Technical Report TR-NCCA-2011-02

THE FORCE DENSITY METHOD: A BRIEF INTRODUCTION

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2011

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ISBN 1-85899-123-4

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Keywords Parameterization, Force Density Method

Abstract
The method of force density was developed in response to the need for computational modelling of structures for the Munich Olympic complex[Lewis, 2003]. The method relies on the assumption that the ratio of tension force to length of each cable can be constant, transforming a system of non-linear equations to a set of linear equations which can be solved directly. In this short note I will give an introduction, description and some results of using the FDM for use as a parameterization technique.

Report date 18 May 2011

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Supplementary notes
Unpublished

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The Force Density Method without the agonizing pain

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Wednesday 18th May, 2011

1 Introduction

The method of force density was developed in response to the need for computational modelling of structures for the Munich Olympic complex[Lewis, 2003]. The method relies on the assumption that the ratio of tension force to length of each cable can be constant, transforming a system of non-linear equations to a set of linear equations which can be solved directly.

The Force Density Method (FDM), first introduced by [Schek, 1974], is commonly used in engineering to find the equilibrium shape of a structure consisting of a network of cables with different elasticity properties when stress is applied. While shape analysis of tensile structures is a geometrically non–linear problem, the FDM linearises the form–fitting equations analytically by using the *force density ratio* for each cable element, $q = F/L$, where F and L are the force and length of a cable element respectively.

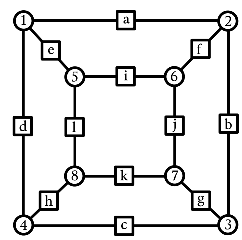
2 The Force Density Method

Given the positions of nodes (vertices) which connect the cables (edges) of our network V, the topology of this network is encoded in the *branch–node matrix* C (see Figure 1. Given a load vector R and the diagonal matrix of force densities Q, the equilibrium location can be deduced by solving for X in

$$(C^T QC)X = R. \qquad (1)$$

The FDM has certain properties which makes it of interest in computer graphics:

- it represents a *minimum energy surface*, and therefore
- it approximates a C^2 continuous surface,
- it depends only on the force density of the edges and the topology of the network, and
- the system is sparse, symmetric and positive definite, quickly solved using the conjugate gradient method.



	C_f				C			
	1	2	3	4	5	6	7	8
a	1	.	.	-1	.	.	.	.
b	.	1	-1	.	.	.	.	.
c	.	.	1	-1	.	.	.	.
d	1	.	.	-1	.	.	.	.
e	1	.	.	.	-1	.	.	.
f	.	1	.	.	.	-1	.	.
g	.	.	1	.	.	.	-1	.
h	.	.	.	1	.	.	.	-1
i	.	.	.	.	1	-1	.	.
j	.	.	.	.	.	1	-1	.
k	.	.	.	.	.	.	1	-1
l	.	.	.	.	1	.	.	-1

Figure 1: Deriving the branch–node matrix. Each row corresponds to an edge, while columns correspond to nodes. Note that the order in which the nodes are specified on the edge will not affect the system.

3 Embedding

In order to use the FDM for 2D embedding, we must be able to constrain nodes on the boundary. For this, the matrix C is separated into two sub–matrices: C_f contains constrained nodes with corresponding position X_f, while C contains those that are free to move. The problem in Equation 1 is reformulated as

$$(C^T QC)X = R - (C^T QC_f)X_f. \qquad (2)$$

Note that for the purposes of embedding, R is typically set to zero.

The FDM is *fold–over free*. This is explained with reference to Figure 2. As the natural rest internal force load of each node is 0, any foldover will result in external forces applied to the nodes which have folded over. As a result it will not be in a state of equilibrium, and this configuration cannot occur.

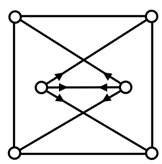


Figure 2: Foldover with FDM cannot occur, as it will not be in a state of equilibrium.

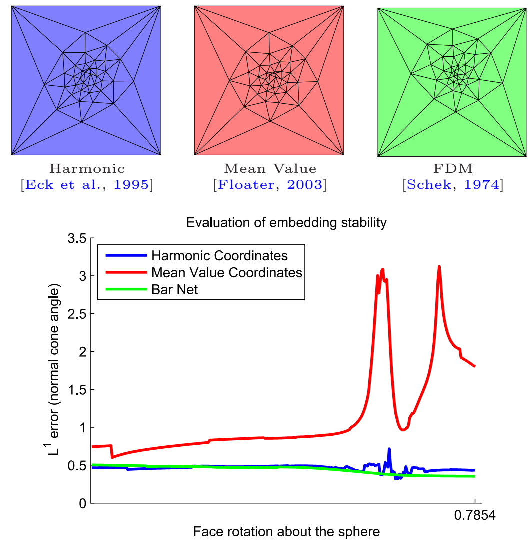


Figure 3: A comparison of the stability under motion of three embedding techniques.

4 Stability under motion

The FDM cannot guarantee any of the commonly advocated properties of embeddings in computer graphics, such as *isometry* (length preserving) or *conformality* (angle preserving). In our application, it is desirable that the path of a vertex or a group of vertices moving across the surface of the shape is mirrored in the embedding. We evaluate this phenomenon by measuring the distortion of these displacement vectors in the embedded space. We call this property *stability under motion*.

In Figure 3, we compare two popular fixed boundary conformal techniques, Harmonic mappings [Eck et al., 1995] and Mean Value Coordinates [Floater, 2003] with the FDM with unit edge forces. The stability under motion of these embedding methods is highly non–linear, and so we evaluate each embedding technique experimentally as follows:

1. Compute the embedding $\mathcal{U}_0 = \mathsf{embed}(\mathcal{M}_0)$.

2. Rotate a set of points on the sphere (in this case, one triangle) in a straight path around the surface of the sphere.

3. For each state of the rotation $\mathcal{M}_i$ compute $\mathcal{U}_i = \mathsf{embed}(\mathcal{M}_i)$.

4. Compute the n displacement vectors in the embedded space. So for each $u_{0,i} \in \mathcal{U}_0$ and $u_{d,i} \in \mathcal{U}_d$ define $\mathbf{d}_i$ as the vector between $u_{0,i}$ and $u_{d,i}$, and
$$D = [\mathbf{d}_1, \ldots, \mathbf{d}_n]^T .$$

5. We use the angle of the normal cone of these displacement vectors $\alpha = \max_{i,j}(\mathrm{acos}(\mathbf{d}_i \cdot \mathbf{d}_j))$ as the distortion error. For this experiment, we evaluate the distortion of only the points moving on the surface.

Figure 3 demonstrates that under all rotations the FDM embedding is stable, even when faces of $\mathcal{M}_d$ overlap. In addition, there is considerably less displacement of surrounding nodes after rotations.

5 Conclusion

I have presented the force density method as a technique to perform mesh embedding using this technique. It is computationally efficient, as it only involves the solution to a sparse linear system, easily solved with the Conjugate Gradient Method or using Cholesky Factorization. In Figure 3 I demonstrate that embedding with FDM is very stable, preventing foldover and discontinuities when parameterizing an unstable triangulation. This is particularly useful when, for example, you need to flatten geometry in a stable fashion (for example, see Zhang et al. [2007]).

References

M. Eck, T. D. DeRose, T. Duchamp, H. Hoppe, M. Lounsbery, and W. Stuetzle. Multiresolution analysis of arbitrary meshes. *Proceedings of SIGGRAPH*, pages 173–182, 1995.

Michael S. Floater. Mean value coordinates. *Computer Aided Geometric Design*, 20(1):19–27, 2003.

Wanda J. Lewis. *Tension Structures: Form and Behaviour*. Thomas Telford, illustrated edition, 2003. ISBN 0727732366, 9780727732361.

J. H. Schek. The force density method for form finding and computation of general networks. *Computer Methods in Applied Mechanics and Engineering*, (3):115–134, 1974.

J. J. Zhang, X. Yang, and Y. Zhao. Bar–net driven skinning for character animation. *Computer Animation and Virtual Worlds*, 18(10):4–5, September 2007.