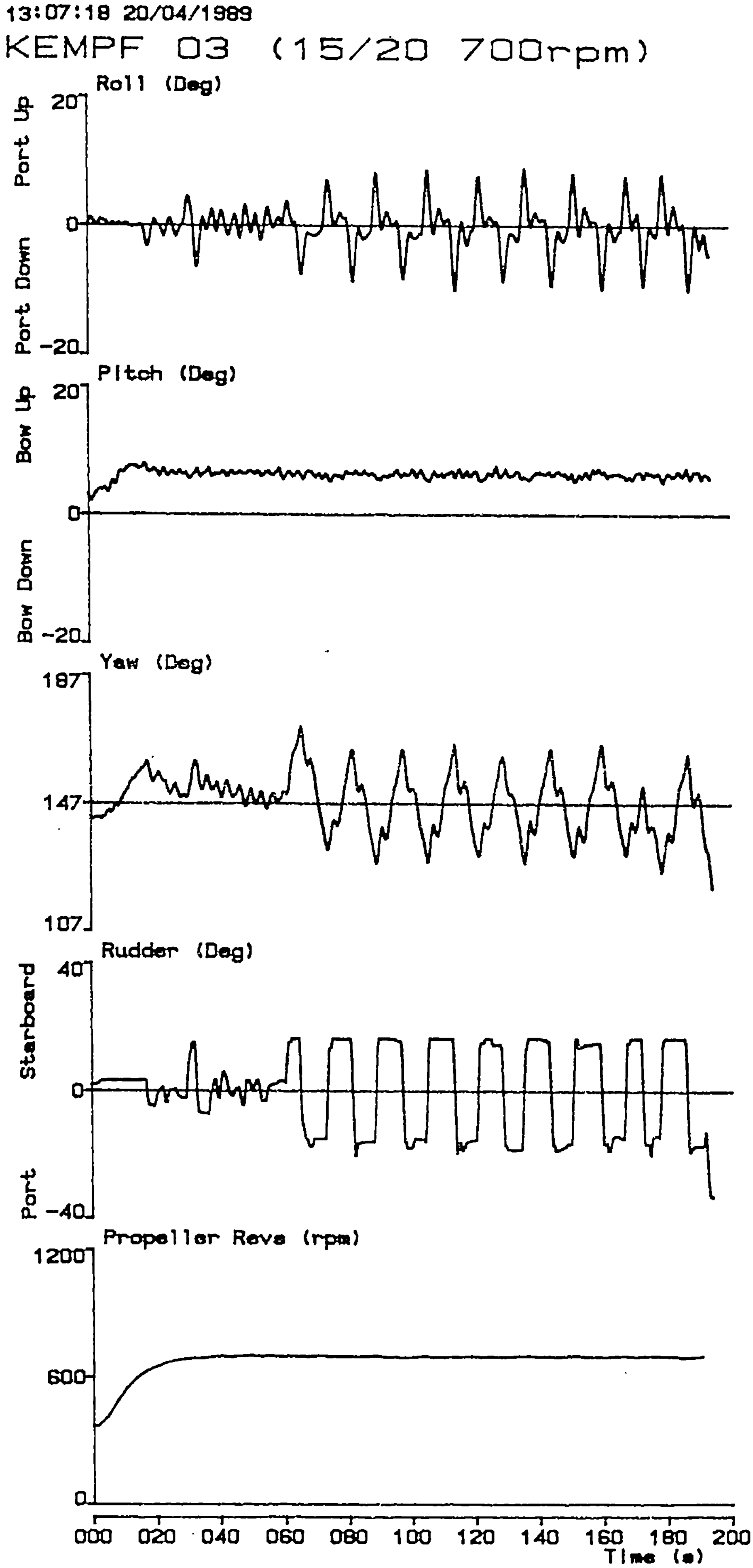
Simulation
KEMPF 01 (10/30 700rpm)
Roll (Deg)
Port Up
Port Down
20
0
-20
Pitch (Deg)
Bow Up
Bow Down
20
0
-20
Yaw (Deg)
224
184
144
Rudder (Deg)
Starboard
Port
40
0
-40
Surge (Kts)
24
12
0
000
020
040
060
080
100
120
140
160
180
200
Time (s)

11:23:03 17/04/1989

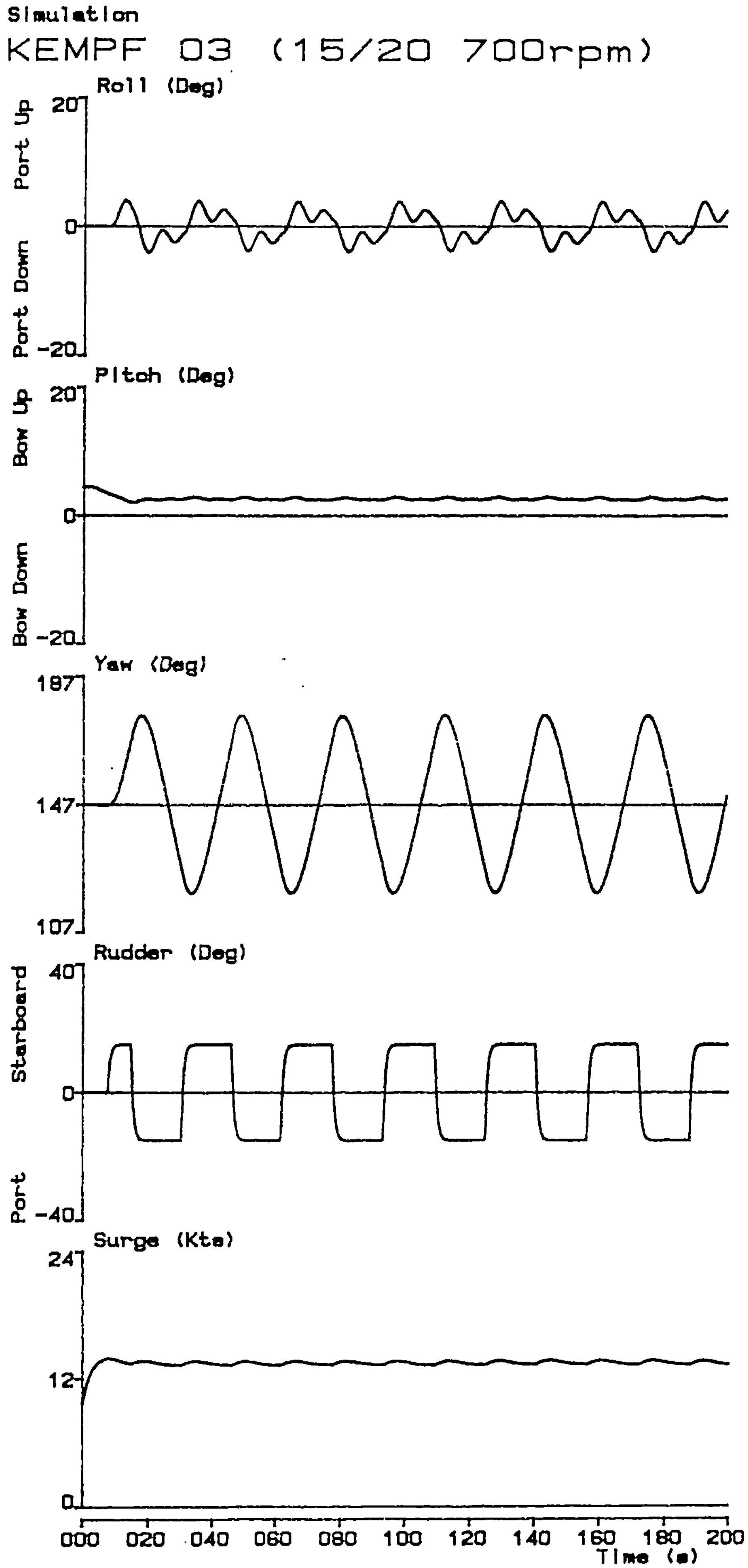
KEMPF 02 (10/30 1050rpm)

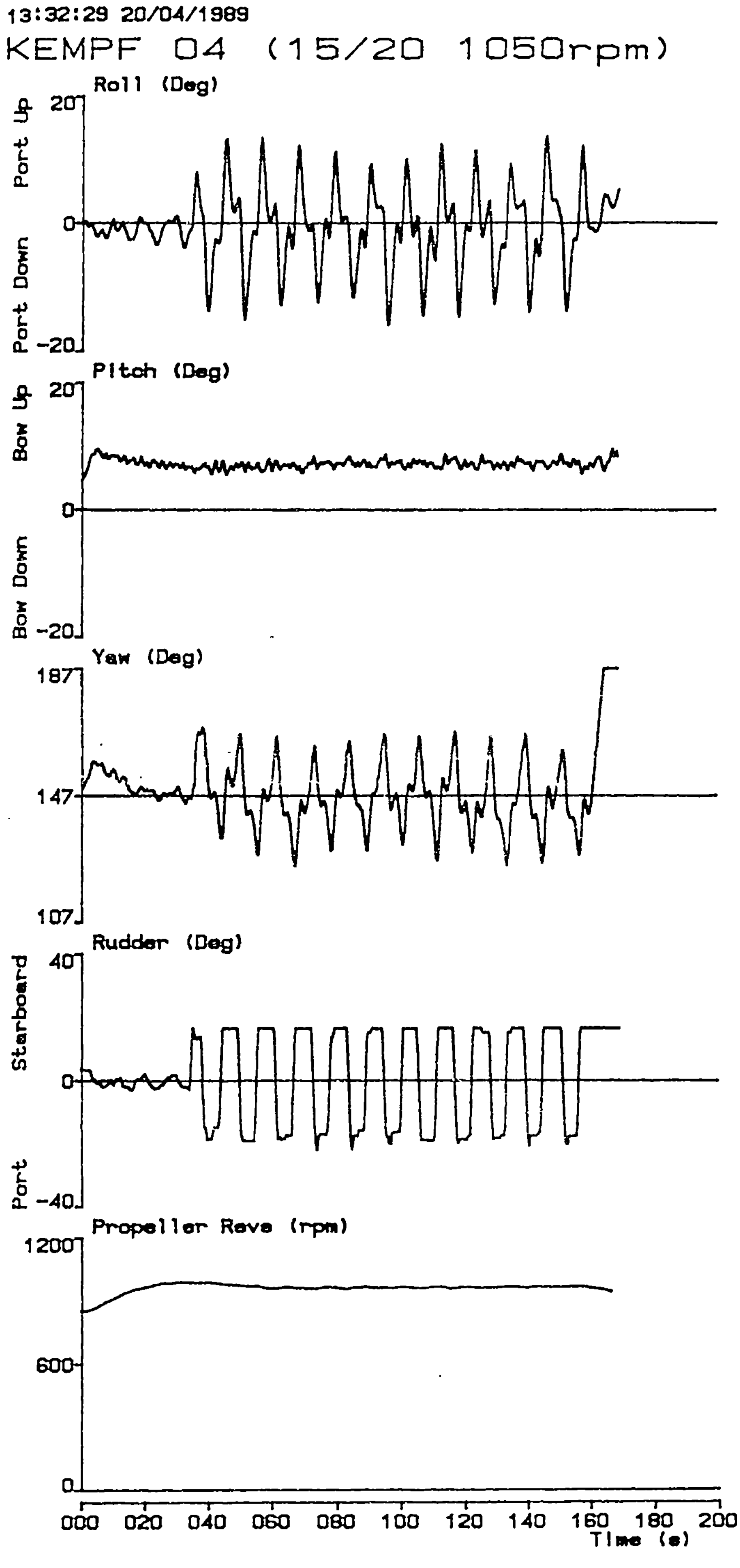



Simulation
KEMPF 02 (10/30 1050rpm)
Roll (Deg)
Port Up
Port Down
20
0
-20
Pitch (Deg)
Bow Up
Bow Down
20
0
-20
Yaw (Deg)
224
184
144
Rudder (Deg)
Starboard
Port
40
0
-40
Surge (Kts)
24
12
0
000 020 040 060 080 100 120 140 160 180 200
Time (s)



13:07:18 20/04/1989
KEMPF 03 (15/20 700rpm)
Roll (Deg)
Port Up
Port Down
20
0
-20
Pitch (Deg)
Bow Up
Bow Down
20
0
-20
Yaw (Deg)
187
147
107
Rudder (Deg)
Starboard
Port
40
0
-40
Propeller Revs (rpm)
1200
600
0
000 020 040 060 080 100 120 140 160 180 200
Time (s)

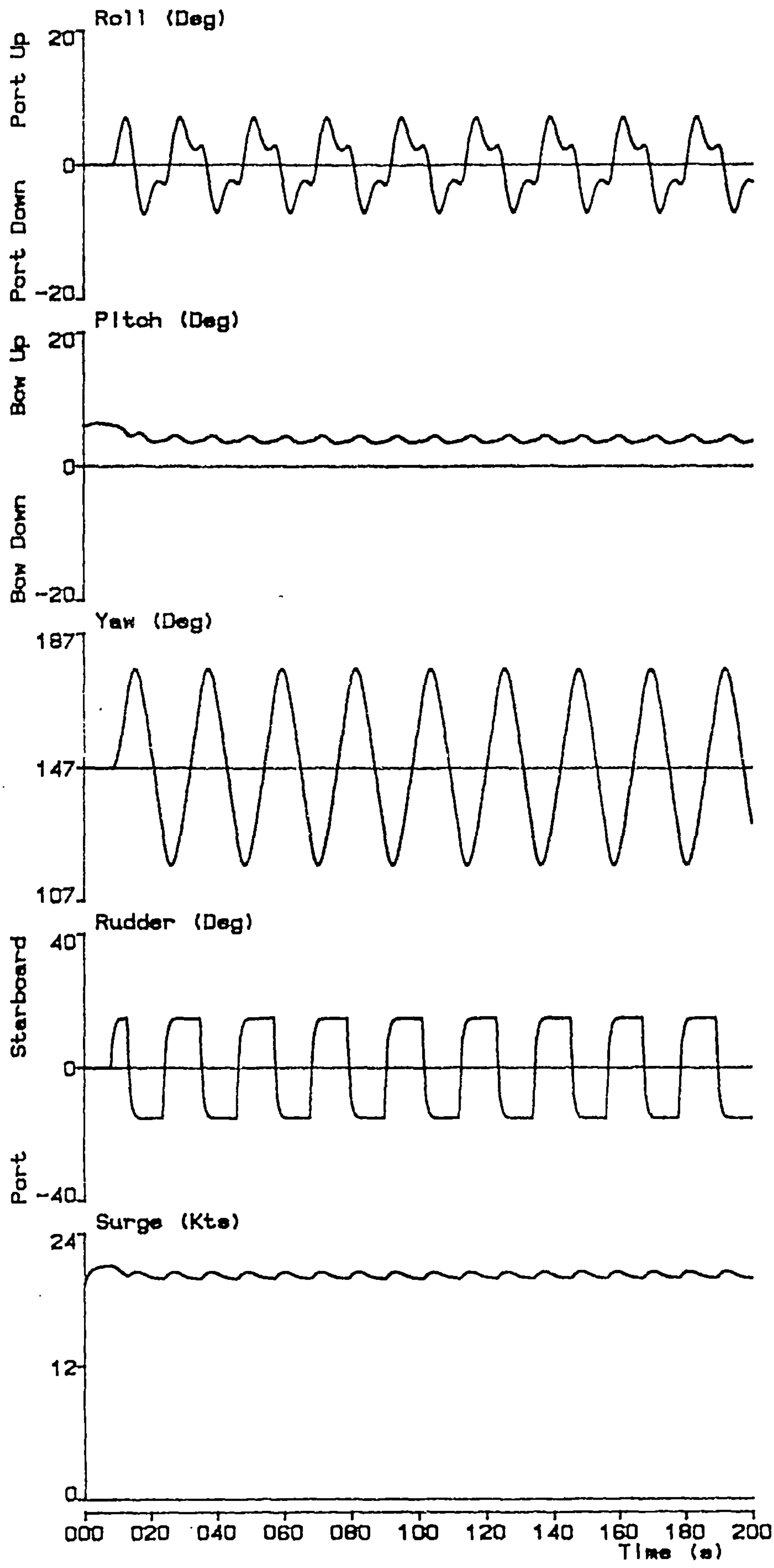

Simulation
KEMPF 03 (15/20 700rpm)
Roll (Deg)
20
Port Up
0
Port Down
-20
Pitch (Deg)
20
Bow Up
0
Bow Down
-20
Yaw (Deg)
187
147
107
Rudder (Deg)
40
Starboard
0
Port
-40
Surge (Kts)
24
12
0
000 020 040 060 080 100 120 140 160 180 200
Time (s)



13:32:29 20/04/1989
KEMPF 04 (15/20 1050rpm)
Roll (Deg)
Port Up
Port Down
20
0
-20
Pitch (Deg)
Bow Up
Bow Down
20
0
-20
Yaw (Deg)
187
147
107
Rudder (Deg)
Starboard
Port
40
0
-40
Propeller Revs (rpm)
1200
600
0
000 020 040 060 080 100 120 140 160 180 200
Time (s)

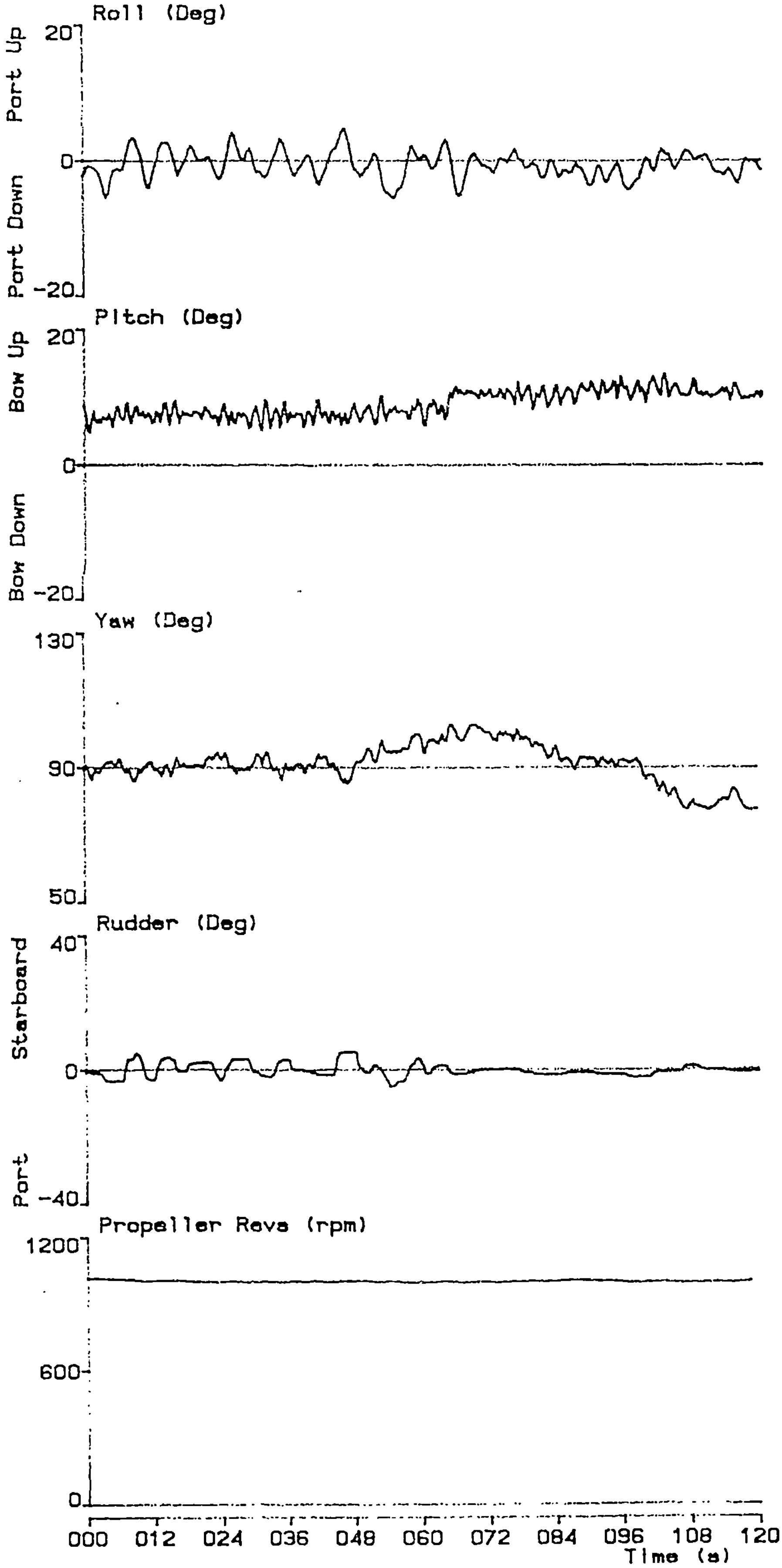
Simulation

KEMPF 04 (15/20 1050rpm)



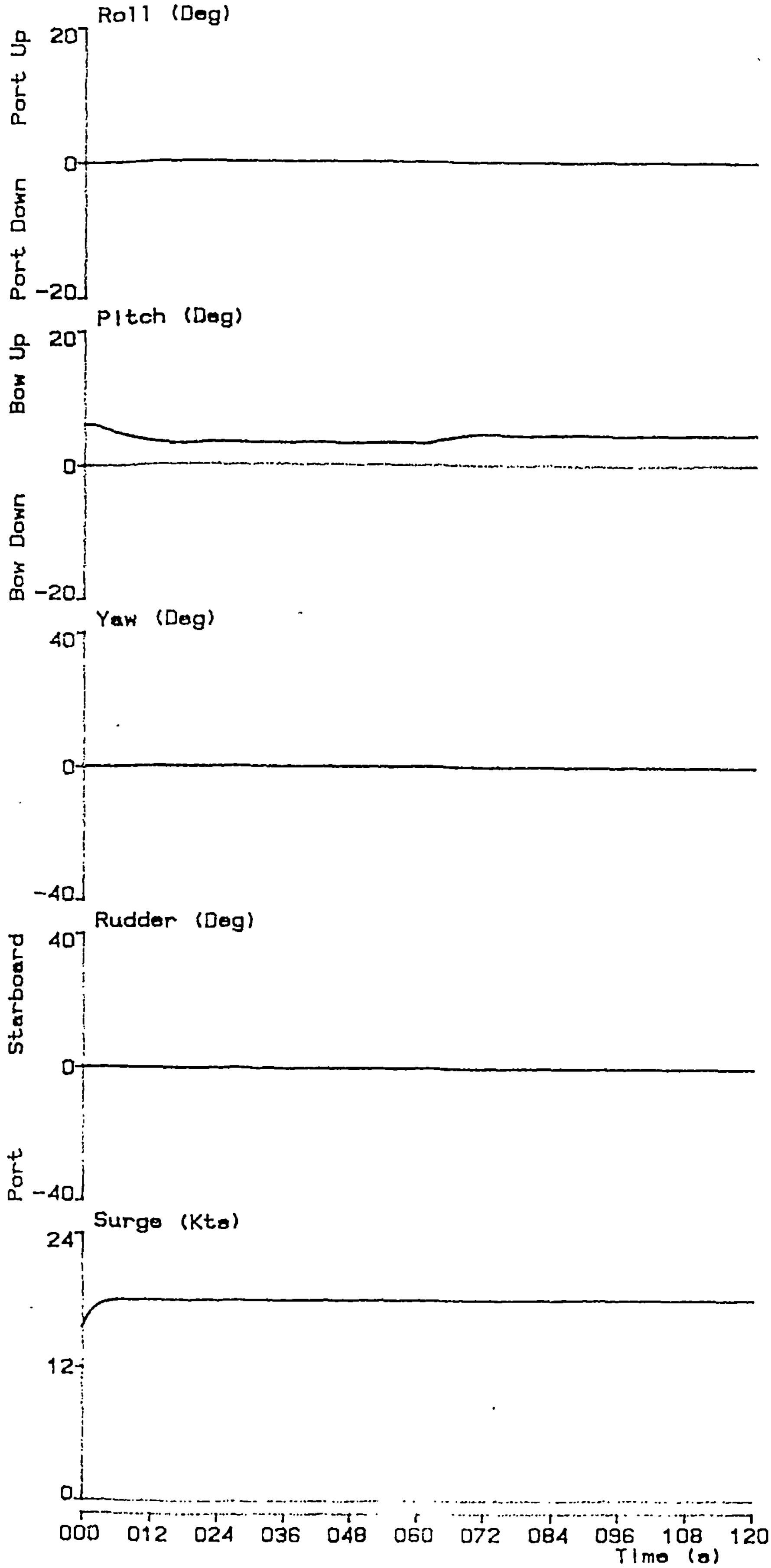
15:13:19 17/04/1989

TRIM TABS SETTINGS



Simulation

TRIM TABS SETTINGS



APPENDIX C

DATA LOGGING INTERFACE

PC-30 Analogue To Digital & Digital To Analogue Conversion Board (Multifunction Board)

I/O Address-space used by the PC-30 board

The base address (BA) can be selected to either 0x0700 or 0x0780.

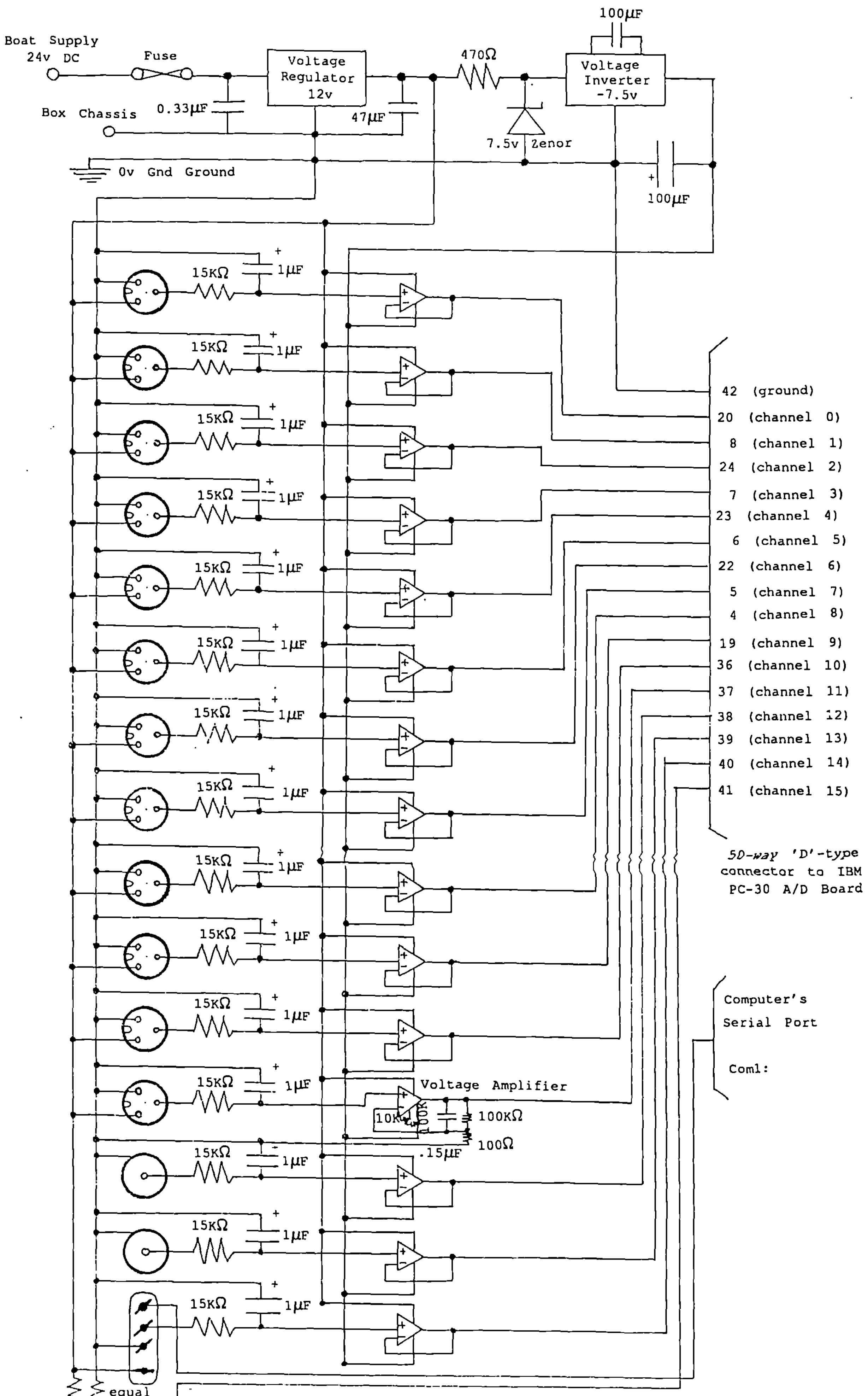
Portaddress	Function	
BA+0x00	Input 8 LSB's from AD 574	
BA+0x01	Input 4 MSB's from AD 574	
BA+0x02	bit0-bit3: Control bits for A/D bit4-bit7: Multiplexer Channel Selection	
BA+0x03	ControlWord for 8255 PPI1 (= 0x92)	
BA+0x04	8253 timer chip	Counter 0
BA+0x05	8253 timer chip	Counter 1
BA+0x06	8253 timer chip	Counter 2
BA+0x07	8253 timer chip	ControlWord
BA+0x08	8255 PPI 2	Port A
BA+0x09	8255 PPI 2	Port B
BA+0x0a	8255 PPI 2	Port C
BA+0x0b	8255 PPI 2	ControlWord
BA+0x0c	D/A-12 1	4 LSB's
BA+0x0d	D/A-12 1	8 MSB's
BA+0x0e		
BA+0x0f		
BA+0x10	D/A-12 2	4 LSB's
BA+0x11	D/A-12 2	8 LSB's
BA+0x12		
BA+0x13		
BA+0x14	D/A-8 1	8 bit
BA+0x15	D/A-8 2	8 bit

Port Addresses used for the AD 574

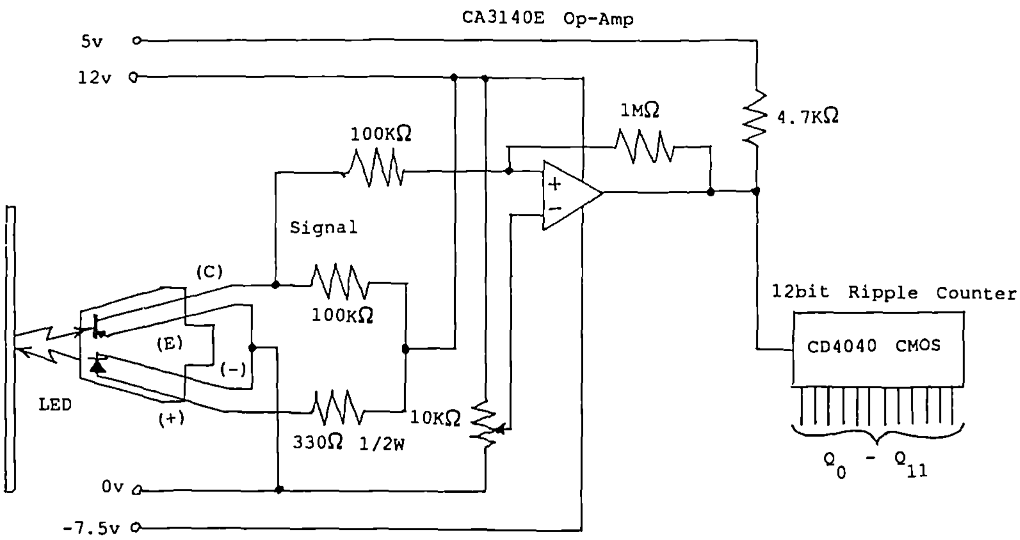
	Address	Function	
Port A	BA	bits A0-A7	= 8 LSB's
Port B	BA+1	bits B0-B3	= 4 MSB's
		bit B4	= TTL trigger input
Port C	BA+2	bit C0	= software clock
		bit C1	= 1 -> software clock
		bit C2	= 0 -> timer clock
		bit C3	= interrupt enable (C3=1)
		bits C4-C7	= channel address for multiplexer

50 Pin D-Connector

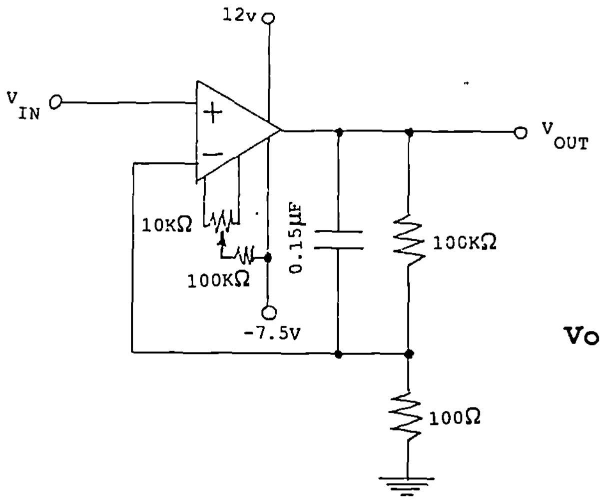
Pin	Function	Pin	Function	Pin	Function
1	O/P D/A-12 1	18	O/P D/A-12 2	34	O/P D/A-8 2
2	+12 volts	19	channel 9	35	O/P D/A-8 1
3	-12 volts	20	channel 0	36	channel 10
4	channel 8	21	O/P timer 2	37	channel 11
5	channel 7	22	channel 6	38	channel 12
6	channel 5	23	channel 4	39	channel 13
7	channel 3	24	channel 2	40	channel 14
8	channel 1	25	port B4	41	channel 15
9	port B7	26	port B6	42	Ground
10	port B5	27	port B3	43	port B0
11	port B4	28	port A7	44	port B1
12	port A6	29	port A5	45	port B2
13	port A4	30	port A3	46	port C3
14	port A2	31	port A1	47	port C2
15	port A0	32	port C7	48	port C1
16	port C6	33	port C5	49	port C0
17	+5 volts			50	port C4



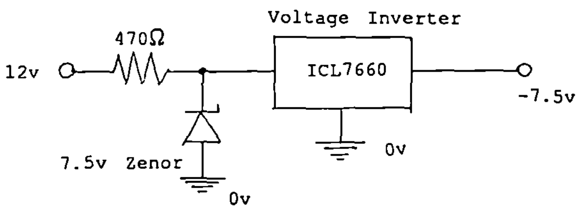
Interface Box



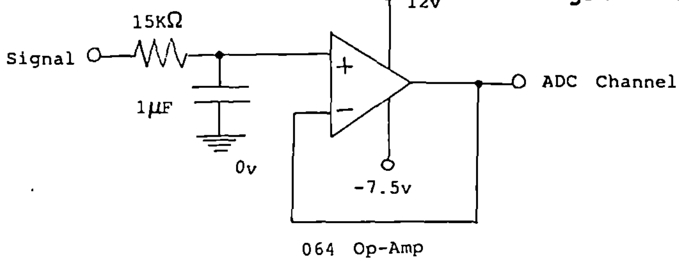
Propeller Revolutions Counter



Voltage Amplifier



Voltage Inverter



Signal Buffer + Anti-Aliasing

<u>Converting Pitch ADC Level To Angles</u>

positive values = bow up ('scending) Vref: μ = 2426

negative values = bow down (pitching) σ = 1

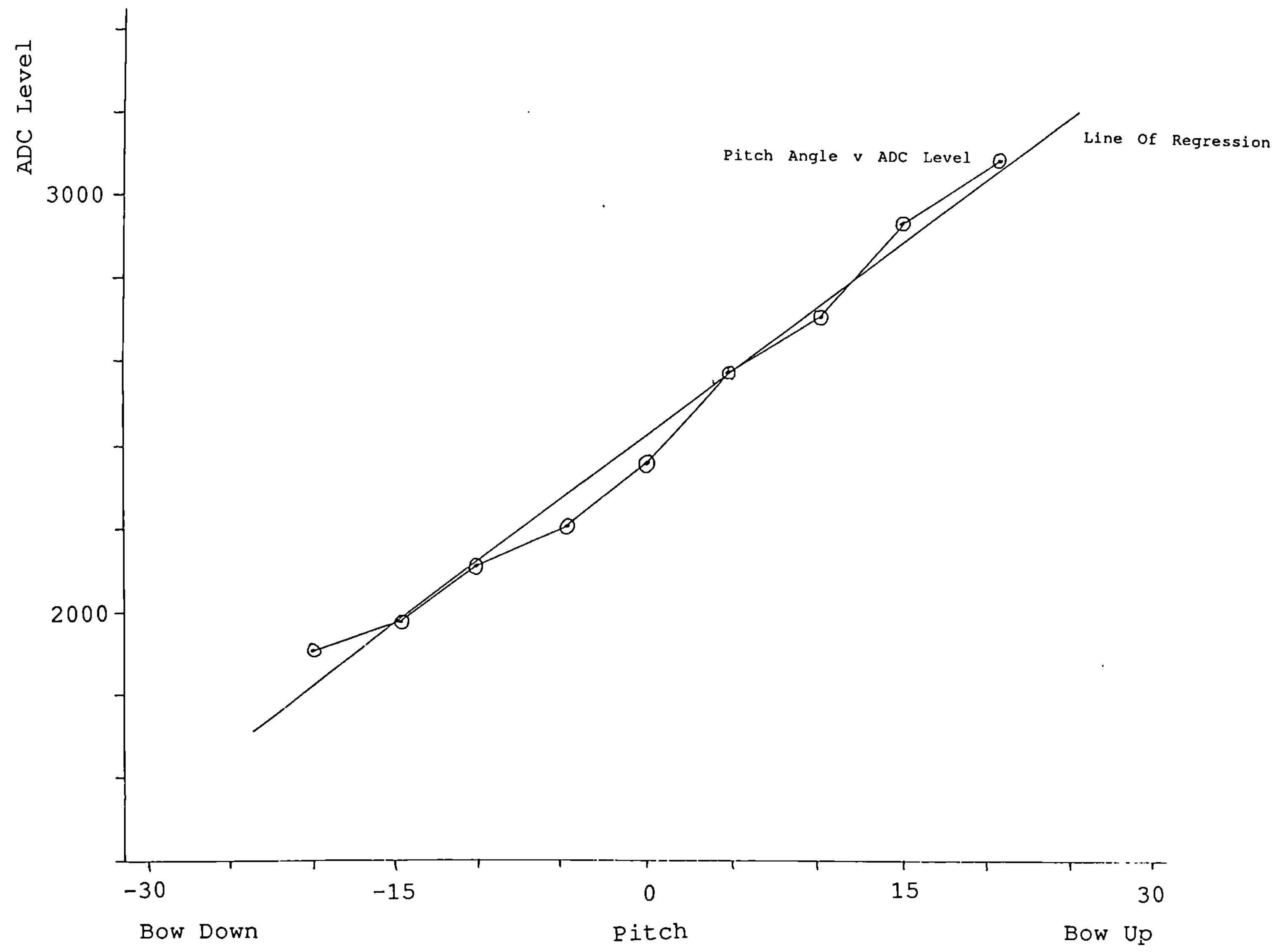
Pitch Angle θ	Mean ADC Level μ	Stnd.Dev of ADC σ	Regressn Values $\hat{\mu}$	$\mu - \hat{\mu}$	$\frac{\mu - \hat{\mu}}{\hat{b}}$
+20°	1902	18	1816	86	2.°85
+15°	1968	9	1967	1	0.°03
+10°	2105	10	2118	-13	-0.°43
+5°	2197	15	2269	-72	-2.°38
0°	2354	2	2420	-66	-2.°19
-5°	2570	3	2571	-1	-0.°03
-10°	2704	5	2722	-18	-0.°60
-15°	2919	2	2873	46	1.°52
-20°	3061	4	3024	37	1.°23

Regressional Analysis As Follows:

n	=	9	$\sum \theta\mu$	=	45300
$\sum \theta$	=	0	$\sum \mu$	=	21780
$\sum \theta^2$	=	1500	$\sum \mu^2$	=	54096576
$\bar{\theta}$	=	0	$\bar{\mu}$	=	2420
$\hat{b}$	=	-30.2	μ	=	$\hat{b}\ (\theta - \bar{\theta}) + \bar{\mu}$

$$\mu = 2420 - 30.2\ \theta$$

$$\theta = (2420 - \mu)\ /\ 30.2$$



Pitch Calibration

Converting Roll ADC Level To Angles

positive values = port up / starboard down   Vref: μ = 2426

negative values = port down / starboard up σ = 1

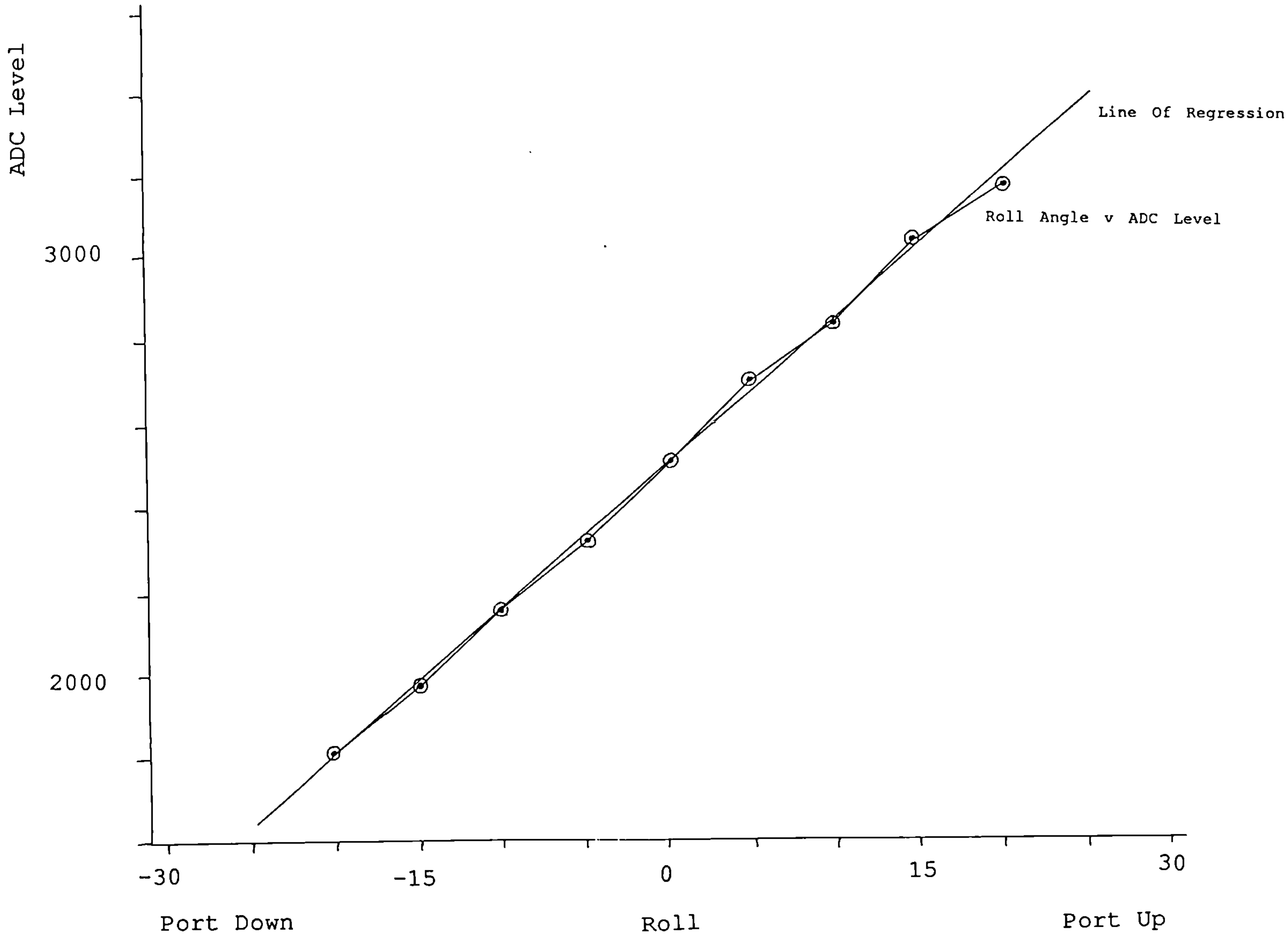
Roll Angle ϕ	Mean ADC Level μ	Stnd.Dev of ADC σ	Regressn Values $\hat{\mu}$	$\mu - \hat{\mu}$	$\frac{\mu - \hat{\mu}}{\hat{b}}$
$+20^\circ$	3181	1	3200	-19	$-0.^\circ55$
$+15^\circ$	3041	2	3025	16	$0.^\circ46$
$+10^\circ$	2848	2	2851	-3	$-0.^\circ09$
$+5^\circ$	2700	2	2677	23	$0.^\circ66$
0°	2505	1	2503	2	$0.^\circ06$
-5°	2311	2	2328	-17	$-0.^\circ49$
-10°	2157	2	2154	3	$0.^\circ09$
-15°	1965	2	1980	-15	$-0.^\circ43$
-20°	1816	2	1805	11	$0.^\circ32$

Regressional Analysis As Follows:

n	=	9	$\sum \phi\mu$	=	52295
$\sum \phi$	=	0	$\sum \mu$	=	22523
$\sum \phi^2$	=	1500	$\sum \mu^2$	=	58191391
$\bar{\phi}$	=	0	$\bar{\mu}$	=	2502.56
$\hat{b}$	=	34.86	μ	=	$\hat{b}\,(\phi - \bar{\phi}) + \bar{\mu}$

$$\mu = 2502.56 + 34.86\,\phi$$

$$\phi = (\mu - 2502.56) / 34.86$$



Roll Calibration

Converting Rudder ADC Level To Angles

positive values = starboard rudder   Vref: μ = 2425

negative values = port rudder σ = 1

Indicatd Rudder	Mean ADC Level μ	Stnd.Dev of ADC σ	Regressn Values $\hat{\mu}$	$\mu - \hat{\mu}$	$\frac{\mu - \hat{\mu}}{\hat{b}}$
+39°	(4095)	(0)	(5041)	-	-
+25°	(4095)	(0)	(4257)	-	-
+20°	3998	4	4013	-15	-0°.28
+15°	3772	3	3797	-25	-0°.46
+10°	3551	1	3553	-2	-0°.04
+5°	3362	1	3337	25	0°.46
0°	3130	2	3120	10	0°.18
-5°	3010	1	2958	52	0°.96
-10°	2784	1	2768	16	0°.30
-15°	2543	2	2552	-9	-0°.17
-20°	2309	2	2335	-26	-0°.48
-25°	2105	2	2119	-14	-0°.26
-36°	1527	2	1470	57	1°.05

Regressional Analysis As Follows:

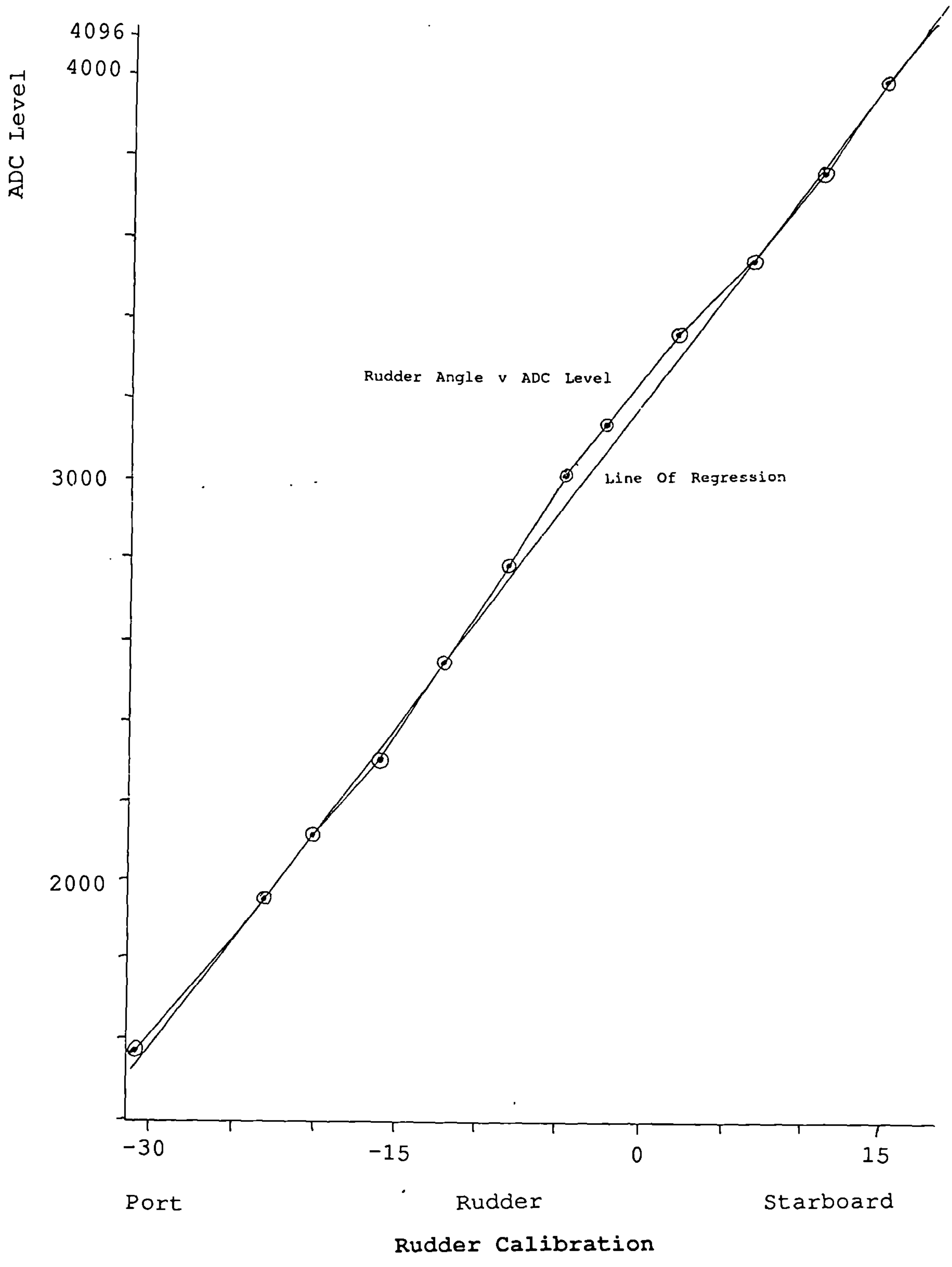
$n = 13$ $\sum \delta\mu = -12428.5$

$\sum \delta = -25.5$ $\sum \mu = 27434$

$\sum \delta^2 = 1278.75$ $\sum \mu^2 = 87164744$

$\bar{\delta} = -2.83$ $\bar{\mu} = 3048.22$

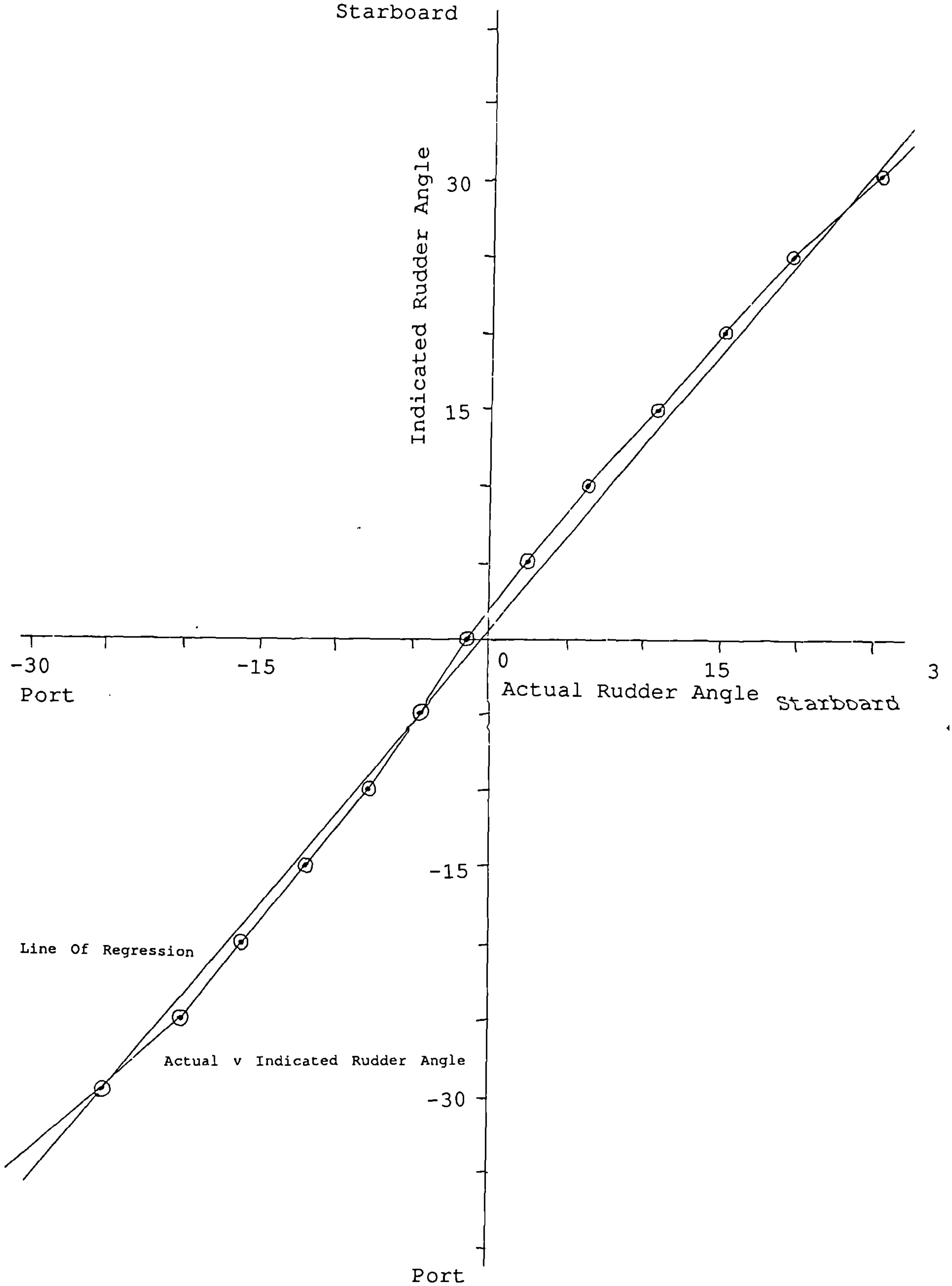
$\hat{b} = 54.12$ $\mu = \hat{b}\,(\delta - \bar{\delta}) + \bar{\mu}$


ADC Level
4096
4000
3000
2000
Rudder Angle v ADC Level
Line Of Regression
-30
-15
0
15
Port
Rudder
Starboard
Rudder Calibration

$$\mu = 3048.22 + 54.12\ (\delta + 2.83)$$

$$\delta = ((\mu - 3048.22) / 54.12) - 2.83$$

Actual Rudder	Indicatd Rudder	Regressn Values $\hat{\mu}$
+34°	+39	1470
+30°	+35	1578
+25°	+30.5	1848
+20°	+25.5	2119
+15°	+20	2390
+10°	+14	2660
+5°	+8	2931
0°	+2	3201
-5°	-5.5	3472
-10°	-12.5	3743
-15°	-18.5	4013
-20°	-25	(4284)
-25°	-29.5	(4554)
-30°	-34	(4825)
-32°	-36	(5312)



Actual v Indicated Rudder Angle

APPENDIX D
GLOSSARY

"*I wish he would explain his explanation*"

Lord Byron

Added Mass (Apparent Mass, Ascension To Mass, Hydrodynamic Mass, Virtual Mass): The apparent increase in mass of a body moving in a fluid. The total hydrodynamic force, per unit acceleration, exerted on a boat in phase with and proportional to the acceleration.
Added Mass Coefficient: A non-dimensional coefficient expressing added mass in relation to the geometry of a body.
Amplitude: The magnitude of the maximum value of a periodic function with respect to the mean value.
Broaching: An involuntary and dangerous change in heading produced by a severe following sea.
Celerity: The phase velocity of a surface gravity wave in deep water (wave speed).
Coupling: The influence of one mode of motion on another.
Damping: A characteristic property of a dynamic system, which dissipates energy and reduces motion.
Damping Force: A force which tends to reduce motion.
Disturbing Force: That part of the exciting force which can be attributed to the forces of nature, that is wind and waves.
Emergence: The vertical distance which an oscillating boat rises with respect to the water surface.
Emmersed: The additional part of a boat which rises above the water surface.
Exciting Force: A force which causes the motion of a boat.
Frequency, Characteristic: The number of cycles occurring per unit of time.
Frequency, Circular: If the motion is cyclic then the circular frequency is the angular velocity.
Frequency of Encounter: The apparent wave frequency due to a combination of the motion of the wave and the motion of the boat.
Green Water: Water shipped on the deck of a boat in heavy seas.
Group Velocity: The average rate of advance of the energy of a finite train of gravity waves.
Heading: The direction assumed by the forward axis or centreline of the boat in the horizontal plane.
Heaving: The vertical motion of the whole boat.
Immersed: The additional part of a boat which is submerged below the water surface.
Impact: The forcible, sudden contact of a boat with the surface of the sea.
Long-Crested Seas: A wave system in which all components advance in the same direction.
Metacentre: The limiting height to which the centre of gravity may be raised without producing initial instability.
Moment of Inertia: The summation of products of elementary masses and the squares of their distance from axes through the centre of gravity of the boat (equal to mass multiplied by the radius of gyration squared).

Period: The length of time for one complete cycle of a periodic quantity.
Period of Encounter: The time interval between successive crests of a train of waves passing a fixed point in a boat, at a fixed angle of encounter.
Phase Angle: The angle between to vectors representing two harmonically varying quantities having the same frequency.
Pitching: The angular motion of a boat about an athwartships axis. Strictly speaking pitching is the bows down inclination (bows up is known as 'scending).
Porpoising: The oscillation of a high-speed craft, primarily in calm water, in which heaving motion is combined with pitching motion. The motion is sustained by energy drawn from the thrust.
Pounding: The impact of the water surface against the side or bottom of a boat hull, whether caused by boat velocity, water velocity or both. Pounding is differentiated from slamming in that the impact, although heavy, is not in the nature of a shock.
Radius of Gyration: The square root of the ratio of the mass moment of inertia to the mass of a body.
Resonance: The dynamic condition of a simple, uncoupled system in which the excitation frequency is equal to the natural frequency.
Restoring (Righting) Force: A force which tends to return the boat to its equilibrium condition from which it has been displaced by an external force.
Rolling: The angular motion of a boat about the longitudinal axis.
Sea Direction:
1. *Beam Sea*: A condition in which a boat and the waves, or the predominant wave components, advance at right angles.
2. *Bow Sea*: A condition in which a boat and the waves, or the predominant wave components, advance at oblique angles. This condition covers the direction between a head sea and a beam sea.
3. *Following Sea*: A condition in which a boat and the waves, or predominant wave components, advance in the same direction.
4. *Head Sea*: A condition in which a boat and the waves, or predominant wave components, advance in opposite directions.
5. *Quartering Sea*: A condition in which a boat and the waves, or predominant wave components, advance at oblique angles. This condition covers the direction between a beam sea and a following sea.
Short-Crested Seas: An irregular wave system in which the components advance in various directions.
Significant Height: The average apparent height of the one-third highest waves in an irregular pattern.
Slamming: A phenomenon described broadly as severe impacting between a water surface and the side or bottom of a boat's hull, where the impact causes a shock-like

blow.

Speed Loss: The decrease in speed, as compared with the speed in calm water, caused directly by wind and waves at a constant setting of the propulsion system.

Speed Reduction: The decrease in speed, as compared with the speed in calm water, caused mainly by reducing the setting of the propulsion system in order to minimise the adverse effects on the boat of wind and waves.

Stabiliser: Equipment used *to reduce the rolling of a* vessel.

Submergence: The vertical distance which an oscillating boat sinks with respect to the water surface.

Surging: The longitudinal motion of a boat.

Swaying: The transverse motion of a boat.

Wave: A disturbance of the water surface that usually progresses across the surface as the result of circular or other local motions of the fluid components.

1. *Amplitude*: The radius of orbital motion of a surface wave particle, equal to one-half of the wave height.
2. *Components*: The infinity of infinitesimal waves of different frequencies and directions that are found by spectral analysis to compose an irregular sea, or the large number of finite waves used to approximate such an irregular sea.
3. *Crest*: The position of maximum upward elevation in a progressive wave.
4. *Frequency*: The reciprocal of the wave period.
5. *Height*: The vertical distance from wave crest to wave trough, equal to twice the wave amplitude of a harmonic wave.
6. *Instantaneous Elevation*: The elevation of a point in a wave system above the undisturbed surface at a given instant in time.
7. *Length*: The horizontal distance between successive wave crests in the direction of advance.
8. *Number*: 2π radians divided by the wave-length.
9. *Period*: The time between the passage of two successive wave crests past a fixed point.
10. *Profile*: The elevation of the surface particles of a wave plotted as a function of space in fixed time.
11. *Slope Of Surface*: The surface slope of a wave profile perpendicular to the crest in space coordinates. The maximum wave slope of a regular harmonic or trochoidal wave is π times the steepness ratio.
12. *Speed or Celerity*: The phase velocity of a surface gravity wave in deep water.
13. *Steepness Ratio*: The ratio of wave height to wave length.
14. *Train*: A continuous sequence of wave crests.
15. *Trochoidal*: A profile closely approximating that of a regular surface gravity wave. It can be geometrically constructed by tracing the locus of a point on the radius of a circle as that circle rolls along the underside of a horizontal line.
16. *Trough*: The position of the maximum downward

elevation in a progressive wave.
Wetness: The quality of a boat's decking with respect to its likelihood of being wet as a result of motions of the boat and waves.
Yawing: The angular motion of a boat about a vertical axis.

APPENDIX E

NOMENCLATURE

A_R	rudder area
A_T	trim tabs area
A_{LS}	projected longitudinal area above waterline
A_{TS}	projected transverse area above waterline
A_{UL}	projected longitudinal underwater area of boat
A_{UT}	projected transverse underwater area of boat
A_{WL}	water-plane area of boat
a_H	ratio of induced rud hydro force to rudder force
a	acceleration in the direction of F
a	semi-longitudinal axis of ellipsoid
b	semi-transverse axis of ellipsoid
B	buoyancy
B	used to denote centre of buoyancy
c	wave celerity
c	semi-vertical axis of ellipsoid
C	criterion for dynamic stability
C_b	centre of buoyancy
C_g	centre of gravity
C_D	drag coefficient
C_L	lift coefficient
C_H	hull flow rectification coefficient
C_P	propeller flow rectification coefficient
C_{PU}	propeller thrust reduction coefficient
C_{PQ}	pitch propeller coefficient
C_{RU}	surge rudder coefficient
C_{RV}	sway rudder coefficient
C_{RP}	roll rudder coefficient
C_{RR}	yaw rudder coefficient
C_{TQ}	pitch trim tabs coefficient
C_{VW}	heave wave coefficient
C_{VP}	roll wave coefficient
C_{VQ}	pitch wave coefficient
C_{FN}	rudder normal coefficient
D	propeller diameter
DAR	propeller developed area ratio
e	the natural number

e_i	Euler parameter (i=0,1,2,3)
$\dot{e}_i$	rate of change of Euler parameter (i=0,1,2,3)
E	energy
E_K	kinetic energy
E_P	potential energy
F	propeller circumscribed area
F_a	propeller developed area
F	total external force applied to the body
F_1	total reaction force
F_2	total remaining contributions to F
F_N	rudder normal force
F_{No}	rudder normal force in open water
G	used to denote centre of gravity
GM	metacentric height
GZ	righting lever
g	acceleration due to gravity
h	water depth
H	the collection of hydrodynamic coefficients
H_{WL}	height of C_g above waterline
I_{ij}	moment or product of inertia (i,j=x,y,z)
I_{pp}	moment of inertia of propeller shaft
J_P	propeller advance constant
J_{pp}	added moment of inertia of propeller
K_Q	propeller torque coefficient
K_T	propeller thrust coefficient
k	radius of gyration
k	radian wave number
KM_L	longitudinal metacentric height
KM_T	transverse metacentric height
L	(when not roll) general symbol for length
L	roll moment about X-axis
L_1	roll moment due to added mass
L_2	remaining roll moment
L_{BP}	boat length between perpendiculars
L_{OA}	boat length overall
LCB	longitudinal centre of buoyancy
LCF	longitudinal centre of floatation

M	used to denote metacentre
M	pitch moment about Y-axis
M_1	pitch moment due to added mass
M_2	remaining pitch moment
m	mass of boat
$\bar{m}$	mass of displaced fluid
MCT 1"	moment to change trim 1 inch
N	yaw moment about Z-axis
N_1	yaw moment due to added mass
N_2	remaining yaw moment
n	number of propeller revolutions
$\dot{n}$	rate of change of propeller revolutions
n_d	demanded propeller revolutions
n_{max}	maximum number of propeller revolutions
p	roll angular velocity about X-axis
$\dot{p}$	roll angular acceleration about X-axis
P	propeller pitch
P_Q	propeller torque
P_T	propeller thrust
Q_E	engine torque
Q_{Emax}	maximum engine torque
q	pitch angular velocity about Y-axis
$\dot{q}$	pitch angular acceleration about Y-axis
r	yaw angular velocity about Z-axis
$\dot{r}$	yaw angular acceleration about Z-axis
R_e	Reynold's number
s	propeller slip-stream ratio
T	period
T_e	period of encounter
t	time
TPI	tons per inch immersion
u	surge velocity directed along X-axis
$\dot{u}$	surge acceleration directed along X-axis
u_r	relative fluid velocity along X-axis
u_s	fluid velocity directed along X-axis
$\dot{u}_s$	fluid acceleration directed along X-axis
V	boat's horizontal velocity

V_b	horizontal speed of boat
V_v	speed of wave
V_r	relative wave velocity
V_E	easterly velocity component
V_N	northerly velocity component
V_P	propeller inflow velocity
V_R	rudder inflow velocity
V_T	trim tabs inflow velocity
V_w	wind velocity
v	sway velocity directed along Y-axis
$\dot{v}$	sway acceleration directed along Y-axis
v_r	relative fluid velocity along Y-axis
v_s	fluid velocity directed along Y-axis
$\dot{v}_s$	fluid acceleration directed along Y-axis
W	weight
w	heave velocity directed along Z-axis
$\dot{w}$	heave acceleration directed along Z-axis
w_P	propeller wake fraction
w_R	rudder wake fraction
w_r	relative fluid velocity along Z-axis
w_s	fluid velocity directed along Z-axis
$\dot{w}_s$	fluid acceleration directed along Z-axis
X	surge force directed along X-axis
X_1	surge force due to added mass
X_2	remaining surge force
x	distance along X-axis
x_b	x coordinate of centre of buoyancy
x_g	x coordinate of centre of gravity
x_P	x coordinate of propellers
x_R	x coordinate of rudder centre of effect
x_T	x coordinate of trim tabs
Y	sway force directed along Y-axis
Y_1	sway force due to added mass
Y_2	remaining sway force
y	distance along Y-axis
y_b	y coordinate of centre of buoyancy
y_g	y coordinate of centre of gravity

y_P	y coordinate of propellers
y_R	y coordinate of rudder centre of effect
y_T	y coordinate of trim tabs
Z	heave force directed along Z-axis
Z_1	heave force due to added mass
Z_2	remaining heave force
z	distance along Z-axis
z_b	z coordinate of centre of buoyancy
z_g	z coordinate of centre of gravity
z_P	z coordinate of propellers
z_R	z coordinate of rudder centre of effect
z_T	z coordinate of trim tabs
α	effective rudder inflow angle
α	wave slope
α_0	ellipsoid numerical constant
α_R	effective rudder inflow angle
α_T	angle of deflection of the trim tabs
β	direction of boat motion w.r.t. its centreline
β_0	ellipsoid numerical constant
β_v	absolute wave direction
γ	absolute wind direction
γ	flow rectification coefficient
γ	surface tension
γ_0	ellipsoid numerical constant
Γ	rudder aspect ratio
Γ_T	trim tabs aspect ratio
δ	rudder angle
$\dot{\delta}$	rate of change of rudder angle
δ_d	demanded rudder angle
δ_e	rudder error (demanded - actual)
δ_{max}	maximum rudder angle (rudder limit)
δ_{dir}	rudder direction (port/starboard/stop)
Δ	displacement of boat
ε	change of inflow angle due to hull and propeller
ζ	wave amplitude
ζ_a	instantaneous wave elevation
η	propeller diameter to rudder height ratio

θ	angle of pitch
λ_{ij}	direction cosine (i,j=1,2,3)
λ_v	wavelength
ν	a rudder inflow rectification angle
π	mathematical constant
ρ, ρ_w	water density
ρ_a	air density
τ_n	propeller time constant
τ_r	rudder time constant
τ_t	trim tabs time constant
ϕ	angle of roll
ϕ	gravitational potential
ϕ_i	components of the velocity potential (i=1,2,3)
χ_i	components of the velocity potential (i=1,2,3)
ψ	angle of yaw (heading)
ψ_d	demanded heading
ψ_e	heading error (demanded - actual)
ω	circular frequency or general angular velocity
$\dot{\omega}$	general angular acceleration
ω_e	wave encounter frequency
∇	underwater volume of the boat
∇	the Del operator

APPENDIX F

CONVERSION TABLE

Multiply	By	To Obtain
atmospheres	10.333	kg per square metre
cubic feet	0.02832	cubic metres
cubic feet	6.2284	gallons (imperial)
cubic feet	28.32	litres
cubic inches	16.39	cubic centimetres
degrees (angle)	60	minutes
degrees (angle)	0.01745	radians
feet	0.3048	metres
foot pounds	5.050×10^{-7}	horse power hours
foot pounds	0.1383	kilogram metres
foot pounds per min	3.030×10^{-5}	horse power
gallons (imperial)	0.1605	cubic feet
gallons (imperial)	4.546	litres
grammes	0.03527	ounces
horse power	33.000	foot pounds per min
horse power	550	foot pounds per sec
horse power	745.7	watts
inches	25.40	millimetres
kilogrammes	2.2046	pounds
kilogramme force	9.807	Newtons
kilometres	3280.84	feet
kilometres	0.6214	miles (statute)
knots	1.689	feet per sec
knots	0.5144	metres per sec
knots	1.15078	miles per hour
metres	3.2808	feet
miles (statute)	5820	feet
miles (statute)	1.6093	kilometres
miles (nautical)	1.852	kilometres
miles (statute)	1609.3	metres
miles (nautical)	1852.0	metres
miles (statute)	1760	yards
miles per hour	1.6093	kilometres per hour
miles per hour	0.86898	knots
Newtons	0.2248	pounds force
Newtons	0.1020	kilogrammes force

pascals	0.0209	pound force per sq ft
pascals	0.102	kg force per sq metre
pounds per sq inch	703.1	kg per sq metre
pounds forces	4.448	Newtons
slug	14.594	kilogrammes
square feet	144	square inches
square feet	0.09290	square metres
temp (°C) +17.8	1.8	temp (°F)
temp (°F) -32	5/9	temp (°C)
tons (long)	2240	pounds
tons (long)	1016.05	kilogrammes
tonnes (metric)	0.9842	tons (long)
watts	1.341×10^{-3}	horse power
yards	0.9144	metres

APPENDIX G

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Notations

Conf.	= Conference
Congr.	= Congress
Dept.	= Department
Inst.	= Institute
Int.	= International
Jnl.	= Journal
Lab.	= Laboratory
No.	= Number
Proc.	= Proceedings of the
Pt.	= Part
Publ.	= Publication
Publr	= Publisher
Rep.	= Report
Symp.	= Symposium
Tech.	= Technology / Technological
Trans.	= Transcript / Transactions / Translation
Univ.	= University
Vol.	= Volume

APPENDIX H

ABBREVIATIONS

AICA	= Association Internationale pour le Calc. Analogique
ARMA	= Auto-Regressive Moving Average
ATMA	= Association Technique Maritime et Aeronautique
ATTC	= American Towing Tank Conference
BEC	= Bassin d'Essais des Carenès (Paris Model Basin)
BMF	= British Marine Federation
BMT	= British Maritime Technology
BSHC	= Bulgarian Ship Hydrodynamics Centre
BSRA	= British Ship Research Association
CAORF	= Computer Aided Operations Research Facility
CASSIM	= CArdiff Ship SIMulator
CGI	= Computer Generated Imagery
CNDCT	= Conselho Nacional de Desenvolvimento Cientifico e Technologico (National Council for Scientific & Technological Development)
DL	= Davidson Laboratory (SIT)
DMI	= Danish Maritime Institute
DSRL	= Danish Ship Research Laboratory
DTMB	= David Taylor Model Basin
DTMBRDC	= David Taylor Model Basin Research and Development Centre
DTNSRDC	= David Taylor Naval Ship Research and Development Center
DUT	= Delft University of Technology
ENSDGM	= Ecole Nationale Supérieure Du Génie Maritime
EP	= Ecole Polytechnique
HSMB	= Hydronautics Ship Model Basin
HSVA	= Hamburgische Schiffbauversuchsanstalt
ICCAS	= International Conference on Computer Applications in Shipping and shipbuilding
IEE	= Institution of Electrical Engineers
IFAC	= International Federation for Automatic Control
IFIP	= International Federation for Information Processing
IfS	= Institut für Schiffbau
IHI	= Ishikawajima-Harima heavy Industries
IME	= Institute of Marine Engineers
IMO	= International Maritime Organisation
IMSF	= International Marine Simulator Forum
IPE	= Instituto de Pesquisas Espaciais (Institute for Space Research)
IPT	= Instituto de Pesquisas Tecnologicas (Technological Research Institute)
ISP	= International Shipbuilding Progress
ISSC	= International Ship Structure Congress
ISSOA	= International Symposia on Ship Operation Automation
ITTC	= International Towing Tank Conference
JBE	= Journal of Basic Engineering
JFM	= Journal of Fluid Mechanics
JIMP	= Joint International Manoeuvring Program

JMES	=	Journal of Mechanical Engineering Science
JMR	=	Journal of Marine Research
JNav	=	Journal of Navigation
JSNA	=	Journal of the Society of Naval Architects of Japan
JSR	=	Journal of Ship Research
LAHPMM	=	Large Amplitude Horizontal Planar Motion Mechanism
LIT	=	Lund Institute of Technology
MARCIS	=	MARine Coefficient Identification System
MARIN	=	MAritime Research Institute Netherlands
MARINTEK	=	Norwegian Marine Technology Research Institute
MARSIM	=	MARine SIMulation
MIT	=	Massachusetts Institute of Technology
MSMS	=	Multi-Ship Manoeuvring Simulator
MT	=	Marine Technology
NA	=	Naval Architect
NAVSEC	=	NAVal Ship Engineering Center (US Navy)
NECIES	=	NorthEast Coast Institute of Engineers and Shipbuilders
NEJ	=	Naval Engineers Journal
NL	=	The Netherlands
NMEA	=	National Marine Electronics Association
NMI	=	National Maritime Institute
NMR	=	Norwegian Maritime Research
NPL	=	National Physical Laboratory
NSMB	=	Netherlands Ship Model Basin
ONR	=	Office of Naval Research (US Navy)
PID	=	Proportional, Integral & Derivative
PMB	=	Paris Model Basin
PMM	=	Planar Motion Mechanism
PNS	=	Pseudo Noise Sequences
PRBS	=	Pseudo Random Binary Sequences
RINA	=	the Royal Institution of Naval Architects
ROV	=	Remotely Operated underwater Vehicles
SGIHE	=	South Glamorgan Institute of Higher Education
SIT	=	Stevens Institute of Technology
SNAJ	=	Society of Naval Architects of Japan
SNAME	=	Society of Naval Architects and Marine Engineers
SOHM	=	Successive Order Heightening Method
SSC	=	Ship Structure Committee (US government)
SSPA	=	Statens Skeppsprovningsanstalt (Swedish State Shipbuilding Experimental Tank)
STG	=	Schiffbautechnische Gesellschaft
USA	=	United States of America
UWIST	=	University of Wales Institute of Science and Technology
WADC	=	Wright Air Development Center

APPENDIX I

COLOPHON

The text in this document was produced entirely using the ChiWriter wordprocessing package for IBM PC's. The plots were created using a Roland XY-plotter with a dedicated driver utility written by the author. All drawings and diagrams were hand produced.