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Conference:  *the 2nd Int. Conf. on Machine Intelligence* (ACIDCA-ICMI'05)

Type:  invited paper

Venue:  Tunisia

Data:  November 5-7, 2005

# A Compact and Complete AFMT Invariant with Application to Face Recognition

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***Abstract*— In this paper, we present a complete set of hybrid similarity invariants under the Analytical Fourier-Mellin Transform (AFMT) framework, and apply it to invariant face recognition. Because the magnitude and phase spectra are not processed separately, this invariant descriptor is complete. In order to simplify the invariant feature data for recognition and discrimination, a 2D-PCA approach is introduced into this complete invariant descriptor. The experimental results indicate that the presented invariant descriptor is complete and similarity-invariant. Its compact representation through the 2D-PCA preserves the essential structure of an object. Furthermore, we apply this compact form into ORL, Yale and BioID face databases for experimental verification, and achieve the desired results.**

## I. INTRODUCTION

In pattern recognition, an object description needs to be invariant under certain Euclidean transformations. In general, invariant description is achieved with the geometric moments, complex moments, and orthogonal moments, which are widely applied to invariant pattern recognition, object classification, pose estimation and image retrieval [8]. It is important that the invariants fulfill certain criteria such as low computational complexity, numerical stability and completeness [1-3]. The first two criteria are intuitively easy to be understood while the third criterion is generally ignored. Indeed the completeness is one of the most important criteria for an invariant descriptor. A complete invariant means that two objects have the same shape if and only if their invariant descriptions are the same. Whereas the invariant property is only relative to a certain transformation. It may only contain partial information of an object and not the whole information. Thus, invariant descriptions based on partial information of an object are usually prone to an incorrect recognition. It is clear that the complete set of invariant descriptions can effectively be used for accurate object discrimination.

Nevertheless, many familiar invariant descriptors are incomplete [4], such as geometric moments, complex moments, Legendre moments and Zernike moments. Although several sets of rotation and scaling invariant descriptors have been designed under the Fourier-Mellin transform framework [9], the completeness property could not usually be satisfied. This is because the phase spectrum was always ignored. In order to overcome this problem, Ghorbel in [3] proposed a complete set of rotation and scaling invariants under the analytical Fourier-Mellin transform (AFMT) based on the complete complex spectra. Furthermore, Brandt and Lin in [2,6] presented two phase invariants with respect to translation and developed a complete set of hybrid similarity invariants based on the Fourier-Mellin transform modulus (Note that this hybrid similarity invariant is the combination of a translation invariant with a rotation and scaling one.). We have analyzed the strengths and weaknesses of this hybrid similarity invariant descriptor in [5,7]. Its main problem is the scaling invariant problem under the Fourier-Mellin transform scheme, which can indeed be suppressed under the AFMT scheme. In this paper, we will devise a new complete set of hybrid similarity invariants under the AFMT scheme.

Recent applications have motivated a renewal of invariant recognition, for example, face recognition and image retrieval. The complete and compact invariant description can effectively improve the recognition performance. Since the completeness guarantees that no information is lost while the compactness reduces the redundancy data so as to improve the robustness of recognition. In general, the compact representation of features is fulfilled by the orthogonal subspace projection approach, for example, Principal Component Analysis (PCA), Independent Component Analysis (ICA) and kernel PCA. Turk and Pentland in [10] proposed the eigenface representation for face recognition based on PCA. [11,12] further analyzed how to measure the eigenface representation (i.e. by using the distance from feature space (DFFS) and the distance in feature space (DIFS)). Nevertheless, in the PCA-based face recognition, images are usually converted into 1D vectors. The obtained vectors of faces lead to a high dimensional vector space. This dimension problem brings about a large computational burden. In order to overcome this problem, Yang et al. proposed the 2D-PCA approach in [13], which is applied directly on 2D images so as to avoid this curse of dimension problem. In this paper, we will employ the 2D-PCA approach to our presented complete invariant descriptions, so as to get a compact and complete invariant descriptor.

The paper is organized as follows: a complete set of similarity invariants is first presented in Section 2. Then, the 2D-PCA based compact representation of complete invariants is presented in Section 3. Experimental results and analysis are shown in Section 4. Finally, our conclusions and future works are presented in Section 5.

## II. COMPLETE INVARIANTS

Invariant analysis is usually performed with respect to the similarity transformation group of rotation, translation and scaling. Because the group of Euclidean transformations (rotation, translation and scaling) is non-commutative, the translation invariant is usually separated from the rotation and scaling invariant. This section will first give a brief introduction to the complete invariant descriptors presented in [2,3,6]. This is then followed by our proposed hybrid complete invariants under the AFMT.

### A. *Translation*

**Taylor Invariant Descriptor [2]:**

With this invariant descriptor, the basic idea is to eliminate the linear part of the phase spectrum by subtracting the linear phase from the phase spectrum. Let $F(u,v)$ be the Fourier transform of an image $I(x,y)$, and $\phi(u,v)$ be its phase spectrum, i.e. $F(u,v)=|F(u,v)|\exp(j\phi(u,v))$. The following complex function is called the Taylor invariant,

$$F_T(u,v) = \exp(-j(au+bv))F(u,v) \tag{1a}$$

where, $a$ and $b$ are respectively the derivatives with respect to $u$ and $v$ of $\phi(u,v)$ at the origin (0,0), i.e. $a = \phi_u(0,0)$, $b = \phi_v(0,0)$. Because the Taylor invariant does not satisfy the property of reciprocal scaling, it can be **modified** as follows:

$$F_T(u,v) = \left(u^2+v^2\right)F(u,v)\exp(-j(au+bv)) \tag{1b}$$

It is proven that this complete translational invariant defined in Eq.(1b) is complete, rotationally symmetric and reciprocally scaled.

### B. *Rotation and Scaling*

**AFMT Complete Invariants [3]:**

In [3], the Analytical Fourier-Mellin Transform (AFMT) was adopted to construct a complete invariant to rotation and scaling. Different from the **Fourier-Mellin transform**, the AFMT adopts the **polar coordinate** instead of the **Log-polar coordinate**. It is defined by,

$$AF(u,v) = \frac{1}{2\pi}\int_{R^+}\int_0^{2\pi} I(r,\varphi)\exp(-jv\varphi)r^{c-ju-1}d\varphi dr, \quad c>0.$$

The inverse Analytical Fourier-Mellin transform is defined by,

$$I(r,\varphi) = \int_R \sum_{v\in Z} AF(u,v)\exp(jv\varphi)r^{-c+ju}du.$$

Under the AFMT, the rotation and scaling transform have a new representation, which is different from the translation property of the Fourier transform. The rotation and scaling in a polar coordinate, i.e. $I_1(r,\varphi) = I_0(\lambda r,\varphi+\beta)$, are transformed through the AFMT as follows,

$$AF_1(u,v) = \lambda^{-c+ju}\exp(jv\beta)AF_0(u,v) \tag{2}$$

It can be noted that the magnitude spectrum is no longer invariant to scaling because of the $\lambda^{-c}$ term. Several of the AFMT numerical algorithms were presented in [8].

The basic idea of the AFMT complete invariant in [3] is to eliminate the scaling term $\lambda^{-c+ju}$ and the linear phase $\exp(jv\beta)$ in Eq(2). On this basis, the **AFMT complete invariant** with respect to rotation and scaling is defined as follows,

$$AI(u,v) = AF(0,0)^{\frac{-c+ju}{c}}\exp(-jv\arg(AF(0,1)))AF(u,v). \tag{3}$$

### C. *Translation, Rotation and Scaling*

When considering the translation, rotation and scaling together, we can combine the translational invariant of Eq.(1) with the rotation and scaling invariant of Eq.(3) to construct a hybrid complete invariant. Note that the basic property of Eq.(2) is satisfied in the complex domain. Thus, the AFMT invariant can be applied to the Taylor spectra. But then, due to the reciprocal scaling property of the Fourier transform, i.e. $F(I(\lambda x,\lambda y)) = \lambda^{-2}F\left(\lambda^{-1}u,\lambda^{-1}v\right)$, when the property of Eq.(2) is applied in the polar domain of the Fourier spectra, it would be modified as follows,

$$AF_1(k,w) = \lambda^{c-2-jk}\exp(jw\beta)AF_0(k,w)$$

The AFMT invariant of Eq.(3) is also modified as follows,

$$AI(k,w) = |AF(0,0)|^{\frac{-c+2+jk}{c-2}}\exp(-jw\arg(AF(0,1)))AF(k,w)$$

By the combination of these invariants, we can construct a complete set of hybrid similarity invariants under the AFMT scheme as follows,

$$S(\cdot) = AI(F_T(\cdot)). \tag{4}$$

In addition, the intensity change usually results in the magnitude spectrum change of $AI(k,w)$. In order to overcome the illumination change problem, the magnitude of $AI(k,w)$ is quantified in the range [0,1].

## III. THE PROPOSED 2D-PCA BASED COMPLETE INVARIANT DESCRIPTIONS

The 2D-PCA approach was first employed to real images in [13]. Indeed, because a 2D complex spectrum is conjugate and symmetric, and the energy distribution is concentrated at the origin of the 2D spectral plane, applying the 2D-PCA to a complex spectrum can more effectively compress data compared to the application of the 2D-PCA to a real image.

For a 2D complex spectrum, $A \in C^{m\times n}$, one can project it onto a unitary vector, $X \in C^n$, i.e. $Y = (A-A_0)X \in C^m$, where $A_0$ is a standard form. In general, the trace of the covariance matrix of the projected feature vector can be used to characterize the total scatter of the projected sample difference. Let the covariance matrix of the projected vectors be,

$$\begin{aligned} B &= (Y-A_0X)(Y-A_0X)^H \\ &= (A-A_0)XX^H(A-A_0)^H \end{aligned}.$$

For a given unitary vector $X$, the covariance matrix $B$ should be a square matrix with $rank = 1$. Thus, we have, $tr(B) = X^H(A-A_0)^H(A-A_0)X$. Denote,

$$G = (A-A_0)^H(A-A_0),$$

which is called the spectral covariance matrix. Because $G^H = G$, thus, the eigenvalues of $G$ are real, and the eigenvectors corresponding to the different eigenvalues are orthogonal. Applying the PCA to $G$, we can obtain a set of orthonormal eigenvectors, which span the unitary subspace of $G$, i.e. using SVD, $G$ is decomposed as $G = U\Sigma U^H$ , and the optimal projection matrix $U_k$ consists of $k$ orthonormal eigenvectors corresponding to the first $k$ largest eigenvalues of $\Sigma$. The obtained set of projected feature vectors, $Y_i = (A - A_0)X_i, i = 1...k$ , is namely the set of the principal component vectors of the sample $(A - A_0)$ . These principal vectors form an $m\times k$ matrix $Y = (Y_1,...Y_k)$ .

The compact representation of the sample $A$, can hence be written as follows,

$$A' = YU_k^H + A_0 . \tag{5a}$$

However, if the 2D-PCA is introduced into statistic recognition, i.e. there is a training set of samples, the similar result can be achieved. Herein, the standard form $A_0$ is replaced by the mean $\overline{A}$ of the training set. The spectral covariance matrix $G$ can be evaluated by,

$$G = \frac{1}{M}\sum_{i}^{M}(A_i - \overline{A})^H(A_i - \overline{A}) .$$

Applying the PCA to $G$, we can obtain a similar form of the compact representation as follows,

$$A' = YU_k^H + \overline{A} . \tag{5b}$$

The similarity metric of this compact form can adopt the residual reconstruction error defined as follows,

$$\varepsilon = \left|A - A'\right|_2 = \left|(A - \overline{A})(I - U_k U_k^H)\right|_2 \tag{6}$$

## IV. EXPERIMENTS AND ANALYSIS

In this section, we pay attention to the following issues,

(1) completeness: When applying the hybrid invariant descriptor of Eq(4) to an object, one should obtain the complete invariant descriptions except that the similarity transform is eliminated;
(2) sensitivity: When the compact form of Eq(5) is applied to a hybrid complete invariant descriptor, the obtained compact form should be insensitive to the size of the complete invariant descriptions;
(3) recognition performance: When applying the 2D-PCA based complete invariant descriptor of Eq(5) to invariant recognition, such as face verification, we will evaluate its performance using a Receiver Operating Characteristic (ROC) curve.

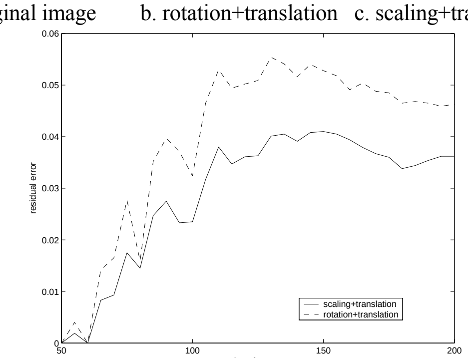
The first two issues can be illustrated by the variance of the residual reconstruction error. For comparison, we performed experiments on two scenarios, i.e. “rotation + translation” and “scaling + translation”. When the coefficient $k$ of the 2D-PCA is fixed, one can compute the corresponding residual errors while increasing the size of the complete invariant descriptions as shown in Fig.(1).

In Fig.(1d), one can note that the quantity of the residual error is very small. Thus the invariant descriptions of Eq(4) is complete. Furthermore, one can also note that the residual error rises with an increase of the size of the complete invariant descriptions. The residual error in Eq(6) results from numerical computation, interpolation and the loss of some image details. Theoretical completeness of the invariant set is best attained with a large set of invariant descriptions. However, the variance of the residual error is very small. Thus, the compact form of Eq(5) is insensitive to the size of the hybrid invariant descriptions.

With the evaluation of the performance, we tested our approach on ORL, Yale and BioID face databases [14-16]. For convenience, we only carried out verification tests, i.e. conduct a one to one comparison between a probe sample and a known template during each computation. The sample set includes 70 persons. Each person has 10 frontal face images with a slight pose and expression variance or illumination changes. All the face images are cropped to the same size, and preprocessed through the illumination correction approach in [12]. Note that in these face images of the used databases, the change in orientation and scaling is small compared to the one shown in Fig.1.

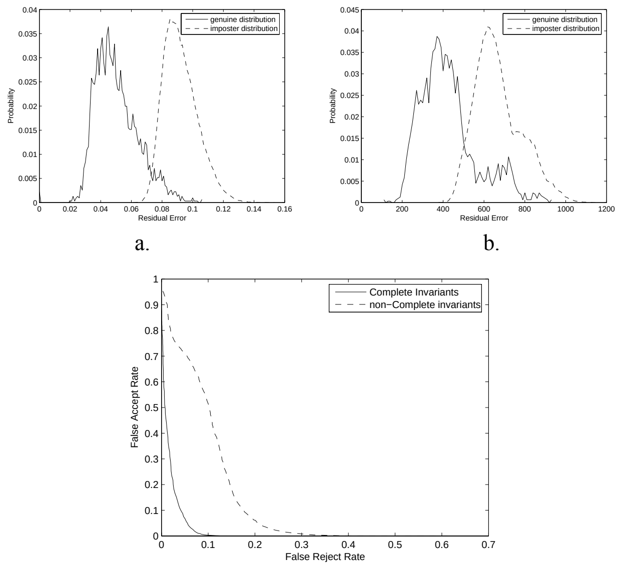
For comparison, we employed Eq(5a) to the hybrid complete invariants of Eq(4) and the original real images respectively, and generated their corresponding ROC curves. The genuine and imposter distributions are shown in Fig.(2a,2b). It is clear that the overlapping region (i.e. false probability) of the complete invariants is smaller than the one of the original images. Furthermore, their corresponding ROC curves relating to the False Acceptance Rate and the False Rejection Rate were depicted by changing the threshold as shown in Fig.(2c). It can be noted that the 2D-PCA based compact and complete invariant descriptor can effectively improve the accuracy of face recognition. For different applications, one can determine a desired threshold accordingly.

a. original image b. rotation+translation c. scaling+translation




d. residual error curve

Fig.1 (a-c) the original image is transformed by “rotation+translation” and “scaling+translation” respectively; (d) using Eq(6) to compute the residual error, where the $k$ of 2D-PCA is 30.




a. b.

c.

Fig.2 Recognition Performance results. a) are the genuine and imposter distribution curves of complete invariant descriptions; b) a) are the genuine and imposter distribution curves of original images; c) are their ROC curves relating to False accept rate and False reject rate.

## V. CONCLUSIONS

In this paper, we first presented a complete set of similarity invariants under the analytical Fourier-Mellin transform framework. Then we applied the 2D-PCA to this complete invariant descriptor to obtain a compact representation of the complete invariants. Moreover, we applied this 2D-PCA based complete invariant descriptor to face recognition. The experimental results indicate that the proposed 2D-PCA based complete invariant descriptor is robust to the slight pose and expression variance or illumination changes of faces.

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