An analysis between implied and realized volatility in the Greek Derivative market

George Filis
University of Portsmouth,
Portsmouth Business School,
Department of Economics, UK.
e-mail: George.Filis@port.ac.uk., tel: 0044 (0) 2392844828.

Abstract

In this article, we examine the relationship between implied and realised volatility in the Greek derivative market. We examine the differences between realised volatility and implied volatility of call and put options for at-the-money index options with a two-month expiration period. The findings provide evidence that implied volatility is not an efficient estimate of realised volatility. Implied volatility creates overpricing, for both call and put options, in the Greek market. This is an indication of inefficiency for the market. In addition, we find evidence that realised volatility ‘Granger causes’ implied volatility for call options, and implied volatility of call options ‘Granger causes’, the implied volatility of put options.

JEL: C22, C32, G10
Keywords: Implied volatility, realised volatility, Athens derivatives exchange, Granger causality

1. Introduction

In this study, we examine the relationship between implied volatility and realised volatility. There are several criticisms of implied volatility derived from the Black-Scholes Option Pricing Model (BSOPM). The most important criticisms concern the ability of implied volatility to predict realised volatility accurately (Jackwerth and Rubinstein, 1996). The mis-estimation of realised volatility can cause mis-pricing in the option market. Through this study, we add to the existing literature by using implied volatility for both call and put options, whereas most of previous studies have concentrated on call option implied volatility. The Greek derivative market is a new market with only 5 years of life. Any evidence of mis-pricing in the option contracts can cause serious problems to the underlying market and the option market. In addition any mis-estimation of the realised volatility could provide evidence that the market is not efficient. According to the BSOPM, if the market is efficient, then implied volatility should be an unbiased and efficient predictor of realised volatility. In this study we use implied volatility for the at-the-money call and put index options of the Greek derivative market (i.e. the Athens Derivatives Exchange – ADEX) and we test it against realised volatility.

2. Background of study

Many researchers (Cox et al, 1976, Hull and White, 1987, Kon, 1984, Rubinstein, 1985, Bodurtha and Courtadon, 1984, Heston, 1993b, Madan et al, 1998, Jiang and Van Der Sluis, 2000, Heston and Nandi, 2000), since the appearance of the BSOPM, have tried to relax some of the model's assumptions. One of these assumptions is constant volatility. These researchers have tried to show that stochastic volatility is independent of the stock price and so should be valued independently. In the case where the volatility is truly uncorrelated with the stock price, then BSOPM provides wrong estimates for the at-the-money options (overvaluation) and additionally for the deep out-of-the-money and in-the-money options (undervaluation).

Specifically, Hull and White (1987) tried to incorporate stochastic volatility into the BSOPM due to the volatility smiles that the constant volatility assumption caused. Hull and White showed that Black-Scholes volatility should be replaced by a stochastic volatility term, which would be instantaneously uncorrelated with the underlying asset. They argued that the mean variance ($\overline{V}$) of the stock over some interval of time [0,T] would equal the integral:

$$\overline{V} = \frac{1}{T}\int_0^T \sigma^2(t)dt \qquad \textbf{(1)}$$

Despite the fact that several variations of the model have been developed, there are still pricing problems with the BSOPM. The main reason for option mispricing is the implied volatility. Is implied volatility the correct measure to use? How can a market predict volatility if it is not efficient? These are some important considerations that arise from the model.

Further questions on the issue of implied volatility were posed by Chance (2003) such as: *"How can the option market tell us that there is more than one volatility for the underlying asset?"* and he replies: *"It does not"*. It can be realised from the above question how important the implied volatility problem is. Chance argues that the BSOPM is incorrect as it provides more than a single volatility for an option with the same underlying asset but different type (i.e. call or put), different expiration dates and exercise prices. To give an example of the implied volatility problem, assume that we know the volatility but not the option price. In this case, we would estimate the implied option price from the volatility. However, we would get more than one option price. How can an asset have more than one price at a specific time? Simply, it cannot (Chance, 2003).

Furthermore, according to Jackwerth and Rubinstein (1996), the BSOPM exhibits bias in the at-the-money option prices. Two reasons can explain such bias. The first one is that the implied volatility of the at-the-money option rarely equals the historic volatility. The second reason is the one we have mentioned before, i.e. the different implied volatilities for the same underlying asset in options with different strike prices and expiration dates (Rubinstein, 1994, Jackwerth and Rubinstein, 1996, Chance, 2003).

The gap between implied and realised volatility could also be considered as market inefficiency. If the market is efficient, then it should be able to predict the realised volatility, thus there should not be any significant difference between the implied and realised volatilities.

In addition, several other studies have also found evidence that implied volatility is a biased and inefficient predictor of realised volatility (Christensen and Prabhala, 1998, Neely, 2002, Doran and Ronn, 2004, Becker et al, 2006). The same conclusion was reached by Szakmary et al (2003), who studied 35 stock markets for the information content of implied volatility. Their findings are very significant due to the number of stock markets under examination. Overall, they concluded that there is no significant information incorporated in implied volatility.

Finally Koopman et al (2005) using the S&P 100 index from October 2001 until November 2003, found evidence that the realised volatility model performs significantly better than the implied volatility model, with regard to volatility forecasts.

3. The Athens Derivatives Exchange

The Athens Derivatives Exchange (ADEX) began to trade option contract on the high capitalization index (FTSE/ATHEX 20) of the Athens Stock Exchange (ATHEX) in September 2000.

[Table 1 HERE]

Table 1 shows that the market has significantly increased its operations from one year to the next. There is a huge increase in the transaction values of the market and in the number of investors. Since 2000, there ha been an increase of 6.7 times in the number of investors that trade in derivatives and the transaction values have increased by 94 times.

However, it is clear that the market is very new as it only trades 10 derivative products and the number of investors and the transaction values are very small compared to the traditional derivative exchanges such as CBOE and LIFFE.

Furthermore, by the time ADEX started to trade options in 2000, the ATHEX was still an emerging market. The ATHEX became a mature market in 2001. So, it is clear that there could be important implications in the underlying and the derivative market, if there is evidence of volatility mis-estimation.

4. Data and methodology

In this study we use daily data from the ADEX and the ATHEX in order to calculate the at-the-money call and put implied volatilities for the index options with two months expiration period and the realised volatility. The data are from January 2000 until January 2003. The underlying reason for choosing the at-the-money index options with two months expiration period is simply because they are the most heavily traded options in the Greek market and thus they will provide the most significant results.

We calculate the implied volatilities from the Black-Scholes Option Pricing Model using the following approximation for call option:

$$\sigma_{ICt} \approx \frac{\sqrt{2\pi / t}}{Se^{-yt} + Xe^{-rt}} \left(C - \frac{Se^{-yt} - Xe^{-rt}}{2} + \sqrt{\left(C - \frac{Se^{-yt} - Xe^{-rt}}{2} \right)^2 - \frac{\left(Se^{-yt} - Xe^{-rt} \right)^2}{\pi}} \right) \quad \textbf{(2)}$$

where, Se^{-yt} is the index price level discounted with the annualized daily dividend yield, Xe^{-rt} is the discounted strike price and C is the call premium for the at-the-money index option with two months expiration date. The implied call option volatility will be shown as *IVC*.

For the put option's implied volatility we use:

$$\sigma_{IPt} \approx \frac{\sqrt{2\pi / t}}{Se^{-yt} + Xe^{-rt}} \left(P - \frac{Xe^{-rt} - Se^{-yt}}{2} + \sqrt{\left(P - \frac{Xe^{-rt} - Se^{-yt}}{2} \right)^2 - \frac{\left(Xe^{-rt} - Se^{-yt} \right)^2}{\pi}} \right) \quad \textbf{(3)}$$

where, Se^{-yt} is the index price level discounted with the annualized daily dividend yield, Xe^{-rt} is the discounted strike price and P is the put premium for the at-the-money index option with two months expiration date. The implied call option volatility will be shown as *IVP*.

The realised volatility will be calculated based on the following formula:

$$\sigma_{Rt} = \sqrt{\frac{1}{T}\sum_{i=1}^{T}\left(r_i - \overline{r}_T\right)} \tag{4}$$

where T is the number of days to expiration, r_i is the return on a particular day, $\overline{r}$ is the average daily return over the option's life. Additionally, the realised volatility has been annualized using the following formula:

$$\sigma_{ARt} = \sigma_{Rt} \times \sqrt{250} \tag{5}$$

We use the number 250 to annualize the realised volatility, as the number of trading days for the each of the years was 250. The annualized realised volatility will be shown as *RV*. We use a single realised volatility, which is tested against the call and put implied volatility, as according to the BSOPM and the put-call parity, call and put options with the same underlying and expiration period, should have the same volatility.

Using the above calculation for realised volatility, we compute an *ex-post* measure of volatility, whereas the calculation of implied volatility represents *ex-ante* implied volatility. This approach will allow us to test the predictive ability of implied volatility.

5. Empirical findings

5.1. Summary statistics and correlation matrix

In Table 2 we present the summary statistics of the time series that will be used in the study. *RV* is the realised volatility, *LRV* is the log realised volatility, *IVC* is the implied volatility for the call options, *LIVC* is the log implied volatility for the call options, *IVP* is the implied volatility for the put options and *LIVP* is the log implied volatility for the put options.

[Table 2 HERE]

It is important to notice that the implied volatilities have higher average values than the realised volatilities. The logged values also show the same pattern, i.e. higher mean values for the implied volatilities and lower for the realised volatility. Overall, all variables (except *LIVP*) show non-normality, as evidenced by the measures of skewness and kurtosis, and the Jarque-Bera test statistic.

Table 3 shows the correlation coefficient between the variables.

[Table 3 HERE]

As was expected, all coefficients are positive and they show moderate to strong correlation among the variables. In addition, all correlation figures are highly significant. It is very interesting that the realised volatility exhibits higher correlation with the call option implied volatility than with the put option implied volatility for both the level and logged values. Furthermore, the correlation between the two implied volatilities (call and put) is moderately positive. However, if the implied volatility estimation was an efficient measure, then the two implied volatilities should exhibit very high positive correlation.

5.2. Testing median differences between implied and realised volatility

We use the Wilcoxon Signed Rank test (due to the non-normality of the data) to check whether the average (i.e. median) logged implied volatilities ($\overline{LIVC}$ and $\overline{LIVP}$) are significantly different from the average logged realised volatility ($\overline{LRV}$). The results are shown in Table 4.

[Table 4 HERE]

As we can observe, the W-statistics for both pairs of data are highly significant at the 1% level. So, we are able to say that there is a significant difference between the realised and implied volatilities for both call and put options. A reason for this result could be that the implied volatility is not an efficient predictor of the realised one. However, it could also be an indication of inefficiency in the Greek market. In addition, such a significant difference between implied and realised volatilities is an indication of mis-pricing with respect to realised volatility.

5.3. Testing implied volatility for bias and inefficiency

The information content of implied volatility can be assessed by estimating a regression equation using realised volatility as the dependent variable and implied volatility as the independent variable (Christensen and Prabhala, 1998). So, we estimate the following regression equations:

$$LRV_t = a_0 + a_1 LIVC_t + e_{t1} \qquad \textbf{(6)}$$

$$LRV_t = b_0 + b_1 LIVP_t + e_{t2} \qquad \textbf{(7)}$$

Based on these equations, we are able to examine the following hypotheses. The first concerns the information content of implied volatility. If implied volatility contains information about future volatility, then we should have $a_1 \neq 0$ and $b_1 \neq 0$. In addition, we should find that $a_1 = 1$ and $b_1 = 1$ if implied volatility is an unbiased estimator of realised volatility and the constants should not be significantly different from zero (i.e. $a_0 = 0$ and $b_0 = 0$). Finally, if implied volatility is an efficient estimator, then the error terms should be white noise, i.e. they should have a mean of zero and they should be uncorrelated. However, we will perform an ADF unit root test prior the regression estimation. The ADF-test results for unit root in the variables are shown in Table 5.

[Table 5 HERE]

From the ADF unit root test, we are able to find strong evidence that the implied and realised volatilities for put and call options are stationary.

Following the ADF unit root test, we perform the regression analysis. Based on the observation from 1/2000 until 1/2003, the regression results indicate that the estimated coefficients of *LIVC* and *LIVP* are significantly different from zero, which suggests that they do contain some information regarding future volatility.

[Table 6 HERE]

However as the variables are significantly different from one and the constants are significant different from zero, we can argue that the implied volatilities are biased predictors. Furthermore, the constant term is negative for both regressions. This finding implies that when the implied volatility (either for the call or put options) is low, the realised volatility is higher and vice versa.

The R-squared is higher for the call implied volatility equation compared to the put implied volatility equation. This is an indication that the predictive power of the call options is higher than the predictive ability of the put options. If the market was able to predict volatility correctly, then there should not be any difference in the predictive abilities between call and put implied volatilities. In addition, if implied volatility was an unbiased predictor of realised volatility, then again there should not be any difference in the R-squared of the two regressions.

The two implied volatilities are inefficient predictors as well. The error terms for both regressions exhibit positive autocorrelation (Durbin Watson statistic is 0.32 and 0.07 respectively), i.e. they are not white noise.

From the regression results, we argue that implied volatility is not a good and efficient predictor of realised volatility. This mis-estimation of realised volatility could create mispricing problems to the options. Figures I and II show this effect.

[Figure 1 HERE]

[Figure 2 HERE]

The above figures indicate the both call and put options are mainly overpriced with respect to the implied volatility. Based on the above scatter diagrams, we can specifically notice there are some days where the mis-pricing seems to be very significant. This is another indication that the implied volatility causes problem to the option pricing and that the market may be inefficient.

5.4. Granger Causality results

Correlation does not necessarily imply causation. In Table 2 we observed that there was a positive correlation among the *LRV*, *LIVC* and *LIVP*. However, we need to identify any causality among them. So, in this part we perform a Granger causality test.

In order to run the test we need first to estimate the optimum number of lags. From the VAR lag order selection criteria we find that the optimum number of lags is three. Table 7 reports the lag order selection criteria.

[Table 7 HERE]

If implied volatility can predict future volatility then we expect to find that *LIVC* and *LIVP* “Granger cause” *LRV*. Further, we expect to find that there is a multi-directional causality between the *LIVC* and *LIVP*, according to the literature.

[Table 8 HERE]

Table 8 reports the F-statistic of the Granger causality test. Our findings are far from expected. From the above results it is clear that *LRV* “Granger causes” *LIVC*, and *LIVC* “Granger causes” *LIVP*. These results indicate that implied volatility cannot cause realised volatility. Furthermore, the uni-directional causality that is observed from *LIVC* to *LIVP* shows that implied volatility is not an efficient predictor, as there should be a multi-directional causality due to the fact that the implied volatility for both call and put options with the same expiration date and underlying asset, should be the same and due to the put-call parity that should hold.

6. Conclusion

Overall the results indicate that implied volatility is a biased and inefficient predictor of the realised volatility. These results support the empirical findings of the past literature. Yet the significance of the evidence is also important due to two reasons. Firstly, in this study we use both the call and put option implied volatilities and we test them against the realised volatility, the first such study for an emerging market, such as the Greek derivative market. During the period of the study, the Greek market was an emerging market and thus option mis-pricing due to implied volatility could create serious problems for the market. Furthermore, the bias and the inefficiency that the implied volatility exhibits could also be interpreted as a market anomaly or inefficiency. Additional tests should be performed in the Greek market, using additional data, in order to assess whether the current mis-estimation of realised volatility is due to implied volatility weaknesses or due to the emerging status of the market.

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TABLES

Table 1: ADEX statistics

	2000	2001	2002	2003
Number of investors	3,181	9,133	15,482	21,256
Number of products	5	7	8	10
Transaction values (nominal values in million €)	276.12	2,255.1	5,774.86	25,983.9
Source: Athens Derivatives Exchange				

Table 2: Summary statistics

	RV	*LRV*	*IVC*	*LIVC*	*IVP*	*LIVP*
Mean	0.263	-1.377	0.322	-1.165	0.304	-1.230
Median	0.253	-1.371	0.317	-1.147	0.288	-1.244
Maximum	0.409	-0.893	0.703	-0.351	0.748	-0.289
Minimum	0.138	-1.975	0.182	-1.698	0.145	-1.931
Std. Dev.	0.076	0.298	0.085	0.259	0.090	0.288
Skewness	0.155	-0.098	0.708	0.121	1.008	0.099
Kurtosis	1.605	1.593	3.573	2.290	4.869	2.864
Jarque-Bera	42.539	42.035	48.703	11.702	157.5759	1.215
Probability	0.000	0.000	0.000	0.002	0.000000	0.544
Sum	131.658	-688.931	161.307	-582.581	152.247	-615.460
Sum Sq. Dev.	2.903	44.343	3.644	33.642	4.109	41.453
Observations	749	749	749	749	749	749

Table 3: Correlation matrix

	RV	*LRV*	*IVC*	*LIVC*	*IVP*	*LIVP*
RV	1.000	0.993*	0.736*	0.765*	0.605*	0.583*
LRV		1.000	0.746*	0.783*	0.581*	0.560*
IVC			1.000	0.988*	0.509*	0.486*
LIVC				1.000	0.505*	0.485*
IVP					1.000	0.979*
LIVP						1.000

* significant at 1% level

Table 4: Wilcoxon signed rank test results - Implied vs Realised Volatility

	Wilcoxon W-statistic	prob.
$\overline{LIVC}$ vs $\overline{LRV}$	10.524	0.000
$\overline{LIVP}$ vs $\overline{LRV}$	6.784	0.000

Table 5: ADF unit root test

	ADF-statistic
LRV	-8.02*
LIVC	-11.98*
LIVP	-10.42*

*significant at 1% level

Table 6: Volatility regression results

	$LRV_t = a_0 + a_1 LIVC_t + e_{t1}$		$LRV_t = b_0 + b_1 LIVP_t + e_{t2}$	
Independent Variables:	Coefficient	prob.	Coefficient	prob.
C	-0.329**	0.000	-0.664**	0.000
*LIVC**	0.899**	0.000		
*LIVP**			0.579**	0.000
R-squared	0.61		0.31	
Durbin-Watson	0.32		0.07	
F-statistic	792.88**	0.000	228.18**	0.000

* the *LIVC* and *LIVP* coefficients are also significant different from 1, at 5% and 1% level, respectively:

LIVC t-statistic = -3.13

LIVP t-statistic = -10.94

The calculation was based on the following formula:

$$t - stat = \frac{coefficient - 1}{st.error}$$

** variables are significant at 1% level

Table 7: Lag order selection criteria

Lag	LogL	LR	FPE	AIC	SC	HQ
0	104.049	NA	0.000133	-0.410	-0.385	-0.400
1	1940.088	3642.222	7.92E-08	-7.837	-7.735	-7.797
2	1977.995	74.73535	7.04E-08	-7.955	-7.776	-7.884
3	2015.863	74.19752	6.26E-08*	-8.072*	-7.816*	-7.972*
4	2022.447	12.81892	6.32E-08	-8.062	-7.729	-7.932
5	2027.369	9.524160	6.43E-08	-8.046	-7.636	-7.885
6	2037.301	19.09786	6.41E-08	-8.050	-7.563	-7.859
7	2047.490	19.46657*	6.38E-08	-8.054	-7.491	-7.833

* indicates the best lag order

Table 8: Granger causality test

	Granger statistic
LRV Granger causes LIVC	7.63**
LRV Granger causes LIVP	1.15
LIVC Granger causes LRV	1.81
LIVC Granger causes LIVP	3.68*
LIVP Granger causes LRV	0.11
LIVP Granger causes LIVC	0.54

*significant at 5% level
**significant at 1% level

FIGURES

Figure 1: Call option mispricing

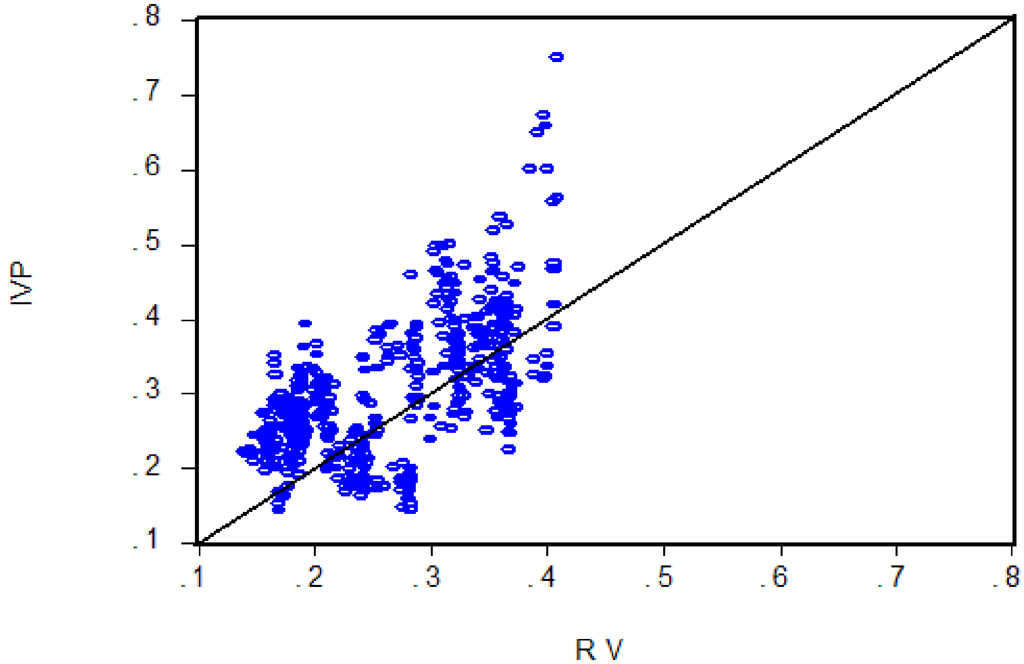

Figure 2: Put option mispricing