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Which effective viscosity?

N. Petford*

School of Conservation Sciences, Bournemouth University, Poole, Dorset BH12 5BB, UK

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ABSTRACT

Magmas undergoing shear are prime examples of flows that involve the transport of solids and gases by a separate (silicate melt) carrier phase. Such flows are called multiphase, and have attracted much attention due to their important range of engineering applications. Where the volume fraction of the dispersed phase (crystals) is large, the influence of particles on the fluid motion becomes significant and must be taken into account in any explanation of the bulk behaviour of the mixture. For congested magma deforming well in excess of the dilute limit (particle concentrations >40% by volume), sudden changes in the effective or relative viscosity can be expected. The picture is complicated further by the fact that the melt phase is temperature- and shear-rate-dependent. In the absence of a constitutive law for the flow of congested magma under an applied force, it is far from clear which of the many hundreds of empirical formulae devised to predict the rheology of suspensions as the particle fraction increases with time are best suited. Some of the more commonly used expressions in geology and engineering are reviewed with an aim to home in on those variables key to an improved understanding of magma rheology. These include a temperature, compositional and shear-rate dependency of viscosity of the melt phase with the shear-rate dependency of the crystal (particle) packing arrangement. Building on previous formulations, a new expression for the effective (relative) viscosity of magma is proposed that gives users the option to define a packing fraction range as a function of shear stress. Comparison is drawn between processes (segregation, clustering, jamming), common in industrial slurries, and structures seen preserved in igneous rocks. An equivalence is made such that congested magma, viewed in purely mechanical terms as a high-temperature slurry, is an inherently non-equilibrium material where flow at large Péclet numbers may result in shear thinning and spontaneous development of layering.

Keywords: viscosity, rheology, fluid motion, magma, engineering, shear rate.

Introduction

While significant progress has been made in quantifying the flow properties of silicate melt in terms of viscosity and density over a wide compositional range (see Shaw, 1965; Giordano *et al.*, 2008 amongst many others), a fundamental treatment that captures simultaneously the rheology and flow behaviour of magma as a fluid-particle suspension remains elusive. This lack of understanding of what may appear at first glance a relatively trivial problem arises from the fact that magmas are complex multiphase flows with time- and rate-dependent properties (Fig. 1). Especially problematic is that factors that govern the flow of magma, including heat transfer, phase transitions, coupled deformation of both solid and fluid phases, seepage phenomena and chemical processes, occur simultaneously and interdependently. Of critical importance is that for multiphase materials, the viscosity is not a single-valued function nor necessarily isotropic, but is instead a macroscopic property derived from the suspension microstructure. It is this microstructure-viscosity connection that defines the complex rheological properties of magma, which in turn controls its rate and style of emplacement, both within and on the surface of the earth and other planets (e.g. Dingwell and

* E-mail: npetford@bournemouth.ac.uk
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Webb, 1990; Kargel *et al.*, 1991; Melnik and Sparks, 2005; Reese and Solomatov, 2006; Hale *et al.*, 2007; Cordonnier *et al.*, 2008; Whittington *et al.*, 2009).

More generally, a better understanding of the motion of particle-fluid mixtures is important for a large number of applications in chemical engineering, polymer science and biophysics, where the complexity of phenomena has attracted much attention. A crude classification of the subject is carried out by the specification of: (1) the solids volume content of the suspension, magmatic or otherwise, defined as the volume occupied by the particles as a fraction or percentage of the total volume of the suspension; (2) the hydrodynamics of particle-particle interactions; (3) the mean shear rate of the flow; and (4) the mean particle size. As to (1), this review is concerned primarily with estimates of flow rheology where the magma crystal content (solidosity) is equal to or exceeds 40% by volume. Such materials are referred to in the micromechanics literature as 'congested' or as 'dense slurries', and magmas during flow where cooling and crystallization dominate, provide excellent natural examples (e.g. Marsh, 1996). With respect to (2), neutral (non-colloidal) particles in Newtonian fluids are envisaged, and (3) the shear rate is sufficiently small so that no turbulent effects, associated with fluid inertia, take place. Finally, (4) the particles (crystals in suspension) are sufficiently large (>0.1 mm) that Brownian motion can be ignored. Despite these restrictions, which correspond to the high-Péclet, low-Reynolds number limit, there is potential to gain insight into a wide range of classical magmatic-emplacement phenomena by treating the problem as one would a dense suspension.

This review is split broadly into two parts – a background introduction to the rheology and flow of both dilute and dense suspensions that includes a survey of some of the better known empirical expressions put forward to capture the important effect of increasing viscosity due to particle loading, and a second, more discursive section that looks afresh at the origin of layering and other structures preserved in igneous rocks using concepts informed by fluid-mechanical studies of dense slurries. The first part in particular draws heavily on previous but highly relevant treatments of suspension flow by Zapryanov and Tabakova (1999) and Stickel and Powell (2005). Interested readers should look to these works and references therein for further in-depth analysis (see also Jeffrey and Acrivos, 1996). Other relevant studies dealing explicitly with magma rheology include McBirney and Murase, 1984; Ryerson *et al.* 1988; Pinkerton and Stevenson, 1992; Lejeune and Richet, 1995; Lavalleé *et al.*, 2007. Finally, this review deals exclusively with two-phase flow and ignores the complicating effects of gas bubbles on magma rheology known to provide an additional degree of freedom on microstructre evolution (e.g. Rust and Manga, 2002; Pal, 2003).

Background

Stickel and Powell (2005) provide a highly accessible introduction to dense-suspension rheology based on dimensional analysis and the conservation of linear momentum. For completeness, some of their analysis is reproduced here. The viscosity of a suspension, magmatic or otherwise, is a function of a range of variables including particle diameter (d), dynamic melt viscosity (η_0), deformation rate (expressed either as shear rate or shear stress), thermal energy and time (see Table 1 for a list of symbols and meaning). In general, these terms can be brought together such that the suspension viscosity, also referred to as the effective viscosity, η_e, is a function of:

$$\eta_e = f[(\phi,d,\rho)(\eta_0,\rho_0,kT,\tau\dot{\gamma},t)] \quad (1)$$

where all the terms inside the first bracket relate to properties of the suspension, and those in the second the fluid phase. Conveniently the variables in equation 1 can be arranged into the following dimensionless groups: $\eta_r = \eta/\eta_0$, $\phi = (4\pi/3)\phi d^3$, $\rho_r = \rho/\rho_0$ and $t_r = tkT/\eta_0 d^3$. Note that fluid deformation can be defined either in terms of shear stress (τ) or shear rate, the latter defined as the time rate of shear strain ($\dot{\gamma}$) with units $1/s$. Strain (ε) is defined non-dimensionally as $\varepsilon = t\dot{\gamma}$. In this study, both shear rate and shear stress are used as the deformation term. A further simplification can be made by assuming that the dense suspension is neutrally buoyant (settling due to mass density is inhibited to some extent by particle interactions), and is steady state, such that

$$\eta_e = f(\phi, Pe, Re) \quad (2)$$

Two important dimensionless numbers with bearing on the suspension rheology during shear are the particle Reynolds ($Re_{\dot{\gamma}}$) and Péclet ($Pe_{\dot{\gamma}}$) numbers

$$Re_{\dot{\gamma}} = \frac{\rho_0 d^2 \dot{\gamma}}{\eta_0} \quad (3)$$

and

$$Pe_{\dot{\gamma}} = \frac{6\pi\eta_0 d^3\dot{\gamma}}{kT} \quad (4)$$

The ratio of the Péclet and Reynolds numbers is the Schmidt number $Pe_{\dot{\gamma}}/Re_{\dot{\gamma}}$. For a given rate of shear, Sc >>1 tends to indicate Newtonian behaviour. Together, these dimensionless numbers provide insight into the rheology and kinds of structures that may form in dense magma slurries undergoing shear.

Time dependency

Suspensions with volume concentrations in excess of ~40% are candidate examples of non-Newtonian fluids (e.g. Zapryanov and Tabakova, 1999). The degree of non-linearity can be related to the degree of hydrodynamic diffusion (diffusion due to interaction with other particles as opposed to thermal motion), to advection of particles in the flow through the Pe number, which must be of O(1) for flow to disrupt significantly the suspension microstructure and

TABLE 1. Selected list of symbols and units used in text.

Symbol	Meaning	Unit
A_0	pre exponential factor in equation 8	
A	parameter in equation 9	Pa
A	kinetic parameter in equations 14 and 15	Pa^m
B	parameter in equation 9	
d	mean particle radius	m
Bg	Bagnold number	
C	parameter in equation 9	
c	proportionality constant for the particle pressure	
g	acceleration due to gravity	$m\ s^{-2}$
E	activation energy	$J\ mole^{-1}$
h	mean gap width between neighbouring grains	m
k	Boltzmann constant	JK^{-1}
n	particle number density	
y	position	m
m	exponent in equations 14 and 15	1–3
T	temperature	ºC or K
ϕ	solidosity (particle volume fraction)	
t	time	s
R	packing factor = $1/\phi_{max}$	
$\tilde{p}$, $\bar{p}$	particle pressure	Pa
R	gas constant	$J\ mole^{-1}\ K^{-1}$
ϕ_{max}	maximum packing fraction	
ϕ_{m0}	packing limit at low shear	
$\phi_{m\infty}$	packing limit at high shear	
ϕM	maximum obtainable value of ϕ	
u	fluid velocity	$m\ s^{-1}$
Pe	particle Péclet number	
Re	particle Reynolds number	
ρ	solid or melt phase density	$kg\ m^{-3}$
η_0	melt viscosity (dynamic)	Pa s
η_e	effective viscosity	
η_r	relative viscosity = η_e/η_0	
τ	shear stress	Pa
$\dot{\gamma}$	shear strain rate	s^{-1}
σ_{ij}	stress tensor	Pa

bring about time-dependent rheology. Time-dependent deviation from simple Newtonian flow in congested slurries is referred to generally as power-law flow and results in two broad kinds of behaviour (Fig. 1), pseudoplastic (shear-thinning) or dilatant (shear thickening). An approximate mathematical description of power-law fluids is captured by the Ostwald de Waele relationship which links shear stress with shear strain rate ($\tau \sim \dot{\gamma}^n$) *via* an exponent n where $n < 1$ = pseudoplastic, $n = 1$ (Newtonian) and $n > 1$ = dilatant (e.g. Bird *et al.* 1960). Pseudoplastic, or shear-thinning fluids that also show reversible behaviour, meaning they thicken when shear is removed, are known as thixotropic (e.g. Barnes, 1997). Other words used to describe time-dependency include shear rejuvenation (viscosity decrease), and aging (viscosity increase). This latter case where the viscosity increases with shear strain rate ($n > 1$) appears to be less common (at least in industrial materials so far tested) and the thickening effect, also reversible, is referred to as negative thixotropy or rheopexy (Chhabra and Richardson, 1999). There is textural evidence that some crystal-rich magmas may behave as shear-thickening fluids (Smith, 2000). While the terminology can be confusing, it is important to grasp the essential feature that both shear thinning and dilatant fluids reflect the underlying control of microstructure – defined as the spatial and temporal distribution of the particles – on the effective viscosity in congested suspensions. Both underscore the importance of kinematic history in influencing the mixture rheology and ultimately emplacement style and structures formation (e.g. segregation effects) during shear. We will return to this point from a magmatic perspective in the discussion section.

Yield strength

Finally, note that the relationship between yield strength materials (Bingham fluids), which appear to support a shear stress without flowing, and power-law fluids remains murky. At the risk of adding yet more confusing terminology, unlike Bingham fluids, their power-law counterparts appear to show no inherent yield strength (see Fig. 1), yet crystal-rich magmas are often regarded by petrologists as Bingham materials. While this may (or may not) be true (see McBirney and Murase, 1984; Kerr and Lister, 1991; Lejeune and Richet, 1995 for points for and against), it is worth noting that laboratory measurements on suspension yield strengths have long proved challenging, often resulting in a wide range of values for the same material (Nguyen and Boger, 1992). Some of this discrepancy may be due to shear localization (i.e. deforming a non-homogeneous material), but

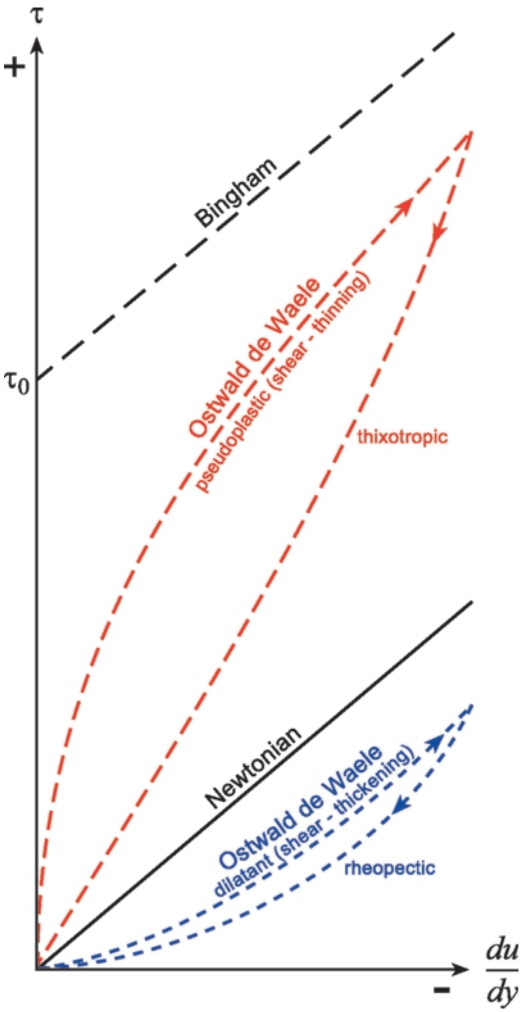

FIG. 1. Summary plot showing variations in rheological behaviour relevant to congested magma around the limiting (ideal) case of Newtonian flow where the shear stress (τ) is proportional to rate of shear strain (*du/dy*). The plot shows two variants of Ostwald de Waele flow, pseudoplastic (shear thinning), and dilatant (shear thickening). Both classes show reversible (thixotropic and rheopectic) behaviour defined by hysteresis loops. Bingham fluids possess an apparent inherent yield strength (τ_0).

also that the material is thixotropic. Indeed, there is growing evidence of an underlying physics linking yield strength and shear-thinning fluids as different manifestations of the same time-dependent process (Møllera *et al.* 2006).

Conceptual model of flowing magma

Bearing in mind the points made about particle microstructure and shear history and the complexity therein, Fig. 2 shows a simple illustration of the set-up. A key feature of dense-slurry flow (dense in this context refers to particle number density not mass density), is that particles in shear cannot move in straight lines. Instead, they need fluctuational motion in order to overtake one another. The idea of particle fluctuations is a very important one, governing both the local microstructure and through this the macroscopic flow properties of sheared suspensions. It is a topic we shall return to later on. Intuitively from Fig. 2 it is clear that as more particles are added, the more difficult it will be for the fluid phase to move in anything close to a straight line, while at the same time the increased volume concentration of particles will lead to interactions that include a greater likelihood of collisions. Adding more particles in this way results in an increase in the bulk viscosity (η) of the suspension. If we assume that there exists some maximum packing of the particles (ϕ_{max}) above which no further loading can take place, then we can express this relationship mathematically as:

$$\lim_{\phi \to \phi_{max}} \eta = \infty \tag{5}$$

The concept of maximum packing fraction determines when the effective viscosity of a suspension becomes infinite, i.e. the curves

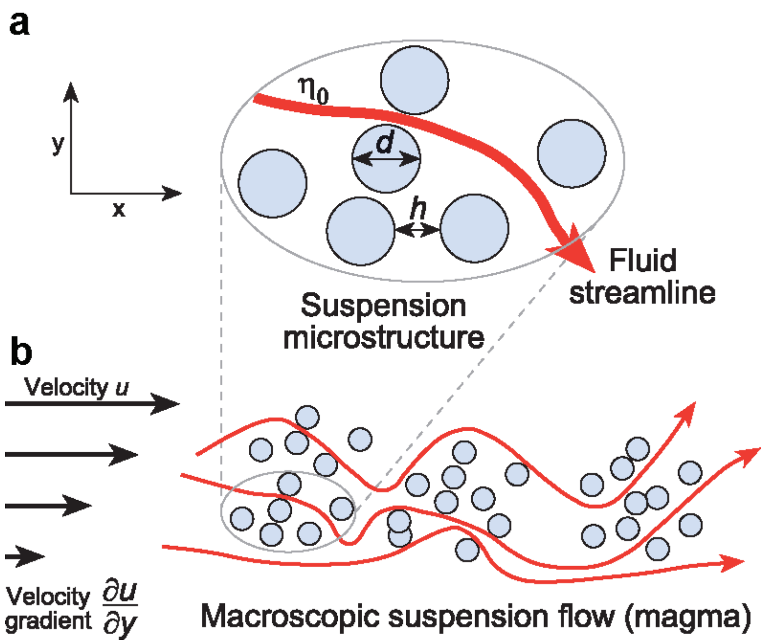

FIG. 2. Component materials and scaling relationships in a congested magma suspension undergoing shear. (*A*) The microstructure (orientation and relative position of grains) plays a governing role in the larger-scale (macroscopic) rheology of the flow. Suspensions are regarded as 'dense' when particle-particle interactions are significant when the gap width (*h*) between grains is equal to or less than the mean particle size (*d*). The particles are carried in a fluid with a viscosity η_0. For magma this fluid is a silicate melt (*B*). The macroscopic properties of the flow during shear are a function of the upscaled microstructure. In congested slurries, melt streamlines (red arrows) cannot flow in straight lines and these become more tortuous as the particle number increases. The cumulative effect of adding more particles is to increase the effective viscosity of the flow up to a critical maximum packing fraction (Table 2).

TABLE 2. Summary of the relationship between packing variables (monodisperse spheres), maximum packing fraction (ϕ_{max}) and the constant R in the Einstein-Roscoe equation 6.

Packing	Expression	ϕ_{max}	R (= $1/\phi_{max}$)
Loose	–	0.0555	18.2
Tetrahedral lattice	$\frac{\pi\sqrt{3}}{16}$	0.3401	2.94
Cubic lattice	$\frac{\pi}{6}$	0.5236	1.90
Hexagonal lattice	$\frac{\pi}{3\sqrt{3}}$	0.6046	1.65
Disordered dense packing	–	0.6400	1.56
Body-centred cubic packing	–	0.6800	1.47
Hexagonal close packing	$\frac{\pi}{3\sqrt{2}}$	0.7405	1.35

describing the effective viscosity become asymptotic at ϕ_{max}. The various kinds of packing arrangements for spheres are listed in Table 2. For reference, the maximum theoretical packing of spherical particles of equal size in a contained volume is $\pi/3\sqrt{2} \approx 74\%$, while for dense, disorderd packing of hard spheres the critical density is ~0.64 (Anikeenko and Medvedev, 2007). While particle geometry is clearly important, experimental rheologists like spheres, and most published analogue models, and the empirical formulae on which they are based, are constructed using monodisperse assemblages, meaning spherical particles of equal size. This is clearly a convenience and in no way a true reflection of the shapes or particle-size distributions in natural magmas (Fig. 3). However, it is a necessary first-stage simplification. Some experimental work has been done on non-spherical particles (rods, oblate and prolate spheroids) and this is discussed later. The reduction in effective viscosity at fixed particle concentration with increasing shear rate is shown schematically in Fig. 4.

What the (petrology) textbooks say

Most igneous petrology textbooks now have a standard section on the physical properties of magmas that provide an overview on the viscosity and density of silicate melts, and the role of crystals and dissolved gas in governing magma transport (e.g. Best and Christiansen, 2001). In essence, when dealing with suspensions, magmatic or otherwise, three viscosity terms come into play: the fluid (melt) phase, the effective or apparent viscosity of the suspension and the relative viscosity (η_r). In dealing with the role of suspended crystals, the impression given generally is that the effect of increasing solids content can be modelled straightforwardly using an expression commonly referred to as the

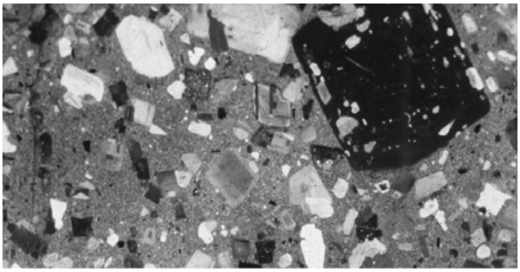

FIG. 3. A frozen-in magmatic slurry. Wide angle ($\times$ 1) thin-section view of a granite porphyry showing euhedral to subhedral feldspar and quartz phenocrysts set in a fine-grained groundmass. Rapid quenching has preserved the near original grain size distribution which does not equate well to a monodisperse particle-rich fluid, the general basis for most effective viscosity formulations (e.g. Table 3).

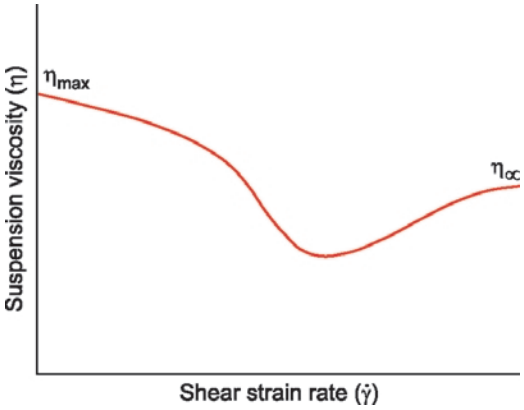

FIG. 4. Generalized plot showing the effect of shear strain rate on suspension (effective) viscosity in a congested flow. The reduction in viscosity (η) from an initial maximum value with increasing shear is a common characteristic of particle-rich fluids that show shear-thinning (pseudoplastic, Fig. 1) behaviour. With increasing shear the viscosity increases to an assumed steady-state far-field value (η_∞).

Einstein-Roscoe (ER) equation relating the change in relative viscosity η_r to increasing particle concentration via:

$$\eta_r = \eta_0\,(1-\phi R)^{-2.5} \quad (6)$$

where R is a constant that defines the packing ratio of the particles in the suspension (see Table 2). Since its introduction into the geological literature (Shaw, 1965; Marsh, 1981), ER has been used routinely by petrologists to assess changes in effective viscosity due to crystallization in both the plutonic and volcanic realm (e.g. Wickham, 1987; Vigneresse, 2008). What is generally less well known is that this and other similar expressions (see Table 3) are derived for dilute or semi-dilute systems where particles generally comprise <20% of the total suspension volume, and where single or two-particle interactions dominate. There are in fact *several hundred* empirically-derived expressions in the literature that show through extrapolation from low concentrations how the relative or effective viscosity of a suspension might change with increasing particle (volume or number) content (Table 3). All seem valid for the case in hand so why choose one over any other? Is one clearly better than the rest? Why has ER become the main choice for geologists who want to model magmatic processes? Indeed, are any of them any good when applied to a material as complex as magma? You know now where the title for this review came from.

The essence of the problem is this. Suspensions contain two independent but mutually coupled parts – the liquid (or melt) phase, also referred to in engineering treatments of two-phase flow as the carrier phase, continuous phase, single phase or solvent, and the solid particles themselves. In a magma, these will be predominantly crystals of various size and shape, either precipitated directly form the melt due to cooling or decompression effects (Blundy and Cashman, 2001), or entrained xenocrysts from elsewhere. More often than not it is a mixture of the two.

Although the types of primary crystals contained in magma clearly depend on the initial melt composition, crystal chemistry does not have a significant effect on overall magma rheology as the particles themselves are generally hard (meaning they are essentially undeformable on short timescales). Crystal shape is important however, as it impacts directly on the microstructure (see below) of the suspension. Other complicating factors likely to be of local consequence in the magmatic realm include surface-tension effects, Van der Waals forces, the elasticity of particles during collision, grain-boundary roughness and the presence of thin thermal or chemical boundary layers surrounding growing crystals where average melt properties are likely to differ. Only the effect of particle roughness on magma rheology will be considered further. An additional point to bear in mind when examining different effective viscosity formulae is that for good reason they are often normalized to a theoretical maximum packing fraction ϕ_{max} (Table 2). However, where magma or lower-temperature materials such as concrete are flowing, and particles start to interact with each other *via* clumping, jamming or dilation in a non-constant shear field, the idea of R in equation 6 as a constant is not well justified. Indeed, this is seen by some as a serious flaw in predictive models of suspension rheology (Wildemuth and Williams, 1984).

Finally, we may well ask why it is so important to pin down as precisely as possible the effective or relative viscosity of magmas? One practical reason is to do with the rate of magma transport in dykes and sills. It is well known that the speed of a fluid moving in a conduit of half-width w is inversely proportional to the fluid viscosity. As an illustration, Table 4 summarizes the range in

TABLE 3. Summary of selected expressions from the geological and engineering literature used to predict suspension viscosity (η). See effective.viscosity@bournemouth.ac.uk for more information on eqution variables and a link to downloadable calculations in Excel™ and Matlab.

Expression	Reference	Notes
1. $\eta = \eta_0(1 + \frac{5}{2}\phi)$	Einstein (1906)	Dilute: first theoretical formulation of effective viscosity. Valid for ultradilute concentrations.
2. $\eta = (1 + \frac{1/2[n]\phi}{1-\phi/\phi_{max}})^2$	Eilers (1943) (in Stickel and Powell, 2005)	Dilute-congested: Obeys both high and low concentration limits, normalized to ϕ_{max}. n = 2.5.
3. $\eta = \exp[\frac{2.5\phi}{1-k\phi}]$	Mooney (1954)	Congested: coefficient 2.5 from (1). Coefficient k determined experimentally.
4. $\eta = (1 - \frac{\phi}{\phi_{max}})^{-[n_0]\phi_{max}}$	Krieger and Dougherty (1959)	Congested: predicts viscosity increase by adding particles to a pre-formed suspension. n = 2.5.
5. $\eta = \eta_\infty + \frac{\eta_0-\eta_\infty}{1+aD}$	Krieger and Dougherty (1959)	Bimodal particle distribution that explicitly factors in shear rate.
6. $\eta = 1 + 2.5\phi + 10.05\phi 2 + 2.73 \times 10^{-3}\ \exp(16.6\phi)$	Thomas (1965)	Dilute-congested: informs Costa (2005).
7. $\eta = \eta_0(1 - \phi R)^{-2.5}$	ER (after Shaw, 1965)	Dilute-congested: common usage in petrology.
8. $\eta = 1 + \frac{9}{8}\frac{(\phi/\phi_{max})^{1/3}}{1-(\phi/\phi_{max})^{1/3}}$	Frankel and Avricos (1967)	Congested: based on lubrication theory.
9. $\ln\eta = \frac{\alpha d}{\left(\frac{\phi_{max}}{\phi}\right)^{0.33}-1} - 0.15$	Sherman (1968) in McBirney and Murase (1984)	α constant (0.011) varying with mean particle size *d*.
10. $\eta = (1 - \phi/\Psi_M)^{-5/2},\ \Psi_M = 1 - \phi[1 - \phi_0]/\phi_0$	Spera (2000)	Modification to ER using bimodal particle size factor Ψ_M.
11. $\eta = \frac{1+\left(\frac{\phi}{\phi_*}\right)^\delta}{\left(1-\alpha\,\mathrm{erf}\left\{\frac{\sqrt{\pi}\phi}{2\alpha\phi_*}\left[1+\left(\frac{\phi}{\phi_*}\right)^\gamma\right]\right\}\right)^{B\phi_*}}$	Costa (2005)	Variables α, δ, γ from experiment, B = 2.5.
12. $\log\eta = -0.993 + 8974/T - 0.543\ \log\dot{\gamma}$	Lavallée *et al.* (2008)	Non-Newtonian flow law for crystalline dome magmas valid for $10^{-6} < \dot{\gamma} < 10^{-3}\ s^{-1}$.
13. $\eta = \frac{3\eta_0\phi N_c}{40}\frac{d}{h}$	Davis *et al.* (2008)	Congested: based on Torquato *et al.* (2000) where $h << d$ (see Fig. 2).
14. $\eta = \eta_0\left(1 - \phi\frac{\phi_{m0}+A\tau^{-m}\phi_{m\infty}}{\phi_{m0}\phi_{m\infty}(1+A\tau^{-m})}\right)^{-2.5}$	Petford, this study, after Wildemuth and Williams (1984)	Congested: maximum packing fraction shear-rate dependent.

relative viscosity predicted from a random selection expressions listed in Table 3 as a function of increasing degree of congestion. The effect of increasing particle loading is clear (viscosity increases) and the mean flow velocity will reduce accordingly. However, growing uncertainty in the value of η at high particle concentrations means that calculated flow velocities are highly variable (all else being equal). Using the median relative flow velocity for each value of ϕ, an estimate of the overall percentage drop in velocity from the initial value can be made. The estimated drop-off in flow velocity is striking (although not unexpected), as is the large variation at fixed ϕ. Even if we stick with just ER, the relative viscosity, and hence the estimated magma velocity, is rather sensitive to the choice of (monodisperse) packing fraction used (Table 5).

Effective viscosity and suspension microstructure

The problem of how to treat the effect of particle loading on the viscosity of a suspension is not unique to geology. In the first theoretical investigation of its kind, Einstein (1906) showed that for dilute mixtures ($\phi < 0.03$) consisting of individual, smooth, equal-sized particles where the grain packing is so remote that their motion is effectively that of a single particle in an infinite fluid, the effective viscosity (η_e) of the suspension is:

$$\eta_e = \eta_0\left(1 + \frac{5}{2}\phi\right) \tag{7}$$

This insightful relationship linking particle content and effective viscosity forms the reference point from which most subsequent studies of suspension rheology, both experimental and more latterly numerical, are based. Many second-order extensions to equation 7 have been proposed subsequently as refinements (see for example Batchelor and Green 1972*a*,*b*), and the coefficient 5/2 turns up frequently as an exponent (2.5) in treatments of concentrated suspensions. However, for the latter case, simple extrapolation of dilute systems to model the macroscopic properties of a suspension as the volume fraction or particle number density expands is problematic for several reasons. One is that for dilute systems, second-order (and higher) refinements fail to capture the true nature of congested suspensions because the inevitable interactions between particles during flow are not accounted for. This is particularly acute in narrow channels, where dispersed particles are forced to come very close together so that particle-particle interactions become dominant, pointing again to the microstructure as a fundamental property of congested suspensions.

In igneous rocks, the suspension microstructure during flow is preserved (to some degree) in the final rock texture. Here, good progress has been made in imaging igneous textures in two- and three dimensions (e.g. Higgins, 2000; Mock and Jerram, 2005). The challenge now is to back out from these geometric data information that can be used either to model quantitatively the flow of magma directly, or to cross-check theoretical models and simulations where the suspension microstructure can be predicted.

Melt viscosity

A significant difference between modelling magma as a suspension as opposed to other geological materials such as sediments or pyroclastic flows relates to the viscosity of the suspending liquid or carrier phase (melt). Silicate melts are complex solutions in their own right

TABLE 4. Effect of variable predictions of suspension viscosity (from Table 3) on percentage reduction in mean magma flow velocity (constant melt viscosity) in a 1 m wide dyke.

ϕ	η	% reduction in relative velocity		
0.4	1–18	(0)	80	(94)
0.5	1–35	(0)	94	(96)
0.6	3–40	(70)	95	(97)
0.7	15–1000	(93)	98	(99)

TABLE 5. The effect of a single-valued maximum packing value on relative viscosity calculated using ER for a melt viscosity of 1 Pa s.

ϕ	η_r (ϕ_{max} = 0.52)	η_r (ϕ_{max} = 0.74)
0.4	39	6.98
0.45	150	10.4
0.51	19,500	18.6
0.60	∞	64.2
0.73	∞	47,100

whose viscosities are strongly dependent on chemical composition, temperature changes on cooling (Webb and Dingwell, 1990; Giordano *et al.*, 2008), and deformation rate which also induces thermal effects due to viscous dissipation (Tuffen and Dingwell, 2005; Hess *et al.* 2008). Thus, unlike sediments transported in water or air, where the fluid phase can be treated as a simple isoviscous fluid, the physical properties of the melt phase may exert a strong influence on overall suspension rheology that requires careful consideration. Indeed, for magmas, the melt phase itself may be non-Newtonian due to polymerization at high silica contents and/or local gradients in the concentration of dissolved volatiles. This is just one reason why magma may rightly be called the mother of all multiphase problems.

All fluids, irrespective of composition, deform continuously in response to an applied shearing stress, some more quickly than others. Thus, where a large shearing stress is applied to a fluid and the rate of deformation is rapid, that fluid is said to have a low dynamic viscosity (sometimes expressed as the kinematic viscosity which is the product of the dynamic viscosity divided by the fluid density). Where the dynamic viscosity of the fluid is proportional to shear stress divided by the rate of shear strain, the material is referred to as Newtonian. This is the most likely general case for silicate melts at low (dilute) crystal factions and low gas contents. Another important influencing factor, especially relevant in the volcanic realm, is the strain rate (Lavallée *et al.* 2008). Here, in a further complication, as melt deformation approaches the melt relaxation timescale, non-Newtonian behaviour is observed (Dingwell, 1995).

As the viscosities of silicate melts have been studied extensively over the last four decades, we will not dwell here in any detail other than to note one important recent development that has challenged a long-held notion relating to temperature effects. It is well known that silicate melt viscosity is temperature dependent and this behaviour follows a general Arrhenian relationship of the form:

$$\log \eta = \log A_0 + E/(2.0303RT) \qquad (8)$$

(Shaw, 1972). Indeed, until very recently, most melt viscosity models used this approximation exclusively (e.g. Scarfe and Cronin, 1986, Dingwell *et al.* 1993). However, using datasets that span a large compositional range, Giordano *et al.* (2008) have shown that silicate melts do not always follow simple Arrhenian behaviour and that their temperature-dependent viscosity is better modelled using the Vogel-Tammann-Fulcher (VTF) viscosity equation:

$$\log \eta_0 = A + (B/T - C) \qquad (9)$$

where A is a constant independent of composition and B and C are compositionally adjustable parameters comprising linear combinations of oxide components.

Along with temperature, dissolved volatiles (mostly H_2O but also CO_2 and other minor dissolved gas species) can play a significant role in reducing melt viscosity at fixed temperature and crystal content. The relationship between melt viscosity, temperature and H_2O is shown in Fig. 5 for a magma (SiO_2 56 wt.%), calculated using equation 9. As expected, both plots show melt viscosity increasing with decreasing temperature (Fig. 5*a*) and H_2O content (Fig. 5*b*). The combined effects of co-variation in $T - X - \phi$ are shown in Fig. 6 where the relative viscosity is calculated using ER and melt viscosity from equation 9. Changes in melt viscosity due to increasing H_2O content (Fig. 6*a*) and temperature (Fig. 6*b*) are relatively small compared to the three orders of magnitude increase in relative viscosity with particle number, clearly the single most important determinant in governing overall viscosity in congested magmas based on predictions using ER.

Congested suspensions

As the concentration of suspended particles (crystals) is raised above some critical point, magma rheology may deviate markedly and abruptly from a simple linear response to applied stress and strain rate. It then enters a far more interesting and complex rheology space where small changes in stress or strain rate can inflict large changes in flow behaviour. In this non-Newtonian state, magma is susceptible to a range of threshold-type behaviours that include development of a yield strength, particle jamming, dilatancy and structures formation including bifurcation, clumping and layering.

We are now in a position to consider how best to model the complex rheology of congested magmas. Rather than attempt third-order (and higher) corrections to the viscosity term as extrapolated from dilute suspensions, a more sensible way to proceed might be to tackle the problem by deriving formulations for the effective viscosity of densely packed suspensions directly. An example of a formula derived explicitly for concentrated suspensions ($0 \leqslant \phi \leqslant \phi_{max}$ is from Thomas (1965) which contains an exponential term:

$$\eta = 1 + 2.5\phi + 10.05\phi^2 + 2.73 \times 10^{-3} \exp(16.6\phi) \qquad (10)$$

but does not include explicit reference to the viscosity of the fluid phase and as such lacks utility with respect to the dilute case. For magmas, where melt viscosity is a key parameter, neglect of this term renders the formula incomplete. While this may be acceptable in some engineering applications, it is not so for magmas. Costa (2005) proposed a modified form of ER that attempts to deal explicitly with high particle concentrations. While successful in capturing some aspects of the

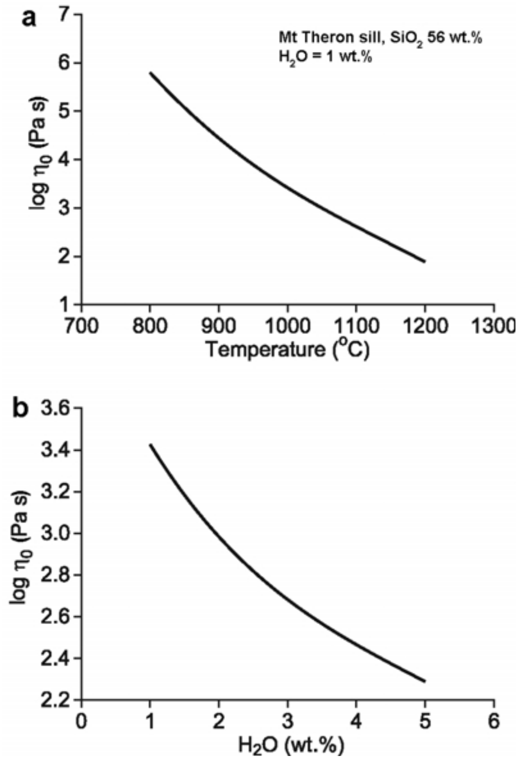

FIG. 5. Plots showing the relationship between melt viscosity (η_0), temperature and H_2O content calculated from equation 9 for a basaltic andesite sill from Mt Theron, Antarctica (data from Leat, 2008). (*a*) The melt viscosity drops from ~6 to 2 log Pa s over a temperature interval of 400ºC (H_2O fixed at 1 wt.%). (*b*) Increasing the melt water content from 1 to 5 wt.% results in a fall in melt viscosity from 3.4 to 2.3 Pa s (after Giordano *et al.*, 2008) http://www.eos.ubc.ca/~krussell/VISCOSITY/grdViscosity.html.

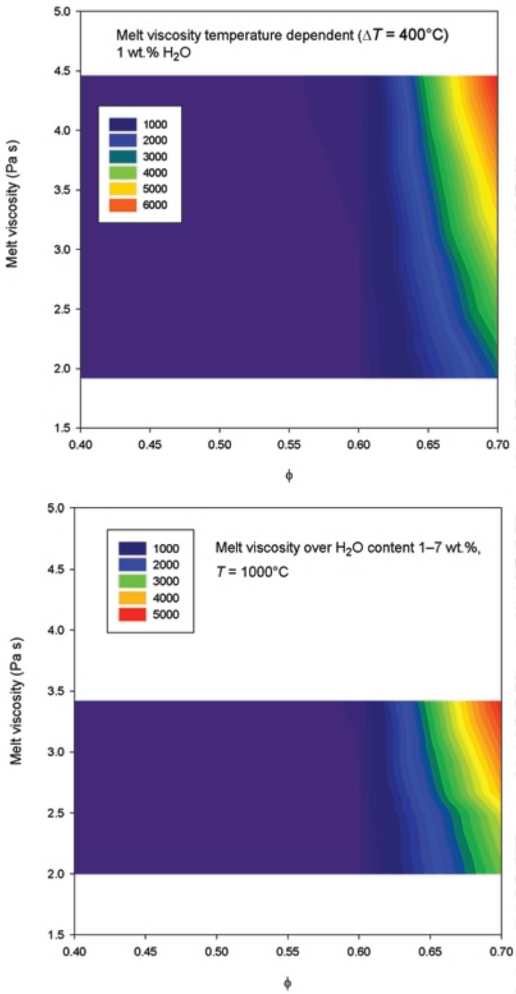

FIG. 6. Contour plots showing co-variation between melt viscosity, particle concentration (ϕ) and relative viscosity calculated using equation 6. Relative viscosity is shown as contours, with hottest colours corresponding to the highest values. Melt composition as in Fig. 5. Read the plots by matching the change in colour (relative viscosity) vertically at a given interval of ϕ. (*a*) Melt viscosity (equation 9) as a function of T (1 wt. % H_2O) over the interval $900 < T^oC < 1200$. (*b*) Melt viscosity as a function of H_2O (1–7 wt.%) at fixed T = 1000ºC. The effect of both temperature and H_2O seen through changes in melt viscosity are minimal in comparison with the extreme effect of increased crystal loading at values of $\phi > 0.6$.

relationship between solids fraction and viscosity, in its original form the Costa (2005) equation suffered several weaknesses including a tendency to saturate at high values of ϕ (the result of a non-linear term in the error function), meaning that the predicted effective viscosity at high particle contents reached a plateau value that was unrealistic. A more recent form, reported in Caricchi *et al.* (2007), is:

$$\eta = \frac{1+\left(\frac{\phi}{\phi*}\right)^{\delta}}{\left(1-\alpha \mathrm{erf}\left\{\frac{\sqrt{\pi}\phi}{2\alpha\phi*}\left[1+\left(\frac{\phi}{\phi*}\right)^{\gamma}\right]\right\}\right)^{b\phi*}} \quad (11)$$

However, like many other empirically-derived formulae based on incremental refinements, it is non-unique. Note that four key variables in the parameterization, α (range 0–1), δ (a track of increasing effective viscosity with increasing concentration), $\phi*$ (equivalent to the maximum packing fraction) and γ a measure of the speed of rheological transition due to particle loading (not to be confused with the shear rate) are unknowns and need to be supplied by experiment. B is the Einstein coefficient 5/2. In practice, this means simply that equation 11 can be tuned to fit any experimental data where the required variables have been measured, but cannot predict these values from first principles (see Fig. 8). That is, it says nothing fundamental about the physics of the problem.

From engineering, a different approach to modelling congested mixtures has been proposed by Torquato *et al.* (2000), based on the lubrication limit concept of particle interactions. This limit dominates when the surface contact distance between two particles (h) is much less than their diameter (d) (see Fig. 2). An approximate expression linking the ratio h/d to the solidosity is:

$$\frac{h}{d} = \frac{(1-\phi)^3}{12\phi(2-\phi)} \quad (12)$$

From equation 12 it is possible to derive an expression for the effective viscosity of a suspension based on lubrication theory where:

$$\eta = \frac{3\eta_0\phi N_c}{40}\frac{d}{h} \quad (13)$$

and N_c is the number of nearest neighbours in a 'cell' of clusterd particles (Davis *et al.* 2008). While N_c is the only modelling parameter in the theory, which was derived for oscillated slurries (see also discussion section), and it includes specific reference to fluid viscosity, the ratio d/h is a major unknown, and in magma is likely to fluctuate with time not only because of shear effects but also with crystallization.

Which effective viscosity?

Figure 7 shows mixture viscosity as a function of particle concentration for both initially dilute and congested suspensions. Curves (taken from Table 3) include the popular ER equation and several others especially relevant to congested mixtures including those of Thomas (1965) and Krieger and Dougherty (1959). Similar comparisons already exist in the literature (e.g. Rutgers, 1962; Thomas, 1965; Stickel and Powell, 2005) where the common factor is always the striking variation and scatter seen in the data at higher values of ϕ. It is this simple fact – the large differences between the values of suspension viscosities at high particle contents – that makes it difficult to select one single relationship that best describes the increasing viscosity of a suspension, magmatic or otherwise. Theoretical approaches may help, but they again suffer from several problems, the most fundamental of which is the determination of the microstructure during shear. This requires solution to the hydrodynamic equation for many-body interactions and other forces such as particle lubrication. Perhaps the

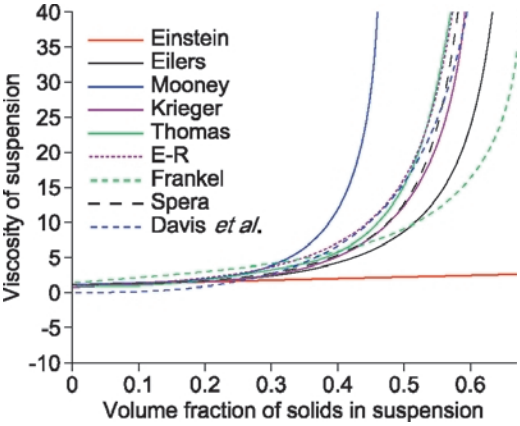

FIG. 7. Compilation plot of selected effective viscosity equations. Individual curves show a similar rapid increase in viscosity with increasing volume fraction of suspended solids ($0 < \phi < 0.3$) but collectively diverge where $\phi > 0.4$.

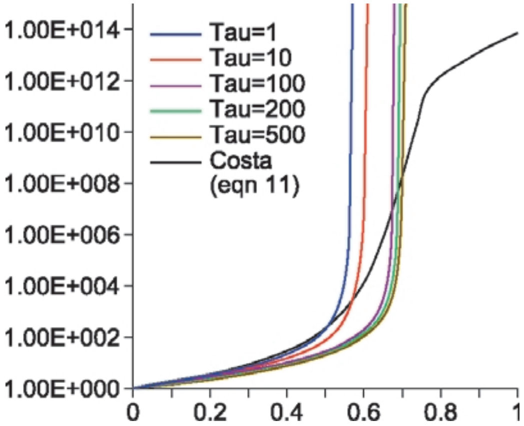

FIG. 8. Graph showing the effects of shear stress (τ, Pa) on relative viscosity where the maximum packing fraction (ϕ_{max}) is shear-rate dependent, predicted from equation 15. In this example, increasing the shear stress allows the suspension viscosity to remain finite between the limiting values ϕ_{m0} = 0.55 and $\phi_{m\infty}$ = 0.7. Within these limits, shear stress only exerts an influence at low values (up to 500 Pa), above which the suspension viscosity becomes insensitive to this parameter. Other variables in the model are η_0 = 1 Pa s, A = 1, m = 1. Also shown for comparison is a modified Costa (2005) curve (see equation 11) using experimentally-derived values for low-melt-fraction rocks (from van der Molen and Paterson, 1979) where α = 0.999918, ϕ = 0.673, γ = 98937, δ = 16.9386; B = 2.5.

biggest hurdle to developing a complete (quantitative) description of suspension flow is that the microstructure cannot be specified *a priori* and must instead be recovered directly from the analysis. As the microstructure determines the macroscopic flow behaviour, could this be done then the effective viscosity (and other relevant parameters including permeability, etc.) would be known for certain. Numerical simulation of the many-particle problem holds most promise here (e.g periodic lattice models) and Stokesian dynamics simulations (Brady and Bossis, 1988). Interestingly, these authors found that a bimodal microstructure comprising a random distribution of larger and smaller particle sizes resulted in lower effective viscosities at large particle concentrations when compared to monodisperse systems. This situation is more comparable with magmas (Fig. 3) where a range in crystal size distributions is the norm. One tantalizing consequence is that the effective viscosities of magmas as bimodal suspensions may be lower than those estimates based on monodisperse spheres (see also Spera, 2000). If so, then crystal-rich magmas may actually be more fluid at higher crystal contents (where the fluidity is expressed as $1/\eta_e$) than the curves in Fig. 7 would suggest. Clearly, more work is needed on the rheology of bimodal suspensions to make sure the viscosity values as applied to magma are not overestimates. Also, by recourse to Table 3 and Fig. 7, an approach that attempts to optimize the problem through a systematic analysis of the key variables controlling magma viscosity and their co-relationships (melt viscosity, volatile content, temperature, shear-rate and particle concentration and size distribution) may also shed light on which effective viscosity model, ER or otherwise, best captures the rheology of deforming magma where variables are constrained by experiment (Picard *et al.* 2008).

In summary, it appears that even those expressions derived directly from initially congested mixtures appear to lack the sophistication required to capture fully the rheology of magma during shear, either because they cannot account for gradients in melt viscosity, shear dependency, or both. One is drawn to the conclusion that an empirical, ‘top down’ approach to the problem will only ever result in incremental refinements and that a new approach that tackles the problem at a fundamental ‘bottom-up’ level is needed.

Towards a temperature- and shear-dependent magma viscosity

As a first step towards deriving a more complete description of the temperature- and shear-rate-dependent viscosity of magma, the following analysis is offered. It is fully acknowledged that, by default, its empirical base renders it approximate at best. We begin with ER as an initial start point as it incorporates the necessary reference to melt viscosity (η_0). As shown in Fig. 5, this term can be approximated independently as a function of T and X using equation 9. The next step is to accommodate the effects of an evolving microstructure as shear commences, i.e. to move away from a single valued maximum packing fraction (R) and derive an expression where the relative viscosity is itself a function of shear stress. To this end, Wildemuth and Williams (1984) proposed the following relationship for a shear-dependent maximum packing fraction:

$$\phi_{\max} = \left[\frac{1}{\phi_{m0}} - \left(\frac{1}{\phi_{m0}} - \frac{1}{\phi_{m\infty}}\right)\left(\frac{1}{1 + A\tau^{-m}}\right)\right]^{-1} \quad (14)$$

where ϕ_{m0} and $\phi_{m\infty}$ are limiting values relating to the maximum packing value and τ is the shear stress. The final step is to combine these descriptions to form a single equation such that $\eta_r = f(\phi, \eta_0, \Delta\phi_{\max}, \tau \text{ or } \dot{\gamma})$. This has been done by substituting equation 14 into equation 6 to give the following expression of relative viscosity:

$$\eta_r = \eta_0 \left(1 - \phi \frac{\phi_{m0} + A\tau^{-m}\phi_{m\infty}}{\phi_{m0}\phi_{m\infty}(1 + A\tau^{-m})}\right)^{-2.5} \quad (15)$$

that retains the original ER melt viscosity term but where R in equation 6 is now replaced by $\phi_{\max}$ that is shear-dependent. At this time, the values of A and m are unknown for magmas and require validation by experiment, but from work on low-temperature suspensions $m > 0$ and $0.6 < A < 33$ Pa (Wildmeuth and Williams, 1984). An example of how equation 15 works is shown in Fig. 8. Here, the effect of increasing shear stress (0−500 Pa) on relative viscosity where the packing fraction $\phi_{\max}$ is now an interval between the low and high shear limits ϕ_{m0} and $\phi_{m\infty}$ respectively, is seen. By allowing for limited changes in particle microstructure during shear, the suspension continues to flow until a terminal packing volume ϕM is reached. It also points to the existence of a gradient in effective viscosity in magmas flowing in parallel-sided conduits. It is noteworthy that from the parameters used in the formulation, the greatest effect is where the shear stress is relatively low (<500 Pa), after which the shear-dependency effect becomes insensitive to further deformation. In simple conduit flow, shear stress is highest at the walls and tends to zero in the centre of the flow. From Fig. 8 it would thus appear that the mid-part of the flow, away from the walls where the magnitude of the shear stress is finite but non-zero, would contain those regions of magma most sensitive to shear-induced changes in maximum packing fraction.

Discussion: magmas as high-temperature slurries

This section of the review is more open ended and speculative. Its purpose is simply to raise awareness of ideas arising from the study of congested suspensions (dense slurries) that are beginning to make headway in the granular and micromechanics literature. Despite their potential relevance, the concepts of particle pressure and migration, jamming, force chains and clustering have yet to make a significant impact in the field of magma rheology. It may be that these ideas are not considered widely relevant, or that their technical nature, of which a flavour is given below, is off-putting. However, for those like me who view magmas as exotic, high-temperature slurries, they have much to offer. One potential obstacle in thinking about magma as dynamic, multiphase fluid is that petrologists have, with good reason, stuck close to the rules of equilibrium thermodynamics of phase equilibria and nucleation in seeking to explain the origin of complex and sometimes contradictory field relationships preserved in igneous rocks. However, from a purely mechanical perspective, non-equilibrium behaviour during flow and transport of congested magma is not only expected, it provides an internal constraint with potential to reveal new and unexpected insight.

Magmatic layering in the Basement Sill, Antarctica

The Basement Sill is a ~300 m thick intrusion that forms part of the Ferrar dolerite Large Igneous Province exposed in spectacular detail in the McMurdo Dry Valleys, Antarctica. The system comprises a vertical stack of four interconnected sills linked to surface flows of the Kirkpatrick flood basalts. The lowermost intrusion, the Basement Sill, offers unprecedented exposure through a remnant magmatic slurry consisting of abundant orthopyroxene (opx) phenocrysts (Fig. 9) and showing well developed mesoscale rhythmic layers of plagioclase. The overall geometry of the axially confined slurry is tongue-like, with the sill margins relatively aphyric (Marsh, 2004). Critically, the nature and degree of exposure allows petrographical and structural observations to be made that have bearing upon flow rheology and emplacement. These include those variables set out in equation 1 (grain size, grain roughness, number density, particle mass density and estimated fluid properties of the carrier phase), in this case basaltic silicate melt preserved in the chilled margin of the sill. We thus have all the ingredients needed to apply some key aspects of the theory of dense suspension flow to the transport and deformation of congested magma.

FIG. 9. Photo montage from the central region of the Basement Sill, Antarctica showing the nature of the densely packed, granular OPX tongue (insert) and rhythmic layering defined by thin bands of plagioclase (main picture). The layering may reflect particle-fluid segregation during shear thinning accompanying high particle Péclet number flow. If so, the layering is a primary feature related to magma emplacement as opposed to post-emplacement crystallization processes.

The particle Reynolds and Péclet numbers (equations 3 and 4) provide an entry point (e.g. Sumita and Manga, 2008). By supplying appropriate values for crystal size (d = 1 mm), melt viscosity (η_0 = 1 Pa s), temperature (1500 K), melt density (ρ_0 = 2700 kg/m^3) and shear rate ($\dot{\gamma}$ = 0.1 s^{-1}) obtained from a numerical model for the emplacement of the Basement Sill (Petford and Marsh, 2008), both the dimensionless particle Reyonlds ($Re_{\dot{\gamma}}$) and Péclet ($Pe_{\dot{\gamma}}$) numbers can be obtained. For the above parameters we get $Re_{\dot{\gamma}}$ ~3×10^{-4} and $Pe_{\dot{\gamma}}$ = 9×10^{10}. For comparison, using the same variables but with water as the liquid phase and a particle size of 0.1 mm, gives $Re_{\dot{\gamma}}$ ~10^{-3} and $Pe_{\dot{\gamma}}$ ~10^5. These large differences in the numerical value of the dimensionless numbers reflect the important role of particle size and carrier phase viscosity in their formulation. Critically, for suspensions to behave as non-Newtonian fluids and exhibit time-dependent behaviour (e.g. shear thinning or shear thickening) – important flow transitions that govern the onset of macroscopic structure development – shearing forces must be capable of deforming the particle microstructure. This happens where $Re_{\dot{\gamma}}$ $O(1)$ (Zapryanov and Tabakova, 1999). Based purely on dimensional analysis, suspensions are also generally shear-thinning where $Pe_{\dot{\gamma}}$ >> 1. Shear thinning comes about in congested slurries where the particles arrange themselves into layers generally parallel with the mean flow direction. This phenomenon has been observed both experimentally and in computer simulations (Stickel and Powell, 2005). Shear thinning comes about in the suspension due to changes in the microstructure, which evolves progressively to a more ordered state with increasing shear rate. This disorder–order transition is manifest as layering, aligned generally in the flow direction (Foss and Brady, 2000). The relationship between $Re_{\dot{\gamma}}$, $Pe_{\dot{\gamma}}$ and melt viscosity appropriate for a wide range of magma compositions is shown in Fig. 10.

By extension, the large Péclet numbers derived for the Basement Sill imply that during flow, the opx tongue magma slurry was, for part of its history just prior to cooling, flowing as a shear-thinning fluid characterized by the development of layering (Fig. 9). The origin of igneous layering and the controversies surrounding it need no introduction (e.g. Wager and Brown, 1968), but whether formed by magmatic sedimentation or fluid processes, most commentators agree that its development depends on protracted cooling, slow crystallization and the action of gravity on the component parts (e.g. Wadsworth, 1985). However, once magma in the Basement Sill had stopped flowing it would have cooled below its solidus in ~1 y (Marsh, 2004). Yet it still displays many of the classical features seen in much larger layered igneous intrusions (see Parsons, 1987). Perhaps then the observed layering is a direct manifestation of the shear-thinning effect in high *Pe* number fluids? The reduction in effective viscosity brought about by a disorder–order transition in the opx grains comprising the particle microstructure (Foss and Brady, 2000) would allow the congested magma to continue to move freely, providing a critical shear rate was maintained. The link between grain-scale segregation effects and macroscopic structures formation in the flow can now be made. Based on the analysis presented above and the short timescales to solidification, the tentative conclusion drawn is that the plagioclase-rich layers are a primary segregation feature that formed due to flow and shear of the magma and then frozen in during cooling. A similar explanation could apply equally well to other layered intrusions.

Shear-aided particle migration

A common observation from field studies of sills and dykes is the concentration of crystals towards the centre of the intrusion (e.g. Philpotts, 1990). This is generally referred to in petrology literature as flow differentiation or flow segregation (e.g. Komar, 1972; Best and Christiansen, 2001) and is widely recognized as a consequence of emplacement of crystal-rich magma. However, the physical process responsible for the phenomena has remained unclear. A similar effect is seen in industrial slurries. For example, it has been observed in both physical (Happel and Brenner, 1965) and numerical experiments on pipe flow (Nott and Brady, 1994), that the particles migrate towards the centre of the pipe, that is, the region where the shear rate is least (e.g. Koh *et al.* 1994; Krishnan *et al.* 1996). In continuum theories of migration dense suspensions either a diffusive formulation (Leighton and Acrivos, 1987), or a particle-pressure formulation, McTigue and Jenkins (1992), may be used. The latter theory employs a particle pressure derived from the so-called granular 'temperature', a scalar quantity that is a measure for the fluctuational motion of the particles in the flowing mixture. The necessity

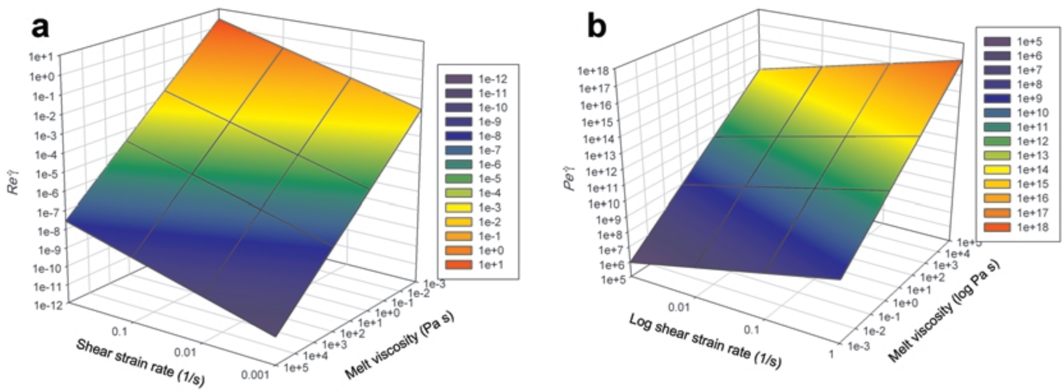

FIG. 10. Plots showing the relationship between melt viscosity, ranging over eight orders of magnitude, log shear strain rate ($\dot\gamma$) and the dimensionless particle Reynolds ($Re_{\dot\gamma}$) and Péclet ($Pe_{\dot\gamma}$) numbers as defined in equations 3 and 4 with d = 1 mm, ρ_0 = 2700 kg/m^3, and T = 1500 K. (*a*) $Re_{\dot\gamma}$ decreases with increasing shear strain rate and increasing melt viscosity. Colour bar shows changing $Pe_{\dot\gamma}$ number. (*b*) $Pe_{\dot\gamma}$ increases with increasing shear strain rate and melt viscosity. Colour bar shows changing $Pe_{\dot\gamma}$.

of fluctuations is obvious in dense flowing mixtures in shear as no interacting particle pair can persist in a linear path; they need to avoid one another (see Fig. 2).

The theory requires a balance equation for the fluctuational energy. This balance equation is coupled to a Fourier-type 'granular heat' equation. The physical concepts behind the theory are therefore quite similar to those of a dense gas, see Chapman and Cowling (1990) and also to those of dry dense granular flow (Jenkins and Mancini, 1989; Gidaspov, 1994). The key particle stress parameter is obtained from a presumed asymmetry in the approach and departure of particle pairs; the particle pressure is then proportional to the root of the granular temperature. The bulk fluid motion is not considered, but may be added by phenomenological means as a refinement. Boundary conditions for the slip velocity of particles and the granular heat flux at the solid boundary are also required. The latter have been derived for a smooth boundary (McTigue and Jenkins, 1992), but not for rough boundaries, a more realistic natural situation where magma is flowing in dykes and sills with irregular walls. A solution to this problem was obtained for the first time by Petford and Koenders (1998) for granitic magmas flowing vertically in a dyke. They showed that the accompanying fluctuation in velocity intensity resulted in a granular velocity fluctuation intensity that was greatest closest to the dyke walls, as expected from theory. The outcome supports the notion that high granular 'temperatures' are obtained near the dyke walls, while low ones appear at the centre, thus allowing diffusion of suspended crystals towards the axis of the conduit (see also Petford, 2003). Specimen results are shown in Fig. 11 for three initial solidosities (ϕ) of 10, 20 and 25%. While various aspects of the theory need further investigation, verification and refinement (as shown by Shapley *et al.* (2002), a key assumption is the isotropy of the granular temperature), the final position-dependent solidosity is greatest at $y = 0$ (the dyke centre). A further measure of the ability of the flow to settle into a solidosity distribution is given by the Bagnold number (*Bg*), defined experimentally (Bagnold, 1954) as:

$$Bg = \frac{\rho d^2 \sqrt{\phi}}{\eta_0} \dot{\gamma} \qquad (16)$$

The Bagnold number gives an impression of the ratio of collisional to viscous forces in a sheared suspension and is useful in distinguishing between macroviscous and grain inertia domains, with the caveat that in the original experiments, only a single particle size was used.

Table 6 shows the order of magnitude vales of the Bagnold number for three different types of magma, defined by their dynamic viscosity and flowing at a constant shear rate of 0.01 s^{-1}. The small values (<<1) even at high packing contents imply that viscous forces dominate. According to Iverson (1997), $Bg < \sim 40$ defines the macroviscous regime while $Bg > \sim 450$ indicates collision-dominated flow. In comparison with more classical inertial-type theories for dry granular flows, the low *Bg* number approximations point to viscous forces (interstitial fluid viscosity) playing a dominant role in governing inertial effects in magmas, reflecting the lubrication theory scaling factor $\eta_0 d/h$ (McTigue and Jenkins, 1992).

Particle pressure

The particle migration effects described above are associated with some form of partial stress in the

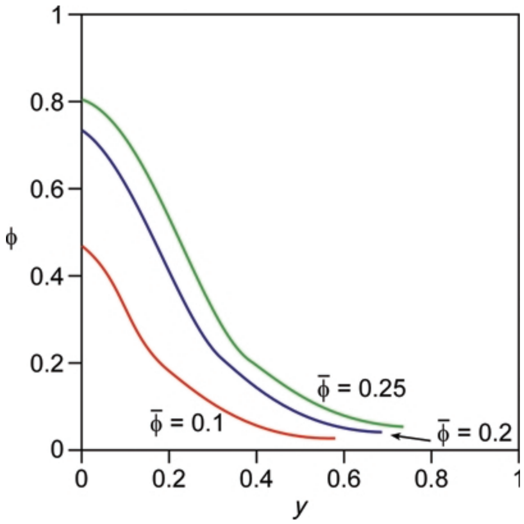

FIG. 11. Simulation plot showing the final position-dependent concentration of particles (ϕ) in the cross-stream direction (y) of a flow with three initial average particle concentrations of 10, 20 and 25%. Maximum concentrations in the centre of the flow ($y = 0$) range from 45 to 80%, indicating that particle migration (flow differentiation) has occurred in the simulation (from Petford and Koenders, 1998).

TABLE 6. Bagnold number (Bg) for selected magmas, terrestrial and planetary with a grain size = 1 mm flowing in a channel 1 m wide. The low values of Bg imply that viscous forces dominate during flow.

Basalt ($\eta_0 = 10^1$ Pa s)	Bg
$\phi = 0.0555$	5.9×10^{-7}
$\phi = 0.7405$	2.1×10^{-6}
Rhyolite ($\eta_0 = 10^5$ Pa s)	
$\phi = 0.0555$	5.9×10^{-11}
$\phi = 0.0745$	2.1×10^{-10}
Cryomagma ($\eta_0 = 10^{-1}$ Pa s)	
$\phi = 0.0555$	3.5×10^{-5}
$\phi = 0.7405$	1.29×10^{-4}

solid suspension phase. The concept of a particle pressure in a dense slurry was introduced by McTigue and Jenkins (1992). It arises when there is the possibility of net momentum transfer due to an asymmetry in the particle-particle interaction, in other words if approaching particles and departing particles do not sense equal-magnitude (opposite sign) forces. The nature of this symmetry-breaking effect relates to particle roughness (see p. 185). The total stress in a dense slurry (σ_{ij}) is a function of the intergranular stress and the fluid stress ($\bar{t}_{ij}$):

$$\sigma_{ij} = -\tilde{p}_{ij}\delta + \bar{t}_{ij} \tag{17}$$

where $\tilde{p}$ is the particle pressure:

$$\tilde{p} = \alpha^{4}\frac{\eta_0}{d}(\phi x)\sqrt{T} \tag{18}$$

It is the particle pressure that makes a dense slurry different from a fluid (e.g. He *et al.*, 2001). The particle pressure can lead to a fluctuating effective viscosity through a time-dependent microstructure, governed overall by the rate of shear. It has been shown recently that for densely packed suspensions, in addition to collisions, the stiffness is solidosity and stress-dependent and in an oscillated slurry the fluctuations in the strain and the fluctuations in the solidosity correlate in such a way as to give a non-vanishing mean value. This is a quadratic effect, proportional to the square of the solids' velocity fluctuations, but the coupling constant to the quadratic term is large, so that even for small fluctuations (corresponding to strain amplitudes of less than say 0.1) a substantial effect may be expected (e.g. Davis and Koenders, 2006).

Oscillating suspensions

Even if the particles experience enduring solid contacts during the agitation the intergranular forces are asymmetric due to frictional effects (Santra *et al.* 1996; Rosato *et al.* 2002). Therefore, in densely packed assemblies at low isotropic stress there will also be a particle pressure associated with oscillation. In a recent paper by Davis and Koenders (2006), these mechanisms are described; a mechanism including the making and breaking of contacts in the granular matrix is also included.

The collisional effect contributes to the intergranular stress particle pressure that can be written with a different formulation from that of equation 18 such that the particle pressure is proportional to the magnitude of the velocity gradient amplitude $|\partial\tilde{v}/\partial y|$. In this case $\bar{p}$ has the form:

$$\bar{p} = c\eta_0\left|\frac{\partial\tilde{v}}{\partial y}\right|\frac{d}{h} \tag{19}$$

where c is a coefficient of order of magnitude 0.1−0.8. Details of this theory are described in Jeffrey and Onishi (1984). Recently, Davis *et al.* (2008) applied the concept of particle pressure in oscillating slurries to the magmatic realm. Here it was shown that magma excited by earthquake activity would respond by partially fluidizing a dense crystal mush at base of a magma chamber on a timescale of seconds. A corresponding instantaneous decrease in pressure in the melt phase would result in *in situ* bubble formation and potential large-scale destabilization of the mush. The excess pressure due to new bubble formation in the magma after the earthquake provides a mechanism for promoting chamber-wide instability through over-pressurization of the magma. If the mush represents a cumulate layer, then large-scale disruption of the layer might be expected, similar to the deformation of wet sediments during seismic loading (e.g. Sumita and Manga, 2008). The resulting structures could include melt pipes and channels, disrupted layering and synmagmatic microfaulting. All these features have been described in some form from layered igneous intrusions around the globe (Parsons, 1987). Perhaps more significantly, segregation in oscillated slurries is near-instantaneous, providing a novel mechanism for extracting trapped intercumulus melt from the crystalline matrix.

Rough particles and cluster formation

In papers by, amongst others, Davis (1992), Zeng *et al.* (1996), Da Cunha and Hinch (1996) and Patir and Cheng (1978), it has been shown that the particle-particle interaction in a Newtonian fluid for rough particles (that is, spherical particles with asperities) is such that the particles may touch. The lubrication force is therefore finite as $h \rightarrow 0$. In the real world, crystals in magmas often have asperities; therefore, in order to describe realistic flow, some way of accommodating particle roughness and their hydrodynamic interactions is required (see Jenkins and Koenders, 2005). When particles touch they come together at a finite relative velocity; on impact a collision takes place, necessitating the consideration of two solid bodies in interaction. The latter introduces an asymmetry between incoming and outgoing force between a particle pair, which leads to the transfer of momentum during an interaction that produces particle stress. One consequence of an interactive force that is finite is that clusters of groups of particles can be created that move together like a 'super particle'. The implications of 'super-particle' formation on the macroscopic flow properties and rheology of magma remain to be explored. However, the idea is not so different from that proposed to account for layering in crystallizing magma chambers where clusters of crystals, as opposed to individuals, overcome density constraints and sink or float accordingly (Wager and Brown, 1968).

Jamming

Recent work by Cates *et al.* (1998) on sheared granular materials has shown that the close packing of congested particles can result in a local increase in rigidity referred to as jamming. However, sheared suspensions cannot become permanently jammed, and sudden changes in shear stress can produce non-equilibrium transitions from solid to fluid-like states. Suspensions showing this kind of behaviour are referred to as fragile, and it is tempting to conjecture that highly congested (locally rigid) magmas undergoing shear behave in an analogous fashion. Again, such transitional behaviour introduces new levels of time-dependent complexity that are extremely difficult to deal with quantitatively in macroscopic treatments of suspension flow. Ultimately, all natural magmas will 'jam' due to cooling and crystallization. But the fact that during flow, the packing density of the microstructure may fluctuate rapidly and unpredictably between solid- and fluid-dominated states may help explain often contradictory field relations observed for example during magma mingling (e.g. Fernandez and Gasguet, 1994; Hallot *et al.*, 1996) where ductile and brittle behaviour appears to have occurred simultaneously on small lengthscales (Fig. 12).

Reynolds dilatancy

Dilatancy in granular materials (Reynolds, 1885) can be thought of as the reverse of jamming. Instead of particles clustering closer together during shear, they move apart from each other. The resulting volume expansion (Reynolds dilatancy) is a curious and still poorly understood effect that is nonetheless an intrinsic feature of congested suspensions that reasonably include magmas (e.g. Mead, 1925; Petford and Koenders, 2003). Note that Reynolds dilatancy is not the same as the reversible shear-thickening behaviour shown in Fig. 1 and the situation envisaged here is one where magma is now mostly at rest and has started to develop a rheology typical of solidification fronts in cooling intrusions (Marsh, 1996).

The role of Reynolds dilatancy is to make new pore space available for the pore fluid, in this case viscous silicate melt. In deforming magma, the

FIG. 12. Triangular-shaped microdioritic enclave in pink granophyre host showing apparent simultaneous brittle and ductile rheology, with knife sharp left and right side margins (brittle failure) and a cuspate 'base' indicative of viscous mingling. This large rheological contrast on a small spatial scale may reflect abrupt changes in rigidity around a critical particle-melt threshold (jamming) in both enclave and host magma in response to an applied strain (Elizabeth Castle, Jersey, Channel Islands, UK).

effect of dilatancy is to reduce the intergranular pressure so that fresh melt is sucked into the expanding microstructure. In static magma undergoing cooling and crystallization, dilatancy will draw melt from the surroundings into the solidification front (for rheological definitions of static magma see Marsh, 1996). For essentially static magma this idea is not new. Emmons (1940) recognized that dilatant rifting of a dense crustal mush would draw melt into the voids in a process later referred to by Carmichael *et al.* (1974) as 'autointrusion'. A comprehensive analysis of Reynolds dilatancy and its implications for magma chamber processes is given in Petford and Koenders (2003), Koenders and Petford (2005) and Koenders and Petford (2007) for both pure and simple shear, based on an exact analytical solution to Biot's equations, modified to include dilatancy. The dilatancy phenomena at high temperatures has been observed experimentally in sheared metal alloys (Gourlay and Dahle, 2007) and more tentatively in high-*PT* deformation studies of silicate-FeS mixtures (Fig. 13). Other petrological consequences of shear-induced dilatancy in magmas with well-developed solidification fronts include the local suspension of non-buoyant crystals due to melt flow, particle rotation (fabric alignment), melt segregation pods comprising evolved liquid (Marsh, 2004) and local resorbtion and irregular zoning in crystals (Petford and Koenders, 2003).

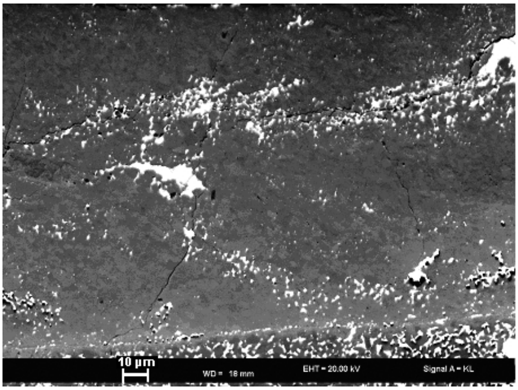

FIG. 13. Reynolds dilation. Section from a melting-deformation experiment (D-DIA, strain rate 6×10^{-4} s^{-1}, T = 1250ºC, P = 3 GPa) showing FeS melt (light grey) frozen into polycrystalline alumina pistons used to shear the sample (powdered olivine-FeS mix) located at the bottom of the picture. The texture is interpreted to show liquid FeS sucked out of the deforming sample and into the partially dilating piston. Similar textures preserved in igneous rocks may have an equivalent origin.

Cryomagmas

Thus far we have focused exclusively on high-temperature silicate systems. However, low-temperature forms also exist, and despite their exotic nature, this class of materials should behave in an analogous fashion during shear. While little is known at present about the rheology and flow properties of cryomagmas (Lewis, 1972; Kargel *et al.*, 1991; Mitchell *et al.* 2008), there is growing evidence that the icy satellites of Jupiter, Saturn and Neptune (Europa, Ganymede, Enceladus, Titan, Triton) have undergone periods of resurfacing in the recent geological past and may be cryomagmatically active today (Manga and Wang, 2007; Mitri *et al.* 2008). Table 6 suggests that despite the low viscosity of the carrier phase (a mixture of water ± ammonia), shear of particle-rich cryomagmas will be dominated by viscous forces (Petford, 2005). It is thus not unreasonable to suspect that many, if not all, of the processes and structures seen preserved in silicate rocks – including magma mixing – will also be present in volcanic and plutonic cryomagmas. That said, field verification may not be immediately forthcoming.

Summary

This review has addressed some of the problems currently hindering a more complete development of the physics underpinning magma rheology. While much progress has been made in understanding the viscosity of the carrier (melt phase), uncertainty still remains about how best to characterize the effective viscosity of highly concentrated suspensions, magmatic or otherwise. Numerical techniques for simulating multiphase flows that require computation of particle and fluid trajectories simultaneously and iteratively, are still in a relatively unadvanced state. The majority of empirical expressions that predict changes in the effective or relative viscosity of suspensions with increasing particle content have been developed by engineers on systems where the liquid viscosity is small (generally water or oil) and the particle size is also small (microns) and monodisperse. While deeply instructive, it is not yet clear how well their predictive, 'top-down' power holds up when translated into the realm of igneous geology where the viscosity of the melt phase is temperature and rate dependent and the crystalline phase is non-uniform in size.

Using the well known Einstein Roscoe equation, a new expression for predicting the relative viscosity of magma has been derived that makes some allowance for changes in the particle microstructure during shear. Initial results suggest that the analysis has predictive power where limiting values of particle packing fraction at low and high shear are known or can be approximated. Shear strain and its influence on microstructure development is also a key parameter in controlling high permeability pathways during the ascent of bubble-rich magma in volcanic conduits (e.g. Gonnermann and Manga, 2003; Okumura *et al.* 2008). A key future goal in developing a more complete understanding of magma rheology will be to link the development of particle microstructure in a temperature dependent carrier phase with the macroscopic flow behaviour. This upscaling will require a much better understanding of particle-particle interactions at the grain and cluster scale and how potential feedback mechanisms are associated with processes of jamming and dilatancy. Further complications arising due to interactions between particles and walls, and when phase changes take place in the carrier fluid, also require attention.

From a field perspective it may be useful to think of magma as a class of high-temperature slurry. At high crystal loads ($0.4 < \phi < 0.7$), a migration effect is expected during shear arising from fluctuations in particle velocity, resulting in flow differentiation. A further consequence of shear in dense magmatic suspensions at high particle Reynolds numbers is a processes of structures formation (alternating particle-rich and melt-rich bands) that have the potential to be preserved as igneous layering. Other mechanical instabilities, including clumping, jamming and dilatancy, can also be expected to influence the macroscopic flow properties that translate into fluctuations in velocity and associated thermal effects. Despite all this, conclusions drawn from uncritical application of fluid dynamical equations for suspension flow are unlikely to tell the full story. Indeed, for magmas, where the carrier phase viscosity is high ($\eta_0 >>1$ Pa s) and particle diameter large ($d > 1$ mm), the high Schmidt numbers ($Sc > 10^7$) indicate by recourse to conventional theory that flow may remain Newtonian over a larger window of shear rates that would be the case for low-temperature, low-viscosity suspensions. A move from incremental refinements of phenomenological effective viscosity models to one that deals with the problem from first principles is needed. Computer simulations that take a 'bottom-up' approach using *ab initio* molecular dynamics simulations first proposed by Heyes *et al.* (1980) may show the way.

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