Smart Passive Adaptive Control of Laminated Composite Plates (Through Optimisation of Fibre Orientation)

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ABSTRACT

In the classical laminate plate theory for composite materials, it is assumed that the laminate is thin compared to its lateral dimensions and straight lines normal to the middle surface remain straight and normal to the surface after deformation. As a result, the induced twist which is due to the transverse shear stresses and strains are neglected. Also, this induced twist was considered as an unwanted displacement and hence was ignored. However, in certain cases this induced twist would not be redundant and can be a useful displacement to control the behaviour of the composite structure passively. In order to use this induced twist, there is a need for a modified model to predict the behaviour of laminated composites. A composite normally consists of two materials; matrix and fibres. Fibres can be embedded in different orientations in composite lay-ups. In this research, laminated composite models subject to transfer shear effect are studied. A semi analytical model based on Newton-Kantorovich-Quadrature Method is proposed. The presented model can estimate the induced twist displacement accurately. Unlike other semi analytical model, the new model is able to solve out of plane loads as well as in plane loads. It is important to mention that the constitutive equations of the composite materials (and as a result the induced twist) are determined by the orientation of fibres in laminae. The orientation of composite fibres can be optimised for specific load cases, such as longitudinal and in-plane loading. However, the methodologies utilised in these studies cannot be used for general analysis such as out of plane loading problems. This research presents a model whereby the thickness of laminated composite plates is minimised (for a desirable twist angle) by optimising the fibre orientations for different load cases. In the proposed model, the effect of transverse shear is considered. Simulated annealing (SA), which is a type of stochastic optimisation method, is used to search for the optimal design. This optimisation algorithm is not based on the starting point and it can escape from the local optimum points. In accordance with the annealing process where temperature decreases gradually, this algorithm converges to the global minimum. In this research, the Tsai-Wu failure criterion for composite laminate is chosen which is operationally simple and readily amenable to computational procedures. In addition, this criterion shows the difference between tensile and compressive strengths, through its linear terms. The numerical results are obtained and compared to the experimental data to validate the methodology. It is shown that there is a good agreement between finite element and experimental results. Also, results of the proposed simulated annealing optimisation model are compared to the outcomes from previous research with specific loading where the validity of the model is investigated.

ACKNOWLEDGEMENTS

I would like to thank my PhD supervision team, Prof. Siamak Noroozi, Dr. Philip Sewell, Prof. John Vinney and also, Dr. Mehran Koohgilani for their on-going excellent support. Additionally, I would like to thank all of the helps of other staff at Bournemouth University. I also, like to dedicate this work to my family, especially my parents, who have been very supportive all the time.

PUBLICATIONS

The scientific papers which have been published in peer review journals and presented at international conferences during this research are listed below. The published journal papers which are relevant to the PhD topic are presented in Appendix A.

[1] **Khandan R**, Noroozi S, Sewell P, Vinney J. 2012 "The Development of Laminated Composite Plate Theories – A Review" Journal of Materials Science 47(16): 5901-5910. (Appendix A1)

[2] **Khandan R**, Noroozi S, Vinney J, Sewell P, Koohgilani M. 2012 "A Semi-Analytical Model for Buckling of Laminated plates with the NKQ Method" Applied Mechanics and Materials 232:68-72. (Appendix A2)

[3] **Khandan R**, Noroozi S, Sewell P, Vinney J, Koohgilani M. 2012 "Optimum Design of Fibre Orientation in Composite Laminate Plates for Out-plane Stresses" Advances in Material sciences and Engineering 232847: 1-11. (Appendix A3)

[4] **Khandan R**, Noroozi S, Sewell P, Vinney J, Koohgilani M. "A Semi-Analytical Model for Deflection Analysis of Laminated plates with the Newton-Kantorovich-Quadrature Method" Materials Research - Ibero-american Journal of Materials. (Appendix A4)

[5] **Khandan R**, Sewell P, Noroozi S, Vinney J, Ramazani MR. “FE Design Tool for Laminated Composite Plates” Applied Mechanics and Materials Journal. (Appendix A5)

[6] **Khandan, R.**, Noroozi, S., Sewell, P. and Koohgilani M. 2012 "Optimisation of Fibre Orientations in Composite Material by Applying Simulating Annealing and Genetic Algorithm Methods" Majlesi Journal of Materials Engineering. (In press)

[7] **Khandan, R.**, Noroozi, S., Sewell, P. and Vinney, J., Ramazani MR. 2010 "Optimum Design of Fibre Orientation Angles in Composite Laminate Plates for Minimum Thickness" ASME 2010 International Mechanical Engineering Congress & Exposition. Vancouver, British Columbia, Canada.

[8] **Khandan, R.**, Noroozi, S., Sewell, P. and Vinney, J. 2010 "Smart Passive Adaptive Control of Composite Plates" ASME 2010 International Mechanical Engineering Congress & Exposition. Vancouver, British Columbia, Canada.

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LIST OF ACRONYMS AND TERMINOLOGIES:

SMA	Shape memory alloys
UD	Unidirectional lamina
ER	Electrorheological
CLPT	Classical Laminate Plate Theory
HOSDT	Higher Order Shear Deformation Theory
σ_{ij}	Stress tensor
C_{ijkl}	Fourth-order stiffness matrix
ε_{ij}	Strain tensor
ESL	Equivalent single layer
t	Thickness
u	Displacement components along the X
v	Displacement components along the Y
w	Displacement components along the Z
FSDT	First-Order Shear Deformation Theories
ϕ_x	rotations about the x axe
ϕ_y	rotations about the y axe
h	Thickness of lamina
TSDT	Third order shear deformation theory
TSDPT	Trigonometric shear deformation plate theory
HSDPT	Hyperbolic shear deformation plate theory
LT	layerwise theory
ZZ	Zig-Zag theory
$\boldsymbol{\gamma_{yz}}$	transverse shear strains in yz plane
$\boldsymbol{\gamma_{xz}}$	transverse shear strains in xz plane
RMVT	Reissner mixed variational theorem
FGM	Functionally graded material
FEM	Finite Element Method
EKM	Extended Kantorovich method
NKQ	Newton-Kantorovich-Quadrature
$\boldsymbol{\bar{Q}_{ij}}$	principal stiffness components

$\boldsymbol{N_{xx}}$	Force in X direction
$\boldsymbol{N_{yy}}$	Force in Y direction
$\boldsymbol{N_{zz}}$	Force in Z direction
$\boldsymbol{n_k}$	Number of plies in the k^{th} lamina
$\boldsymbol{A_{ij}}$	Components of extensional stiffness matrix
$\boldsymbol{\theta_k}$	Fibre orientation in the k^{th} lamina
$\boldsymbol{M_{xx}}$	Moment in X direction
$\boldsymbol{M_{yy}}$	Moment in Y direction
$\boldsymbol{M_{zz}}$	Moment in Z direction
MTEKM	Multi-term extended Kantorovich method
$\boldsymbol{V_r}$	Shear force in xz direction
$\boldsymbol{V_q}$	Shear force in yz direction
S	Simply supported boundary condition
F	Free boundary condition
C	Clamped supported boundary condition
DoF	Degree of freedom
Π_{mp}	Potential energy functional
T_x	Tractions applied at upper surfaces in x axe
T_y	Tractions applied at upper surfaces in y axe
T_z	Tractions applied at upper surfaces in z axe
N_t	Number of simulation test
$\Delta\theta_t$	steps of changing in fibre orientation
n	number of layers of lamina
SA	Simulated annealing
GA	Genetic algorithm
[A]	Extension-shear stiffness matrix
[B]	Extension-bending coupling stiffness matrix
[D]	Bending-torsional stiffness matrix
[E]	Effect of transverse shear and normal stresses
I_F	Safety factor
F_{1t}	Tensile stress limit in x direction

F_{1c}	Compressive stress limit in x direction
F_{2t}	Tensile stress limit in y direction
F_{2c}	Compressive stress limits in y direction
F	Optimisation penalty function
$\boldsymbol{P_{MS}}$	Penalty values calculated based on the maximum stress criterion
$\boldsymbol{P_{TW}}$	Penalty values calculated based on the Tsai–Wu criterion
$\boldsymbol{SF_{MS}}$	Safety factors according to the maximum stress
$\mathbf{SF_{TW}}$	Safety factors according to the Tsai–Wu
$\boldsymbol{\alpha_{max}}$	Maximum acceptable twist
$\boldsymbol{\alpha}$	Induced twist of the plate

CHAPTER 1:

Introduction

1.1 Introduction

A structural composite is a material system consisting of two or more constituent materials with significantly different properties which remain separate within the final structure or product. The mechanical performance and properties of them are designed to be superior to those of the constituent materials acting independently. One of them which is called reinforcement is usually stiffer and stronger. The less stiff and weaker part is called the matrix which bond the reinforcement together, as it is shown in Figure (1.1) (Daniel and Ishai 2006).

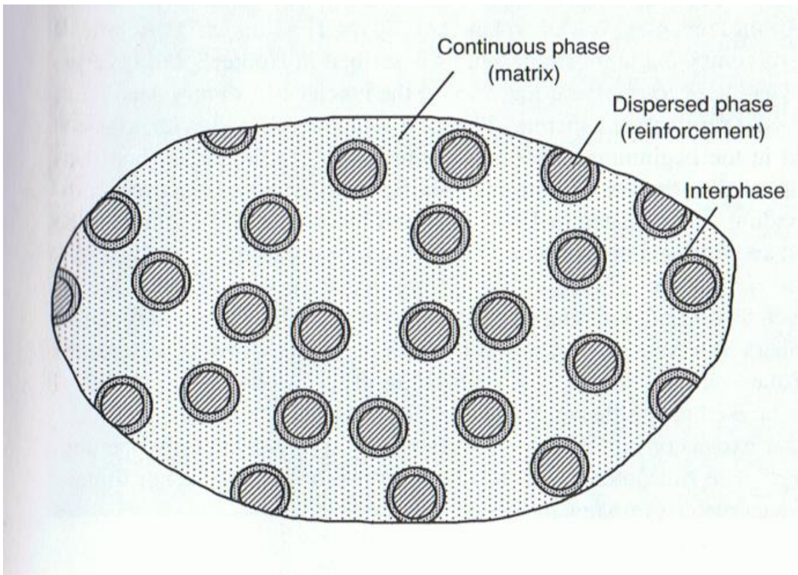

Figure1.1 Phase of a composite material (Daniel and Ishai 2006)

Conventional materials can be classified into three general categories: metals, polymers and ceramics. Composites are combinations of two or more materials from one or more of these categories. In the Stone Age primitive man relied primarily on ceramics (stone) for tools and weapons and on natural polymers and composites (wood). The use of metals began with gold and was preceded with copper, bronze, and iron. Metals, especially steel and aluminium became dominant starting in the last century and continue to the present. A new trend is currently taking place presently where polymers, ceramics, and composites are increasing in relative importance. Whereas, in the early years man used natural forms of these materials, the newer developments and applications emphasise man-made or engineered materials (Astrom 1992).

Fibres normally have a very high length-to-diameter ratio. Short fibres, called whiskers, exhibit better structural properties than long fibres. To gain full understanding of the behaviour of the fibres, matrix materials and other properties of fibre-reinforced materials, it is necessary to know certain aspects of material science. Since the present study is entirely devoted to mechanics aspects and analytical methods of fibre-reinforced composite materials in macroscopic scale, no attempt is made here to present some aspects such as the molecular structure or inter-atomic forces that hold the matter together. (Reddy 2004)

Below is an explanation of some basic material definitions which are used frequently in this research:

Homogeneity: A material is called homogeneous if its properties are the same at every point or are independent of location. The concept of homogeneity is associated with a scale or characteristic volume and the definition of the properties involved. Depending on the scale or volume observed, the material can be more homogeneous or less homogeneous. If the variability from point to point on a macroscopic scale is low, the material is referred to as quasi-homogeneous (Daniel and Ishai 2006).

Heterogeneity/ Inhomogeneity: A material is heterogeneous or inhomogeneous if its properties vary from point to point, or depend on location. As in the case above, the concept of heterogeneity is associated with a scale or characteristic volume. As this scale decreases, the same material can be regarded homogeneous, quasi-homogeneous, or heterogeneous.

Isotropy: Many material properties, such as stiffness, strength, thermal expansion, thermal conductivity, and permeability are associated with a direction or axis (vectorial or tensorial quantities). A material is isotropic when its properties are the same in all directions or are independent of the orientation of reference axes.

Anisotropy/Orthotropy: A material is anisotropic when its properties at a point vary with direction or depend on the orientation of reference axes. If the properties of the material along any direction are the same as along a symmetric direction with respect to a plane, then that plane is defined as a plane of material symmetry. A material may have zero, one, two, three, or an infinite number of planes of material symmetry through a point. General anisotropic materials have no planes or symmetry. Isotropic materials are the opposite of anisotropic materials. It means isotropic materials have an infinite number of planes of symmetry. A special type of anisotropic composite material is one that has at least three mutually perpendicular planes of symmetry. The intersections of these planes define three mutually perpendicular axes, called principal axes of material symmetry or simply principal material axes.

Lamina and laminate characteristic: A lamina, or ply, is a plane (or curved) layer of unidirectional fibres or woven fabric in a matrix. In the case of unidirectional fibres, it is also referred to as unidirectional lamina (UD). The lamina is an orthotropic material with principal material axes in the direction of the fibres (longitudinal), normal to the fibres in the plane of the laminate (in-plane transverse), and normal to the plane of the lamina. These principal axes are designated as 1, 2, and 3 or (X, Y, Z) respectively. In the case of a woven fabric composite the warp and the fill directions are the in-plane l and 2 principal directions, respectively. A laminate is made up of two or more unidirectional laminae or plies stacked together at various orientations. The laminae (or plies, or layers) can be of various thicknesses and consist of different materials. Since the orientation of the principal material axes varies from ply to ply, it is more convenient to analyse laminates using a common fixed system of coordinates (X, Y, Z) as shown in Figure (1.2) The orientation of a ply is given by the angle between the reference x-axis and the major principal material axis (fibre orientation or warp direction) of the ply, measured in a anticlockwise direction on the x-y plane (Daniel and Ishai 2006).

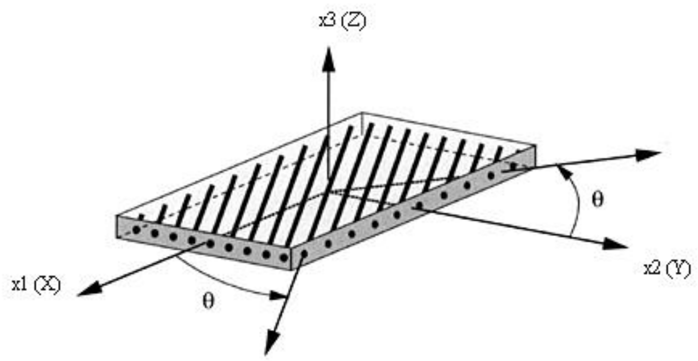

Figure 1.2 Coordinate system

1.1.1 Smart Materials

Active (smart) materials are a class of materials that have the potential and ability to change mechanical force and motion into other forms of energy. Unlike a simple transducer, many of these materials are capable of transforming energy from one type to another. For example, from heating to deformation or deformation to electricity or motion and mechanical motion into energy, which can provide so many new and creative sensing and actuation possibilities in the applications such as wireless ID tags and remote sensors. They also can be used for inventing novel instruments and structures which are lighter and more accurate compared to current ones (Khandan et al. 2009). For example, the conventional use of measurement instrumentations is often obtrusive and interfering with the normal act of the structure or machine. These types of measurements more likely need further instrumented locations to ensure robust, adaptive and repeatable sensing capabilities which are hard to adapt with the original structure or machine. Active materials have been used in these situations to improve the behaviour of the structure by allowing actuation and sensing to become fully integrated into a structure or machine tool in an unobtrusive manner. Active materials can exhibit piezoelectric or magnetostrictive effects. Shape memory alloys (SMA), ionic polymers and electrorheological (ER) fluids are types of active materials. Generally some advantages of using smart materials in devices are their compactness, higher operating frequency, low-power consumption, ease of integration into critical structural areas and low-weight (Park, et al. 2007; Khandan, et al. 2009). Anisotropic composite materials have the potential to be added to this category as elastic-coupling, warping and twist of these materials can potentially be used to control the behaviour of the structure passively.

1.1.2 Composite Materials

Figure (1.3) shows the recent English language refereed journal publications related to multifunctional materials and structures (Gibson 2010). The increased interest in multifunctional materials and structures is driven by the need for the development of new materials and structures that simultaneously perform multiple structural functions or combine non-structural and structural functions. It is also shows the application of composite materials is increasing exponentially.

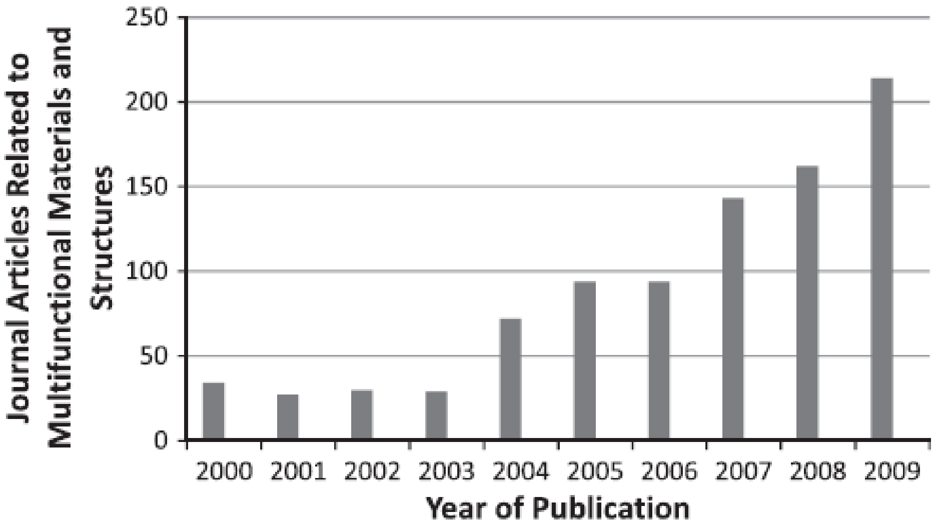

Figure 1.3 Recent English language refereed journal publications related to multifunctional materials and structures (Gibson 2010)

In the Figure (1.4) the type and classification of composite materials is shown. This research focuses on multidirectional continuous fibre composites as the fibre direction can be controlled and optimised. In the other composite types fibre direction cannot be utilised as it does not exist (unidirectional orientation) or it is not predictable (random fibre orientations).

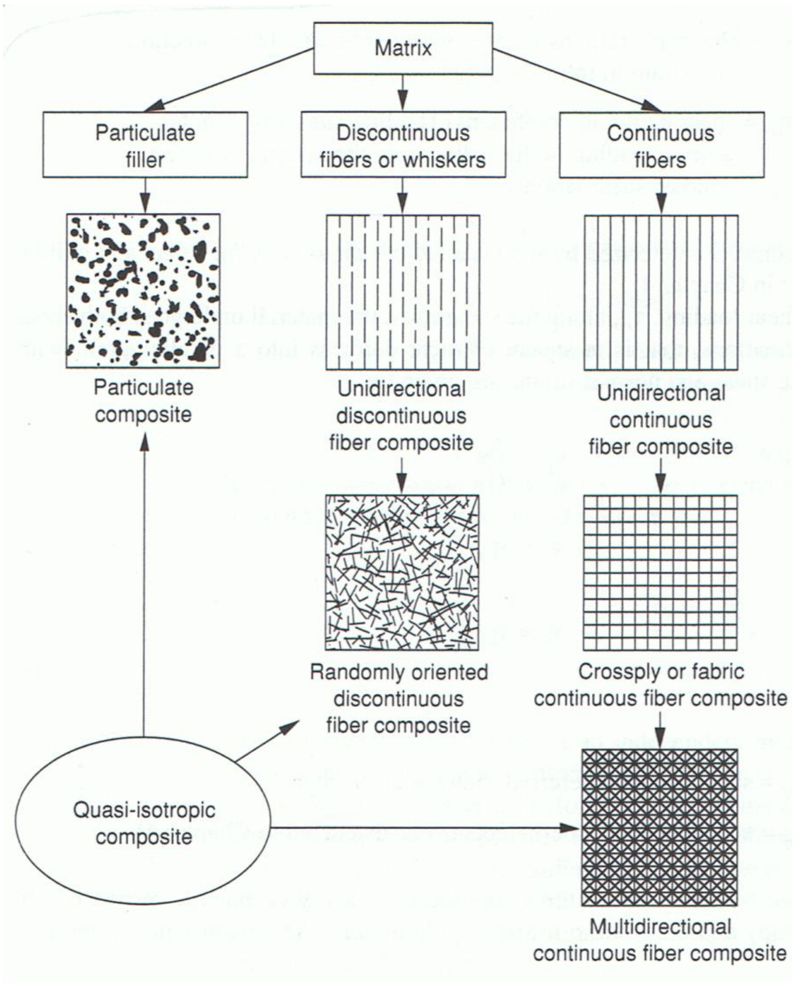

Figure 1.4 Classification of composite material systems (Daniel and Ishai 2006).

1.1.3 Composite Modelling

Composites can be analysed from a micro or macro mechanical point of view. When viewed on the scale of fibre dimensions, composites have the advantage of high-stiffness and high-strength fibres. The usually low fracture toughness of the fibre is enhanced by the matrix ductility and the energy dissipation at the fibre/matrix interface. The stress transfer capability of the matrix enables the development of multiple-site and multiple-path failure mechanisms. On the other hand, the fibres exhibit a relatively high scatter in strength. Local stress concentrations around the fibres reduce the transverse tensile strength appreciably. Conventional materials are more sensitive to their microstructure and local irregularities that influence the brittle or ductile behaviour of the material. Their homogeneity makes them more susceptible to flaw growth under long-term cyclic loading (Daniel and Ishai 2006). However, in this research we are looking at the modelling of composite materials in macro mechanical point of view.

In macro mechanical analysis, where the material is treated as quasi-homogeneous, its anisotropy can be used to its advantage. The average material behaviour can be controlled or predicted from the properties of the constituents. However, the anisotropic analysis is more complex and more dependent on the computational procedures. On the other hand, the analysis for conventional materials is much simpler due to their isotropy and homogeneity. The analysis of composite structures requires the input of average material characteristics. These properties can be predicted on the basis of the properties and arrangement of the constituents. However, experimental verification of analysis or independent characterisation requires a comprehensive test program for determination of a large number (more than ten) of basic material parameters. On the other hand, in the case of conventional isotropic materials, mechanical characterisation is simple, as only two elastic constants and two strength parameters suffice.

In order to analyse and understand stress and the nature of stress concentration in composite laminates, the three-dimensional (3D) stress and elasticity boundary value problem must be solved. This problem involves a series of coupled equations. Unfortunately, exact solutions to this problem are, as yet, unavailable. Therefore, various researchers have presented a variety of approximate models to calculate the transverse/interlaminar stresses (Kant and Swaminathan 2000).

It is important to note that in most laminated composite models the laminate is considered to be sufficiently long. Thus, the effect of the boundary conditions, loading points in other locations of the laminated plate can be neglected. This is one of the assumptions normally considered in laminated composite formulation to simplify the model. The same assumption was considered by Wang and Choi (Wang and Choi 1982a; Wang and Choi 1982b; Wang and Choi 1983). An appropriate and accurate formula for interlaminar stress analysis of laminated composites must consider the continuity conditions and transverse shear effect for interlaminar stresses and displacements on the composite interface. It is also desirable if the formulation and equations are variationally consistent so that they can be used for finite element methods (FEM). Lu and Liu (1990) developed an interlaminar shear stress continuity theory to prove the importance of these points. The model was developed by considering the mentioned conditions and deriving the interlaminar shear stress could not be directly estimated from the constitutive equations. However, the interlaminar normal stress directly from the constitutive equations as deformation in the thickness direction was neglected.

Study in this field is still continuing. A comprehensive review of methods of modelling laminated composites is presented in Chapter 2.

1.1.4 Composite Structures

Analysing and modelling of composite structures can be very complicated. In order to be able to analyse the behaviour of these structures, we need to break them down to simpler parts such as a beam and plate. In this research we focus on investigating laminated composite plates. Plates are an important part of any complex structure and developing those help to improve the whole composite applications and structures. Analysing the composite plates is the first step towards understanding and modelling more complicated composite structures and products.

Laminated plate models are vital to provide an accurate evaluation of laminated composite plates. Numerous laminated plate theories have been developed and proposed in different literatures (Zhang and Yang 2009). A review of various equivalent single layer and layerwise laminated plate theories was presented by Reddy and Robbins (1994). A study of laminated theories and models based on a displacement hypothesis is presented by Liu and Li (1996) that includes shear deformation and layerwise theories. An overall review of models for laminated and sandwich plates is presented by Altenbach (1998). Similar work investigating displacement and stress-based refined shear deformation theories of anisotropic and isotropic laminated plate is presented by Ghugal and Shimpi (2001). In this work various equivalent single layer and layerwise theories were discussed. A historical study of approaches to model multi-layered plates and shells up to 2003 is presented by Carrera (2003). Also, a review of shear deformation plate and shell models is explored by Reddy and Arciniega (2004). A survey of the theories focused on estimation of transverse/interlaminar stresses in laminated composites is given by Kant and Swaminathan (2000), and a selective literature survey on the free-edge effect since 1967 is presented by Mittelstedk and Becker (2004).

None of the previous work studies general and out of plane loads by considering the transverse shear effect, warping and twist behaviour of the laminated composite plate.

In this research the transverse shear effect for laminated composite plate under general loads is considered.

1.1.5 Optimisation in Composites

The advantage of composite materials is that they provide excellent mechanical properties. However, in order to use this advantage, the optimisation of size and shape and the proper placement of fibres within the material are essential. This potentially gives a good opportunity to tailor the material properties. However, it increases the complexity of the design problem. This complexity exists, not only because of numerous design variables, but also because of having variable-dimensional and multimodal optimisation problem costly derivatives (Ghiasi et al. 2009).

Optimum strength designs of continuous fibre-reinforced composite laminates have been used since the early days of these materials. The first research to investigate the fibre orientation of a unidirectional lamina yielding maximum strength under in-plane stress conditions has been carried out by Sandhu and Brandmaier (1969). Brandmaier found that the strength of a unidirectional lamina under in-plane stresses could be maximised analytically with respect to the fibre orientation (Brandmaier 1970). The results based upon Tsai-Hill failure criterion indicated that the optimum fibre orientation depended upon the stress state and the relative value of the transverse and in-plane shear strengths of the lamina. When the strength of a multidirectional composite laminate is to be maximised, more complicated and explicit optimisation techniques are needed (Wang and Karihaloo 1999). Chao et al. (1975) were the pioneers that sought the optimum strength design of multidirectional laminates using a search technique. Subsequently, many studies have been devoted to the optimum strength design of multidirectional laminate. Among these are the works by Park (1982), Fukunaga and Chou (1988), Miravete (1989), Fukunaga and Vanderplaats (1991), Haftka and Gurdal (1992), Gurdal et al. (1999), Spallino et al. (1999), Weaver (2002), Chattopadhyay et al. (1999), Luersen and Le Riche (2004) and Ghiasi et al. (2008).

Previous studies have shown that composite fibre angles can be optimised by different optimisation methods for specific load cases such as longitudinal or in-plane loading (Fukunaga and Vanderplaats 1991; Wang and Karihaloo 1996; Tabakov and Walker 2007; Akbulut and Sonmez 2008).

In this work, a stochastic optimisation method is used to optimise the orientation of fibres in laminated composites by considering different loadings and the transverse shear effect.

1.2. Applications

Composites are used in a very wide range of applications. They include aerospace, aircraft, automotive, marine, energy, infrastructure, armour, biomedical, and recreational (sports) applications. It is expected that the outcome of this research has the potential to be applied in these applications.

Aerospace structures, such as space antennae, mirrors, and optical instrumentation, make use of lightweight and extremely stiff graphite composites. A very high degree of dimensional stability under severe environmental conditions can be achieved because these composites can be designed to have nearly zero coefficients of thermal expansion.

The high-stiffness, high-strength, and low-density characteristics make composites highly desirable in primary and secondary structures of both military and civilian aircraft. The Boeing 777, for example, uses composites in fairings, floor beams, wing trailing edge surfaces, and the empennage (Figures 1.5 and 1.6) (Mabson et al 2001). The strongest sign of acceptance of composites in civil aviation is their use in the new Boeing 787 "Dreamliner" and the worlds largest airliner, the Airbus A380 (Figure 1.7). Composite materials, such as carbon/epoxy and graphite/titanium, account for approximately 50% of the weight of the Boeing 777, including most of the fuselage and wings. Besides the advantages of durability and reduced maintenance, composites afford the possibility of embedding sensors for on-board health monitoring (Daniel and Ishai 2006).

Figure 1.5 Boeing 777 (Mabson et al 2001)

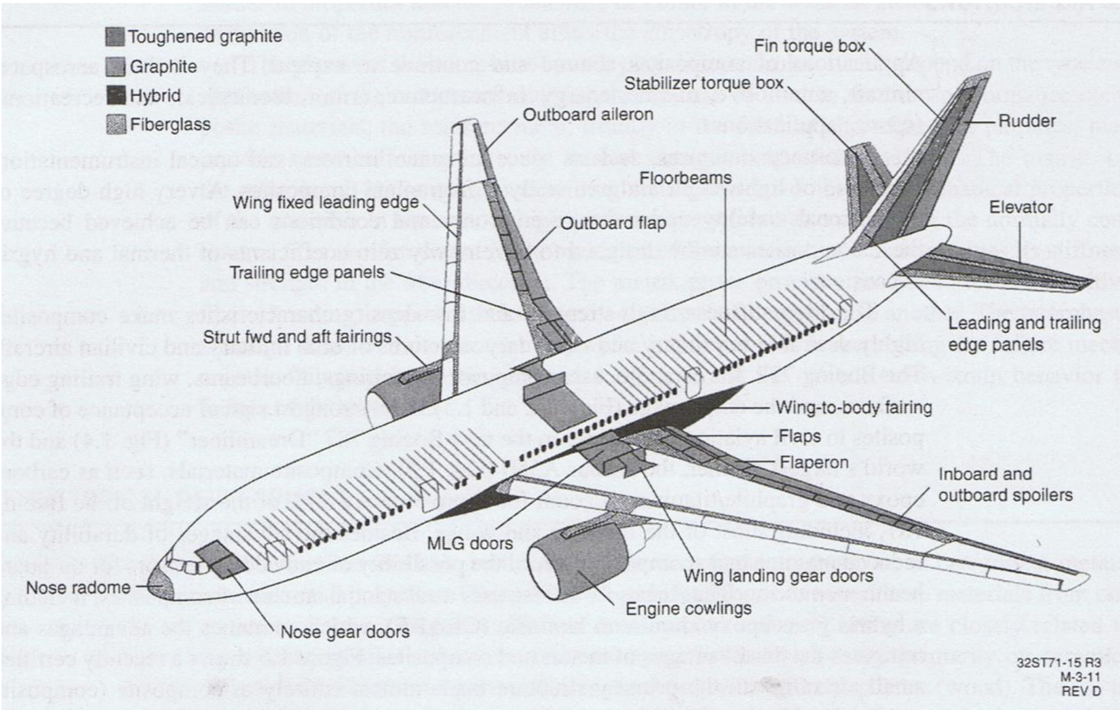

Figure 1.6 Diagram illustrating usage of composite materials in various components of the Boeing 777aircraft (Mabson et al 2001)

Figure 1.7 Airbus A380 (Daniel and Ishai 2006)

In the energy production field, carbon fibre composites have been used in the blades of wind turbine generators that significantly improve power output at a greatly reduced cost (Figure 1.8). In offshore oil drilling installations, composites are used in drilling risers like the one installed in the field in 2001 (Daniel and Ishai 2006).

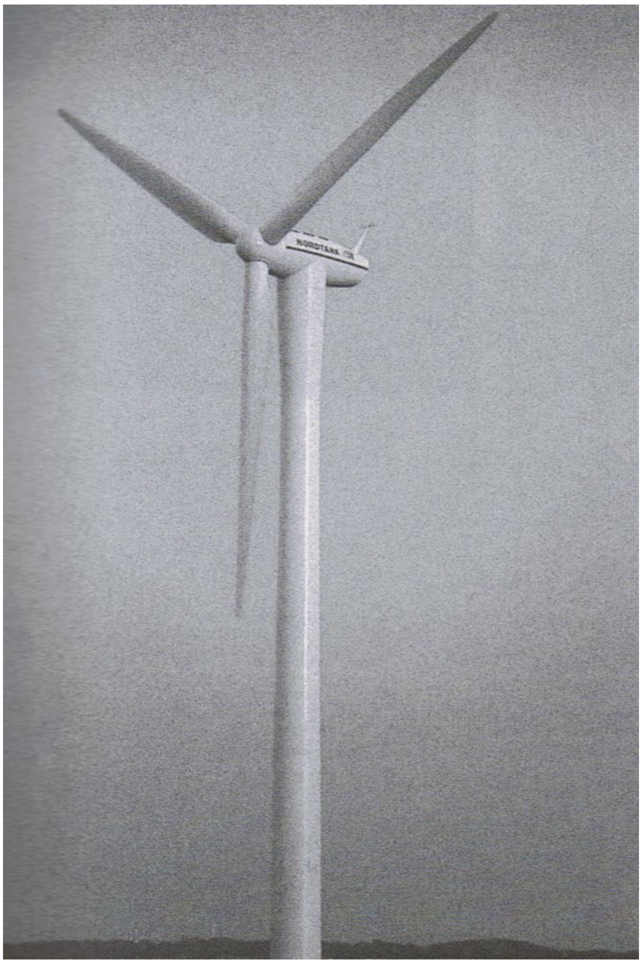

Figure 1.8 Composite wind turbine blade used for energy production (Daniel and Ishai 2006).

One of the wide applications of laminated composite is in biomedical devices. One application could be in the design of prosthetic feet. The desire of individuals with a lower-limb amputation to participate in sports and the high demands of athletics, have resulted in the development of composite prosthetics which are capable of storing energy during stance and returning it to the individual in late stance to assist in forward propulsion. However, the static and dynamic characteristics (natural frequencies, mode shapes and damping) of them depends on materials properties. As it was mentioned earlier the fibre orientation can be used as a design parameter to control and improve the behaviour of them. The evaluation of such parameters is urgently required when the use of energy-storing-and-returning (ESR) feet have been investigated through legal and justice systems to determine participation of amputee athletes using them at the Olympic Games (Noroozi et al. 2012). The research in this area is continuing. The two types of prosthetic which are investigated at Bournemouth University are shown in Figure (1.9). (Noroozi et al. 2012)

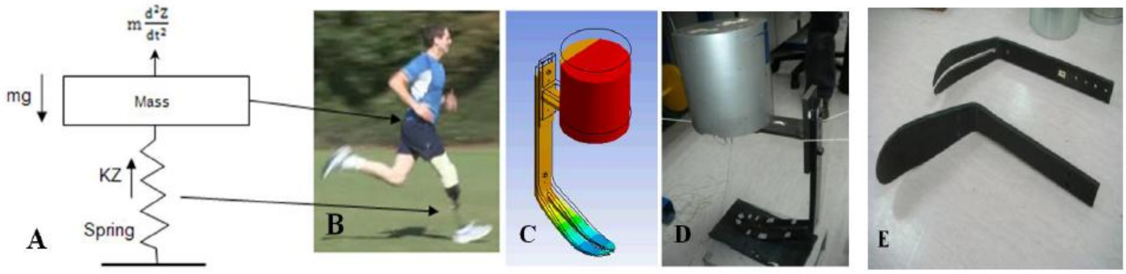

Figure 1.9 A) Mass spring typical system B) Runner C) equivalent system D) mass over an Elite Blades E) an example of Elite Blades (Noroozi et al. 2012)

Composites are used in various forms in the transportation industry, including automotive parts and automobile, truck, and railcar frames. Also ship structures incorporate composites in various forms, thick-section glass and carbon fibre composites and sandwich construction. The latter consists of thin composite sheets bonded to a thicker lightweight core. Applications include minesweepers and corvettes (Figure 1.10). Composite ship structures have many advantages such as insulation, lower manufacturing cost, low maintenance, and lack of corrosion (Daniel and Ishai 2006). A composite panel representative of a boat's hull structure is instrumented and calibrated to function as a complex load cell capable of measuring external normal loads which is shown in Figure (1.11). The panel is divided into 16 patches and 12 loading positions are picked to be loaded utilising weights. Generated superimposed data from these readings are used to train an Artificial Neural Network. Preliminary results from static loading of a composite panel indicate very promising results with a very low error margin (Ramazani et al. 2012). This research on this composite panel is continuing at the Design Simulation Research Centre (DSRC) at Bournemouth University.

This applications show the importance of composite in various products which are being used widely everyday. Therefore, improvement and development in composite modelling can potentially lead to better and more efficient design of composite products. This research mainly focuses on induced twist deformation in composite plate and how this induced twist can be used and controlled by optimising the fibre orientations in laminae.

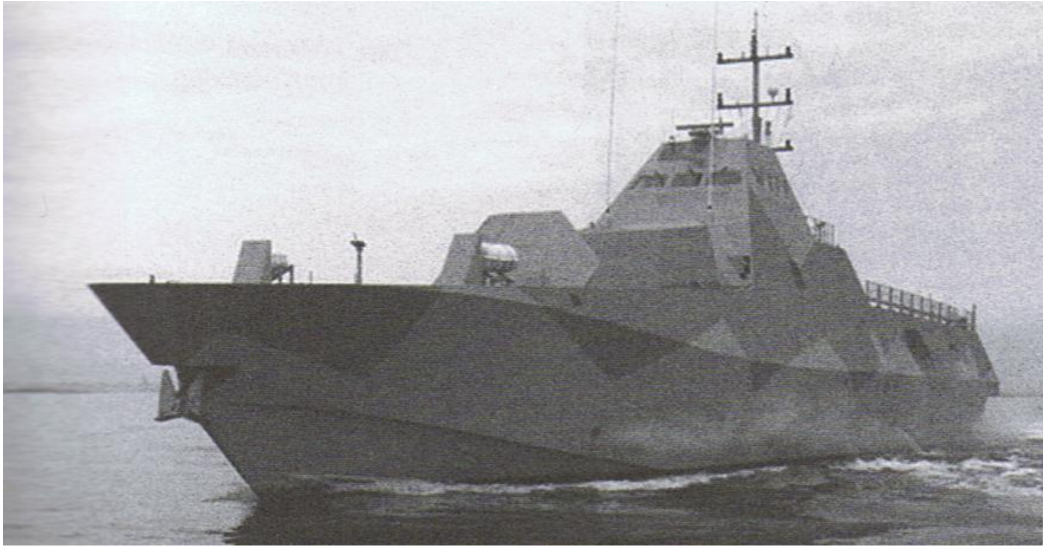

Figure 1.10 Royal Swedish Navy Visby class corvettes (Daniel and Ishai 2006)

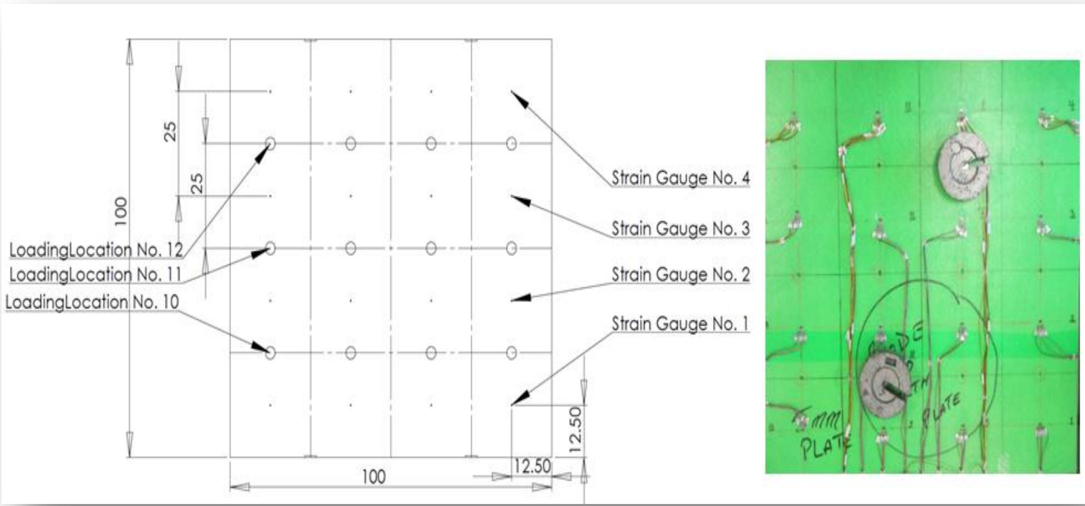

Figure 1.11 Panel drawing with the location of gauges and loading (Ramazani et al. 2012)

1.3. Research Question and Objectives

Fibre-reinforced composite materials for structural applications are often made in the form of a thin layer, called lamina. A lamina is a macro unit of material whose material properties are determined through appropriate laboratory tests. Structure elements, such as bars, beams or plates are then formed by stacking the layers to achieve desired strength and stiffness. Fibre orientation in each lamina and stacking sequence of the layers can be chosen to achieve desired strength and stiffness for specific application. It is the purpose of the present study to investigate the equations that describe appropriate kinematics of deformation, govern force equilibrium, and represent the material response of laminated structures under general loads by controlling the fibre orientations in plies. Analysis of structural elements made of laminated composite materials involves several steps. The analysis requires knowledge of anisotropic elasticity, structural theories (i.e., kinematics of deformation) laminates, analytical or computational methods to determine solutions of the governing equations, and failure theories to predict modes of failures and determine failure loads.

Controlling the fibre orientations in laminated composites can potentially lead to designing passive smart structures. On the other hand, fibre orientations can be used as a design parameter to control the composite structure behaviour, because changing fibre orientations directly affects material properties (stiffness matrix).

The fibre orientation in each layer of a composite can range from -90 to +90 degrees and the angle directly effects the behaviour of the structure. The ability to create a wide range of fibre orientation in each layer gives so many options to design the structure with composite. Therefore, using an optimisation method is unavoidable in order to find the best fibre orientations for the composite structures (products) depending on the final structures (products) requirements.

The initial idea of this research is formed from previous work which has been done by Maheri et al. (2007) on controlling the composite structure (wind turbine blade) passively. A passive control approach reduces the size of the controlling system, its cost of maintenance and the initial cost. The idea was to have an adaptive and smart structure to employ the structure itself as the controller (e.g. for the wind turbine blade to sense the wind velocity and adjust its aerodynamic characteristics to affect and increase the efficiency of the wind turbine performance). The anisotropy of composites produces shear coupling and twist in composite structures. Considering this shear coupling and transverse shear effect on modelling of composite structures with complex profile such as a wind turbine blade is very complicated. Therefore, the problem is broken down to a thin wall beam which is placed and joint inside the wind turbine blade. So each deformation in beam is directly followed by the blade. In two opposite sides of thin wall beam a composite panel is considered. The combined effect of twisting in opposite surface of the beam which is enhanced from composite panels causes shear coupling on the blade. On the other hand for example by applying uniform distribution loads on a cantilever anisotropic composite plate (Fixed in one side and free the other sides), unlike conventional materials, twist deformation is induced as well as bending. The amount of this twist can be controlled by changing the fibre orientations in composite plates. Because changing fibre orientations changes the stiffness matrix in Hook's law and any deformation (including twist) can be calculated from Hook's law.

Therefore, the main question (research question) which this work seeks to answer is:

"How can a laminated composite plate be designed and controlled by using the induced twist deformation through optimisation of the fibre orientations of the laminae"

The research question is very general and broad, therefore to find answer for it, a set of key subquestions and issues must be studied. On the other hand, finding answers for the following questions are the objectives of this research.

- What are the current methods which can model the induced twist deformation in laminated composite plate?
- Is there any possibility to modify the constitutive equation for composite in order to predict the behaviour of laminated composite more accurate?
- How precise the applied model can predict the behaviour of laminated composite plate in comparision with experimental results? (On the other hand the validity of the method must be checked)
- How the fibre orientations in laminated composite plate can be optimised to achieve certain induced twist displacement?

The following milestones are set to answer the above questions and achieve research objectives.

- Studying and evaluating the current composite models subject to how accurate they can predict the induced twist deformation in composite structure.
- Modification of current analytical and semi-analytical models of composites (if possible), to predict the behaviour of laminated composite plate more accurate.

- Validation of the applied method must be checked. Therefore, a series of experimental tests should be undertaken to compare the experimental and simulation results.
- An optimisation algorithm is requied to find the suitable fibre orientations in laminae to achieve desirable induced twist deformation.

Each milestone is studied and reffered in one chapter. The milestones can be seen in Figure (1.12) where the structure of the research is shown.

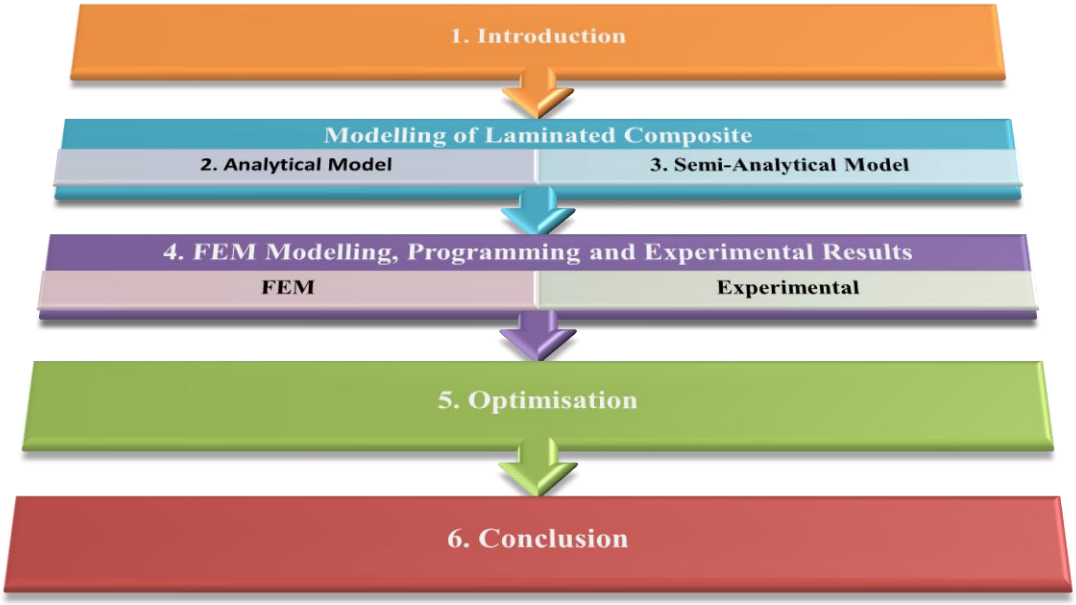

Figure 1.12 Structure of this research

In the first step an appropriate and accurate model for laminated composite plate should be chosen. In Chapter 2 an overall review of analytical model for laminated composites are presented. Advantages and disadvantages of each one are discussed. Semi-analytical models are explained in Chapter 3. Also, a new semi-analytical model based on Newton-Kantorovich-Quadrature (NKQ) method is proposed.

In the next step, some experimental tests on laminated composite plates have been done and the results compared with finite element analysis to validate the results. In order to solve the composite models accurately, using numerical methods such finite element (FE) is unavoidable. After investigation about different FE software packages

Abaqus is chosen because of more accurate results, being user friendly and the ability to write programs (in Python language) and easily link this program to Abaqus's model and solver. In order to answer the effect of fibre orientations, it is necessary to have the FE results for all possible fibre orientations. As it is mentioned, the orientations can be varied from -90 to +90 degrees in each layer. When the number of layers is increasing, consequently the number of FE tests is increasing and it is almost impossible to run the program each time manually. Therefore the written Python program is used to run these FE tests for the different orientations for each ply iteratively. It is tried to expand this Python program as a design tool. In the next step, some experimental tests on laminated composite plates have been done and the results compared with finite element analysis to validate the results. Finite element simulation of laminated composite plate with Abaqus, Python program, manufacturing the composite samples and the comparison of FE and experimental results are explained and discussed in Chapter 4.

In Chapter 5 the results of FEM are used to optimise the fibre orientations in laminated composite plate. The optimisation model tries to find the appropriate fibre orientations for laminated composite plate depending on the requirements of the structure or product. Different optimisation models have been introduced to solve this challenging problem, which is often non-linear, non-convex, and multidimensional. The stochastic non-linear optimisation methods are utilised for this problem as they can avoid the local minimums. One of the best algorithms in this category is the simulated annealing (SA) method which is used to solve this problem in this research. The effect of transverse shear, which causes twisting, in laminate composite plate under different types of loading (in plane and out of plane) are considered in the proposed model and the results are compared with previous works. The conclusion of this research and possible future works are presented in Chapter 6.

In summary, the novelty of this work and what distinguished this study from current researches are:

- In current studies, the effect of transverse shear is neglected. Therefore, the induced twist in laminated composite structures is ignored and considered as unwanted displacement. This work suggested that the induced twist can be a useful displacement to control the behaviour of composite structures.

- A comprehensive study in modelling of composite materials subject to transverse shear effect is done and also a semi analytical model is proposed which can consider the transverse shear effect in constitutive equations of composite materials.

- Fibre orientation in laminated composite plate is considered as a design parameter which can control the transverse shear and as a result can control the induced twist displacement in laminated composite plate. The fibre orientations are optimised by modifying the current penalty function for SA and proposing a new penalty function which is able to control the amount of induced twist in laminated composite.

1.4. Methodology

Here, how the study was carried out and what procedures were used, are concisely explained and in the following chapters these methods and techniques will be expanded. As it is mentioned earlier, in this research we try to present a way to design and control composite plate by using induced twist displacement. Fibre orientation of laminated composite is the design parameter that can control the induced twist deformation in composite products, because the stiffness matrix of composite materials is depend on the fibre orientations of laminae. To answer the research question four milestones and objectives were set.

- Investigate and evaluate the current composite models
- Modification of current analytical and semi-analytical models
- Check the validation of the applied method
- Optimisation algorithm to find the suitable fibre orientations in laminae to achieve desirable induced twist deformation.

Because the nature and type of these objectives are different, each objective is considered as a smaller research in this work and investigated in individual chapter. For example, modification of semi-analytical model involves mathematical calculations and computations study whereas to validate the model a series of experimental test is required. Also, optimisation algorithms and methods are explored separately from constitutive equations and models of composite materials to avoid confusion.

In the literature review, laminated composite plate theories have been reviewed. Also, they have been categorised based on how accurate they estimate the induced twist in laminated composites. The advantages and disadvantages of each model have been explained. In the single layer theory approach, layers in laminated composites are assumed to be one equivalent single layer whereas in the discrete theory approach each layer is considered individually in the analysis. In discrete layer models (layerwise theories) each layer is analysed independently in order to achieve more accurate results and to assure the compatibility of the displacements and the inter-laminar equilibrium equations. Furthermore, it is discussed, because of some simplification assumptions (normal lines to the mid-plane before deformation remain straight and normal to the plane after deformation), classical laminate plate theory cannot be used in this research as it would not capable of estimating the transverse shear and as a result any induced twist in the laminated composite plate. Therefore, FE packages such as NX which were using this model to solve the composite problems were not used in this research. Different models try to overcome the limitations of CLPT in order to predict the transverse shear more accurately by using more terms in Taylor series and as a result increasing the order of constitutive equations for the model, such as first order shear deformation theory, third order shear deformation theory, etc. The FE package (Abaqus) which is used in the following chapters to model the laminated composite plate in this research, is considering the transverse shear in modelling of composite materials.

One aim of this research is to modify the current model in a way that it presents the induced twist more accurate. Semi-analytical models are explained in Chapter 3 and a new semi-analytical model by using the Newton-Kantorovich-Quadrature method is proposed. The key advantage of this method, compared to current models, is that it can potentially be used for more complicated cases such as out of plane loading and different boundary conditions with the expense of increasing processing time. The model can predict the transfer shear and as a result induced twist of laminated composite more precise in comparision with current models. The non-linear equations are solved by Newton-Kantorovich-Quadrature (NKQ) method. This method breaks down the laminated composite plate equations into a series of sequential equations and attempts to solve iterative integral equations. The convergence of the proposed method is compared with other semi-analytical methods (EKM and MTEKM). Various numerical examples with different boundary conditions and loadings are studied. Originally the aim of investigating on semi-analytical model was to find a semi-analytical method which is able to model the laminated composite plates for out of plane loading and considering the transverse shear effect. The proposed model is theoretically can be used for these purposes. However, higher processing time and lower accuracy are two critical disadvantages of this semi-analytical model in comparison with numerical models and make this model impractical. Therefore, this model is not used in this research and instead the numerical method and finite element method is used in this work. FEM in Abaqus is used in this work due to its wide material modelling and analysing capability especially in terms of considering the transverse shear effect in comparison with other FE software. Abaqus is also easy to define fibre orientations for lay-ups in the laminate composite plate. How to model and simulate the laminated composite plate in Abaqus is explained. In order to answer the effect of fibre orientations on composite plate behaviour, it is necessary to have the FE results for all possible fibre orientations. When the number of layers is increasing, as a result the number of FE tests which is needed is increasing as well and it is very time-consuming and hard to set up and run the program for each test manually. So, the written Python program is used to run these FE tests for the different orientations iteratively. An expanded and modified version of Python program is presented as a design tool. Some experimental tests on laminated composite plates have been performed with Vacuum bagging process. To minimise the error, it is necessary to perform the test for few times. In this research, six identical plates are manufactured for each test and the average of these results is considered as a final result for that sample. The plates are under the constant load and square laminated composite plates are fabricated (500 x 500 mm). They are fixed in one side and the displacement is measured on the opposite free side. The plates are under their own weight and in FE this load is consider as a uniform distribution pressure on the plate. The displacement is measured by Laser Displacement Measurement Sensors with accuracy of .001 mm. The validity of the model is verified by comparison of experimental outcomes verses model results. The results of FEM displacement is generated and saved in a database which included all the results of the laminated composite plates for all possible different orientation for each ply. In order to achieve the certain induced twist (depending on the designers requirements), using an optimisation method is unavoidable because the fibre orientation in each layer of composite can be in a wide range of [-90 +90] degree and it means the number of combinations of different orientations for laminated plate are considerably large. The SA optimisation method is proposed and used to solve this challenging problem, which is normally non-linear due to anisotropy. The validity of the optimisation model is verified by another GA and SA models which were proposed recently. A new penalty function was proposed for SA in order to overcome the difficulties and shortcomings faced by the previous SA model where the effect of transverse shear (induced-twist) was ignored. By the proposed model, the effect of transverse shear and, as a result, out of plane loadings can be solved as well as the in-plane loadings. In this work the Tsai-Wu and maximum stress failure criteria are chosen for laminated composite. By using the proposed optimisation method for fibre orientations of laminae, the amount of induced twist in laminated composite plate can be controlled.

CHAPTER 2:

Literature Review

(Analytical Modelling of Laminated Composite Plate)

2. Modelling of Laminated Composite

Modelling of laminated composite plates can be categorised into three different types:

- Analytical models
- Semi-analytical models
- Numerical models

Analytical models are the constitutive equations which are explaining the behaviour of the materials. These equations derive from Hook's law and users normally simplify them. Shortcomings and errors which are caused by this simplifying can be neglected in conventional materials but cannot be ignored in anisotropic materials as it makes considerable differences on the behaviour of such materials. Considering the anisotropy effect on composite materials, leads to very complicated equations which are not possible to solve them by hand. Semi-analytical and numerical methods are used to solve these equations. Semi-analytical models try to find mathematical methods to estimate the acceptable solutions for these equations. Although the semi-analytical results are not as accurate as numerical results, they help to understand the physical interpretation of each parameter in equations and the effect of them in the final result. Analytical models for laminated composite plates are investigated in this chapter and semi-analytical and numerical models are explained in the following chapters.

In this chapter previously developed analytical models are studied comprehensively and the advantages and disadvantages of them are investigated.

Composite theories that have been proposed and developed are explained and their suitability and functionality assessed in this section. The particular focus is on normal stresses and the through-thickness distributions of transverse shear. These are important for composite plates as these are the parameters which affect composites behaviour under different loadings. Therefore, it is essential, to understand and calculate transverse shear and normal stress through the thickness of the plate accurately. Here previous laminated composite plate theories are categorised and reviewed in a general sense, i.e. not problem specific, and the advantages and disadvantages of each model are discussed. It starts with displacement-based theories from very basic models such as Classical Laminate Plate Theory (CLPT) to more complicated and Higher Order Shear Deformation Theory (HOSDT). Models are furthermore, categorised by how the models consider the overall laminate. Here the theories are divided in to two parts; Single Layer

Theory (SLT), where the whole plate is considered as one layer, and Layerwise Theory (LT), where each layer is treated separately. The models based on Zig-zag and Discrete theories are then reviewed and finally the mixed (hybrid) plate theories are studied.

2.1. General Introduction

It is important to explain, in studying composite models, why focus is placed on how the model considers the transverse shear of composite materials. The normal stresses and through-thickness distributions of transverse shear for composite materials are important for many reasons. For example, in laminate composite plate stress-induced failures occur through three mechanisms. If the in-plane stress gets too large, fibre breakage or material yield occurs. However, normally before the in-plane stresses exceed the fibre breakage point, inter-laminar shear stress failure occurs when one layer slips tangentially relative to another. Alternatively, transverse normal stress may increase enough to cause deboning failure where two layers pull away from each other. Therefore, it is essential, to understand and calculate transverse shear and normal stress through the thickness of the plate accurately (Fox 2000).

Generally, two different approaches have been used to study laminated composite structures analytically: single layer theories and discrete layer theories. In the single layer theory approach, layers in laminated composites are assumed to be one equivalent single layer whereas in the discrete theory approach each layer is considered in the analysis. Also, plate deformation theories can be categorised into two types: stress based and displacement based theories (Aydogdu 2009).

A review of displacement based theories is given below: Displacement based theories can be divided into two categories: Classical laminated plate theory and shear deformation plate theories. Normally laminated composite plate theories are described in the classical laminated plate theory, first-order shear deformation theory, global higher-order theory and global–local higher shear deformation theory (Wu. et al. 2005).

The most general form of the linear constitutive equations for infinitesimal deformations is:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \qquad i, j, k, l = 1,2,3 \tag{2.1}$$

which is referred to as the generalised Hooke's law, where C is the fourth-order tensor of material parameters and is termed stiffness tensor. There are, in general, 34=81 scalar components of a fourth-order tensor. The number of independent components of C are considerably less because of the symmetry of σ, symmetry of ε, and symmetry of C. Since the strain tensor is symmetric, $\varepsilon_{kl} = \varepsilon_{lk}$, then C_{ijkl} must be symmetric in the last two subscripts as well, further reducing the number of independent material stiffness components to 6×6=36. If we assume that the material is hyperelastic, i.e., there exists a strain energy density function $U_0(\varepsilon_{ij})$ such that:

$$\sigma_{ij} = \frac{\partial U_0}{\partial \varepsilon_{ij}} = C_{ijkl}\varepsilon_{kl} \tag{2.2}$$

$$\sigma_{ij} = \frac{\partial^2 U_0}{\partial \varepsilon_{ij}\partial \varepsilon_{kl}} = C_{ijkl} \tag{2.3}$$

Since the order of differentiation is arbitrary, it follows that $C_{ijkl} = C_{klij}$. This reduces the number of independent material stiffness components to 21 or on the other hand:

$$\sigma_i = C_{ij}\varepsilon_j \tag{2.4}$$

Or;

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} \tag{2.5}$$

$C_{ij} = C_{ji}$ lead to 6+5+4+3+2+1=21 independent stiffness coefficients for the most general elastic material.

When there is no preferred directions in the material (the material has infinite number of planes of material symmetry), the number of independent elastic coefficients reduces to 2. These materials are called isotropic materials. Figure (2.1) shows the difference in behaviour between isotropic and anisotropic materials under uniaxial load. Because properties of materials depend on the direction in anisotropic materials the deformation is not the same as isotropic materials.

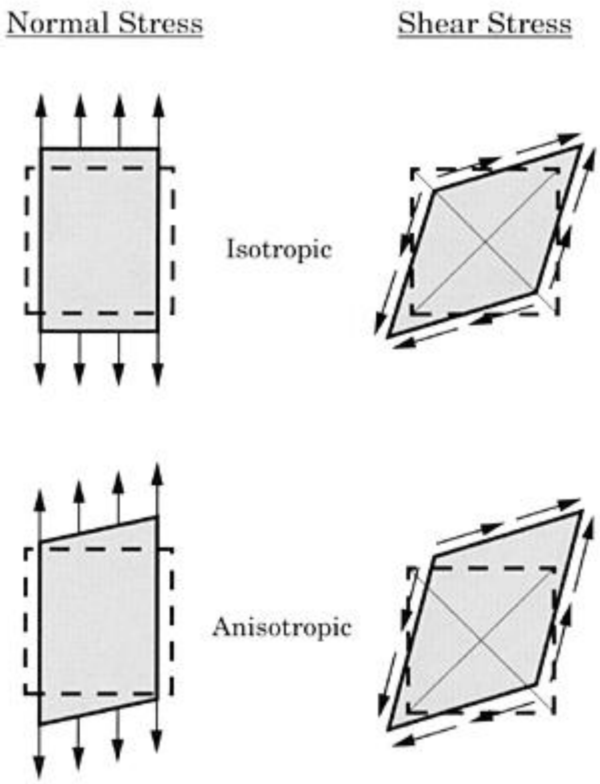

Figure 2.1 The behaviour of isotropic and anisotropic materials under uniaxial load.

2.2. Classical Laminated Plate Theory (CLPT)

The simplest equivalent single layer (ESL) laminated plate theory, based on the displacement field, is the classical laminated plate theory (CLPT) (Reissner 1961; Stavsky 1961; Dong 1962; Yang 1966; Ambartsumyan 1969; Whitney1969 and Reddy 2004). The two dimensional classical theory of plates was initiated by Kirchhoff (1850) in the 19th century, and then was continued by Love (1934) and Timoshenko (1934) during the early 20th century. The principal assumption in CLPT is that normal lines to the mid-plane before deformation remain straight and normal to the plane after deformation. Although this assumption leads to simple constitutive equations, it is the main deficiency of the theory. The effect of the transverse shear strains on the deformation of the elastic two-dimensional structure is ignored and some of the deformation mode constraints by reducing the model to a single degree of freedom (DoF) results are neglected. This is a consequence of the basic assumptions made. It is also worth mentioning that neglecting shear stresses leads to a reduction or removal of the three natural boundary conditions that should be satisfied along the free edges. These boundary conditions being the normal force, bending moment and twisting couple (Reissner 1945).

Despite its limitations, CLPT is still a common approach used to get quick and simple predictions especially for the behaviour of thin plated structures. The main simplification is that three-dimensional thick structural plates or shells are treated as two-dimensional plate or shells located through mid thickness which results in a significant reduction in the total number of variables and equations, consequently saving a lot of computational time and effort. The governing equations are easier to solve and present in closed-form solutions, which normally provides more physical or practical interpretation. This approach remains popular as it is well-known and has become the foundation for further composite plate analysis methods.

This method works relatively well for structures that are made out of a symmetric and balanced laminate, experiencing pure bending or pure tension. The error induced/introduced by neglecting the effect of transverse shear stresses becomes trivial on or close to the edges and corners of thick-sectioned configurations. The induced error increases for thick plates made of composite layers, for which the ratio of longitudinal to transverse shear elastic modulii is relatively large compared to isotropic materials (Reddy 1984). It neglects transverse shear strains, underpredicts deflections and overestimates natural frequencies and buckling loads (Cosentino and Weaver 2010).

The ESL theories are generally developed by assuming the form of the displacement field or stress field as a linear combination of unknown functions and the thickness coordinate (Mindlin 1951):

$$\rho_i(x,y,z,t) = \sum_{j=0}^{N} (z)^j \rho_i^j(x,y,t) \tag{2.6}$$

where ρ_i is the ith component of displacement or stress, (x, y) the in-plane coordinates, z the thickness coordinate, t the time, and ρ_i^j are functions to be determined.

When ρ_i are displacements, then the equations governing ρ_i^j are determined by the principle of virtual displacements (or its dynamic version when time dependency is to be included which is not a subject here) (Reddy 2004):

$$0 = \int_0^T (\partial U + \partial V - \partial K)dt \quad (2.7)$$

where δU, δV, and δK denote the virtual strain energy, virtual work done by external applied forces, and the virtual kinetic energy, respectively.

The classical laminated plate theory which is an extended version of the classical Kirchhoff's plate theory for laminated composite plates. It is based on the displacement field.

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0}{\partial y} \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \quad (2.8)$$

where (u_0, v_0, w_0) are the displacement components along the (x, y, z) coordinate directions, respectively, of a point on the midplane (z=0). Figure (2.2) shows the deformed geometries of composite plate for CLPT.

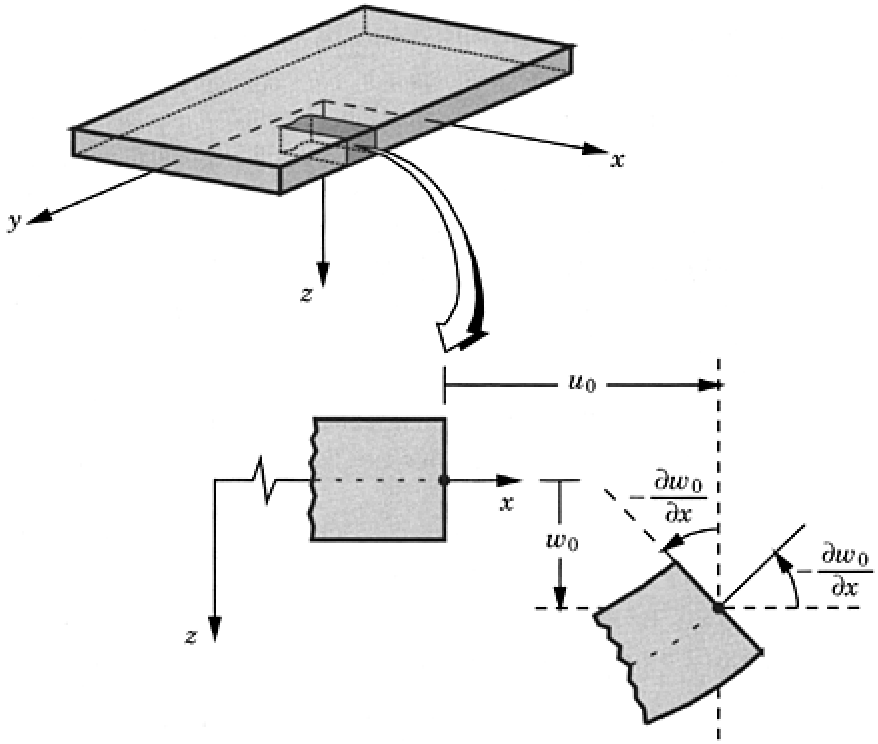

Figure 2.2 Deformed geometries of composite plate for CLPT (Reddy 2004)

Composite plates are, in fact, inherently subjected to transverse shear and normal stresses due to their discontinuous through-thickness behaviour and their global anisotropic nature (Cosentino 2010) in order to achieve better predictions of the response characteristics, such as bending, buckling stresses, etc. A number of other theories have been developed which are presented in following sections (Noor 1972; Pagano 1972; Noor 1975; Aydogdu 2006).

2.3 First-Order Shear Deformation Theories (FSDT)

Several theories have been proposed to analyse thicker laminated composite plates in order to consider the transfer shear effect. Most of these theories are extensions of the conventional theories developed by Reissner (1945) and Mindlin (1951), which are known as the shear deformation plate theories. These theories are based on the assumption that the displacement w is constant through the thickness while the displacements u and v vary linearly through the thickness of each layer (constant cross-sectional rotations w_x and w_y). Generally these theories are known as First-Order Shear Deformation Theories (FSDT) (Onsy 2020). According to this theory, transverse straight lines before deformation will still be straight after deformation but they are not normal to the mid-plane after deformation. This theory assumes constant transverse shear stress and it needs a shear correction factor in order to satisfy the plate boundary conditions on the lower and upper surface. The shear correction factor adjusts the transverse shear stiffness and thereby, the accuracy of results of the FSDT will depend significantly on the shear correction factor (Wu 2005; Aydogdu 2009). Further research has since been undertaken to overcome some of the limitations of FSDT without involving higher-order theories in order to avoid increasing the complexity of the equations and computations (Levinson 1980; Murthy 1981; Reddy 1984).

Bhaskar and Varadan (1993) used the Laplace transform technique to solve the equations of equilibrium. From the results they observed that the dynamic magnification factor for the deflection and the in-plane stress is close to 2.0, and that the interlaminar stresses can reach higher values depending on the geometry of the plate and type of loading (Kant 2000).

Onsy et al. (2002) presented a finite strip solution for laminated plates. They used the FSDT and assumed that the displacements u and v vary linearly through the thickness of each layer and are continuous at the interfaces between adjacent layers. They also assumed that the displacement w does not vary through the thickness. These assumptions allow for a more realistic situation (compared with CLPT) where the shear strains across the interfaces between adjacent laminae are not continuous. This model is more realistic compared with CLPT as the displacement results allow for the warping of the composite cross-section and the shear strain field is discontinuous at the linear interface. Two-node and three-node finite strip elements are developed as a result of their work (Onsy et al. 2002). Equations (2.8), for first order shear deformation theory, is presented as:

$$
\begin{aligned}
u(x, y, z, t) &= u_0(x, y, t) + z\phi_x(x, y, t) \\
v(x, y, z, t) &= v_0(x, y, t) + z\phi_y(x, y, t) \\
w(x, y, z, t) &= w_0(x, y, t)
\end{aligned}
\qquad (2.9)
$$

where ϕ_x and ϕ_y are rotations about the y and x axes, respectively. The FSDT extends the kinematics of the CLPT by including a transverse shear deformation in its kinematic assumptions. As it was mentioned the transverse shear strain is assumed to be constant with respect to the thickness coordinate (Reddy 2004). Figure (2.3) shows deformed geometries of composite plate for FSDT.

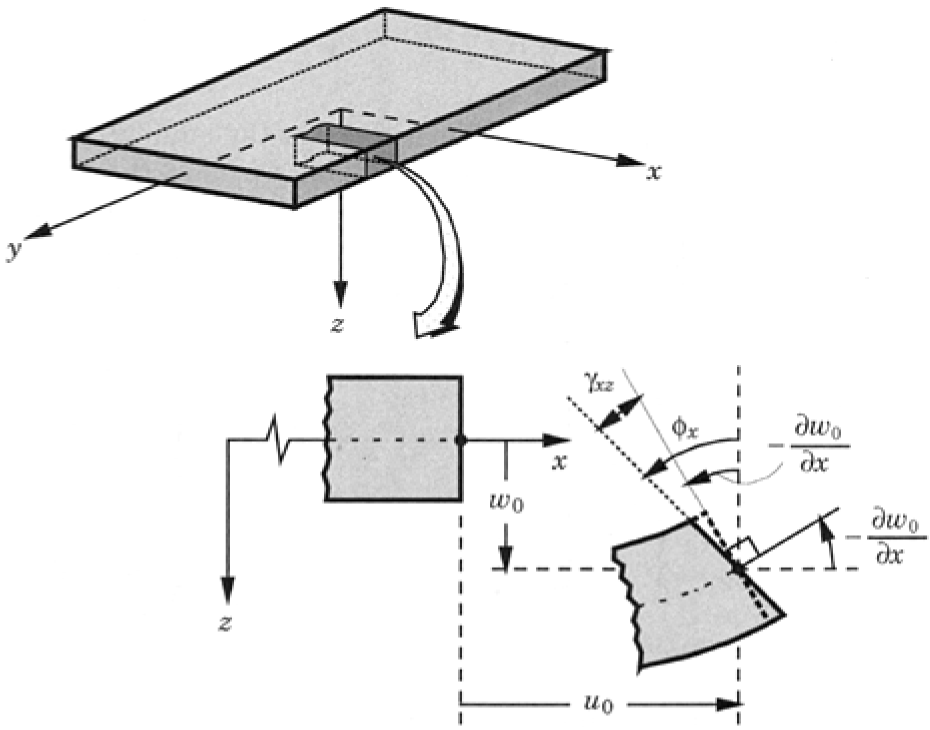

Figure 2.3 Deformed geometries of composite plate for FSDT (reddy 2004)

In order to calculate transverse shear more accurately, to satisfy all boundary conditions and to analyse the behaviour of more complicated thick composite structures under different loads, the use of higher order shear deformation theories is inevitable.

2.4 Higher Order Shear Deformation Theories (HOSDT)

The limitations of the classical laminated plate theory and the first order shear deformation theory led researchers to develop a number of global Higher Order Shear Deformation Theories (HOSDT) (Wu et al. 2005).

The higher order models are based on an assumption of nonlinear stress variation through the thickness. These theories are capable of representing the section warping in the deformed configuration. However, some of these models do not satisfy the continuity conditions of transverse shear stresses at the layer interfaces. Although the discrete layer theories do not have this concern, they are significantly more labour intensive and computationally slow when solving such problems due to the fact that the order of their governing equations depends on the number of layers (Karama at al. 1998). This is the main advantage of HOSDT compared to discrete layer theories which will be presented in following sections.

Whitney et al. (1973) attempted to examine the problem with interlaminar normal stress. The calculation of interlaminar normal stress was studied by Pagano (1974), a boundary layer theory, by Tang (1975) and Tang and Levy (1975), the perturbation method by Hsu and Herakovich (1977), and approximate elasticity solutions by Pipes and Pagano (1974) was carried out. Later, Pagano (1978), based on assumed in-plane stresses and the use of Reissner's variational principle, developed an approximate theory. In most of these models the laminate is assumed to be reasonably long. The stress singularities were considered in a model presented by Wang and Choi (1982). They used the Lekhnitskii's (1963) stress potential and the theory of anisotropic elasticity and determined the order of stress singularities at the laminate free-edges. The eigen function technique developed by them uses a collocation system at every ply interface in order to satisfy continuity. This limits the application of this method to relatively thin laminate (Kant et al. 2000).

Ambartsumian (1958) proposed a higher order transverse shear stress function in order to explain plate deformation. Soldatos and Timarci (1993), suggested a similar approach for dynamic analysis of laminated plates. Various different functions were proposed by Reddy (1984), Touratier (1991), Karama et al. (2003) and Soldatos (1992). The results of some of these methods were compared by Aydogdu (2006). Swaminathan and Patil (2008) used a higher order method for the free vibration analysis of antisymmetric angle-ply plates (Aydogdu 2009).

Some models just focus on special cases such as buckling, vibration, etc. For example, a 2D higher-order theory is developed by Matsunaga (1994, 1997) to investigate buckling in isotropic plates for in-plane loads. Effects of transverse shear and normal deformations have been predicted in this work. General higher order theories, which consider the complete effects of transverse shear, normal deformations and rotary inertia, have been studied for the vibration and stability problems of specific laminates (Matsunaga 1969, 2000). Thermal buckling, modal vibration properties and optimisation have been focused in angle-ply laminated composite plates (Khandan et al. 2012; Matsunaga 2006; 2007; Fiedler et al. 2010).

2.4.1 Third order shear deformation theory (TSDT)

Generally, researchers who have wanted to simulate plates have used the third-order shear-deformation theories first published by Schmidt (1977) and then developed by Jemielita (1975). A Third order shear deformation theory (TSDT) is presented by Reddy (2004). Some references know this theory as parabolic shear deformation plate theory (PSDPT). Phan and Reddy (1985) applied this theory for free vibration, bending and buckling of composite plates (Aydogdu 2006). The same unknown displacements as those used in first-order shear deformation theories were used. The theory also satisfies transverse shear free conditions at the outer surfaces. The results for the thick laminates prove that the in-plane stresses are much better than those identified using FSDT, but still have errors when comparing with three-dimensional models. It is noteworthy that this theory is not the layer-wise type, therefore, unlike most of the other equivalent single layer theories, it does not satisfy the continuity conditions of transverse shear stresses between layers (Wu et al. 2005).

Another parabolic distribution of shear strains through the laminated plate thickness was proposed by Vuksanovic (2000). It has a cubic variation of in-plane displacement. The results confirm that this model can explain/predict the global laminate response better than previous parabolic methods, but it is complex to accurately compute the interlaminar stress distributions (Wu et al. 2005).

In the third-order theories (Pandya and Kant 1988; Reddy 2004), the in-plane displacements are assumed to be a cubic expression of the thickness coordinate while the out-of-plane displacement is a quadratic expression at most. Based on the model that Vlasov (1957) presented for equations for bending of plates, Carrera (2007) presented a third order theory. The reduced third order model with three displacement variables was obtained by imposing homogeneous stress conditions with correspondence to the plate top-surface. This was further modified in the same research for the non-homogeneous stress conditions. Again, a closed form solution result was presented for both stresses and displacements in the case of harmonic loadings and simply supported boundary conditions. Idlbi et al. (1997) compared TSDT and parabolic shear deformation plate theory with CLPT for the bending of cross-ply plates (Aydogdu 2006).

The third-order laminated plate theory (Reddy 1984) which is based on the displacement field is:

$$u(x,y,z,t) = u_0(x,y,t) + z\phi_x(x,y,t) + z^3\left(-\frac{4}{3h^2}\right)\left(\phi_x + \frac{\partial w_0}{\partial x}\right)$$
$$v(x,y,z,t) = v_0(x,y,t) + z\phi_y(x,y,t) + z^3\left(-\frac{4}{3h^2}\right)\left(\phi_y + \frac{\partial w_0}{\partial y}\right) \qquad (2.10)$$
$$w(x,y,z,t) = w_0(x,y,t)$$

The displacement field accommodates quadratic variation of transverse shear strains/stresses. There is no need to use shear correction factors in a third-order theory. The third-order theories provide an increase in accuracy relative to the FSDT solution, at the expense of an increase in computational effort.

Figure (2.4) shows the differences between CLPT, FSDT and TSDT:

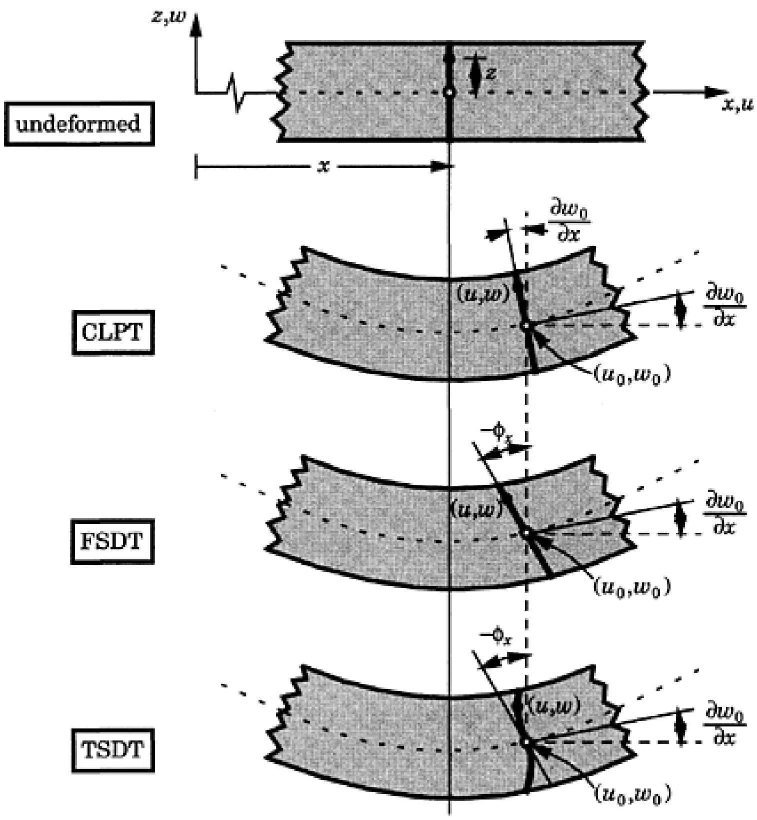

Figure 2.4 Comparing CLPT, FSDT and TSDT

2.4.2. Trigonometric Shear Deformation Plate Theory (TSDPT)

Trigonometric functions are used to describe the shear deformation plate theories called trigonometric shear deformation plate theory (TSDPT). Touratier (1991) chose transverse strain distribution as a sine function.

Stein (1986) developed a 2D theory wherein the displacements are stated by trigonometric series. Trigonometric terms are combined with the usual algebraic through-the-thickness terms assumed for the displacements, to provide a better solution. Later the effects of transverse shear on cylindrical bending of laminated composite plates were studied by Stein and Jegley (1987). They show that this theory calculates the stresses more accurately than other theories. A straightforward method to analyse symmetric laminates under tension/compression using the principle of minimum complimentary energy and the force balance method, based on assumed stress distributions was introduced by Kassapogolou and Lagace (1986, 1987). Afterward, Kassapogolou (1990) generalised and modified this approach for general unsymmetric laminate loads (in-plane and out-of-plane common moment and shear loads). This model was more general; however, it does not solve the weaknesses of inequality in Poisson's Ratios and the coefficients of mutual effect that exists between different plies through the thickness (Kant and Swaminathan 2000). Touratier compared the TSDT proposed by Reddy (1984) with the one suggested by Touratier (1991) using a sine function to illustrate the warping throughout the thickness of the plate fibre during rotation, by considering the transverse shear (IdIbi 1997). Becker (1993) made use of cosine and sine functions for warping deformation of v and w displacement respectively and developed a closed-form higher-order laminated plate theory.

An analytical method was presented by Mortan and Webber (1993), using the same approach as Kassapoglou (1987) and Becker (1993); they considered the thermal effects in their model (Kant and Swaminathan 2000).

An accurate theory for interlaminar stress analysis should consider the transverse shear effect and continuity requirements for both displacements and interlaminar stresses on the composite interface. In addition to accurate interlaminar stress and transverse shear prediction, it is advantageous if the model is variationally consistent in order to use it for finite element formulation. Considering these points, and to obtain the interlaminar shear stress directly from the constitutive equations, Lu and Liu (1990) improved an interlaminar shear stress continuity theory. Although the interlaminar shear stress is obtained directly from the constitutive equations in their model, the deformation in the thickness is neglected and therefore it cannot calculate the interlaminar normal stress directly from the constitutive equations. Despite the conventional analysis for laminated composite materials, where the composite interface is always assumed to be rigidly bonded due to the low shear modulus and poor bonding, the composite interface can be non-rigid. Later, Lu and Liu (1992), in their search to improve their model by investigating the effect of interfacial bonding on the behaviour of composite laminates, developed the interlayer shear slip theory based on a multilayer approach. Finally, a closed-form solution for the general analysis of interlaminar stresses for thin and thick composite laminates under sinusoidal distributed loading was derived by Lee and Liu (1992). Both interlaminar shear stress and interlaminar normal stress at the composite interface were satisfied in this model and also the interlaminar stresses could be calculated directly from the constitutive equations, yet the accuracy of their model was questioned later. Touratier (1991) proposed a theory based on using certain sinusoidal functions for shear stress. Numerical results for the bending of sandwich plates were presented and compared with the other theories. It was shown that this theory is more accurate than both FSDT and some other higher-order shear deformation theories (Kant and Swaminathan 2000).

By considering the following equations:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z w_{0x} + \phi_1(z) u_0(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) - z w_{0y} + \phi_2(z) v_0(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{2.11}$$

where, for TSDPT we have:

$$\phi_1(z) = \phi_2(z) = \left(\frac{h}{\pi}\right)\sin\left(\frac{\pi z}{h}\right) \tag{2.12}$$

2.4.3. Hyperbolic Shear Deformation Plate Theory (HSDPT)

Hyperbolic shear deformation plate theory (HSDPT) was proposed by Soldatos (1992). Timarci and Soldatos (1993) united/combined these shear deformation theories. The basic advantage of this unified theory was the ability to change the transverse strain distribution (Reddy 1984). Ramalingeswara and Ganesan (1996) used parabolic and hyperbolic function to uniform external pressure and a simply supported cylindrical shell for cross ply laminated composite by considering an internal sinusoidal pressure (Kant and Swaminathan 2000). It is also worth mentioning the Karama model where

Karama et al. (2003) proposed an exponential function for the transverse strain in his study of the bending of composite.

In equation (2.11) for HSDPT we have:

$$\phi_1(z) = \phi_2(z) = h\sin\left(\frac{z}{h}\right) - z\cosh\left(\frac{1}{z}\right) \qquad (2.13)$$

2.4.4. Other HOSDT

Rohwer (1992) presented a comparative study of different higher-order shear deformation theories for multilayer composite plates. The advantages and limitations of the various models were shown with the analysis applied on a rectangular plate with a range of thickness, number of layers, edge ratios and material properties. A double Fourier series was used by Kabir (1992, 1994). It was tested for symmetric and antisymmetric laminate plates with simply supported boundary conditions at all edges and +45/-45 orientation angles for laminae. Later, by using the same approach and Kirchhofe's theory, a simply supported laminated plate with arbitrary laminations was presented by Kabir (1996). For validation the results were compared with FSDT based on finite element solutions. Based on Kassapogolou and Lagace's (1986, 1987) model, and using boundary layer theory, Ko and Lin (1992) proposed a model to analyse the 3D stress distribution around a circular hole in symmetric laminate for in-plane stresses. The laminate was subdivided into interior and boundary layer regions and each stress component was introduced by superposition of the interior and boundary layer stress. All the boundary conditions for each ply and the interface traction continuity were satisfied. Later Ko and Lin (1993) extended the work for analysing the complete state of stress around a circular hole in symmetric cross-ply laminates under bending and torsion.

Wang and Li (1992) used 3D anisotropic elasticity and the method of separation of variables to derive the constitutive equations with unknown displacements for each cylindrical lamina of a multilayered shell. They considered mechanical and thermal load with various boundary conditions in their work. The model is able to determine the interlaminar stresses exactly (Kant and Swaminathan 2000). Matsunaga (2002) proposed a global higher-order theory. It was employed to analyse the inter-laminar stress problems of laminated composite and sandwich plates. The advantage of this method is that the total number of unknowns does not increase as the number of layers increases and from the constitutive relations the in-plane stresses can be predicted accurately. By integrating the three-dimensional equilibrium equation, transverse shear and normal stresses can be calculated, which satisfy the free surface boundary conditions and continuity conditions at interfaces (Wu et al. 2005).

A complex function was presented by Soldatos (2004) which only dealt with simple bending of a special kind of transversely inhomogeneous monoclinic plates (class of symmetric cross-ply laminates and functionally graded material). The basis of these methods was the governing equations of the generalised, shear deformable plate theory presented by Soldatos (1997). Later, Soldatos presented a theory which considered the effects of both transverse shear and transverse normal deformation, in a way that allows for multiple, a posteriori, choice of transverse strain distributions to be stated. This function considers the bending-stretching coupling for un-symmetric laminates and inhomogeneous material through-thickness (Soldatos et al. 1997, 2006).

2.5. Layerwise Theory (LT)

In recent years, the layerwise theories and individual layer theories have been presented to achieve more accurate results. Some of this research is carried out by Wu and Chen (1994) and Cho et al. (1991), Plagianakos and Saravanos (2009), and Fares and Elmarghany (2008). These theories require many different unknowns for multilayered plates and are often computationally time-consuming and expensive to obtain accurate results (Nosier et al. 1993; Di-Sciuva et al. 1995).

A number of layerwise plate models, which can represent the zig-zag behaviour of the in-plane displacement through the thickness, have been developed to predict both gross response and the stress distributions (Wu et al. 2005). The basic idea is assuming certain displacement and/or stress models in each layer, followed by equilibrium and compatibility equations at the interface to reduce the number of the unknown variables (Fiedler et al. 2010). For example, Di-Sciuva and Icardi (1995) proposed an eight-noded general quadrilateral plate element with 56 degrees of freedom for anisotropic multi-layered plates based on the third-order zig–zag plate model and demonstrated a good accuracy. Generally in Layerwise theories the number of unknowns increases significantly with the number of layers and consequently the computational weight becomes considerably heavier and higher. To overcome this problem different solutions are suggested. Cho and Parmerter (1993) presented a model in which the number of unknowns is independent of the number of layers. They modified their previous model of a composite plate theory for general lamination configurations by superimposing a cubic varying displacement field on a zig–zag linearly varying displacement. This method satisfies transverse shear stress continuity at the layer interfaces and shear-free surface conditions.

There is a lack of accuracy in Equivalent Single Layer (ESL) for thick laminated composite plates, especially in the vicinity of the free edges, corners or other special features such as holes, where the interlaminar shears dominate by the stress field distortion and highly depend on the real stacking sequence. The so-called free edge effect is mainly explained by the mismatch of elastic properties, particularly the Poisson's ratio, at the interface of two consecutive layers. Therefore, some other more accurate layerwise field equations are needed to overcome such limitations. The theories that have been developed to justify through-the-thickness piece-wise behaviour of stresses and displacement are often subject to Zig-Zag theories (ZZ).

The general difference between a single layer plate theory and a multi-layer plate theory is that different tangential elastic compliances of the plies cause the displacement components to show a quick change of their slopes in the thickness direction at each layer interface, the so-called zig–zag effect. To summarise, the in-plane stresses $(\sigma_{11}, \sigma_{22}, \sigma_{12})$ can be discontinuous at each layer interface while the transverse stresses $(\sigma_{13}, \sigma_{23}, \sigma_{33})$, for equilibrium, must be continuous. In ZZ theory the compatibility of the displacements and the inter-laminar equilibrium of the transverse stresses in the thickness direction are assured by defining a new stiffness matrix called C0z (Fiedler et al. 2010).

Lekhnitskii (1968) was one of the pioneers who tried to define a ZZ theory. The main drawback for this approach was the limitation of the approach to multi-layered composite where each layer is isotropic. Ren (1986) later improved this model by using an extension of the theory developed by Reissner (1944) to multi-layered plates. This approach used a Lagrange function with five parameters, which represent the degrees of freedom of the structure. In fact, each degree of freedom is represented by a function for the whole domain. Normally, the DoFs are:

- Two axial in-plane displacements along x and y co-ordinates (namely u_x and u_y)

- Transverse displacement w,
- Two transverse shear strains γ_{xz} and γ_{yz}.

In uncoupled problems the essential number of degrees of freedom is three. On the other hand when it is possible to separate the in-plane from the transverse response in order to solve the transverse equilibrium equations the shear strains are sometimes replaced by the two rotations of the cross-section about the y and x-axes, therefore the number of DoFs reduced to three (Cosentino and Weaver 2010).

Discrete-layer models which categorise Layerwise theories, have been developed by Noor and Burton (1989) and Carrera (1998), which can provide very accurate prediction of the displacement and stress. However, increasing the number of layers lead to increasing the number of unknowns. Due to the number of variables depending on the number of layers, they become impractical for engineering application.

Di-Sciuva (1987, 1993) and then Touratier (1992) proposed simplified discrete layer models with only five essential variational unknowns to consider the warping in the deformed configuration. Like many early methods these models also could not satisfy the compatibility conditions (both at layer interfaces and at the boundaries). Beakou (1991) and Idlbi (1995) modified those models to overcome this limitation and then He (1994) introduced the Heaviside step function which allows automatic satisfaction of the displacement continuity at interfaces between different layers. Later, based on discrete layer theory and Di-Sciuva (1993), He (1994) and Ossadzow et al. (1995) a new model is presented for laminated composite structure by Karama et al. (1998).

Based on Sciuva's theory, new zig-zag models have been proposed by Lee et al. (1990), Di-Sciuva (1992) and Cho and Parmerter (1992). These models contain the same variables as those given by the FSDT and satisfy transverse shear stresses continuity conditions at interfaces. The number of variables does not depend on the layers of laminated plate. Numerical solutions verify that in-plane stresses compared with 3D solutions are very accurate. However, the transverse shear stresses from the direct constitutive equation method cannot be calculated accurately. In order to accurately predict interlaminar stress, the equilibrium equation approach has to be considered (Khandan et al. 2012).

The discrete-layer theories (Sun and Whitney 1973; Srinivas 1973; Seide 1980) are capable of modeling the warpage of the cross-section during bending and of predicting in-plane responses. A refined shear deformation theory (RSDT) with a simplified discrete-layer model is presented by He (1989) to reduce the total number of dependent variables. The transverse displacement is constant across the thickness and the in-plane displacements assumed to be piecewise linear. The transverse shear strains across any two different layers next to each other are assumed to be linearly dependent on each other as real transverse shear stresses are continuous between layers. Based on those assumptions, governing equations have been derived using the principle of minimum potential energy. The difference between this method and FSDT is the set of governing differential equations which are of the 12^{th} order, which means two orders higher than FSDT. However, the variables are still the same. The limitation on the application of this theory is that the thickness of the shell must be small compared to the principal radii of curvature. Thus, the analytical solutions can be obtained for only a few cases. This model have been tested for various problems such as cylindrical bending of an infinitely long cross-ply laminated strip under sinusoidal loading, and bending of simply supported symmetric and antisymmetric cross-ply rectangular plates under sinusoidal transverse loads (He 1992; He et al. 1993). It was shown that RSDT is accurate to predict the stresses and deflections by comparing the numerical results of RSDT with FSDT and 3D elasticity theory. Later, based on the same model, He and Zhang (1996) presented a simple closed-form solution for antisymmetric angle ply, simply-supported rectangular plates subjected to sinusoidal transverse loads and compared their model with CLPT and FSDT.

In order to satisfy the continuity of displacements at the interfaces between two adjacent layers Robbins and Reddy (1993) introduced a layerwise laminate theory by using a one-dimensional Lagrangian interpolation function associated with a series of n nodes. Using the Lagrangian function for the variation of the displacements through the thickness causes calculations to become large when the number of layers increases (Chaudhri 1986; Barbero et al. 1989; Han and Hao 1993).

Wu and Kuo (1992) proposed a local higher-order lamination theory. They used the Lagrange multiplier method and defined the Lagrange multipliers as the interlaminar stresses to evaluate the interlaminar stresses. They also used interlaminar stresses as the primary variables. It guaranteed the equilibrium equation and the displacement continuity constraints at the interface between consecutive layers. Then they used the Fourier series expansion method to analyse the problem (Kant and Swaminathan 2000).

A higher-order layerwise theoretical framework is presented by Plagianakos and Saravanos (2009), which was able to predict the response of thick composite plates. The linear approximation by linear layerwise theories is considered. The displacement in each discrete layer through the thickness of the laminate includes quadratic and cubic polynomial distributions of the in-plane displacements. Interlaminar shear stiffness matrices of each discrete layer are presented by considering the interlaminar shear stress compatibility conditions in order to guarantee the continuity of interlaminar shear stresses through the thickness. The main advantage of this model when compared to linear layerwise theories is in the small number of discrete layers used to model the thick composite laminate through-thickness and in the prediction of interlaminar shear stresses at the interface.

2.6. Mixed Plate Theory

Recently, some researchers have attempted to combine previous models in order to overcome the limitations of each one. Unified equations have been proposed for mixed layerwise and mixed equivalent single layer theories. The main aim is to formulate these unified C0z theories in the most general way for users to be able to choose the approach (equivalent single layer, Layerwise zig–zag, etc); at the same time the order of the expansion of displacements and transverse stresses (Carrera and Ciuffreda 2005).

This class of model has been contemplated over the last few decades. The so-called mixed variational approach-based on the variational principles developed by Hellinger (1914) was proposed and then improved by Reissner (1944). The number of variables that must be computed is at least 2N+1 where N is the total number of layers. The number of variables can be significantly reduced by using a weak form of Hooke’s Law (Carrera 1995), which shows the variables in terms of the three displacements only. Shimpi et al. (2007) derived two novel formulations with only two variables, which work perfectly for moderately thick isotropic plates. However, it requires ad-hoc calculated shear correction factors for transverse shear stresses in multilayered composite plates.

Recently, Tessler et al. (2009) have developed a refined zig-zag theory based on the kinematics of FSDT. The deployment of novel piece-wise linear zig-zag functions provides an accurate and robust approach. The number of computations is relatively low compared to other layerwise theories. By reducing the number of functional degrees of freedom and choosing the appropriate degree of freedom, the function converges quickly and the computational efforts are reduced (Cosentino and Weaver 2010).

Cen et al. (2002) presented a simple displacement-based, quadrilateral 20 DoF (5 DoF per node) bending element based on the FSDT for analysis of arbitrary laminated composite plates by considering the Timoshenko's beam theory for elements and interpolation for shear strain, rotation and in-plain displacement. By proposing a hybrid procedure the stress solutions, especially transverse shear stresses, in comparison with LT are improved.

A Reissner mixed variational theorem (RMVT) based on TSDT was developed for the static analysis of simply-supported, multilayered plates under mechanical loads. Wu et al. (2005) based on the observations of these solutions, developed a model for the static analysis of simply supported, multi-layered composite and functionally graded material (FGM) plates. Reddy's third-order displacement model (Reddy 1990) and a layer-wise parabolic function of transverse shear stresses are used as the kinematic and kinetic assumptions, respectively. A set of Euler–Lagrange equations associated with the possible boundary conditions was derived. The results obtained from the present TSDT based on RMVT are compared with those obtained from the published TSDT based on the principle of virtual displacement for single layer orthotropic, multilayered composite and multilayered FGM plates (Wu and Li 2010).

Another hybrid model, based on FSDT, was presented by Daghia et al. (2008) for the analysis of laminated composite plates. This model considered a new quadrilateral four-node finite element from a hybrid stress formulation involving, as primary variables, compatible displacements and element wise equilibrated stress resultants. By the minimum number of parameters, the transverse stresses through the laminate thickness are reconstructed using three-dimensional equilibrium. Their strategy can be adopted with any plate finite element and does not need any correction factor. However, like other hybrid models, the amount of computational calculations increased drastically.

Demasi (2009) presented mixed plate theories based on the Generalised Unified Formulation (GUF) in five parts. In the first part, the governing equations for GUF are explained. GUF is a recent common method which is categorised as a displacement-based theory. GUF is extended for the first time for a mixed variational statement, Reissner's mixed variational theorem (Reissner 1944). In this technique each of the displacement variables and out-of-plane stresses is independently considered. Also, different orders of expansions for the different unknowns can be chosen. This method is applied to the case of LT. As mentioned before for LT each layer is independently modelled and the compatibility of the displacements and the equilibrium of the transverse stresses between two contiguous layers are enforced a priori. Infinite combinations of the orders used for displacements and out-of-plane stresses can be freely chosen (Demasi 2009 part II). In addition, GUF is applied to mixed higher order shear deformation theories. The displacements have an equivalent single layer description, whereas the stresses are based on a layerwise category. The compatibility of the displacements and the equilibrium of the transverse stresses between two adjacent layers are considered. The displacement-based ‘‘classical” HSDT results will be obtained by neglecting the out-of-plane stresses and using the static condensation technique in this model (Demasi 2009 part III). Furthermore, this model is applied for advanced mixed higher order zig-zag theories by adopting Murakami’s zig-zag function (Demasi 2009 part IV) and finally the numerical results of this model are presented (Demasi 2009 part V).

Bhar et al. (2011) presented a method based on least square of error for accurate evaluation of through-the-thickness distribution of transverse stresses in thick composite and sandwich laminates, using a displacement-based C0 higher-order shear deformation theory (HSDT). This model is different from the conventional method of integrating the 3D equilibrium equations for transverse stress in composite laminates. The numerical results compared well with the results from FSDT and HSDT. They used this method to express the transverse shear stress accurately and furthermore expand it for post-processing techniques.

Based on the least-squares variational principle, which is an alternative approach to the mixed weak form FEM, Moleiro et al. (2008) presented a mixed finite element model for the static analysis of laminated composite plates. This model is a FSDT with displacements and stress resultants as independent variables. By using equal-order C0 Lagrange interpolation functions of high levels along with full integration, the model is proposed and the results for different boundary conditions calculated and compared with previous work by Chou and Kelly (1980). The mixed least-squares model was developed for free vibration and different side-to-thickness ratios (from 10 to 500) (Moleiro et al. 2009).

It is worth noting that Finite Element Methods (FEM) can be used for all the different methods which have been mentioned. (Reddy 1984, 2004; Cho 1994; Ugo 1998; Kant and Swaminathan 2000; Carrera and Demasi 2002; Sheikh and Chakrabarti 2003; Oh and Cho 2004; Han et al. 2004; Zhang and Kim 2005; Kim et al. 2005; Kreja and Schmidt 2006; Ramesh et al. 2008, 2009). However, FEM is not the only solution for these models. Recently, different meshless methods are presented to solve equation for laminated composite plates (Xiang et al. 2011). The developments of element-free or meshless methods and their applications in the analysis of composite structures have been reviewed by Liew et al. (2011) recently.

2.7. Summary

It is shown in the research literature review (this chapter) that current composite models either ignore the induced twist displacement or consider it as an undesirable displacement. It is discussed, because of some simplification assumptions (normal lines to the mid-plane before deformation remain straight and normal to the plane after deformation), some composite models such as classical laminate plate theory are not able to estimate the transverse shear and therefore cannot predict the behaviour of laminated composite structure accurately. Not considering a transverse shear effect leads to ignoring the induced twist displacement in composite products. They are some other models which consider the transfer shear effect. They have been categorised to single layer and discrete layer theories. The advantages and disadvantages of each model have been explained. In the single layer theory approach, layers in laminated composites are assumed to be one equivalent single layer whereas in the discrete theory approach each layer is considered individually in the analysis. In discrete layer models (layerwise theories) each layer is analysed independently in order to achieve more accurate results and to assure the compatibility of the displacements and the inter-laminar equilibrium equations. Furthermore, the laminated composite models for plate can be categorised into two types: stress based and displacement based theories. This research mainly focused on how accurately and efficiently the models can predict the transverse shear effect, as it is using this transverse shear effect to estimate the induced twist in laminated composite plate. However, the induced twist has not been used in any of the mentioned models to control the composite structures. This is the idea behind this work and this research is trying to fill this gap and find a way to control the composite structure by controlling the induced twist displacement. The amount of twist can be controlled by changing the fibre orientations in composite plates. Because changing fibre orientations changes the stiffness matrix in Hook's law, and any deformation (including twist) can be calculated from Hook's law. On the other hand, the aim of this research is to answer "How can a laminated composite plate be designed and controlled by using the induced twist deformation through optimisation of the fibre orientations of the laminae?”

In order to overcome the limitation of the current models of composite materials, a semi analytical model is proposed which can consider the transverse shear effect in constitutive equations of composite materials. Also, in order to answer the above question, fibre orientation in laminated composite plate is considered as a design parameter which can control the transverse shear and as a result can control the induced twist displacement in laminated composite plate. The fibre orientations are optimised by modifying the current penalty function for SA and proposing a new penalty function which is able to control the amount of induced twist in laminated composite plate.

CHAPTER 3:

Semi-Analytical Modelling of Composites

3.1. Semi-analytical Models

The non-linear evaluation of laminated plates has been studied by many researchers. Also, various semi-analytical and numerical methods for the description and response of laminated plates have been developed. A comprehensive study of the analysis for the geometrically non-linear modelling of isotropic and composite laminated plates was given by Chia (1980) and later (Khandan et al. 2012). The common analytical non-linear theories for laminated composites such as classical laminated plate theory and first shear deformation plate theory, which were described earlier, generally use the Rayleigh-Ritz method (Liew et al. 2004) or the Galerkin method (Savithri et al. 1993; Renganathan et al. 2002; Tanriover et al. 2004). The accuracy of these analytical models depends on the trial functions which they choose and they have to satisfy at least the kinematic boundary conditions. There are also some numerical methods to analyse laminated plates for the large deflection including the finite strip method (Akhras et al. 1998; Dawe 2002); the differential quadrature technique (Yang and Zhang 2000); the method of lines (Rabinovitch 2005); Finite Element Method (FEM) (Kant and Swaminathan 2000; Carrera and Demasi 2002; Ramesh et al. 2008, 2009). Different meshless methods are presented to solve the equations for laminated composite plates (Xiang et al. 2011).

Due to numerous computations and the number of unknown variables, numerical methods are needed to solve the problem of laminated plates. However, the analytical and semi-analytical non-linear models are essential and the first step to explain the physical non-linear behaviour of the composite plate structure. Furthermore, these methods generally present reliable and quick solutions for simple problems and give an initial idea in the early stage of the design process. They also provide a means of validating the numerical methods and potentially are the tools to enable the further development of new computational models. Therefore, the development of the semi-analytical methods has been growing rapidly (Shufrin et al. 2008).

In semi-analytical models instead of using the virtual work as it is used in numerical models, the constitutive equations are solved by mathematical estimation techniques. So, the main requirement of semi-analytical models is a mathematical technique to solve (estimate) the complex and non-linear constitutive equations. The Kantorovich method is one of the techniques which were used to solve the constitutive equations. Later, the extended Kantorovich method (EKM) was found to be a valuable technique to offer more accurate approximate solutions for various systems of governing partial differential equations of problems. This technique also converges faster than similar mathematical techniques with less computational efforts and the potential of achieving closed form solutions for simple cases are other advantages of this method (Alijani and Aghdam 2009). The EKM is used for eigenvalue (Kerr 1969), bending, vibration and buckling of thin rectangular plates, bending of variable thickness thin plates, bending of moderately thick rectangular isotropic and orthotropic plates and free-edge stress analysis. Most recent EKM articles also include buckling of laminated and bending of piezolaminated rectangular plates (Alijani and Aghdam 2009).

Recently, EKM was improved by Shufrin et al. (2009). They used extra terms and named it the multi-term extended Kantrovich method. The multi-term extended Kantorovich method assumes a solution of the 2D problem in the form of a sum of products of functions in one direction and the same structure functions in the other direction. As a result, the general form of problem is reduced to a set of non-linear ordinary differential equations in the second direction. Now, solving a set of non-linear ordinary differential equations (even for complicated cases) is possible with any mathematical tool. Then the solution of the 1D problem is used as the assumed functions which can be replaced in the original 2D problem. Subsequently, the 2D equations should be solved. These iterations are repeated until convergence is completed. Unlike most of the other semi-analytical models, the accuracy of this method is independent of the initial chosen functions. This initial function, even if it does not satisfy any of the boundary conditions (Yuan and Jin 1998; Shufrin et al. 2008), does not affect the accuracy of the final solution. Noteworthy, the EKM was applied by Soong (1972) to the large deflection analysis of thin rectangular isotropic plates subjected to uniform loading. Some solutions used only one-term for expansion yielding of isotropic plates. However, it is showed that one term formulation is not enough to predict the behaviour of anisotropic plates (Shufrin et al. 2008).

By presenting new techniques which are solving the non-linear equations much faster and more accurately, the semi-analytical models for composites are developing. Therefore, it is important to apply these new methods in the composite field to keep the modelling of composites up to date. One of these mathematical techniques which is presented recently is Newton-Kantorovich-Quadrature (NKQ) method. NKQ method was proposed recently, by Saberi-Najafi and Heidari (2010), for solving nonlinear integral equations in the Urysohn form. Here, NKQ method is used to present a more accurate semi-analytical model for composites. The main aim of this work was to achieve a semi-analytical approach for the non-linear model of composite plates. As this method is able to solve the general constitutive equations for composites, it is potentially able to consider the out of plane loadings as well as in plane loadings. However, it is not practical to use this model for complex loadings as the computational process increases exponentially and the model cannot converge. The accuracy and convergence of the method has been investigated through a comparison with other semi-analytical solutions and with FEA using a number of numerical examples in order to validate the model.

Governing Equations

3.2. General Governing Equations for Composites

In order to be able to present the semi-analytical model, some governing equations for composites are explained. The state of stress at a point in a general continuum can be represented by nine stress components σ_{ij} $(i, j = 1,2,3)$ acting on the sides of an elemental cube with sides parallel to the axes of a reference coordinate system (Figure 3.1).

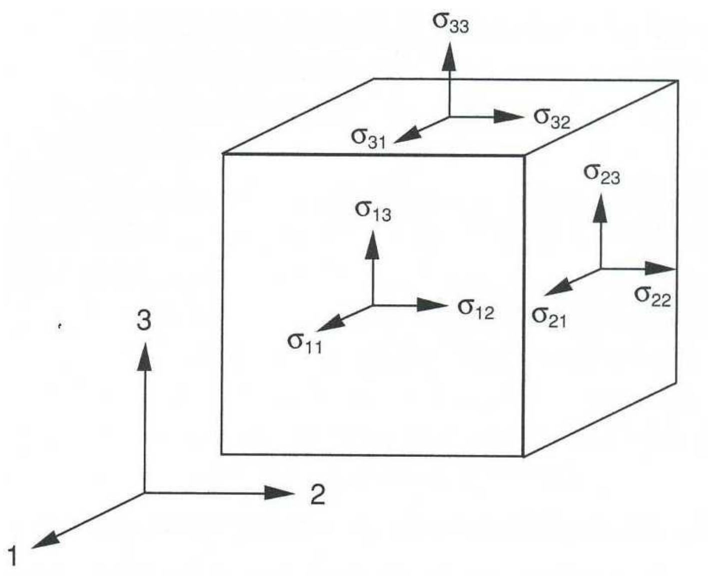

Figure 3.1 State of stress at a point in a general continuum (Daniel and Ishai 2006)

As it was mentioned in the most general case the stress and strain components are related by the generalised Hook's law. Here, two different scenarios are explained which are normally considered to solve composite problems.

3.2.1 In-plane Stress

The classical laminate theory is used to analyse the mechanical behaviour of the composite laminate. It is assumed that plane stress components are taken as zero. The in-plane stress components are related to the strain components as:

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix}_k = \begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\end{bmatrix}\begin{bmatrix}\epsilon_{xx}\\ \epsilon_{yy}\\ \gamma_{xy}\end{bmatrix} \tag{3.1}$$

where k is the lamina number, $\overline{Q}_{ij}$ are the stiffness components, which can be explained in terms of principal stiffness components, Q_{ij}, which are defined in reference (Daniel and Ishai 2006; Khandan et al. 2010).

Stress resultants, or forces per unit length of the cross section, are obtained as:

$$\begin{bmatrix}N_{xx}\\ N_{yy}\\ N_{xy}\end{bmatrix} = \int_{-h/2}^{h/2}\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix}\mathrm{dz}=2\sum_{k=1}^{m} n_k t_0 \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix}_k \tag{3.2}$$

Here m is the number of distinct laminae, n_k is the number of plies in the kth lamina. Here, lamina is meant to be a group of plies with the same orientation angle. Substituting the stress–strain relation given by Eq. (3.1) into Eq. (3.2):

$$\begin{bmatrix}N_{xx}\\ N_{yy}\\ N_{xy}\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12} & A_{16}\\ A_{12} & A_{22} & A_{26}\\ A_{16} & A_{26} & A_{66}\end{bmatrix}\begin{bmatrix}\epsilon_{xx}\\ \epsilon_{yy}\\ \epsilon_{xy}\end{bmatrix} \tag{3.3}$$

where A_{ij}, components of extensional stiffness matrix, are given by:

$$A_{ij} = 2\sum_{k=1}^{m} n_k t_0 (\overline{Q}_{ij})_k \tag{3.4}$$

Principal stress components can be obtained using the following transformation:

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \tau_{12}\end{bmatrix} = \begin{bmatrix} cos^2\theta_k & sin^2\theta_k & 2cos\theta_k sin\theta_k \\ sin^2\theta_k & cos^2\theta_k & -2cos\theta_k sin\theta_k \\ -cos\theta_k sin\theta_k & cos\theta_k sin\theta_k & cos^2\theta_k - sin^2\theta_k \end{bmatrix}_k \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix} \tag{3.5}$$

3.2.2 Out of Plane Stress

In the classical laminate theory, it is assumed that straight lines normal to the middle surface remain straight and normal to that surface after deformation. These assumptions are not valid in the case of thicker laminates and laminates with low stiffness central plies undergoing significant transverse shear deformations. In the following, referred to as first order shear deformation laminate plate theory, the assumption of normality of straight lines is removed. On the other hand straight lines normal to the middle surface remain straight but not normal to that surface after deformation (Reddy 2004). Q_{ij} can be found in Daniel and Ishai (2006), Khandan et al. 2010).:

$$\begin{gathered}\begin{bmatrix}N_{xx}\\ N_{yy}\\ N_{xy}\end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix} dz \\ \begin{bmatrix}M_{xx}\\ M_{yy}\\ M_{xy}\end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix} zdz \\ \begin{bmatrix}V_q\\ V_r\end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix}\tau_{xz}\\ \tau_{yz}\end{bmatrix} dz\end{gathered} \tag{3.6}$$

$$\begin{bmatrix}N_{xx}\\ N_{yy}\\ N_{xy}\\ M_{xx}\\ M_{yy}\\ M_{xy}\\ V_q\\ V_r\end{bmatrix} = \begin{bmatrix}N_1\\ N_2\\ N_{12}\\ M_1\\ M_2\\ M_{12}\\ V_1\\ V_2\end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} & 0 & 0 \\ A_{12} & A_{22} & A_{23} & B_{12} & B_{22} & B_{23} & 0 & 0 \\ A_{13} & A_{23} & A_{33} & B_{13} & B_{23} & B_{33} & 0 & 0 \\ B_{11} & B_{12} & B_{13} & D_{11} & D_{12} & D_{13} & 0 & 0 \\ B_{12} & B_{22} & B_{23} & D_{12} & D_{22} & D_{23} & 0 & 0 \\ B_{13} & B_{23} & B_{33} & D_{13} & D_{23} & D_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & E_{11} & E_{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & E_{12} & E_{22} \end{bmatrix} \begin{bmatrix}\epsilon_1\\ \epsilon_2\\ \gamma_{12}\\ k_1\\ k_2\\ k_{12}\\ \gamma_{13}\\ \gamma_{23}\end{bmatrix} \tag{3.7}$$

where the components of this stiffness matrix are given by:

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} \overline{Q}_{ij}^{m} (1, z, z^2) dz \quad (i, j = 1,2,3)$$
$$E_{ij} = \int_{-h/2}^{h/2} \overline{Q}_{\alpha\beta}^{m} k_i k_j dz \quad (i, j = 1,2 \; and \; \alpha, \beta = i + 6, j + 6). \tag{3.8}$$

The out-of-plane boundary conditions include three cases: simply supported (S), clamped (C), and free (F) edges. The four possible in-plane restraints along the plate edges are shown in Figure (3.2) and they are denoted by a subscript index (Shufrin et al 2008).

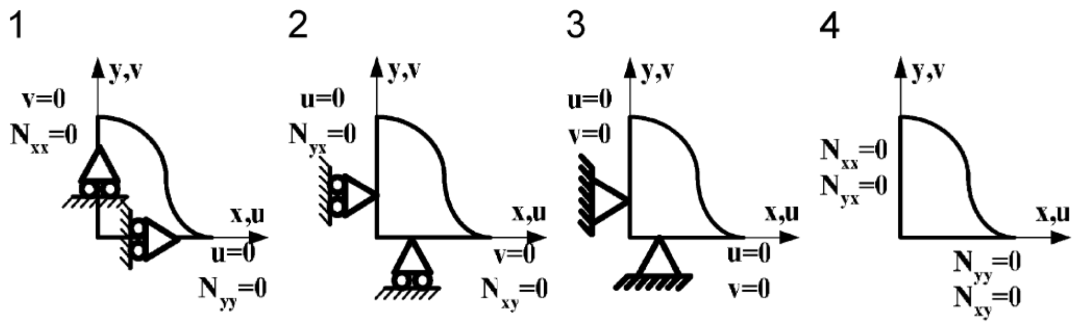

Figure 3.2 Boundary conditions (Shufrin et al 2008)

3.3. NKQ Equations

As it was mentioned in Section 3.1 NKQ method is recently proposed by Saberi-Najafi and Heidari (2010) for solving nonlinear equations in Urysohn form. This technique is used in this research to solve nonlinear equations of composite materials. Here, The NKQ method and related equations are presented. The nonlinear integral equation in the Urysohn form is defined as (Saberi-Nadjafi and Heidari 2010):

$$y(x) = f(x) + \int_{\Omega} K(x, t, y(t)) dt \qquad a \le x \le b \tag{3.9}$$

If $\Omega = (a, x)$, it is named a nonlinear Volterra integral equation and if $\Omega = (a, b)$, it is named the nonlinear Fredholm integral equation. To approximate the right-hand integral in Eq. (3.9), the usual quadrature methods similar to the ones used to approximate the linear integral equations that lead to the following nonlinear systems for Fredholm and Volterra equations are used, respectively. For further information on quadrature methods in this respect, see (Atkinson 1995, 1997; Baker et al. 1982; Delves et al. 1985; Saberi-Nadjafi and Heidari 2010; Jerri 1999; Kondo1991; Kytheet al. 2002).

$$y(x_i) = f(x_i) + \sum_{j=0}^{n} w_j K(x_i, x_j, y(x_j)), \qquad i = 0,1,2,,...,n \tag{3.10}$$

$$\begin{cases} y(x_0) = f(x_0) \\ y(x_i) = f(x_i) + \sum_{j=0}^{i} w_{ij} K(x_i, x_j, y(x_j)), \qquad i = 0,1,2,,...,n \end{cases} \tag{3.11}$$

where $w_{ij}s$ and $w_j s$ are weights of the integration formula.

In the Newton-Kantorovich method, an initial solution for $y(x)$ is considered. The following iteration method is used to solve the following sequence of linear integral equations instead of a nonlinear integral equation. For further information on the Newton-Kantorovich method, see Appell et al. 1991; Saberi-Nadjafi and Heidari 2010; Polyanin et al. 2008).

$$\begin{cases} y_k(x) = y_{k-1}(x) + \phi_{k-1}(x) \\ \phi_{k-1}(x) = \varepsilon_{k-1}(x) + \int_\Omega K'_y(x,t,y_{k-1}(t))\phi_{k-1}(t)dt \\ \varepsilon_{k-1}(x) = f(x) - y_{k-1}(x) + \int_\Omega K(x,t,y_{k-1}(t))dt \end{cases} \tag{3.12}$$

where $K'_y(x,t,y) = \dfrac{\partial}{\partial y} K(x,t,y)$.

In the NKQ method which is used, equations (2.23-2.25) are combined by Saberi-Najafi and Heidari (2010) to solve the nonlinear integral equations.

3.4. Applying the NKQ Method

In order to be able to use the NKQ method for composites, the composite constitutive equations should be in a format which can be used by the NKQ method. Therefore, here, the form of the integral equations that can be solved by the NKQ method is explained and then the composite constitutive equations are represented in the same format. The general form of the nonlinear integral equations of the Urysohn form is:

$$y(x) = f(x) + \int_a^x K(x,t,y(t))dt \qquad a \le x \le b \tag{3.13}$$

By considering the equations (3.10-3.12) and by integrating $\phi_{k-1}(x)$ with $y_k(x) - y_{k-1}(x)$:

$$\begin{cases} y_k(x_0) = f(x_0) \\ y(x_i) = f(x_i) + \sum_{j=0}^{i} w_{ij} K(x_i, x_j, y_{k-1}(x_j)) + \\ \qquad \sum_{j=0}^{i} w_{ij} K'_y(x_i, x_j, y_{k-1}(x_j)) [y_k(x_j) - y_{k-1}(x_j)] \quad i = 1,2,...,n \end{cases} \tag{3.14}$$

Consider:

$$\left(F^{(k-1)}\right)_{i+1} = \begin{cases} f(x_0) & i = 0 \\ f(x_i) + \sum_{j=0}^{i} w_{ij} K(x_i, x_j, y_{k-1}(x_j)) - \sum_{j=0}^{i} w_{ij} K'_y(x_i, x_j, y_{k-1}(x_j)) y_{k-1}(x_j) & i = 1,2,...,n \end{cases} \tag{3.15}$$

$$\left(A^{(k-1)}\right)_{i+1 \quad j+1} = \begin{cases} w_{ij} K'_y(x_i, x_j, y_{k-1}(x_j)) & i = 1,2,...,n \quad j = 0,1,...,n \\ 0 & otherwise \end{cases} \tag{3.16}$$

$$\left(Y^{(k)}\right)_{i+1} = y_k(x_i) \qquad i = 0,1,2,...,n \tag{3.17}$$

This equation can be solved by considering an initial solution $y_0(x)$ and constructing the $Y^{(0)}, A^{(0)}, F^{(0)}$ and also using the following repetition sequence:

$$\left(I - A^{(k-1)}\right) Y^{(k)} = F^{(k-1)} \qquad k = 1,2,...,n \tag{3.18}$$

On the other hand, by considering an initial solution $y_0(x)$, $(Y^{(0)})_i$ would be $y_0(x_i)$ and by using equation (3.15) and (3.16) $F^{(0)}, A^{(0)}$ are obtained respectively. Then by solving the system $\left(I - A^{(0)}\right)Y^{(1)} = F^{(0)}$, $Y^{(1)}$ is obtained. By repeating this procedure and then using equation (3.18), the values of $Y^{(1)}, Y^{(2)}, Y^{(3)}, ...Y^{(m)}$ are calculated for $m \in N$. m is a constant value which can be increased for higher n. Depending on n an approximate solution for equation (3.9) is presented. Noticeably, by increasing m, the solution tends to be more accurate with respect to n. However it is shown that to achieve good results it is not necessary to increase m significantly.

Here the composite constitutive equations should be present in a format that can be solved by NKQ method. This format is derived from the general basic equations for laminated composite plates (Reddy 2004). How to derive the NKQ model for laminated composites is explained by Khandan et al. (2012). (Appendix A4)

3.5. Verification Study

In order to verify the NKQ method a number of numerical examples are solved and compared with previous research. In the first example, a four layer glass/epoxy laminate [0°, 90°]$_s$ with the following ply properties (Herakovich 1988) is studied:

$$\begin{aligned} &E_1 = 43.5GPa, \quad E_2 = E_3 = 11.5GPa, \quad \nu_{12} = \nu_{13} = .27, \\ &\nu_{23} = .4 \quad G_{12} = G_{13} = 3.45GPa \quad G_{23} = 4.12GPa \end{aligned} \tag{3.19}$$

The plate is a square with 0.5m length and 0.01m thickness and it is fixed on one side and uniform distribution loadings (under its weight). A trigonometric function is chosen for initial guess ($y_0(x) = \sin(\pi x / l)$). It is shown in Figure (3.3) that it is converged after five iterations. Aghdam and Falahatgar (2003) used an Extended Kantorovich method EKM for analysing the thick composite plate. By choosing a trigonometric function as an initial guess, the model converges after 4 iterations.

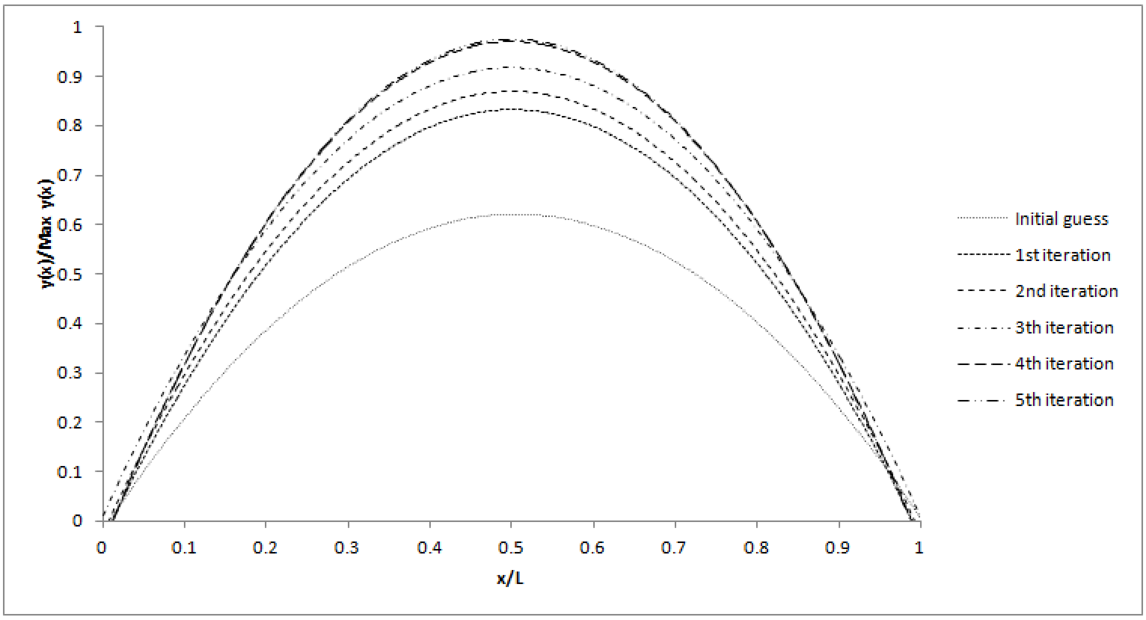

Figure 3.3 Convergence of NKQ method

In Table (3.1) the number of iterations, which are needed for convergence, for three different initial functions are shown. As it is mentioned earlier the initial guess does not have to satisfy the boundary conditions, so any initial function can be selected. Furthermore, as it is shown in Table (3.1) that the NKQ method is almost as quick as EKM and does not significantly depend on the initial value. The proposed model potentially can be used for more complicated cases such as out of plane loading and different boundary conditions. However, it increases processing time dramatically. Therefore as it was mentioned earlier, it is not practical to use this model to solve the complex composite problems.

Table 3.1 Number of iteration for convergence of NKQ and EKM

Initial guess	Number of iteration in order to converge for NKQ	Number of iteration in order to converge for EKM (Aghdam, Falahatgar 2003)
Trigonometric function $y_0(x) = \sin(x)$	5	4
Polynomial function $y_0(x) = 1 + x + x^2$	4	NA
Exponential function $y_0(x) = e(x)$	4	NA

In the next study the material properties are:

$$E_1 = 215GPa \quad E_2 = E_3 = 23.6GPa, \quad \nu_{12} = \nu_{13} = .17, \\ \nu_{23} = .28 \quad G_{12} = G_{13} = 5.4GPa \quad G_{23} = 2.1GPa \tag{320}$$

The plate is a square and each length is 0.25m, the thickness is 0.006m and the lay ups are [0°, 90°, 0°]$_S$. In Table (3.2) the relative error between the NKQ method and FEM (Further details of FEM and how it is applied in this research is explained in the next chapter) for different numbers of iterations are shown for a plate clamped on one side (C) and free (F) on the other three sides (CFFF). This example was then repeated for SSSS (simply supported on all sides) and CCCC boundary conditions and the results are shown in Tables (3.3) and (3.4) respectively.

Table 3.2 Relative error for CFFF boundary condition

Number of iterations	1	2	3	4	5	6	7	8	9	10
Relative error (%) for u	29.4	8.3	3.4	1.9	1.1	0.6	0.3	0.1	0.0	0.0
Relative error (%) for v	34.5	9.2	5.1	2.1	1.2	0.8	0.3	0.1	0.0	0.0
Relative error (%) for w	53.4	11.3	5.4	2.1	1.2	0.8	0.4	0.2	0.0	0.0

Table 3.3 Relative error for SSSS boundary condition

Number of iterations	1	2	3	4	5	6	7	8	9	10
Relative error (%) for u	25.4	9.1	3.9	1.9	1.2	0.7	0.4	0.1	0.0	0.0
Relative error (%) for v	47.2	11.0	5.6	2.5	1.5	1.0	0.5	0.2	0.1	0.0
Relative error (%) for w	76.6	21.2	7.0	2.9	1.5	0.9	0.5	0.2	0.1	0.1

Table 3.4 Relative error for CCCC boundary condition

Number of iterations	1	2	3	4	5	6	7	8	9	10
Relative error (%) for u	92.2	30.4	14.1	8.3	5.3	3.2	1.8	1.0	0.4	0.1
Relative error (%) for v	63.8	19.2	9.2	5.1	3.7	1.9	0.9	0.4	0.1	0.0
Relative error (%) for w	77.1	24.5	11.1	6.6	4.9	2.2	1.3	0.7	0.3	0.1

In Table (3.5), the dimensionless deflection at the centre of the plate is compared between FEM, multi-term extended Kantorovich method (MTEKM) and NKQ method under different levels of loads (Shufrin et al. 2008). As shown, the NKQ results generally show a reasonable agreement with FEM. Semi-analytical models (MTEKM and NKQ) illustrate less than 3% error. The structure is a square plate with CFCC boundary conditions. The angle-ply laminated plate has four symmetric layers [45, -45].

Table 3.5 Dimensionless W/h for CFCC square laminated plate under different loads

Q	MTEKM (W) (Shufrin et al. 2008)	NKQ (W)	ABAQUS	%MTEKM Error (Shufrin et al. 2008)	%NKQ Error
2488	.746	.741	.763	2.23	2.88
4975	1.092	1.090	1.115	2.05	2.24
7463	1.328	1.325	1.335	2.02	1.50
9950	1.512	1.559	1.541	1.81	1.16
12438	1.667	1.667	1.695	1.66	1.66
14925	1.800	1.802	1.827	1.54	1.36
17413	1.919	1.913	1.947	1.43	1.74
19900	2.027	2.025	2.053	1.33	1.36

This technique is also used to propose a semi-analytical model for buckling of composite structures which is explained in appendix (A2).

3.6. Summary

In this chapter semi-analytical models are explained and a new semi-analytical model by using the Newton-Kantorovich-Quadrature method is proposed. The key advantage of this method, compared to EKM, is that it can potentially be used for more complicated cases such as out of plane loading and different boundary conditions with the expense of increasing processing time. The non-linear equations are solved by the Newton-Kantorovich-Quadrature (NKQ) method. This method breaks down the laminated composite plate equations into a series of sequential equations and attempts to solve iterative integral equations. The convergence of the proposed method is compared with other semi-analytical methods (EKM and MTEKM). Various numerical examples with different boundary conditions and loadings are studied. Good agreement between the NKQ model and FEM results are shown to validate the model.

Originally the aim of investigating semi-analytical model was to find a semi-analytical method which is able to model the laminated composite plates for out of plane loading and considering the transverse shear effect. The proposed model theoretically can be used for this purpose. However, higher processing time and lower accuracy are two critical disadvantages of this semi-analytical model in comparison with numerical models and make this model impractical. Therefore, this model is not used in this research; however, as numerical methods are using the results of the analytical and semi-analytical equations, any improvement in analytical and semi-analytical models potentially leads to developing better numerical methods.

CHAPTER 4:

FEM Modelling, Programming and Experimental Results

4. FE Modelling, Programming and Experimental Results

Abaqus FEA package (version 6.10) is used in this chapter to simulate laminated composite plates. The steps which are taken in Abaqus are explained and short description of different parts of Abaqus, which are used in this research to simulate the composite panel, are presented. Also, Python programming is overviewed as this is the link with Abaqus and the final program in this research is written by Python. In order to validate the FE results, some experimental tests have been performed. Some essential notes about the methods of making composites and the materials are studied and finally the results of FE and experimental tests are presented.

4.1. Finite Element

The approximate analytical and semi-analytical methods discussed in Chapter 2 and 3 are often inadequate for assessment of stress and strain in composite structures. The procedure and computation become very time-consuming (almost impossible) when multilayer laminates are involved. A common feature of all analytical methods is that they can only be used for the simple problems (in terms of loadings, boundary conditions and geometries). For thick composite structural laminates, the solution for 3D problem is extremely complicated. Thus a variety of numerical methods, e.g., finite difference and finite element, have been developed to calculate these interlaminar stresses. These models not only give the option of placing an optional mesh close to regions of possible stress concentration but also are able to be used for the accurate evaluation of laminated composite structures having complex geometry, loading and different boundary conditions (Kant and Swaminathan 2000).

The finite element method is a powerful computational technique for the solution of differential and integral equations that arise in various fields of engineering and applied science. This method is a generalisation of the classical variational (i.e., Ritz) (Reddy 2004) and weighted-residual (e.g., Galerkin, least-squares, etc.) methods (Reddy 2004). Since most real-world problems are defined on domains that are geometrically complex and may have different types of boundary conditions on different portions of the boundary of the domain, it is difficult to generate approximation functions required in the traditional variational methods. The basic idea of the finite element method is to view a given domain as an assemblage of simple geometric shapes, called ”finite element”, which it is possible to systematically generate the approximation functions needed in the solution of differential equations by any of the variational and weighted-residual methods. The ability to represent domains with irregular geometries by a collection of finite elements makes the method a valuable practical tool for the solution of eigenvalue problems which are existing in various fields of engineering and science.

It is important to note that any numerical or computational method is a tool to analyse a practical engineering problem and that analysis is not an end in itself but rather an aid to design. The value of the theory and analytical solutions is presented in the preceding chapters, in order to gain insight into the behaviour of laminated composite plates. Here, the numerical modelling of composite plates is presented to help the designer for faster and more accurate analysis of composite plates. However, those who are quick to use a computer rather than think about the problem to be analysed may find it difficult to interpret or explain the computer-generated results. Even to develop proper input data to a computer program requires a good understanding of the underlying theory of the problem as well as the method on which the program is based.

Many studies exist on the modelling and analysis of laminated composite plates and shells using a 2D finite element. Each method is looking from a different perspective and tries to solve a specific problem. An overview of 2D FE models without evaluating them in depth is presented, as FE is just used as a tool in this research and is not the main aim of this work.

Reddy (1985) presented an overview of the literature on FE modelling of laminated composite plates. In his work only research up to the year 1985 were included. Kapania (1989) gave a review of studies up to the year 1989 on the analysis of laminated shells.

Engblom et al. (1989) presented a model in which a shear deformable element is considered in a parametric plate and shell element with respect to the shear effects by allowing mid surface displacements to be independent of the rotations. The model is based on 8-noded quadrilateral geometries with 4 corner nodes and 4 mid-side nodes located at the mid surface of the element with 6 dof per node. The displacement field is expressed in the following form:

$$
\begin{aligned}
u(x,y,z,t) &= u_0(x,y,t) + z\left[n_z\theta_x(x,y) - n_y\theta_z(x,y)\right] \\
v(x,y,z,t) &= v_0(x,y,t) + z\left[n_z\theta_y(x,y) - n_x\theta_z(x,y)\right] \\
w(x,y,z,t) &= w_0(x,y,t) - z\left[n_x\theta_x(x,y) + n_y\theta_y(x,y)\right]
\end{aligned}
\qquad (4.1)
$$

where u_0, v_0, w_0 are the mid-plane displacements respectively and $\theta_x, \theta_y, \theta_z$ are the surface rotations. The transverse stresses are found out by using equilibrium equations and Guass Quadrature is used to obtain the integration within each layer. Hamdallah and Engblom (1990) presented a similar model. However, the plate element developed includes shear effects. Also, the main aim of this model was to analyse 3D structures they introduced a 6th DoF to represent rotations normal to the plane of the element. Their model was an 8-noded quadrilateral plate element with 5 DoF at each of the mid-side and corner nodes. The displacements are similar to Equation (2.9). They used the equilibrium equations for calculating the transverse stresses.

Manjunatha and Kant (1992) formulated stiffness matrix for finite elements based on a set of higher-order theories which considers the effect of transverse shear deformation, transverse normal strain and non-linear variations of in-plane displacements. Therefore, there is no need for shear correction coefficients. The various higher-order theories used in their study are summarised below in the increasing order of their DoFs (Kant and Swaminathan 2000).

Higher-order shear deformation theory (HOST 5 dof/node)

$$\begin{aligned} u(x,y,z) &= z\theta_x(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= z\theta_y(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{4.2}$$

Higher-order shear deformation theory (HOST 6 dof/node)

$$\begin{aligned} u(x,y,z) &= z\theta_x(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= z\theta_y(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_0(x,y) + z^2 w_0^*(x,y) \end{aligned} \tag{4.3}$$

Higher-order shear deformation theory (HOST 7 dof/node)

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\theta_x(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_0(x,y) + z\theta_y(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{4.4}$$

Higher-order shear deformation theory (HOST 9 dof/node)

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\theta_x(x,y) + z^2 u_0^*(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_0(x,y) + z\theta_y(x,y) + z^2 v_0^*(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{4.5}$$

Higher-order shear deformation theory (HOST 11 dof/node)

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\theta_x(x,y) + z^2 u_0^*(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_0(x,y) + z\theta_y(x,y) + z^2 v_0^*(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_0(x,y) + z\theta_z(x,y) + z^2 w_0^*(x,y) \end{aligned} \tag{4.6}$$

Higher-order shear deformation theory (HOST 12 dof/node)

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\theta_x(x,y) + z^2 u_0^*(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_0(x,y) + z\theta_y(x,y) + z^2 v_0^*(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_0(x,y) + z\theta_z(x,y) + z^2 w_0^*(x,y) + z^3\theta_z^*(x,y) \end{aligned} \tag{4.7}$$

where u, v and w are the displacements of a general point (x, y, z) in the laminate in the x, y and z directions, respectively. The parameters u_0, v_0, w_0 and $\theta_x, \theta_y, \theta_z$ are the appropriate 2D terms in the Taylor series and are defined in x-y plane at z=0. The parameters u_0^*, v_0^*, w_0^* and $\theta_x^*, \theta_y^*, \theta_z^*$ are higher-order terms in the Taylor's series expansion which are difficult to interpret in physical terms, except that they represent higher-order transverse cross-sectional deformation modes.

Wu and Kuo (1993) presented a method based on the local higher-order lamination theory to analyse thick laminated composite plates. In their theory, the displacement continuity at the interface between layers is described by considering the potential energy functional of the laminates and applying Lagrange multipliers. The Lagrange multipliers are defined to be the interlaminar stresses ($\tau_{xy}, \tau_{yz}, \sigma_z$) at the interface between layers. The modified potential energy functional which is presented in this model is:

$$\begin{aligned} \Pi_{mp} = &\sum_{i=1}^{N} \iint \int_{-h/2}^{h/2} \frac{1}{2} \left[\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz} \right]_i dz dA \\ &- \iint \left[T_x u^* + T_y v^* + T_z w^* \right] dA \end{aligned} \tag{4.8}$$

where T_x, T_y, T_z are the tractions applied at upper surfaces and i is the i[th] layer. The 9 nodal unknowns in this model are the 3 displacements, 3 rotations and 5 higher-order functions. The local displacement fields are described as:

$$\begin{aligned} u_i(x,y,z) &= u_0(x,y) + z_i\theta_x(x,y) + z_i^2[\psi_x(x,y)] + z_i^3[\phi_x(x,y)] \\ v_i(x,y,z) &= v_0(x,y) + z_i\theta_y(x,y) + z_i^2[\psi_y(x,y)] + z_i^3[\phi_y(x,y)] \\ w_i(x,y,z) &= w_0(x,y) + z_i\theta_z(x,y) + z_i^2[\psi_z(x,y)] \end{aligned} \tag{4.9}$$

$\psi_x, \psi_y, \psi_z, \phi_x, \phi_y$ are the suitable higher-order functions. z is always measured from the middle surface of the i[th] layer. Di-Sciuva's zig-zag model (Di-Sciuva 1993) was explained in Chapter 2. This model is later used in FE. The basic equations were:

$$\begin{aligned} u &= u^0 - zw^0_{,x} + f(z)g_x + \sum \phi_i (z - z_k) Y_i \\ v &= v^0 - zw^0_{,y} + f(z)g_y + \sum \psi_i (z - z_k) Y_i \\ w &= w_0 \end{aligned} \tag{4.10}$$

where,

$$f(z) = z\left(\delta_F - \delta_r \frac{4}{3h^2} z^2\right) \tag{4.11}$$

Y_i is the Heaviside unit function; it takes a value of 0 for $z < z_k$ and the value 1 for $z \geq z_k$. δ_F, δ_r are tracers which identify the contribution brought by the various plate models. This model developed a 4 noded quadrilateral plate element with 10 dof/node using an improved zig-zag model. The displacement field allows a non-linear variation of the in-plane displacements through the laminate thickness and fulfils a transverse stress continuity condition at interfaces. Also, the model satisfies the static condition of zero transverse shear stresses on the bottom and top surfaces for symmetric laminates. The plate model used is based on the following representation of the displacement field across the plate thickness.

Noor et al. (1994) presented a computational model based on FSDT for accurate determination of transverse shear stresses and their sensitivity coeffcients in laminated composite panels for thermal and mechanical loads.

Bose and Reddy (1998) presented FEM of various shear deformation theories for the evaluation of composite plates and also compared transverse displacements and through the thickness distributions of transverse stresses and in-plane stress. Another model which decreases the order of differentiation by one as compared to the standard equilibrium approach was shown by Rolfes et al. (1998). The model needs only quadratic shape functions for analysing the required derivatives at the element level and also the computational effort is low since it must only have stiffness continuity in shape functions in the FE code.

Abaqus software is used to perform the FEA for the tests. Further details of the finite element equations and formulas can be found in (Kant and Swaminathan 2000; Reddy 2004).

4.2. Abaqus

Abaqus is FEA software with the ability to solve a wide variety of problems and simulate them in order to avoid the expense of manufacturing and prototype making. Abaqus is used in the aerospace and automotive industries widely. It is used in this work due to its wide material modelling capability. Abaqus also provides a good collection of multiphysics capabilities, such as piezoelectric, shape memory alloys and coupled acoustic-structural, making it attractive for production-level simulations where multiple areas need to be joined. It was originally released in 1978. It has also been widely used in research of composite materials, as it is accurate and it is an open-source software which gives the option to users to customise the software.

Figure (4.1) shows the components and the main window appearance of Abaqus (Abaqus 6.10). Furthure details of each component are explained in Appendix (B1).

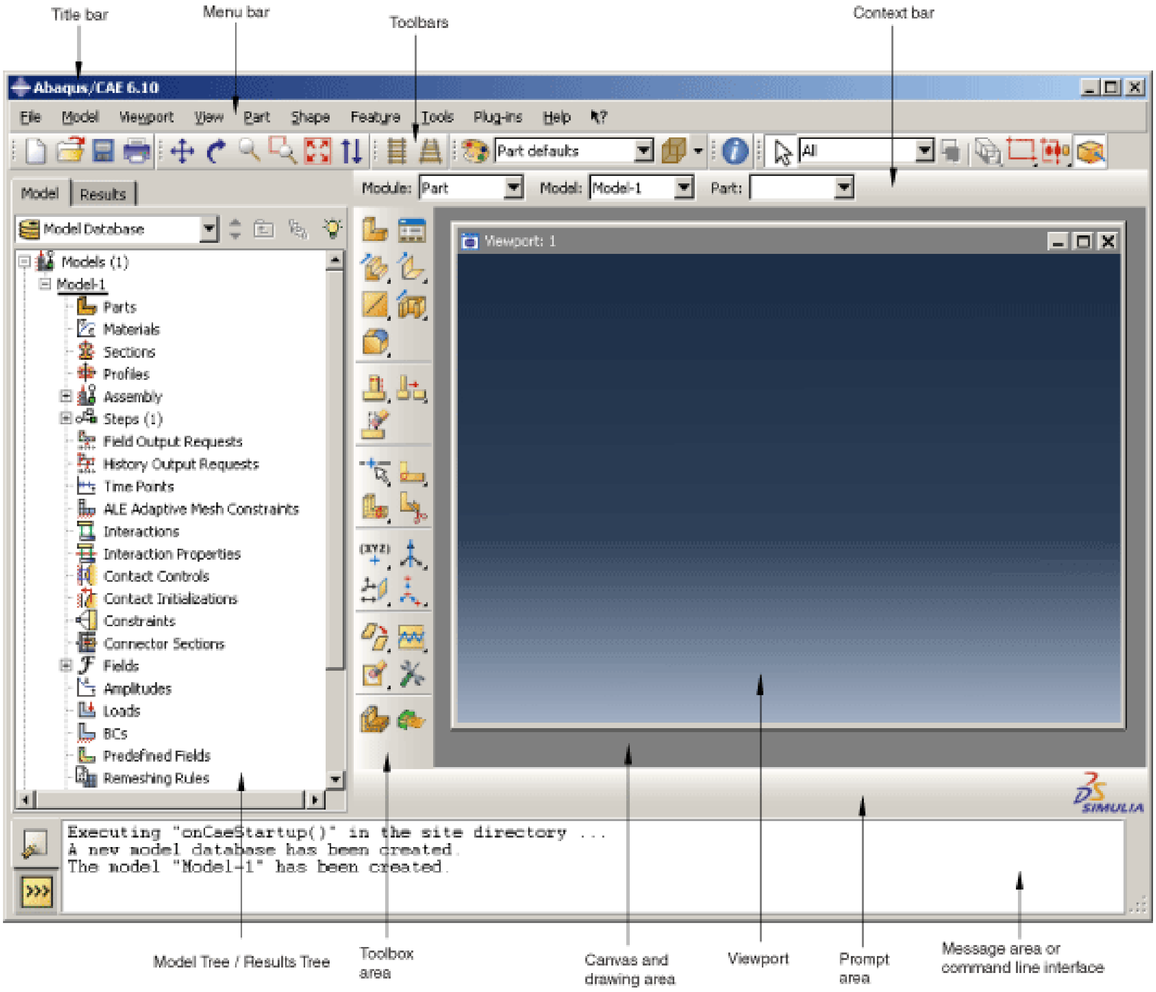

Figure 4.1 Components of the main window (Abaqus 6.10)

4.2.1 Parts

The first step to model the laminate composite plate is to model the plate. From the Model Tree, Figure 4.1, shell part is chosen. In the Parts components the dimension of the panel is defined. As a laminate composite plate is to be modelled, there is no need to define the thickness of the plate at this stage. The thickness of the plate is calculated automatically when identifying the laminate lay-ups. The thickness of each layer is defined and the total thickness of the plate will be the total thickness of the layers.

4.2.2 Materials

The next step in the Model Tree is to define the materials and the lay-ups for laminated composite plate. Composite layups in Abaqus are designed to user to manage a large number of plies in a typical composite model. Designers might find it increasingly difficult to define the model by increasing the number of plies. It can also be cumbersome to add new plies or remove or reposition existing plies. The Abaqus composite layup editor allows designers to easily add a ply, choose the region to which it is applied, specify its material properties, and define its orientation. It is also able to read the definition of the plies in a layup from the data in a text file. This is convenient when the data are stored in a spreadsheet or were generated by a third-party tool.

It is important to specify the correct orientation of the fibres in a ply. Abaqus allows us to define a reference orientation for the layup as well as a reference orientation for each ply in the layup. In addition, we can specify the direction of the fibres in a ply relative to the reference orientation of the ply. By default, the coordinate system of a layup is the same as the coordinate system of the part; similarly, the coordinate system of a ply is the same as the coordinate system of the layup. Figure (4.2) shows the Edit Composite Layup window in Abaqus for one example. For this example, 4 plies are defined. 2 layers are with fibre orientations of +90 degrees and the other two with fibre orientations of +45 degrees. As it can be seen in Figure (4.2), the name of each ply can be given in ply name box. Also, in the region column, the part which these plies apply to is chosen. Each ply can be applied only on one part. In our case as there is only one plate, all the layers applied on part which is explained before. For each layer, the thickness, reference axes and fibre orientations can be defined in Thickness, CSYS and Rotation Angle tabs respectively. By right clicking on each layer, these parameters can be edited and also the plies can be copied, cut, pasted, moved up/down which makes this window really easy to work with by users.

Materials are selected for each layer by clicking on the Material tab which lead to another window (Figure 4.3). Depending on the problem, the user can consider one type of material. In this case as the laminated composite plate is simulated, the Lamina is selected.

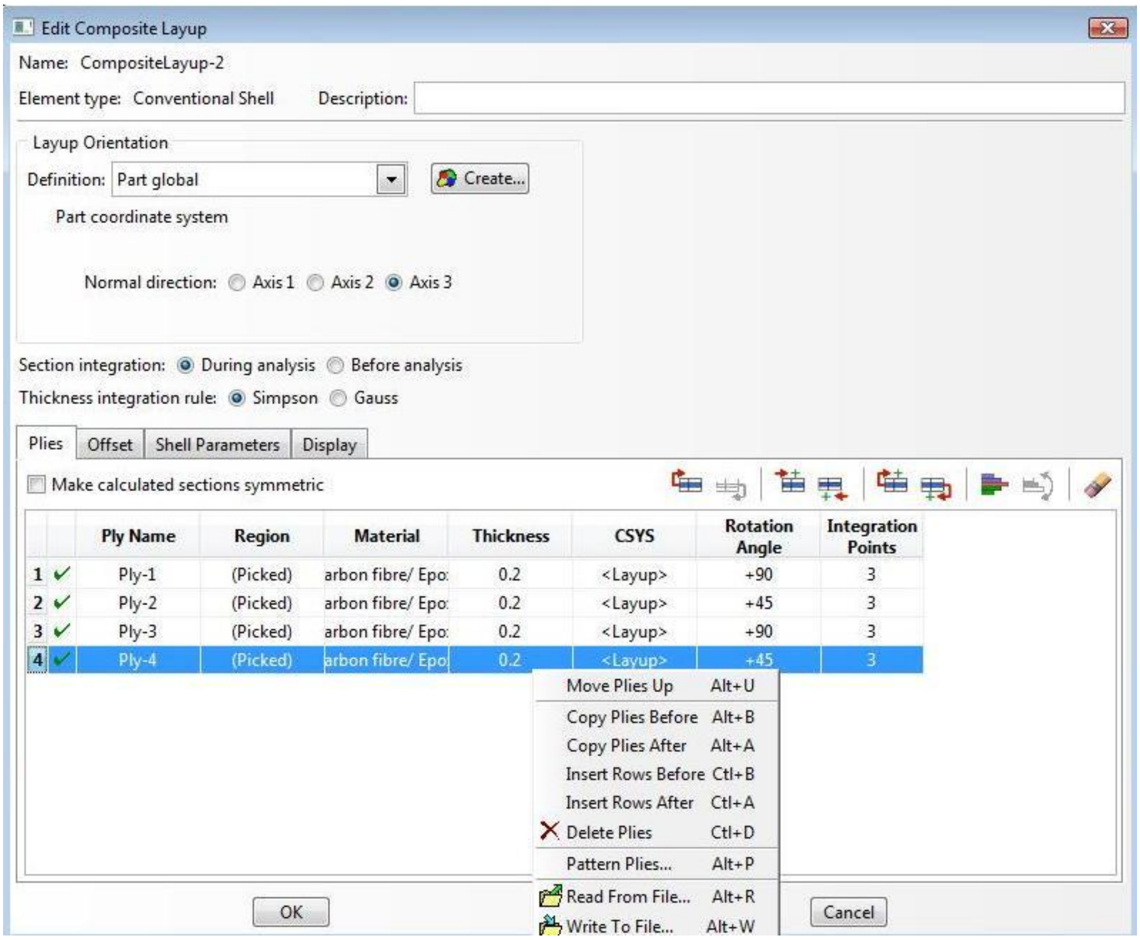

Figure 4.2 The ply table in the composite layup editor

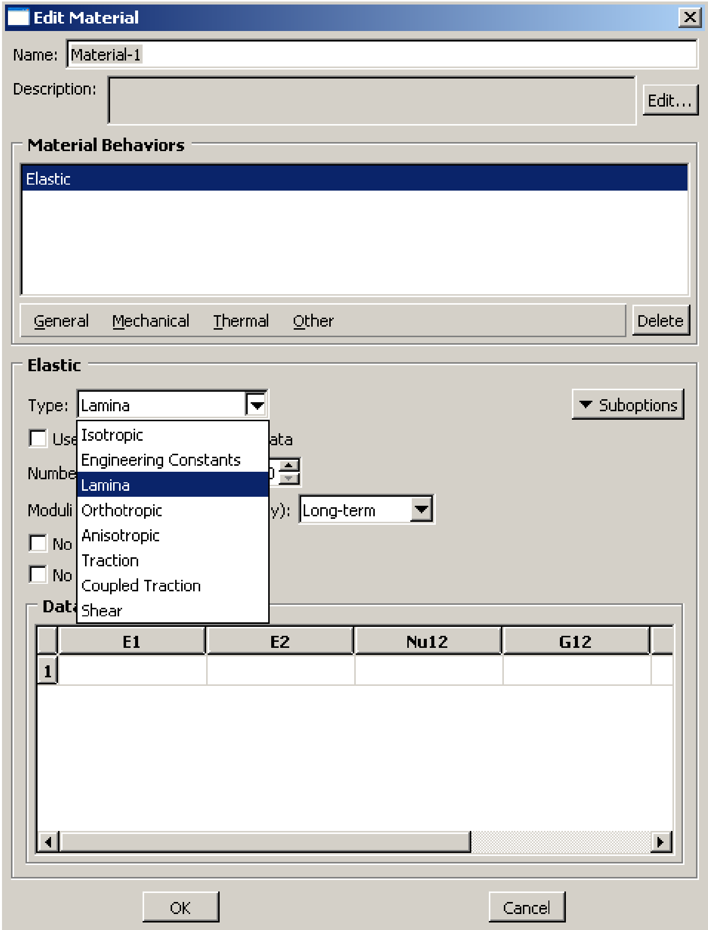

Figure 4.3 Input material properties

4.2.3. Mesh/Element

The next step is to mesh the laminated composite plate. As the plate does not have a complex shape (simple panel) the automatic meshing which is suggested by Abaqus is acceptable here. (The appropriate manual meshing must be considered for more complicated structures.) Choosing the appropriate element for a composite plate is essential. A wide range of elements is available in Abaqus. This extensive element library provides a powerful set of tools for solving many different problems. Each element in Abaqus has a unique name, such as T2D2, S4R, or C3D8I. The element name identifies each of the aspects of an element. Figure (4.4) shows the element families most commonly used in a stress analysis. One of the major distinctions between different element families is the geometry type that each family assumes. For composite material analysis, the shell element is used in this research which is the most common element to analyse laminated composite plates.

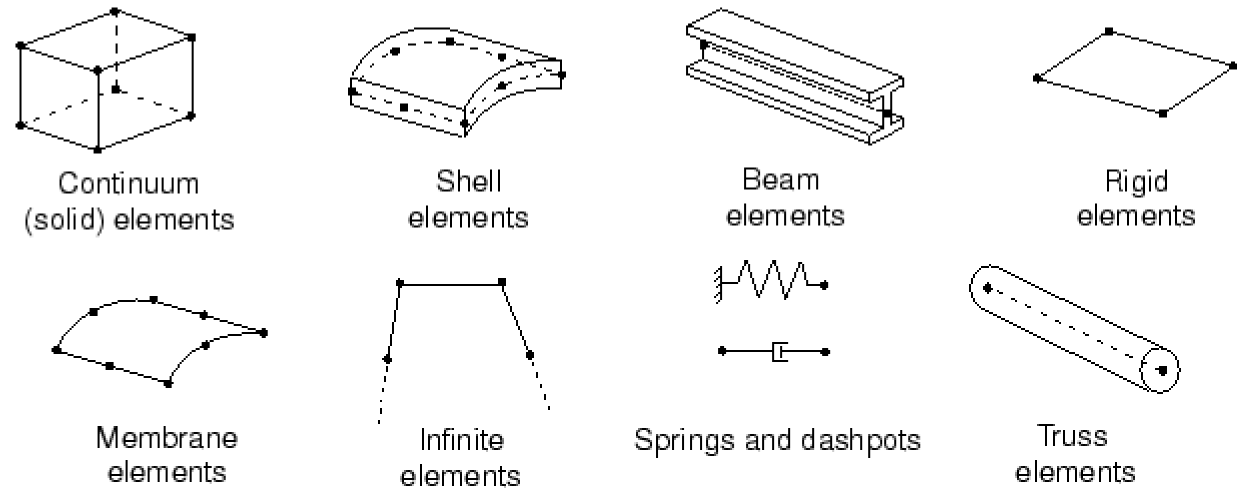

Figure 4.4 Commonly used element families (Abaqus 6.10)

The degrees of freedom (dof) are the fundamental variables calculated during the analysis. For a stress/displacement simulation the degrees of freedom are the translations at each node. Some element families, such as the beam and shell families, have rotational degrees of freedom as well.

Shell elements are used to model structures in which one dimension (the thickness) is significantly smaller than the other dimensions. In Abaqus general three-dimensional shell elements are available with three different formulations: general-purpose, thin-only, and thick-only. Table (4.1) summarises the shell elements available in Abaqus.

Table 4.1 Three classes of shell elements in Abaqus

General-Purpose Shells	**S4, S4R, S3/S3R, SAX1, SAX2, SAX2T, SC6R, SC8R**
Thin-Only Shells	STRI3, STRI65, S4R5, S8R5, S9R5, SAXA
Thick-Only Shells	S8R, S8RT

The three-dimensional elements in Abaqus whose names end in the number "5" (e.g., S4R5, STRI65) have 5 degrees of freedom at each node: u, v, w and two in-plane rotations (i.e., no rotations about the shell normal). However, all six degrees of freedom are activated at a node if required; for example, if rotational boundary conditions are applied or if the node is on a fold line of the shell. In this work S4R5 and S9R5 elements are used, as these elements are suggested for stress-strain analysis of laminate composite plates in Abaqus.

4.2.4. Boundary Conditions/Loading

In the Model Tree loading and boundary conditions can be defined in Loads and BCs respectively. In this research the plate is under uniform distributed loads (pressure). In order to experimental results match the FEA, this load is considered to be equal to the weight of the plate which is manufactured. The final model is shown in Figure (4.5). As it is shown in Figure (4.5), in this case the plate is fixed (clamped) in one side and it is free on the other sides.

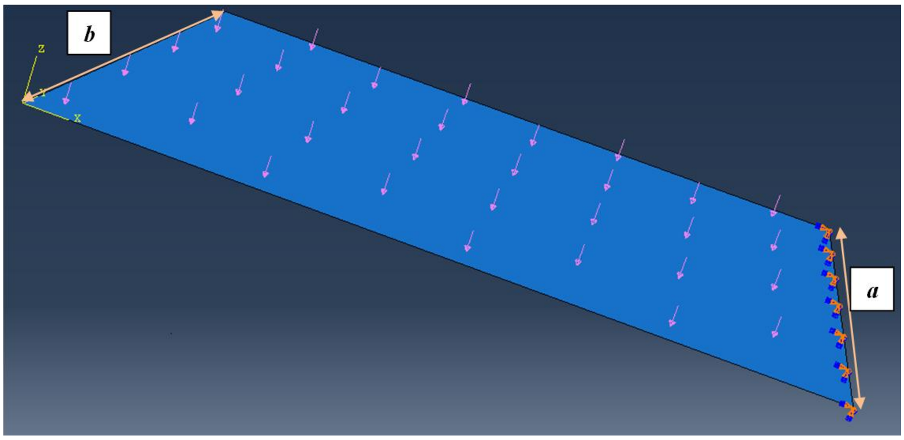

Figure 4.5 Loads and Boundary condition for the laminate composite plate (the plate is fixed in side a and it is free in side b)

4.2.5. Submitting the Job

The final step is to submit the created model as a new job to Abaqus. By clicking the jobs box from Analysis Tree, choosing a name for the job and the location to save the results, the job can be submitted. Two examples of the FEA simulation of the plate are shown in Figures (4.6) and (4.7).

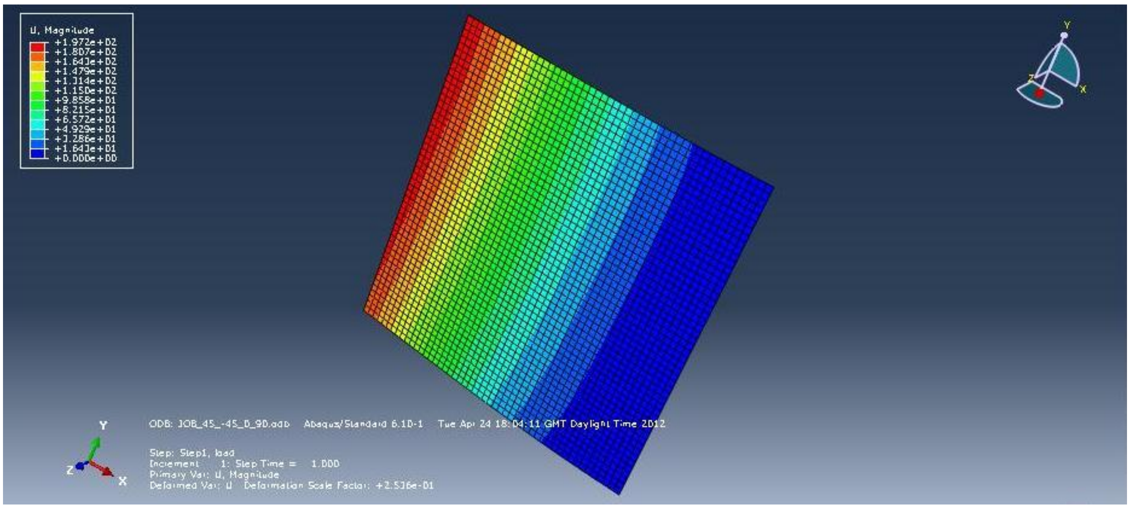

Figure 4.6 FE simulation deformation results for 4 layers plate [45 -45 0 90]

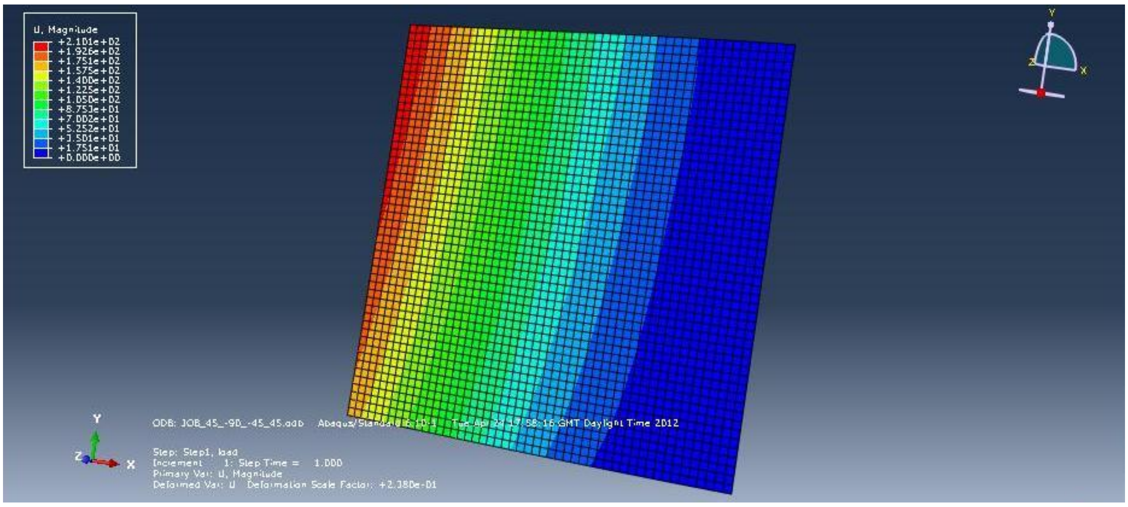

Figure 4.7 FE simulation deformation results for 4 layers plate [45 -90 -45 45]

4.3. Python

In Section 4.2, how to simulate the laminated composite plate in Abaqus is explained. As was mentioned, the fibre orientation in each layer can be changed from -90 to +90 degrees. The aim of this work is to find the best combination of fibre orientations for laminated composite plate depending on design requirements. On the other hand, if designer has a laminated composite plate under certain loads and the plate must have certain behaviour (deformation); what are the suitable fibre orientations for the layers of this plate.

In order to answer this question, it is necessary to analyse the laminated composite plate for all the possible fibre orientations and save these results on a database to be able to find the best fibre orientations from this database. It means that fibre orientations must be changed from -90 to +90 degrees for each layer. This change should be undertaken for all the layers. The number of simulation tests which are needed is shown in equation (4.12)

$$N_t = \frac{90-(-90)}{\Delta\theta_t} \times n = \frac{180n}{\Delta\theta_t} \quad (4.12)$$

where N_t is number of simulation tests which are needed, Δ_t is the step changes in fibre orientation and n is the number of layers. For example, the number of tests for 4 layer laminated composite plate, when the fibre orientation changes by 2 degrees is:

$$\frac{180 \times 4}{2} = 360$$

When the number of plies increases and/or Δ_t decreases, the number of tests which must be simulated, increases. It is almost impossible to set up the simulation tests and run these many tests manually. Therefore, a program is written in Python to perform these simulation tests by automatically changing the fibre orientations for plies and save the results.

Python is a powerful programming language. It has efficient high-level data structures and a simple but effective approach to object-oriented programming. Rather than needing all the desired functionality to be completely built into the language's core, Python is designed to be highly extensible. Also, built-in modules can be written in C or C++. Python can also be used as an extension language for existing modules and applications that require a programmable interface such as Abaqus. The following chart shows the Abaqus scripting interface commands (Figure 4.8). Further details on programming in Python and the command which are used in this research programming is explained in Appendix (B2).

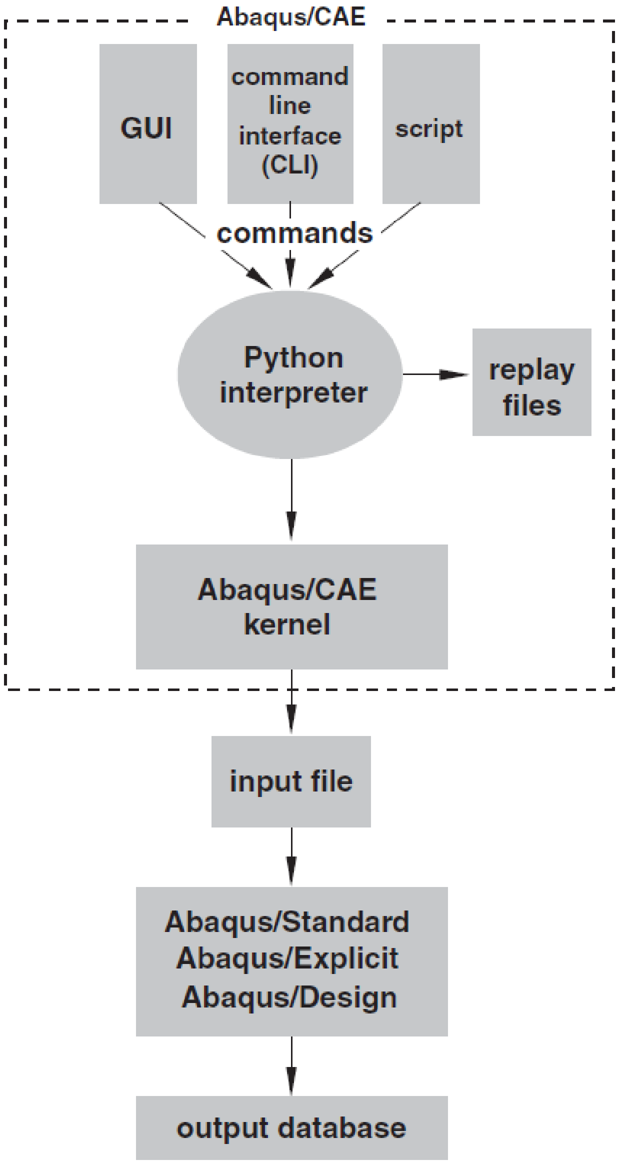

Figure 4.8 Abaqus Scripting Interface commands

After writing a Python program to run iteratively for a laminated composite plate for all different possible fibre orientations the program is expanded and modified in order to be able to be used as a design tool for laminated composite plate. The interface of this tool is shown in Figure (4.9)

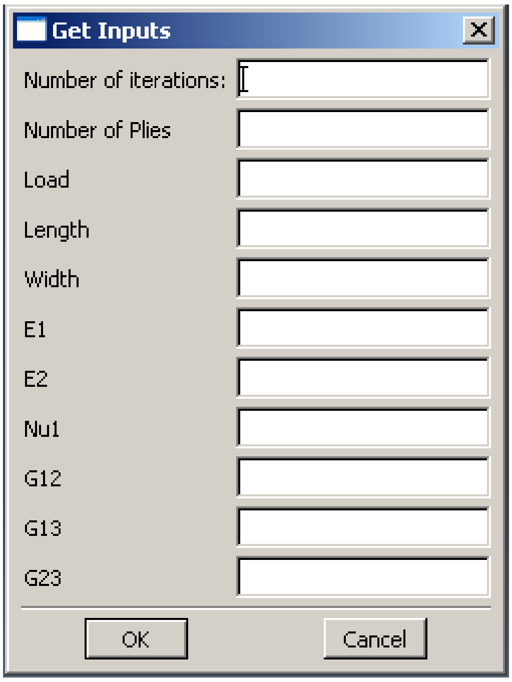

Figure 4.9 Laminated composite plate tool

The input parameters are shown in Figure (4.9). This tool is directly linked with Abaqus. The inputs are regarding the material, geometry and loads where the first two are entered in the tool and the loads and boundary conditions can be defined in the Abaqus program. The output of this tool is a database which contains the results of FEA for all possible fibre orientations, in separate files (the result of each combination of fibre orientations for layers is saved in one file). Figures (4.10) and (4.11) show two different outputs of these files. Later, this database can be used to select, optimise and generally design laminated composite plate. This design can potentially be used by optimising for loads, geometry, thickness or induced twist depending on requirements. The details of the program are explained in Appendix (C).

disp_U_all_JOB_0_45_-45_0 - Notepad

File Edit Format View Help

Node	U[1]	U[2]	U[3]
1	8.941171109e-05	-9.485221468e-03	-6.779534912e+01
2	8.992764197e-05	-9.492676705e-03	-6.596056366e+01
3	9.088285879e-05	-9.499908425e-03	-6.412556458e+01
4	9.232051525e-05	-9.506760165e-03	-6.229040527e+01
5	9.434459207e-05	-9.512807243e-03	-6.045525742e+01
6	9.704372496e-05	-9.517529979e-03	-5.862039948e+01
7	1.004844235e-04	-9.520371445e-03	-5.678623199e+01
8	1.047125188e-04	-9.520756081e-03	-5.495325089e+01
9	1.097561908e-04	-9.518100880e-03	-5.312206268e+01
10	1.156289436e-04	-9.511825629e-03	-5.129337692e+01
11	1.223321015e-04	-9.501350112e-03	-4.946799850e+01
12	1.298567804e-04	-9.486098774e-03	-4.764683533e+01
13	1.381855254e-04	-9.465503506e-03	-4.583089828e+01
14	1.472934237e-04	-9.438998066e-03	-4.402128983e+01
15	1.571490720e-04	-9.406026453e-03	-4.221922302e+01
16	1.677151449e-04	-9.366033599e-03	-4.042600250e+01
17	1.789489179e-04	-9.318470955e-03	-3.864302063e+01
18	1.908025151e-04	-9.262796491e-03	-3.687178421e+01
19	2.032230550e-04	-9.198469110e-03	-3.511388779e+01
20	2.161527373e-04	-9.124955162e-03	-3.337102127e+01
21	2.295286686e-04	-9.041724727e-03	-3.164497375e+01
22	2.432828624e-04	-8.948249742e-03	-2.993762398e+01
23	2.573418897e-04	-8.844008669e-03	-2.825094986e+01
24	2.716266026e-04	-8.728481829e-03	-2.658702278e+01
25	2.860518289e-04	-8.601155132e-03	-2.494800377e+01
26	3.005257749e-04	-8.461517282e-03	-2.333614922e+01
27	3.149495751e-04	-8.309063502e-03	-2.175380516e+01
28	3.292166220e-04	-8.143292740e-03	-2.020340919e+01
29	3.432117810e-04	-7.963708602e-03	-1.868748474e+01
30	3.568104876e-04	-7.769822609e-03	-1.720864487e+01
31	3.698776127e-04	-7.561151870e-03	-1.576959038e+01
32	3.822663566e-04	-7.337222807e-03	-1.437310123e+01
33	3.938165028e-04	-7.097572088e-03	-1.302204323e+01
34	4.043526715e-04	-6.841747090e-03	-1.171935940e+01
35	4.136819043e-04	-6.569310557e-03	-1.046806717e+01

Figure 4.10 Output of the design tool; this notepad file shows the displacement for some nodes of laminate composite plate with the fibre orientations of [0 45 -45 0]

disp_U_end_nodes_JOB_90_-90_0_90 - Notepad

File Edit Format View Help

Node	U[1]	U[2]	U[3]
1	2.700612694e-02	1.314130495e-04	-2.295170441e+02
52	2.703088149e-02	1.307447237e-04	-2.295448303e+02
103	2.705566771e-02	1.295414986e-04	-2.295721741e+02
154	2.708070725e-02	1.276019757e-04	-2.295988922e+02
205	2.710544877e-02	1.248718472e-04	-2.296249237e+02
256	2.712935768e-02	1.213881114e-04	-2.296501617e+02
307	2.715203352e-02	1.172310949e-04	-2.296745605e+02
358	2.717320621e-02	1.124978662e-04	-2.296980438e+02
409	2.719272114e-02	1.072878076e-04	-2.297205658e+02
460	2.721050754e-02	1.016949318e-04	-2.297420807e+02
511	2.722656168e-02	9.580398182e-05	-2.297625427e+02
562	2.724092454e-02	8.968877955e-05	-2.297818909e+02
613	2.725367434e-02	8.341190551e-05	-2.298001099e+02
664	2.726490423e-02	7.702506264e-05	-2.298171387e+02
715	2.727472037e-02	7.056995673e-05	-2.298329468e+02
766	2.728323452e-02	6.407927867e-05	-2.298475037e+02
817	2.729056031e-02	5.757781037e-05	-2.298607788e+02
868	2.729680203e-02	5.108348705e-05	-2.298727417e+02
919	2.730206028e-02	4.460840501e-05	-2.298833618e+02
970	2.730642445e-02	3.815973832e-05	-2.298926086e+02
1021	2.730997838e-02	3.174059384e-05	-2.299004822e+02
1072	2.731279097e-02	2.535075510e-05	-2.299069366e+02
1123	2.731491625e-02	1.898735900e-05	-2.299119873e+02
1174	2.731640264e-02	1.264550428e-05	-2.299155884e+02
1225	2.731728181e-02	6.318812666e-06	-2.299177551e+02

Figure 4.11 Output of the tool this notepad file shows the displacement for some of the nodes on free side of the laminate composite plate with the fibre orientations of [90 -90 0 90]

4.4. Experimental tests

To find how accurate the results of FEA are in comparison with actual behaviour of laminated composite plates and in order to validate the FE results, a series of experimental tests have been performed. In addition, validating the model, helps us to rely on the FE results in more complicated cases which are quite hard to test practically.

4.4.1 Composite Properties

Here, some of the properties of composite material and materials which have been used in the experimental tests in this research are explained. In order to understand the properties of the final composite material it is necessary to examine more closely how the properties of the resin matrix and fibre components combine. Figure (4.12) illustrates the stress/strain curves for a typical composite laminate and its fibre and resin components.

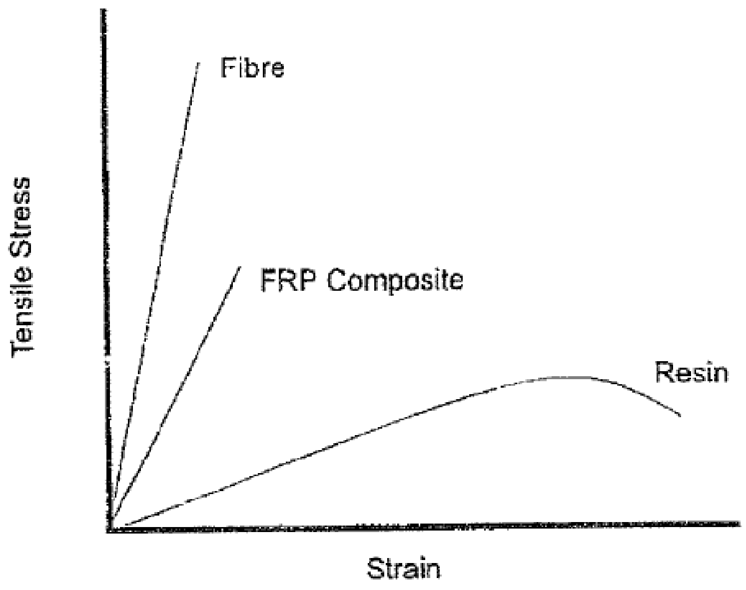

Figure 4.12 Stress/Strain curve of composite components (Gurit 2012)

It can be seen that the properties of the composite fall between those of the fibre and the resin. As the fibre content of the composite increases, the closer its properties will tend towards those of the pure fibre. Note also that for the fibres to be able to achieve their full mechanical properties, the resin must be capable of deforming to at least the same extent as the fibre. The amount of this deformation before failure occurs is known as the elongation to break. Different materials have different elongation to break values. Those for the reinforcements E-glass, S-glass, Aramid and carbon are shown in Figure (4.13).

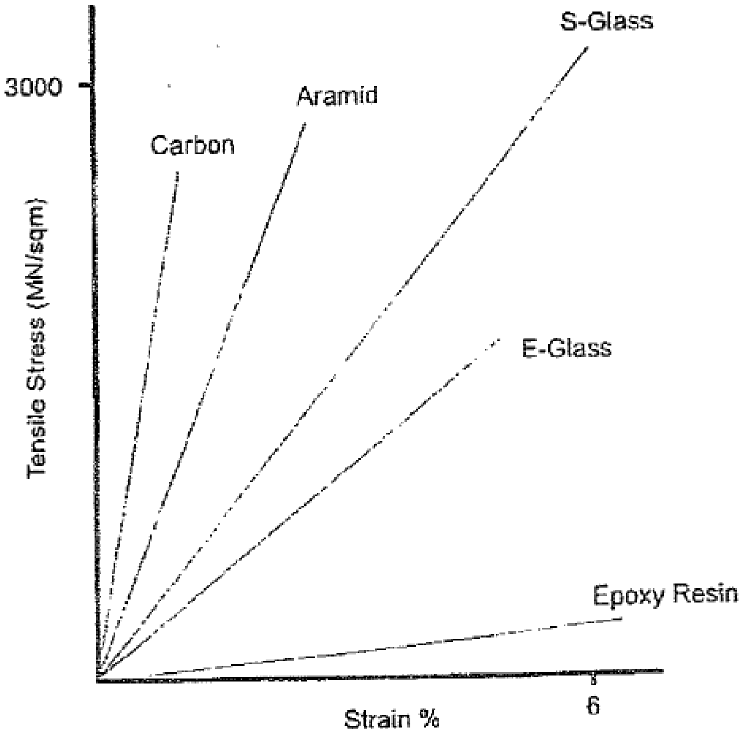

Figure 4.13 Elongation to break curve of fibre reinforcements (Gurit 2012)

4.4.2 Resin

Resin used in a composite material needs good mechanical properties such as toughness, resistance to environmental degradation and good adhesive properties. Here, we just consider the mechanical properties of the resin system. Figure (4.14) shows the stress/strain curve for an ideal resin system. The graph for this specific resin shows a high elongation to break, high stiffness and high ultimate strength which means that the resin is initially stiff but at the same time will not suffer from brittle failure.

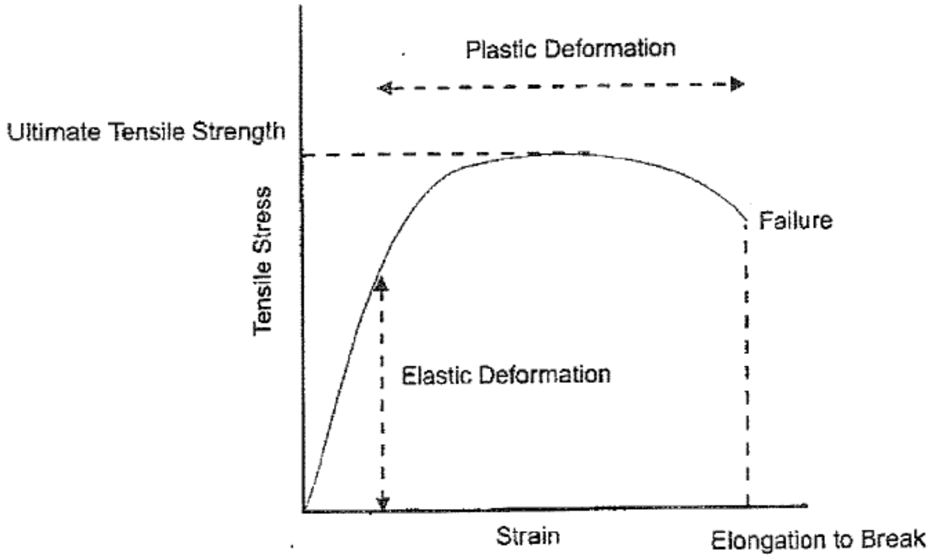

Figure 4.14 Stress/Strain resin curve (Gurit 2012)

High adhesion between reinforcement fibres and resin is essential for any resin. This is to ensure that the loads/forces are transferred completely and also to avoid cracking or fibre/resin debonding when stressed which leads to delamination. The stress/strain curve of the resin also illustrates the materials stiffness. The elongation values describe the resins ability to deform under load, both shock loads and static. The more deformation the resin accepts before failure (its elongation to break) the tougher the material is.

The resins in fibre reinforced composites are sometimes called polymers. Polymers can be categorised in two types, thermosetting and thermoplastic, depending on the effect of heat on their material properties.

Thermoplastics, like metals, are either soft or hard subject to the temperature. They harden with cooling and soften with heating. This process of crossing the softening point on the temperature scale can be repeated as desired without any appreciable effect on the material properties. Typical Fibre Reinforced Thermoplastics would be glass filled nylon and glass filled polypropylene.

Thermosetting materials, however, are the result of a chemical reaction, where the resin and hardener (or resin and catalyst) are mixed and then undergo a non-reversible chemical reaction to form a hard, infusible product. In some thermosetting materials, such as phenolic resins, volatile substances are produced. Other resins such as polyester and epoxy cure by mechanisms that do not produce any volatile by products and thus are much easier to process.

There are three main types of resins which have widely been used in the composites industries; polyester, vinylester and epoxy.

For the experimental tests in this research epoxy is used for the following reasons:

- Epoxies generally have better performance compared to all other resin types in terms of mechanical properties and resistance.
- Epoxy is environment friendly and produce considerably less volatile by products in comparison with polyester and vinylester. Therefore, it is safer to work with epoxy compared to other resins.
- Adhesive properties and resistance to water degradation make these resins ideal for use in the composites industry and especially in marine applications.

The adhesion of the resin matrix to the fibre reinforcement is important. Polyester resins generally have the lowest adhesive properties of the three systems described here. Vinylester resin shows improved adhesive properties over polyester but epoxy systems offer the best performance of all. As epoxies cure with low shrinkage the various surface contacts set up between the liquid resin and the adherents are not disturbed during the cure. Another case where the adhesive properties of epoxy become really useful is in honeycomb core laminates where the small bonding surface area means that maximum adhesion is needed. Figures (4.15) and (4.16) compare the tensile strength and stiffness of three different resins.

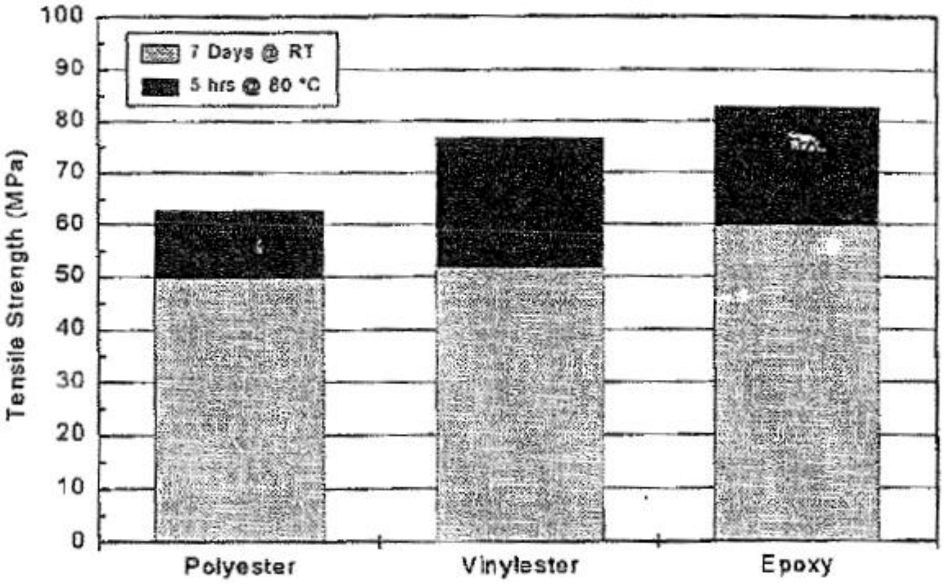

Figure 4.15 Comparative tensile strength of resins (Gurit 2012)

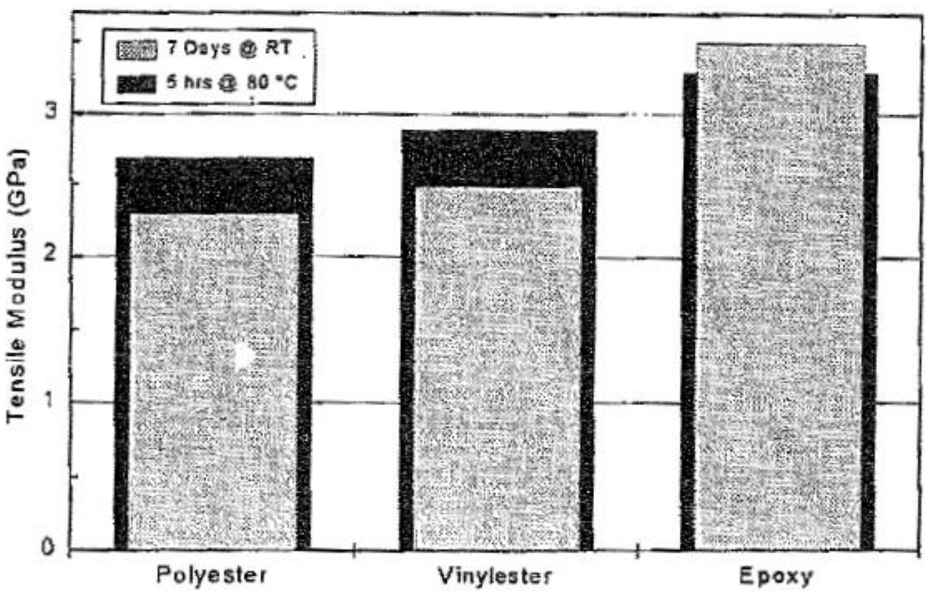

Figure 4.16 Comparative stiffness of resins (Gurit 2012)

The epoxy in this research is the combination of resin SP106 and hardener SP106 which is mixed in a ratio of 5 to 1. (Figure 4.17)

4.4.3 Fibres

Figure 4.17 Resin and hardener SP106

Figure (4.18) shows a sample of resin after mixing with hardener and becomes solid.

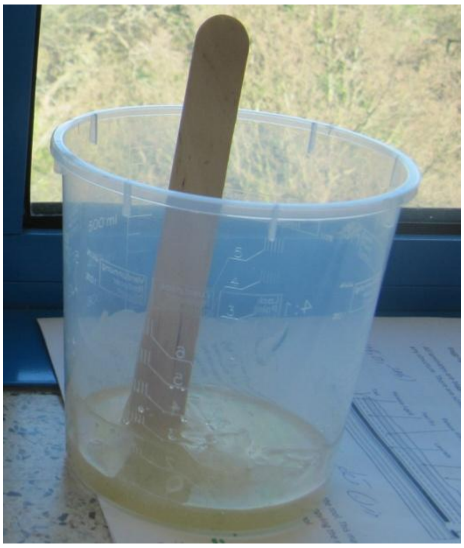

Figure 4.18 Resin SP106 after become solid

4.4.3 Fibres

The materials used for reinforcement have very high tensile and compressive strength, but in bulk, these properties are not obvious because small numbers of random surface flaws in the material cause it to fail under load much earlier than would be predicted from theory. To overcome this problem the materials are manufactured in the form of fibres, because the number of random flaws is few these will appear on only a small number of fibres, so that the majority of fibres exhibit the theoretical properties of the material. To maximise this effect even further, the fibres must be kept apart from each other to prevent the introduction of further flaws from fibre/fibre abrasion. This is one of the basic functions of the matrix system.

The mechanical properties of a composite are largely dictated by the fibre reinforcements and their orientations. High reinforcement content produces high strength (although not necessarily high stiffness). For an efficient composite the elongation of the fibre must be less and its stiffness must be higher, than that of the matrix. Stress transfer along the fibre/matrix interface is particularly important, and there are various methods and procedures for achieving the best possible fibre/matrix bond for a particular combination of fibre and matrix. If it is provided by a commercial company, they normally consider this and it can be bought together.

Fibre diameter is also important as smaller diameter fibres provide higher surface area of fibre per unit weight which spreads any fibre/matrix interface load. In general, regardless of fibre chemical type, the finer the fibre, the more expensive per kg it will be.

In comparison with other type of fibres, carbon fibre is the most common one which has been used in different industries. It is produced by a very controlled oxidation and carbonisation of carton-rich organic precursors which are already in fibre form. Once they were formed, the carbon fibre has a surface treatment or size applied. This is to increase the matrix/fibre bond in the composite and to protect it during handling.

Carbon fibre was first produced in 1971 in the UK at which time the world annual production capacity was about 1/2 a tonne and the price was about £200/kg. In 1996 the annual capacity was about 7000 tonnes in the world and the price for the equivalent (high strength) grade was about £25/kg.

The usual distinction of grades is high strength, high modulus, intermediate modulus and very high modulus. The filament diameter of all types is about 7 microns. Considering Table (4.2), although carbon fibres are relatively expensive and are not as good as Aramid in impact strength, they are much better in all other properties such as, tensile and compressive strength, flexural modulus and fatigue. Therefore, carbon fibre is used in this research. The sample of unidirectional carbon fibres (UT-C300/500) which is used in this work is shown in Figure (4.19).

Table 4.2 Advantage and disadvantage of fibres (A=Excellent B=Average C=Poor)

Property	Aramid	Carbon	Glass
Tensile Strength	B	A	B
Tensile Modulus	B	A	C
Compressive Strength	C	A	B
Compressive Modulus	B	A	C
Flexural Strength	C	A	B
Flexural Modulus	B	A	C
Impact Strength	A	B	B
In-plane Shear Strength	B	A	A
Density	A	B	C
Fatigue	B	A	C
Low Cost	C	C	A

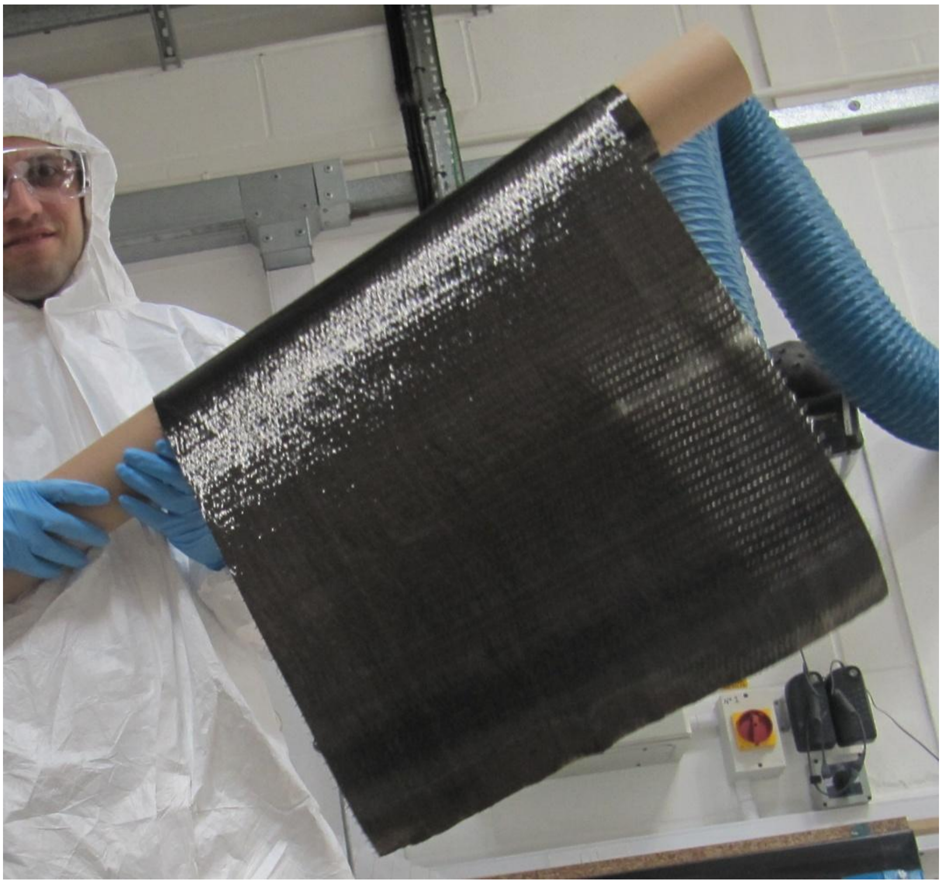

Figure 4.19 unidirectional carbon fibres (UT-C300/500)

4.5. Manufacturing Process

There are many different material options to choose as composite materials. From fibres, cores and resins, where each one has its own unique set of properties such as stiffness, toughness, strength, heat resistance, production, rate cost, etc. However, the final properties of a composite part produced from these different materials are not only a function of the individual properties of the resin matrix and fibre but are also dependent on the way that the materials themselves are designed into the products and the way that they are processed.

Wet lay-up (hand lay-up) is the basic manufacturing process which is used in this work.

In this process resins are soaked into fibres which can be in the form of knitted, unidirectional, woven or bonded fabrics, by hand. This will normally have been done by rollers or brushes, with a special type of roller (nip-roller) to impregnate resin into the fabrics better. Then laminates are left to cure under standard atmospheric conditions.

In this process any kind of resins and fibres can be used. However, Aramid fabrics are not suggested as they can be hard to wet-out by hand. This method is widely used in the production of boats, standard wind-turbine blades and architectural mouldings. Advantage and disadvantage of this method is listed in Table (4.3) (Gurit 2012).

Table 4.3 advantages and disadvantages of Hand lay-up method

Advantages	• **Simple principles to teach** • **Widely used for many years** • **Low cost tooling** • **Wide choice of suppliers and material types** • **Higher fibre contents, and longer fibres than with spray lay-up**
Disadvantages	• Resin mixing, laminate resin contents, and laminate quality are very dependent on the skills of laminators. • Low resin content laminates cannot usually be achieved without the incorporation of excessive quantities of voids. • Health and safety considerations of resins. The lower molecular weight of hand lay-up resins generally means that they have the potential to be more harmful than higher molecular weight products. The lower viscosity of the resins also means that they have an increased tendency to penetrate clothing etc. • Resins need to be low in viscosity to be workable by hand. This generally compromises their mechanical/thermal properties due to the need for high styrene levels.

In order to improve the process, the Vacuum bagging process is normally considered for hand lay-up. Vacuum bagging is an extension of the wet lay-up process, where pressure is applied to the laminate once laid-up in order to improve its consolidation. This method is used in this research and samples are made by the vacuum bagging process. Vacuum bagging is performed by sealing a plastic film over the wet laid-up laminate and onto the tool. Air under the bag is extracted by a vacuum pump and thus up to one atmosphere of pressure is applied to the laminate to consolidate it. By applying this process, the laminate cures relatively faster and also the resin is spread in the laminate almost equally. In this process, extra resin will be absorbed from the laminate, it also results in a better volume fraction of fibre to resin. The main applications are, race car components, core-bonding in production boats and large one-off cruising boats. Advantages and disadvantages of this method are listed in Table (4.4). There are other manufacturing processes for composites which are explained and compared in Appendix (B3) (Gurit 2012)

Table 4.4 advantages and disadvantages of vacuum bagging method

Advantages	• **Higher fibre content laminates can usually be achieved than with standard wet lay-up techniques.** • **Lower void contents are achieved than with wet lay-up.** • **Better fibre wet-out due to pressure and resin flow into bagging materials.** • **Health and safety: The vacuum bag reduces the amount of volatiles emitted during cure.**
Disadvantages	• The extra process adds cost both in labour and in disposable bagging materials A higher level of skill is required by the operators • Mixing and control of resin content still largely determined by operator skill.

All the samples in this research are manufactured by the vacuum bagging process. In vacuum bagging, like other composite manufacturing processes, using a release agent is essential. Before starting to lay up the plies, the release agent is applied to the tool to be sure the part will separate from the tool easily after the process. The plies are placed one by one in the desirable orientation and then get wet by applying resin with a brush. Once the plies are sited, the bag is applied all over the plies. The first item to go down is peel ply. Peel plies are a tightly woven fabric, often nylon, and impregnated with some type of release agent. The peel ply will stick to laminate, but it will pull away easily after the process. Peel ply is an optional item and is used to give the laminate a smooth and nice finish surface. After peel ply a layer of release film is applied. This is a thin plastic which has been treated so it won't bond to the laminate. It is highly stretchable so it can conform to complex geometries. At least one layer of bleeder cloth goes above the release film. Bleeder is a thick, felt-like cloth. Its purpose is to absorb excess resin. The bleeder also acts as a breather, providing a continuous air path for pulling the vacuum. If the bag wrinkles against hard laminate, it will trap air. The breather prevents this from happening. The breather must be thick enough so that it doesn't become fully saturated with resin. A thick breather is also desirable to keep resin from coming in contact with the bag. The bag is the last item to be placed at the top. It is a relatively thick plastic layer. The bag is usually applied along one edge at a time. Starting at one corner the release paper is removed from the tape as it is moving along the edge. It must carefully be applied in order not to get any wrinkles in the bag or it will leak. Also the vacuum part is attached before closing the bag at the corner of the bag.

The sample is now ready to vacuum. The vacuum pressure for all samples was 1 bar and it was applied for 24 hours for each sample in room temperature. Figure (4.20) is a schematic form of the vacuum bagging process. Figure (4.21) shows the items which have been used in this research at BU composite workshop.

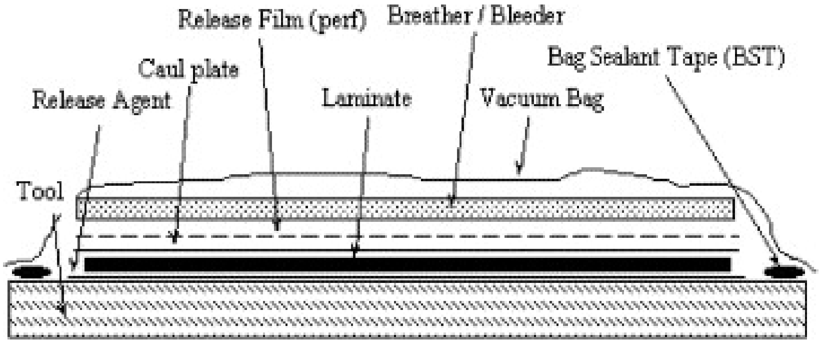

Figure 4.20 Schematic of vacuum bagging process

Figure 4.21 From left to right: release film, vacuum bag, peel ply, unidirectional carbon fibre laminated composite plate tool

Figure (4.22) shows the sample during vacuuming and in Figures (4.23) and (4.24) the final manufactured samples after vacuum bagging are shown. More than 300 samples are prepared in this way. As this process is completely manufacturing by hand (not with the machine mechanism), so the quality of the samples are varied and depend on the experience of the manufacturer. Therefore, not all the samples which are prepared in this way have acceptable quality to be used in the tests. Figure (4.25) shows one of the faulty samples. The reason might be because the fibres didn't become completely wet (not enough resin) or the resin cured more quickly than normal which shows the appropriate ratio of resin to hardener is not used.

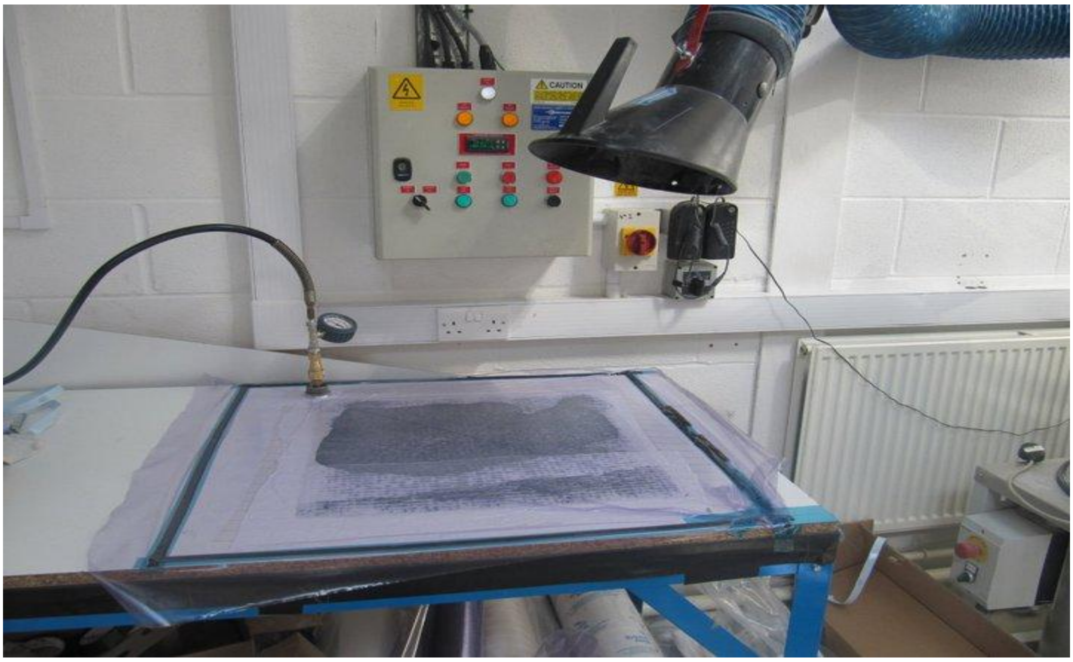

Figure 4.22 Process of making laminated plate

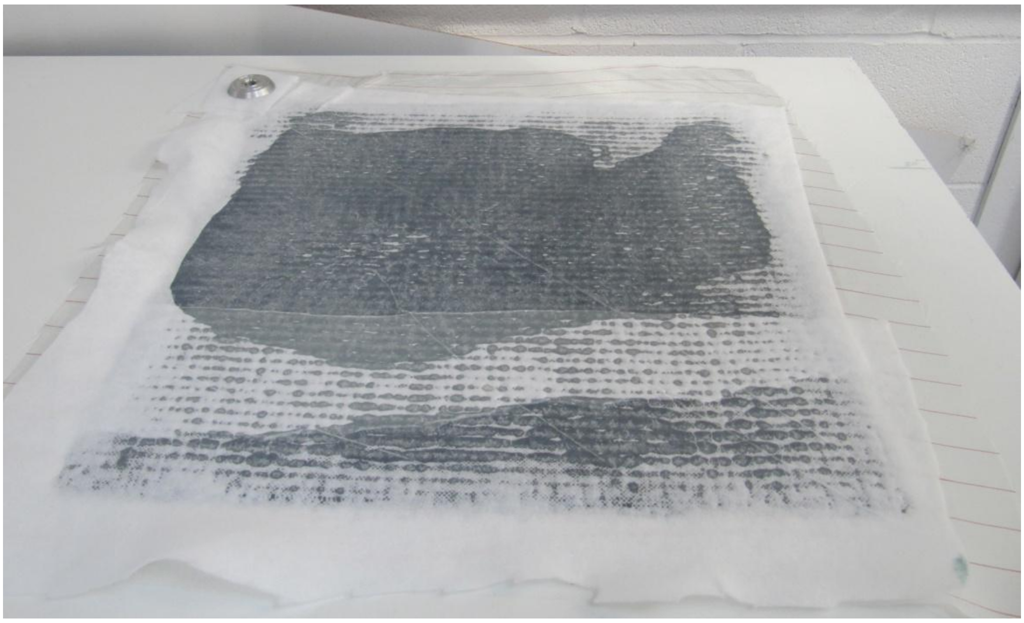

Figure 4.23 Laminated composite plate after vacuuming

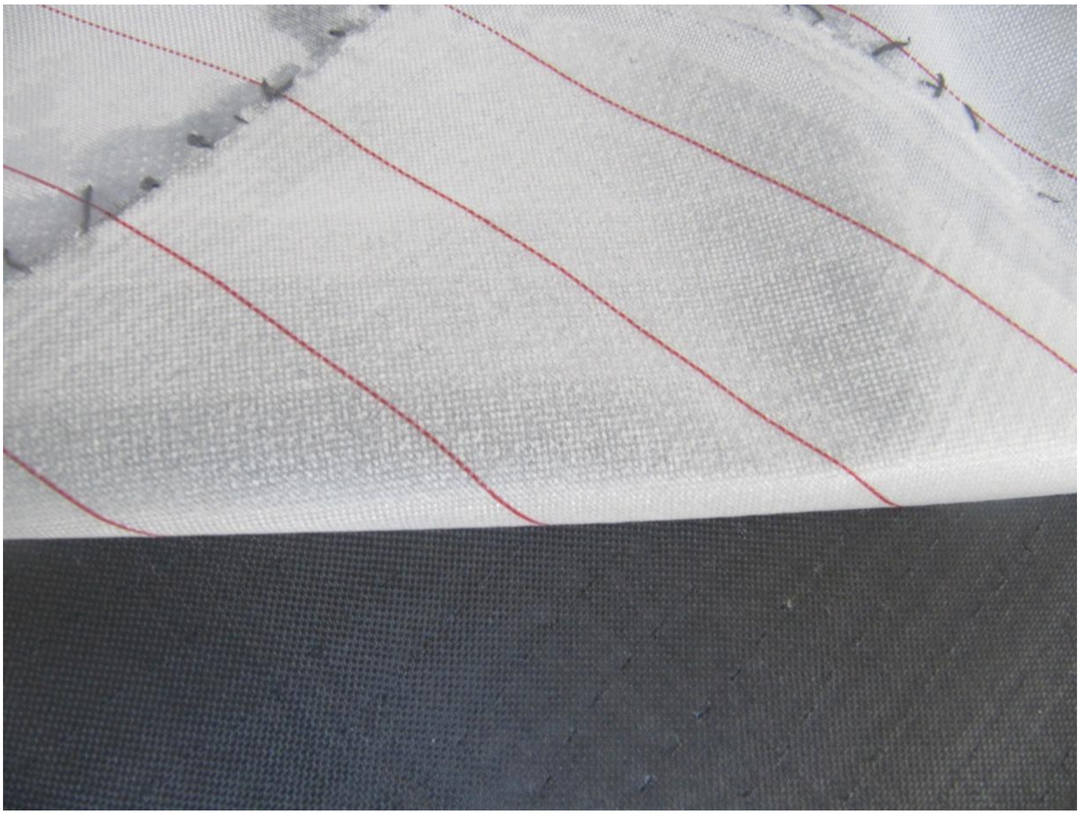

Figure 4.24 Laminated composite plate

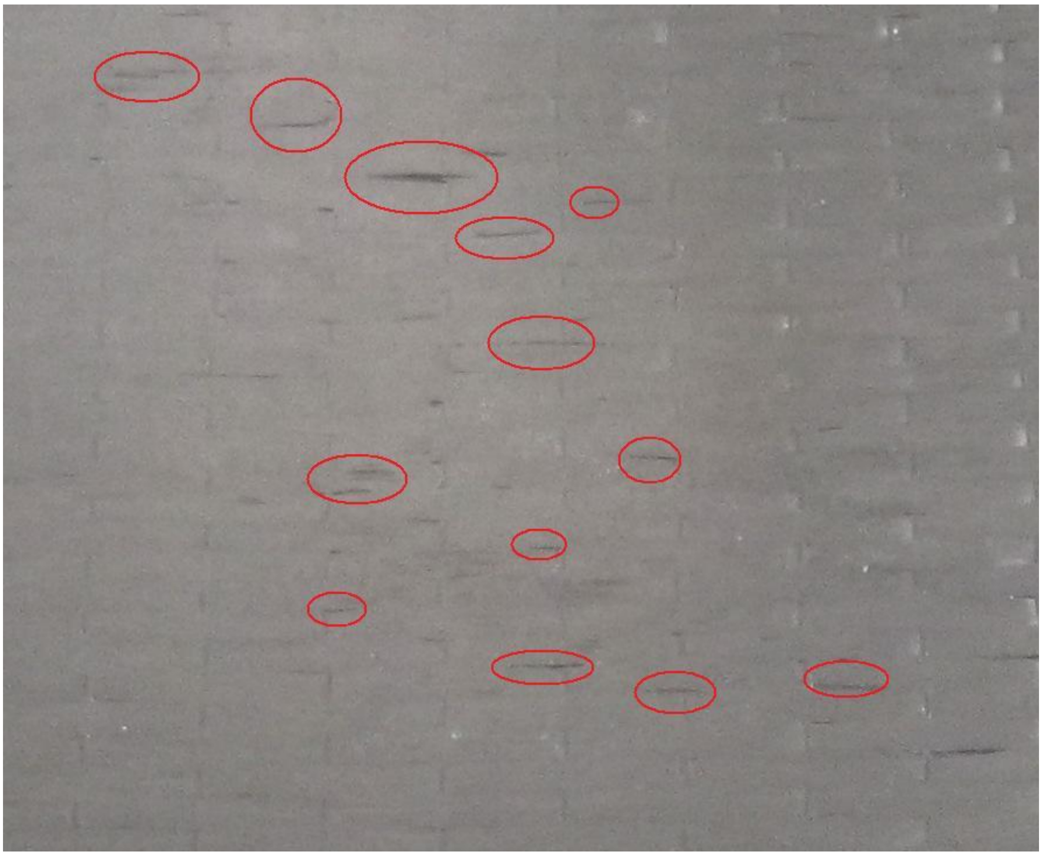

Figure 4.25 Faulty sample (the red parts are dry)

4.6. Experimental Set up

After preparing the laminated composite plates, the behaviour of them (deformation) must be evaluated. In order to minimise error, it is necessary to perform the test several times. Therefore, six identical plates (in terms of fibre orientations, composite materials, loading and boundary conditions) were chosen for each test and the average of these results is considered as a final result for that sample. In Figure (4.26) t_1 to t_6 are the experimental test results for six similar plates under the constant load. Six plates in each case have the same layup and geometry; t_a is the average of t_1 to t_6 and t_F is the FEM results. The size of the laminated composite plates are (500 x 500 mm) and as it is shown in Figure (4.5) they are fixed in one side and the displacement is measured on the opposite free side. The plates are under their own weight which is 270±2 grams. In FE this load is considered as a uniform distribution pressure on the plate. The displacement is measured by Laser Displacement Measurement Sensors with accuracy of .001 mm.

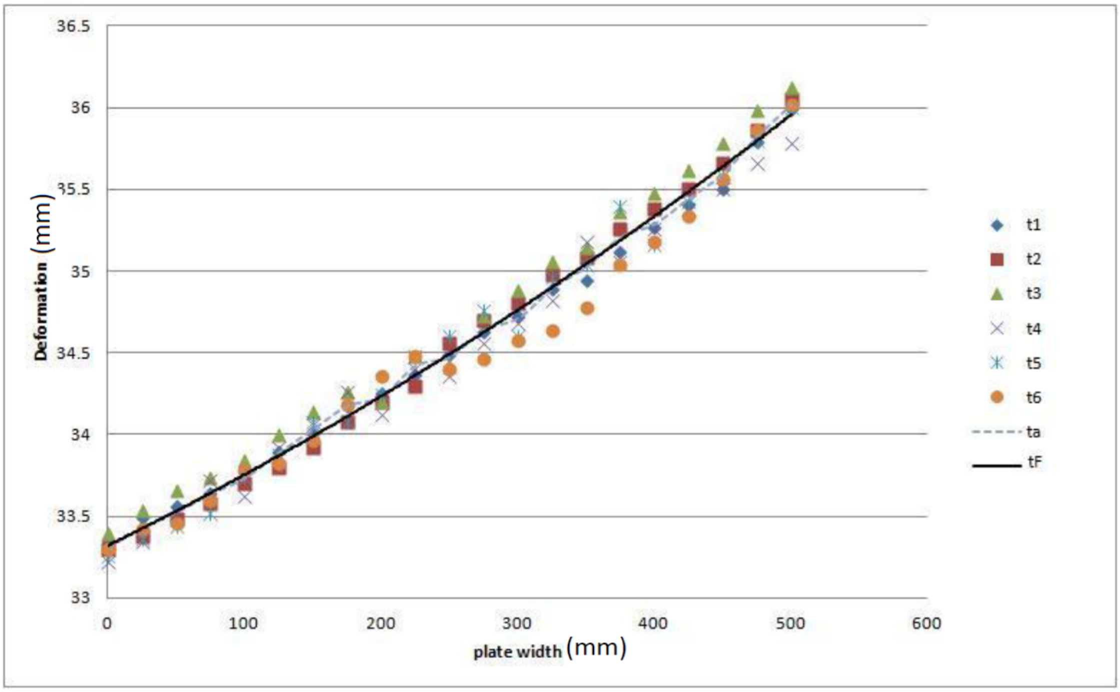

Figure 4.26 An example for one case; t_1 to t_6 are the experimental test results for six similar plates under constant load. t_a is the average of t_1 to t_6 and t_F is the FEM outputs.

4.7. Results

In this section, some of the FE results in this research are compared with current finite element models and later the finite element results are compared with experimental tests.

For the first case study, the effect of number of layers is compared with the work that has been done by Reddy (2004). Non-dimensionalised maximum deflections and stresses for five-layer with [0/90/0/90/0] fibre orientations and a simply supported square laminate plate is considered under uniform distribution loads. Table (4.5) shows the deflection of the plate and stresses for both models. The finite element results by Reddy were obtained with 4×4 mesh of eight-node quadratic elements. The results are in a very good agreement with the Reddy's work. However it is observed that Reddy's FEM is slightly under predicting the results for lower a/h. The ratio of material properties in this test are; (E1=25E2, G12=G13=0.5E2, G23=0.2E2, v12=0.25.)

Table 4.5 Comparison of non-dimensionalised maximum deflections and stresses of simply supported five-layer [0/90/0/90/0] square laminated plates.

a/h	Source	$\overline{w}\times10^2$	$\overline{\sigma}_{xx}$	$\overline{\sigma}_{yy}$	$\overline{\sigma}_{xy}$	$\overline{\sigma}_{xz}$	$\overline{\sigma}_{yz}$
4	(Reddy 2004)	1.5620	0.4339	0.4991	0.0233	0.3033	0.2281
	Abaqus FEM	1.5629	0.4345	0.4501	0.0235	0.3033	0.2283
10	(Reddy 2004)	0.6212	0.4986	0.4078	0.0219	0.3435	0.1984
	Abaqus FEM	0.6212	0.4989	0.4082	0.0219	0.3434	0.1984
20	(Reddy 2004)	0.4796	0.5239	0.3722	0.0214	0.3592	0.1827
	Abaqus FEM	0.4799	0.5239	0.3727	0.0212	0.3590	0.1825
100	(Reddy 2004)	0.4331	0.5345	0.3573	0.0211	0.3655	0.1761
	Abaqus FEM	0.4341	0.5343	0.3577	0.0210	0.3655	0.1762

In order to show that this test is repeatable another test has been performed. This time different fibre orientations are considered. Here, a three-layer [0/90/0] square plate is considered (Table 4.6). The layers are of equal thickness, with the same material properties as the previous example. The same locations and non-dimensionalisation are used.

While the classical laminate plate theory underpredicts deflections for small values of a/h, the stresses predicted are in general agreement with the first-order shear deformation theory. Also, the transverse shear stresses predicted through equilibrium equations, for the laminates studied so far, are very close to those predicted by the elasticity theory (Reddy 2004). Again there is a very good agreement between this work and Reddy's work.

Table 4.6 Comparison of non-dimensionalised maximum deflections and stresses of simply supported three-ply [0/90/0] square laminated plates.

a/h	Source	$\bar{w} \times 10^2$	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{yy}$	$\bar{\sigma}_{xy}$	$\bar{\sigma}_{xz}$	$\bar{\sigma}_{yz}$
10	FEM 4Q8-S (Reddy 2004)	0.6692	0.5098	0.2518	0.0250	0.4060	0.0908
	Abaqus FEM S9R5	0.6692	0.5101	0.2516	0.0251	0.4060	0.0906
20	FEM 4Q8-S (Reddy 2004)	0.4921	0.5281	0.1983	0.0222	0.4176	0.0754
	Abaqus FEM S9R5	0.4923	0.5281	0.1982	0.0222	0.4175	0.0754
100	FEM 4Q8-S (Reddy 2004)	0.4336	0.5346	0.1791	0.0212	0.4215	0.0699
	Abaqus FEM S9R5	0.4336	0.5347	0.1792	0.0212	0.4217	0.0699

It is shown that the Abaqus FEM results have good agreement with Reddy's FEM results. It is important to show that these results also match with the experimental results of laminated composite plate in order to prove the validity of the FE model. A series of experimental tests have been performed in order to validate the current FE method. One random sample is chosen from the experimental tests which were described in Section 4.5. After simulating the same example by Abaqus, the displacement result for the free edge of the plate is compared with experimental results. In this example the laminated composite plate has four layers and fibre orientations are [0/45/0/90]. The comparison of the FEA and experimental results are shown in Figures (4.27) and (4.28).

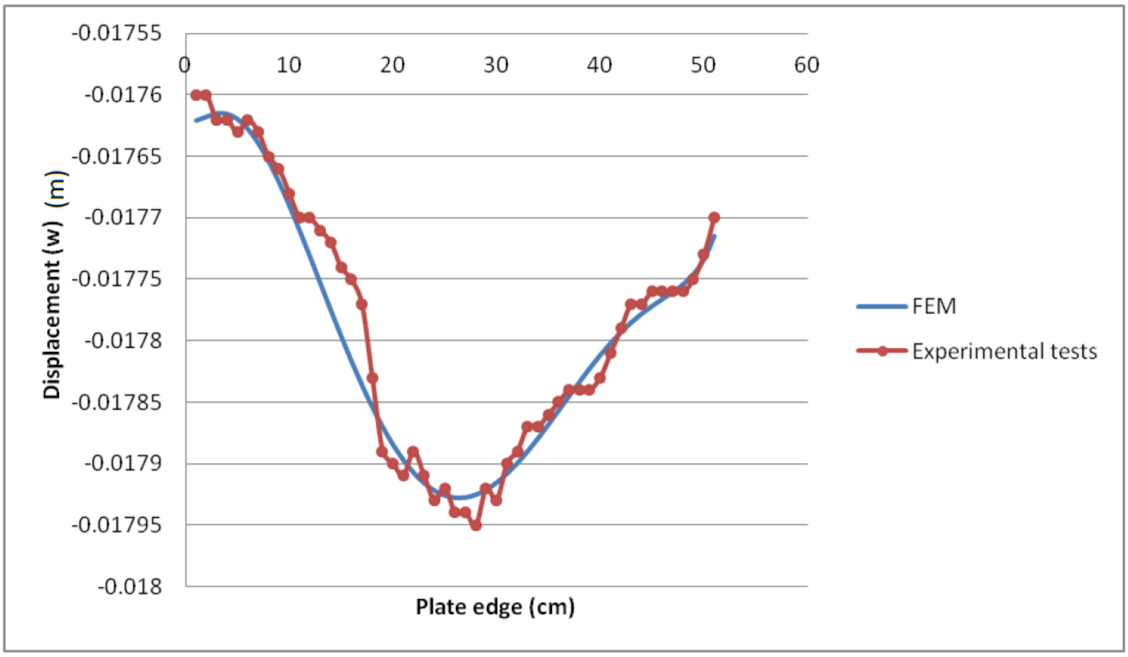

Figure 4.27 Displacement for the free edge of [0/45/0/90] plate with FEM and experiment

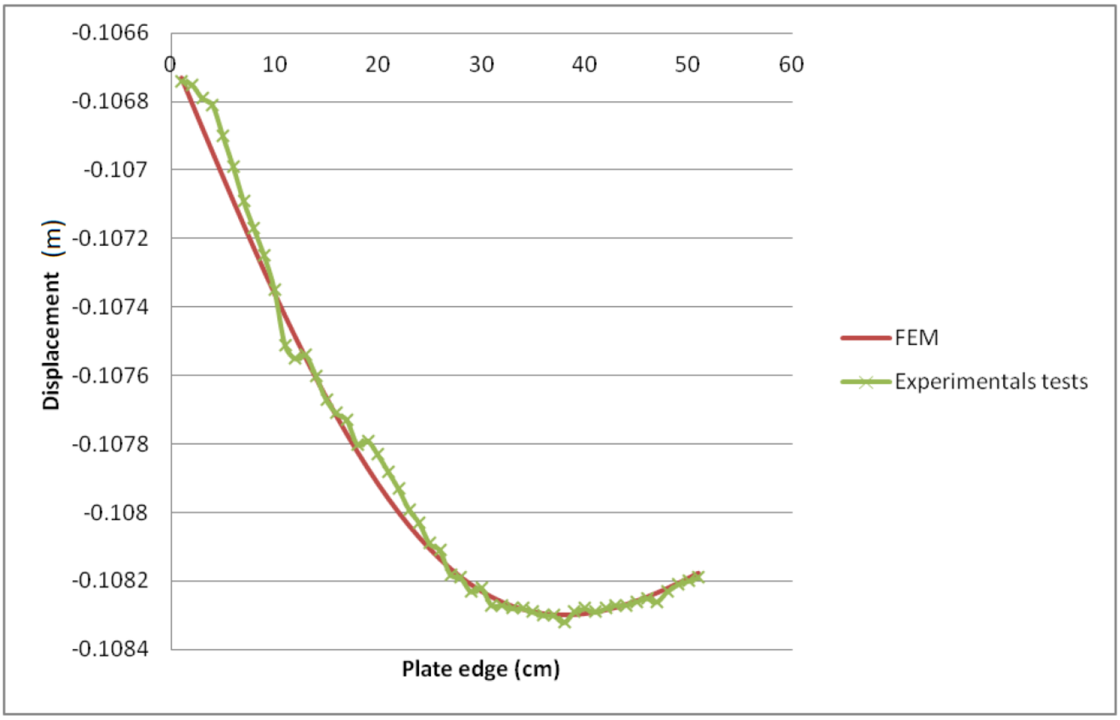

Figure 4.28 Displacement for the free edge of [0/45/0/90] plate with FEM and experiment

180 acceptable samples of which every 6th plate of them has the same fibre orientations, material properties and size were manufactured, as it is mentioned in Section 4.4 and 4.5. Therefore, 30 tests have been performed for laminated composite

plates with fibre orientations. The average displacement from each 6 plates with the same fibre orientations are compared with finite element results. As it is shown in Figures (4.29) to (4.34), the error is less than 2% for all the tests and experimental tests show a very good agreement with the FE results. These tests show the validity of the FE model. In the next chapter, the results of FE are used for different loading and boundary condition to find optimum fibre orientations for laminated composite plate.

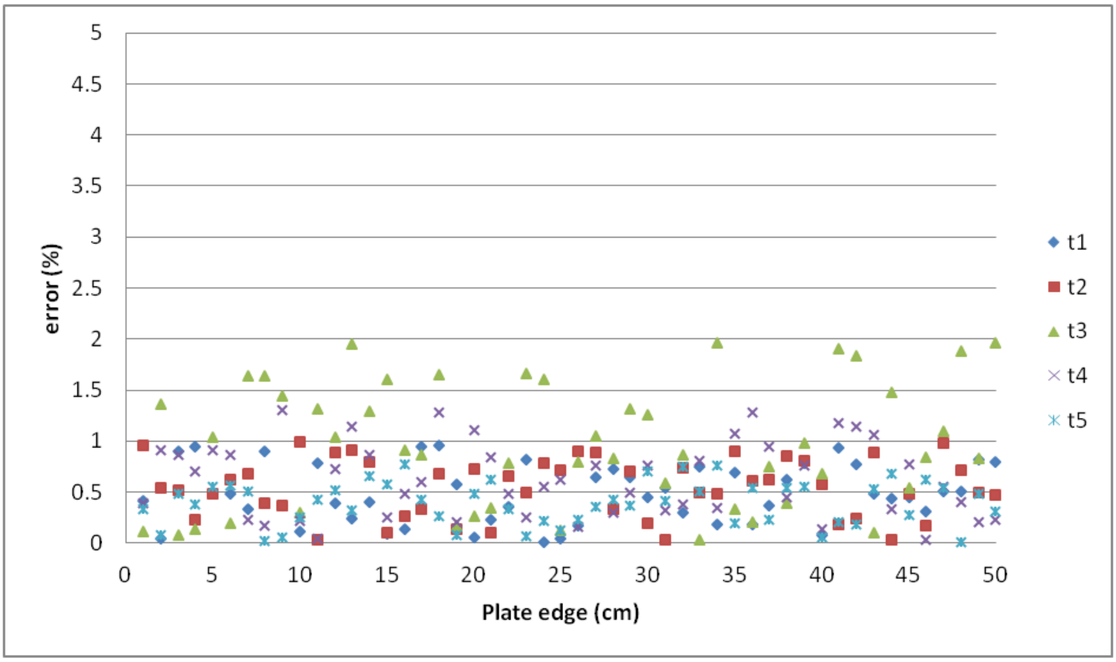

Figure 4.29 percentage error between FEM and experimental tests (t1-t5)

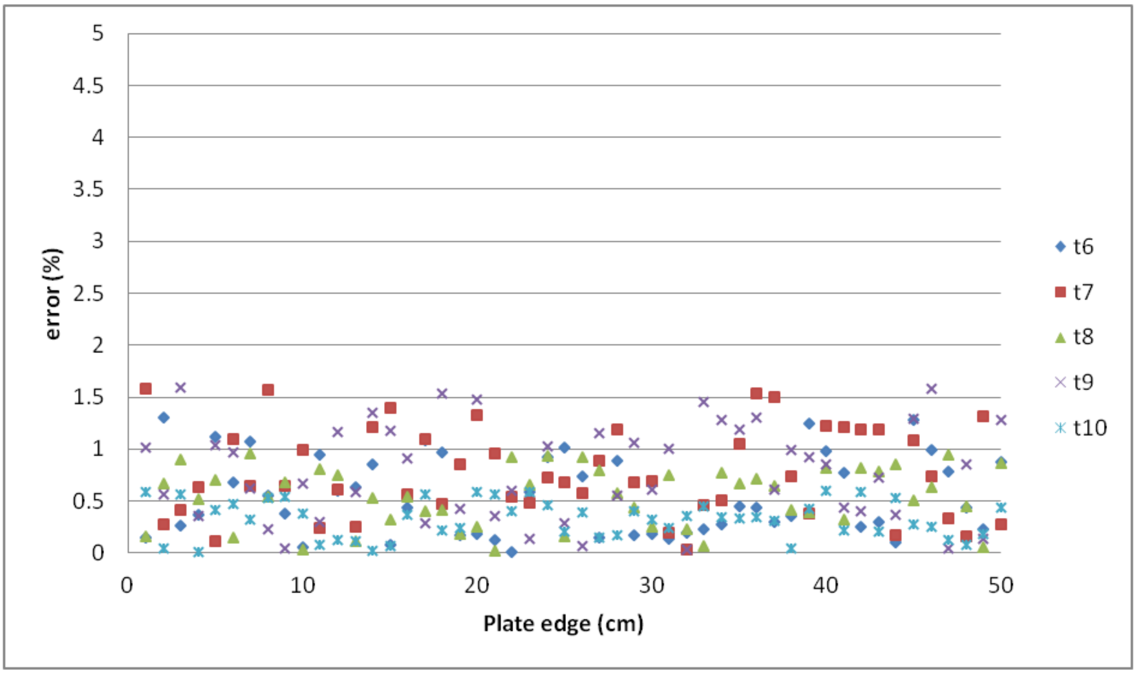

Figure 4.30 percentage error between FEM and experimental tests (t6-t10)

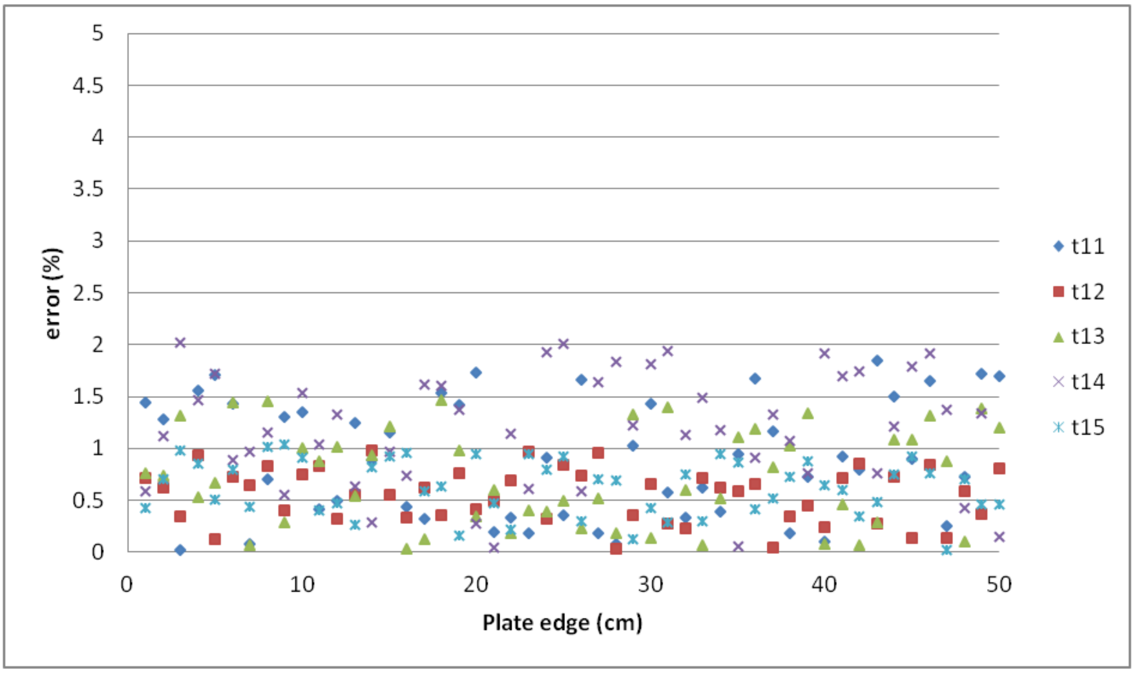

Figure 4.31 percentage error between FEM and experimental tests (t11-t15)

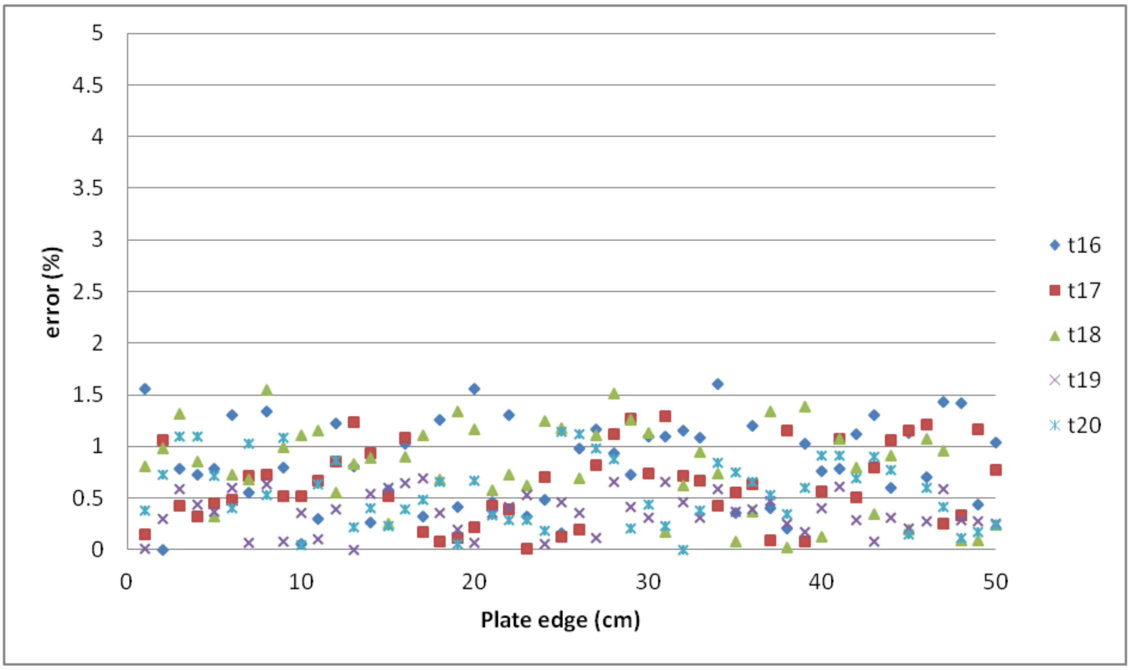

Figure 4.32 percentage error between FEM and experimental tests (t16-t20)

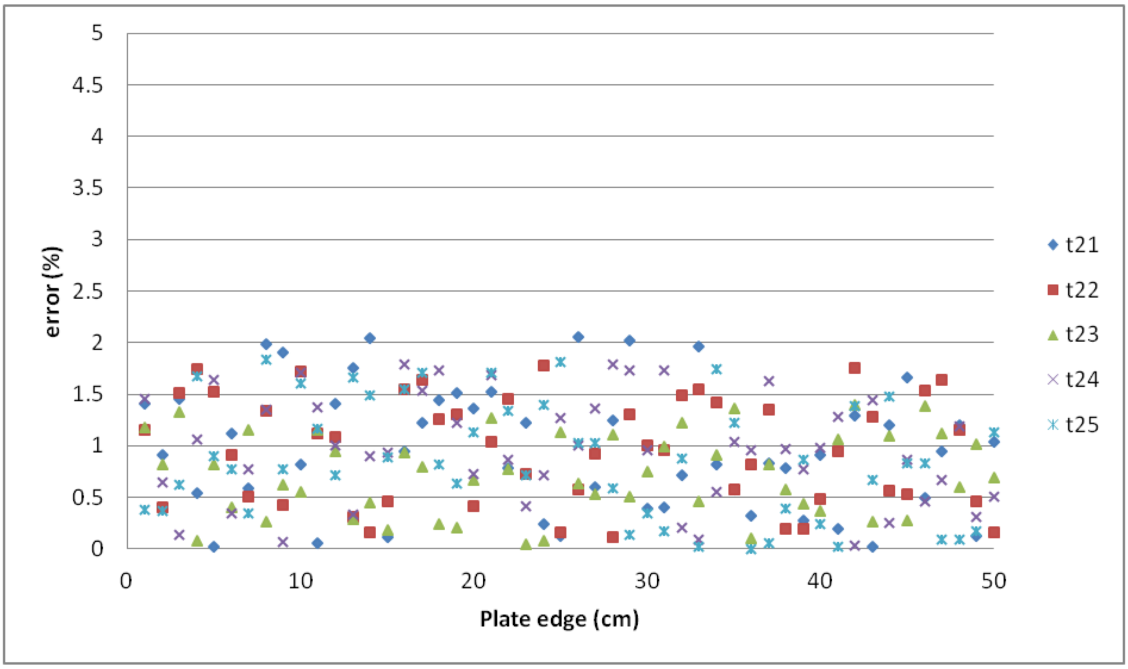

Figure 4.33 percentage error between FEM and experimental tests (t21-t25)

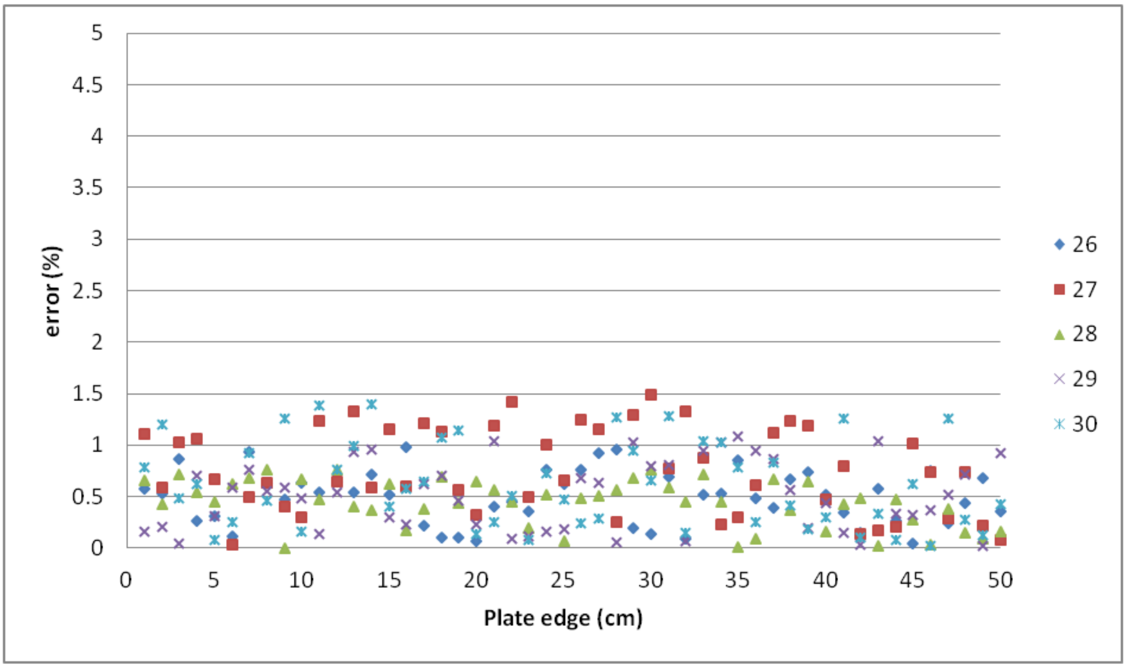

Figure 4.34 percentage error between FEM and experimental tests (t26-t30)

4.8. Summary

In this chapter, finite element modeling for laminated composites is studied. FEM in Abaqus is used in this work due to its wide material modelling and analysing capability especially in terms of considering the transverse shear effect in comparison with other FE software. Abaqus is also easy to define fibre orientations for lay-ups in the laminate composite plate. How to model and simulate the laminated composite plate in Abaqus is explained. In order to answer the effect of fibre orientations on composite plate behaviour, it is necessary to have the FE results for all possible fibre orientations. When the number of layers is increasing, as a result the number of FE tests which is needed is increasing and it is very time-consuming and hard to set up and run the program for each test manually. So, the written Python program is used to run these FE tests for the different orientations iteratively. An expanded and modified version of the Python program is presented as a design tool. Some experimental tests on laminated composite plates have been performed with the Vacuum bagging process. The steps for manufacturing laminated composite plate and the advantages and limitations of the process and the samples are described. In order to check the validity of the Abaqus FE model, the Abaqus's FE results are compared with Reddy's FE and experimental results and it is shown that there is a good agreement between them.

CHAPTER 5:

Optimum Design of Fibre Orientation for Laminated Composite Plate

5. Optimum Design of Fibre Orientation

In this research the main parameter considered for optimisation is desirable twist for laminated composite plates which is the effect of transverse shear. All possible fibre orientations for laminated composite plates are simulated in Abaqus and the results are saved in a database. In this chapter, an optimisation algorithm is presented to find the suitable fibre orientations from this database. Nowadays usually stochastic non-linear optimisation methods are utilised for complicated problems as they can avoid the local optimums. One of the best algorithms in this category is the Simulated Annealing (SA) method which is used in similar problems (Ghiasi et al. 2009). Previous research by Akbulut and Sonmez (2008) shows that using just one failure criterion in composite might lead to inaccurate results therefore two different criteria are used in this research for optimisation at the same time. Here, the Tsai-Wu failure and maximum stress criteria for composite laminate are chosen. Tsai-Wu failure is operationally simple and readily amenable to computational procedures. In addition, this criterion shows the difference between tensile and compressive strengths clearly, through its linear terms. By applying two failure criteria at the same time the results are more reliable. The proposed SA model is compared with a genetic algorithm (GA) and the SA optimisation models. Unlike other optimisations, the proposed SA optimisation model is able to be used for out of plane loadings as well as in-plane loadings.

5.1. ABDE Matrix

The advantage of fibre reinforced composite structures which is highlighted in this research is the ability to control the stiffness by tailoring the layup. By choosing the layer fibre orientations selectively, a composite can be elastically coupled to produce unique structural responses not possible with isotropic homogeneous conventional materials. The layup of the composite can be designed to tailor the stiffness matrix to couple the elastic response of the structure so that the uniaxial load causes twist, bending, shearing. The equations (3.7) show the relations between the force and moment resultants to the strains and deformation. It can be observed that the forces and moments are related to the strains through the terms of the ABDE matrix. Take for example the first equation that relates the load in the x direction (Nx) to the strains. In an isotropic homogenous material, the terms C16, C41, C42, and C46 are 0. Which simply means the axial force Nx is related only to in-plane strains Ex and Ey. A composite laminate on the other hand can be designed so that these terms can be designed to be various non-zero values. This means that these terms couple the axial load with to the shear and bending (elastic coupling).

In equation (3.7) [A] is the extension-shear stiffness matrix, [B] is the extension-bending coupling stiffness matrix, [D] is the bending-torsional stiffness matrix and [E] is the effect of transverse shear and normal stresses. Deformation of a laminated composite plate depends strongly on the fibre orientation angles in various plies as well as the layers sequence. Depending on the construction of the laminate, unique deformation characteristics of laminate composites are (Chandra et al. 1992):

Extension-shear coupling: If the laminate is not balanced, normal stresses acting on the laminate lead not only strains also shear strains. Likewise, shear stresses initiate the shear strain as well as normal strains. Balanced laminate is when for each ply with a fibre orientation of +ϴ, there is an identical ply with fibre orientation of –ϴ in the laminate.

Bending-torsion coupling: [D] is not zero, if for each ply with a fibre orientation of +ϴ above the mid-plane, there is no identical ply with fibre orientation of – ϴ at an equal distance below the mid-plane. In this case, the bending-torsion stiffness matrix [D] causes the bending-torsion coupling. On the other hand, the bending moment acting on the laminate composite plate creates bending as well as twisting.

Extension-bending coupling: If the laminate is not symmetric, the extension-bending coupling matrix , [B], is not zero which means in-plane loads create bending and twisting deformation as well as in-plane deformation. When for each ply with fibre orientation of +ϴ above the mid-plane there is an identical ply with + ϴ at the same distance below the mid-plane the laminate is called symmetric.

The twist in laminate composite plate is induced from the above coupling effects. Figure (5.1) shows the schematic exaggerated warping deformation of a laminate composite plate. Here, the angle of twist is considered as (α).

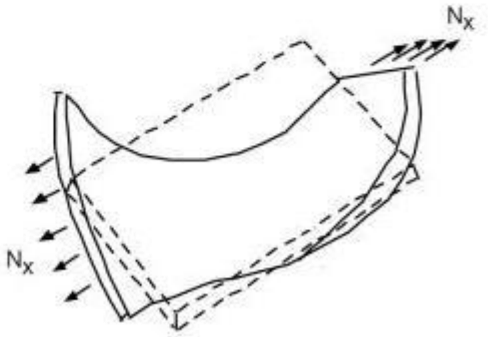

Figure 5.1 Applying a load to unsymmetrically laminated plate causes coupling between extension, shear, bending and twisting. (Chandra et al. 1992)

5.2. Failure Criteria

Here, the stress based criteria are briefly overviewed. It is essential to mention that in this research the delamination and other types of failure are not considered. In all cases tensile values must be positive and compressive values must be negative. The input data for the stress-based failure theories are tensile and compressive stress limits, F_{1t} and F_{1c}, in the 1-direction; tensile and compressive stress limits, F_{2t} and F_{2c}, in the 2-direction; and shear strength (maximum shear stress), F_{12}, in the X–Y plane.

Maximum stress theory

The maximum stress failure criterion is:

$$I_F = \max\left(\frac{\sigma_{11}}{X}, \frac{\sigma_{22}}{Y}, \left|\frac{\sigma_{12}}{S}\right|\right) < 1.0 \tag{5.1}$$

If $\sigma_{11} > 0, \quad X = F_{1t}$ otherwise, $X = F_{1c}$. If $\sigma_{22} > 0, \quad Y = F_{2t}$ otherwise, $Y = F_{2c}$.

Tsai-Hill theory

The Tsai-Hill failure criterion is:

$$I_F = \frac{\sigma_{11}^2}{X^2} - \frac{\sigma_{11}\sigma_{22}}{X^2} + \frac{\sigma_{22}^2}{Y^2} + \frac{\sigma_{12}^2}{S^2} < 1.0 \tag{5.2}$$

If $\sigma_{11} > 0, \quad X = F_{1t}$ otherwise, $X = F_{1c}$. If $\sigma_{22} > 0, \quad Y = F_{2t}$ otherwise, $Y = F_{2c}$.

Azzi-Tsai-Hill theory

The Azzi-Tsai-Hill failure theory is similar to Tsai-Hill theory except the fact that the absolute value of the cross product term is taken.

$$I_F = \frac{\sigma_{11}^2}{F_1^2} - \frac{|\sigma_{11}\sigma_{22}|}{F_1^2} + \frac{\sigma_{22}^2}{F_2^2} + \frac{\sigma_{12}^2}{F_{12}^2} < 1.0 \quad (5.3)$$

This difference just shows up if σ_{11}, σ_{22} have different opposite signs.

Tsai-Wu theory

The Tsai-Wu failure criterion is:

$$I_F = F_1\sigma_{11} + F_2\sigma_{22} + 2F_{12}\sigma_{11}\sigma_{22} < 1.0 \quad (5.4)$$

The Tsai-Wu criterion is used in this work and the coefficients are defined later. Figures (5.2) to (5.4) show the differences between these failure criteria.

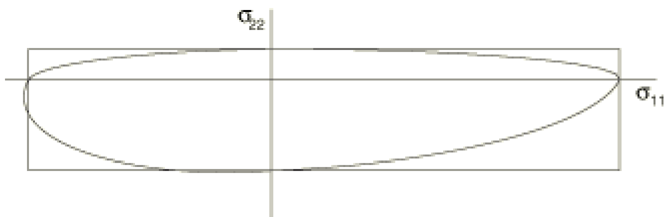

Figure 5.2 Tsai-Hill versus maximum stress failure envelope ($I_F = 1.0$) (Reddy 2004)

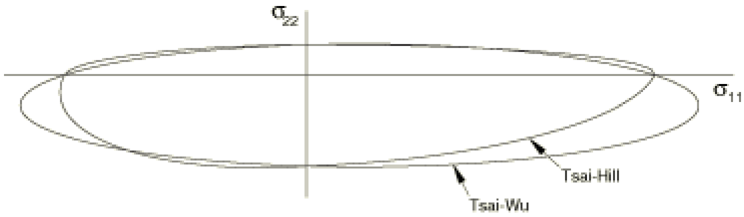

Figure 5.3 Tsai-Hill versus Tsai-Wu failure envelope ($I_F = 1.0$) (Reddy 2004)

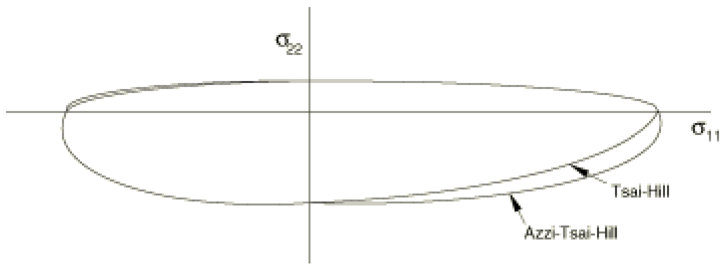

Figure 5.4 Tsai-Hill versus Azzi-Tsai-Hill failure envelope ($I_F = 1.0$)(Reddy 2004)

Here, two failure criteria (Maximum stress and Tsai-Wu) are considered and explained more as they are utilised in the optimisation.

Maximum stress criterion is one of the simplest failure methods to apply. According to this criterion, failure is predicted whenever one of the principal stress components exceeds its corresponding strength. It is expressed in the form of the following sub-criteria:

$$\sigma_1 = \begin{cases} F_{1t} & when\ \sigma_1 > 0 \\ -F_{1c} & when\ \sigma_1 < 0 \end{cases} \tag{5.5}$$

$$\sigma_2 = \begin{cases} F_{2t} & when\ \sigma_2 > 0 \\ -F_{2c} & when\ \sigma_2 < 0 \end{cases} \tag{5.6}$$

$$\sigma_3 = \begin{cases} F_{3t} & when\ \sigma_3 > 0 \\ -F_{3c} & when\ \sigma_3 < 0 \end{cases} \tag{5.7}$$

$$|\tau_4| = F_4 \tag{5.8}$$

$$|\tau_5| = F_5 \tag{5.9}$$

$$|\tau_6| = F_6 \tag{5.10}$$

F_{1t} and F_{1c} are the longitudinal tensile and compressive strengths, F_{2t} and F_{2c} are the transverse longitudinal tensile and compressive strengths and F_6 is the in-plane shear strength. Four additional lamina strength parameters, which are relevant in three-dimensional analysis, are the out of plane or interlaminar tensile, compressive, and shear strengths, F_{3t}, F_{3c}, F_4 and F_5.

The Tsai-Wu failure criterion is one of the most reliable static failure criteria as it provides a simple analytical expression. Tsai-Wu in 3D format can be presented as:

$$f_i\sigma_i + f_{ij}\sigma_i\sigma_j = 1 \tag{5.11}$$

where f_i and f_{ij} are second- and fourth-order strength tensors, and $i, j = 1,2,\ldots,6$ (Reissner 1961). By applying assumptions some of f_i and f_{ij} are identified. Finally it is reduced to a failure envelope for constant values of shear stress $\tau_6 = kF_6$

$$f_1\sigma_1 + f_2\sigma_2 + f_{11}\sigma_1^2 + f_{22}\sigma_2^2 + 2f_{12}\sigma_1\sigma_2 = 1 - k^2 \tag{5.12}$$

or equivalently

$$\frac{\sigma_{11}^2}{F_{1t}|F_{1c}|} + \frac{\sigma_{22}^2}{F_{2t}|F_{2c}|} + \frac{\tau_{12}^2}{F_6} - \frac{\sigma_{11}\sigma_{22}}{\sqrt{F_{1t}F_{1c}F_{2t}F_{2c}}} + \left(\frac{1}{F_{1t}} - \frac{1}{|F_{1c}|}\right)\sigma_{11} + \left(\frac{1}{F_{2t}} - \frac{1}{|F_{2c}|}\right)\sigma_{22} < 1 \tag{5.13}$$

5.3. Simulated Annealing Algorithm

Kirkpatrick et al. (1983) proposed simulated annealing as a powerful stochastic search technique. The method gets its name from the physical process whereby the temperature of a solid is raised to a melting point, where the atoms can move freely and then slowly cooled. The method attempts to model the behaviour of the atoms in forming arrangements in solid material during annealing. Although the atoms move randomly, as their natural behaviour they tend to form lower-energy configurations (Erdal and Sonmez 2005). However, this is a time driven process. When a material is crystallised from its liquid phase, it must be cooled slowly if it is to assume its highly ordered, lowest-energy, perfect crystalline state. At each temperature level during this annealing process, the material should reach equilibrium. As the temperature decreases, the arrangement of the atoms gets closer and closer to the lower energy state. This process continues until the temperature finally reaches freezing point. The temperature is initially assigned a higher value, which corresponds to more probability (chance) of accepting a bad move and is gradually decreased by a cooling schedule rate which is defined by the user. Retaining the best solution is recommended in order to preserve the good solution (Erdal and Sonmez 2005).

At each iteration of the simulated annealing algorithm, a new point is randomly generated. The distance of the new point from the current point, or the extent of the search, is dependent on a probability distribution with a scale proportional to the temperature. Not only does the algorithm accept all new points that lower the objective, but also, with a certain probability, it accepts points that raise the objective. The algorithm avoids being trapped in local minima, by accepting points that raise the objective, and is capable of searching globally for more potential optimums. An annealing schedule is chosen to systematically decrease the temperature as the algorithm proceeds. As the temperature decreases, the algorithm reduces the extent of its search to converge to a minimum.

If a set of configurations is considered, in each iteration the speed convergence would be increased. In this research the SA proposed by Erdal and Sonmez (2005) is applied. The number of these configurations depends on the dimension of the problem.

$$N = 7(n + 1) \tag{5.14}$$

where n is the dimension of the problem, i.e. the number of design variables.

The basic algorithm of SA in MATLAB is shown in Figure (5.5). The algorithm starts from a valid solution and randomly generated new states, for the problem and calculates the associated cost function. Simulation of the annealing process starts at high temperature (which is defined by the user). A new point is randomly selected and the difference in cost function is calculated. If it is lower, then this new point is accepted. This method guides the system towards a minimum which can be a local or global minimum. To avoid a local minimum, an increase of the cost function is accepted with a certain probability.

Figure (5.6) illustrates how this algorithm works. It starts at point *i*, then the new point *k1* is chosen and accepted, but the new point *k2* is only accepted with a certain probability. The probability of accepting a worse state is high at the beginning and decreases at the temperature decreases. For each temperature, the system must reach an equilibrium which means a minimum number of new points must be tried, before the temperature is reducing.

```
Initialization (Current_Solution, Temperature)
Calculation of the Current _Cost
Loop
      New_State
     Calculation of the New_Cost
     If (Current_Cost-New_Cost)<=0 Then
              Current_State=New_State
     Else
            If Exp((Current_Cost-New_Cost)/Temperature)>Random (0,1)
     Then
        -- Accept
           Current_State=New_State
     Else
         -- Reject
     Decrease the temperature
Exit when Stop_Criterion
End Loop
```

Figure 5.5 The basic algorithm of SA

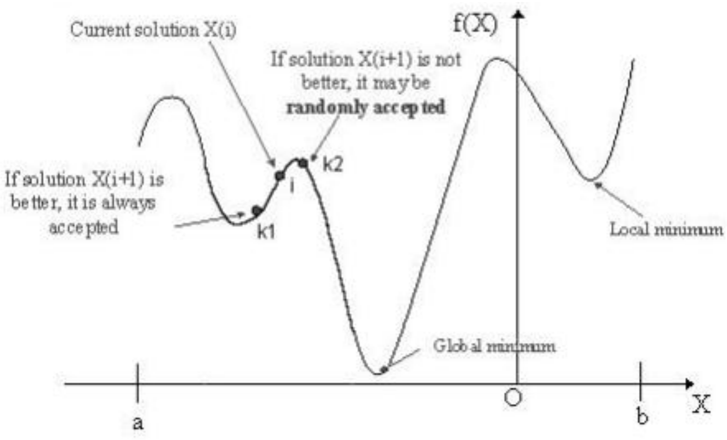

Figure 5.6 Illustrating how the SA algorithm work

5.4. Penalty Function and Optimisation Procedure

In this step a penalty function is expressed and then this function has to be optimised.

$$F = \sum_{k=1}^{m} n_k + w_1 P_{MS} + w_2 P_{TW} - w_1 SF_{MS} - w_2 SF_{TW} + \frac{w_{3i}}{k}\cos(\Delta\theta) \qquad (5.15)$$

n_k is the number of plies in the k^{th} lamina, in which the orientation angle is θ_k; m is the total number of distinct lamina; the second and third terms represent the penalty values introduced to increase the value of the objective function for designs for which failure is predicted and thus to restrict the search to the feasible design space; P_{MS} and P_{TW} are penalty values calculated based on the maximum stress criterion and the Tsai–Wu criterion, respectively. SF_{MS} and SF_{TW} are equal to the safety factors according to the maximum stress and Tsai–Wu criteria, respectively, if they are greater than 1.0, otherwise these terms are equal to zero; w_i are suitable coefficients (Akbulut and Sonmez 2008). This penalty function is the same as the one defined by Akbulut and Sonmez (2008) except the last term of Equation (5.15). In their work, the ply angles are optimised for the in-plane plate and the effect of shear stress and as a result induced twist angle of the plate was neglected. However in this research the induced twist angle is considered. Therefore, the maximum acceptable twist angle is defined and by assuming this maximum twist angle, the appropriate coefficient for w_{3i} is obtained.

Applying several tests by FEM software shows that maximum twist for each material happens at the specific angles$(\theta_{max})_i$ $i = 1,2,3, ...$ i is the number of possible θ_{max}. The range of fibre orientations $[-90, 90]$ is divided into several areas, each θ_{max} is the centre of the area and in each iteration the program finds the θ_k to the range that it belongs to. Then the program works with the appropriate θ_{max} and corresponding w_{3i}. Except for the related w_{3i}, the other w_{3i} variables are equal to zero. $\Delta\theta$ is defined as:

$$\Delta\theta = \theta_{max} - \theta_k \qquad (5.16)$$

So when the ply angle in each layer (θ_k) would be close to θ_{max}, the value of $cos(\Delta\theta)$ is bigger and thus the last term of the penalty function. As a result the penalty function would be larger. For a proper w_3, the amount of induced twist (α) will always be less than the maximum acceptable twist (α_{max}).

The reason that safety factors are used in the designs is that there may be many feasible designs with the same minimum thickness. Of these designs, the optimum is defined as the one with the largest failure load. Accordingly, the objective function is linearly reduced in proportion to the failure margin (Le and Haftka 1995). Similarly in another study (Kere et al. 2003), the margins to initial failure were maximised with the minimum feasible number of laminae. The safety factor of the laminate according to the maximum stress criterion, $\mathrm{SF_{MS}}$, is calculated as follows (Akbulut and Sonmez 2008):

$$SF_{MS}^{k} = \text{min of} \begin{cases} SF_X^k = \begin{cases} F_{1t}/\sigma_{11} & if \;\; \sigma_{11} > 0 \\ F_{1c}/\sigma_{11} & if \;\; \sigma_{11} < 0 \end{cases} \\ SF_Y^k = \begin{cases} F_{2t}/\sigma_{22} & if \;\; \sigma_{22} > 0 \\ F_{2c}/\sigma_{22} & if \;\; \sigma_{22} < 0 \end{cases} \\ SF_S^k = S/|\tau_{12}| \end{cases} \tag{5.17}$$

$$SF_{MS} = \text{min of } SF_{MS}^{k} \quad for\; k = 1,2,\dots,m-1,m \tag{5.18}$$

The safety factor for the k^{th} lamina, SF_{TW}^{k}, according to the Tsai–Wu criterion is defined as the multiplier of the stress components at lamina k, σ_{ij}^{k}, that makes the right hand side of Eq. (5.15) equal to 1 then it turns into:

$$a\left(SF_{TW}^{k}\right)^{2} + b\left(SF_{TW}^{k}\right) = 1 \tag{5.19}$$

where

$$\begin{aligned} a &= \frac{\left(\sigma_{11}^{k}\right)^{2}}{F_{1t}|F_{1c}|} + \frac{\left(\sigma_{22}^{k}\right)^{2}}{F_{2t}|F_{2c}|} + \frac{\left(\tau_{12}^{k}\right)^{2}}{F_{6}^{2}} - \frac{(\sigma_{11}^{k})(\sigma_{22}^{k})}{\sqrt{(F_{1t}F_{1c}F_{2t}F_{2c})}} \\ b &= \left(\frac{1}{F_{1t}} - \frac{1}{|F_{1c}|}\right)\sigma_{11}^{k} + \left(\frac{1}{F_{2t}} - \frac{1}{|F_{2c}|}\right)\sigma_{22}^{k} \end{aligned} \tag{5.20}$$

The root of the above equation gives the safety factor. Because a negative safety factor is not physically meaningful, the absolute value of the first root is considered as the actual safety factor.

$$SF_{Tw}^{k} = \left|\frac{-b \pm \sqrt{b^2+4a}}{2a}\right| \tag{5.21}$$

Then, the minimum of SF_{TW}^{k} is chosen as the safety factor of the laminate.

$$SF_{TW} = \text{min of } SF_{TW}^{k} \quad for\ k = 1,2,\ldots,m-1,m \tag{5.22}$$

In equation (5.21), the $\left|\frac{-b+\sqrt{b^2+4a}}{2a}\right|$ can be considered, as b is always positive and the aim is to find the minimum of SF_{TW}^{k}.

The penalty value due to the violation of the maximum stress and Tsai-Wu criteria are calculated in equations (5.23) and (5.24) respectively:

$$P_x^k = \begin{cases} 0 & if\ SF_x^k \geq 1 \\ (1/SF_x^k) - 1 & if\ SF_x^k < 1 \end{cases}$$
$$P_y^k = \begin{cases} 0 & if\ SF_y^k \geq 1 \\ \left(1/SF_y^k\right) - 1 & if\ SF_y^k < 1 \end{cases} \tag{5.23}$$
$$P_s^k = \begin{cases} 0 & if\ SF_s^k \geq 1 \\ (1/SF_s^k) - 1 & if\ SF_s^k < 1 \end{cases}$$

$$P_{TW}^k = \begin{cases} 0 & if\ SF_{TW}^k \geq 1 \\ \left(1/SF_{TW}^k\right) - 1 & if\ SF_{TW}^k < 1 \end{cases} \tag{5.24}$$

The total penalty value for the laminate due to the violation of the maximum stress and Tsai-Wu criteria are then calculated by summing up the penalty values calculated for each lamina.

$$P_{MS} = \sum_{k=1}^{m} P_x^k + P_y^k + P_s^k \tag{5.25}$$

$$P_{TW} = \sum_{k=1}^{m} P_{TW}^k \tag{5.26}$$

5.5. Numerical Results

Here, the induced twist for different boundary conditions (CFFF and CCCF) are compared between experimental and FE results. In order to validate the presented SA optimisation model, the results of it are compared with another optimisation method (genetic algorithm) and a modified SA which are recently proposed by Hade et al. (2011) and Akbulut and Sonmez (2008) respectively for in-plane stress. Furthermore, the results of the proposed SA optimisation model are shown for out of plane stresses.

As it is mentioned in Chapter 4 in order to validate the FE model some experimental tests have been performed. For each panel six similar laminated plates are manufactured and the results used here is the average of them.

The experimental tests have been carried out for 15 different samples with a uniform distributed loads of P=50 pa for two different boundary conditions. In Figure (5.7) the percentage difference of induced twist angle (α), between tests and FEM results for CFFF boundary condition is shown. The same tests have been performed with CCCF boundary conditions which the results are shown in Figure (5.8). As it is shown in Figures (5.7) and (5.8) there is a very good agreement between FEM and experimental results. The Y axis is zoomed in to 10% to distinguish the differences between each case.

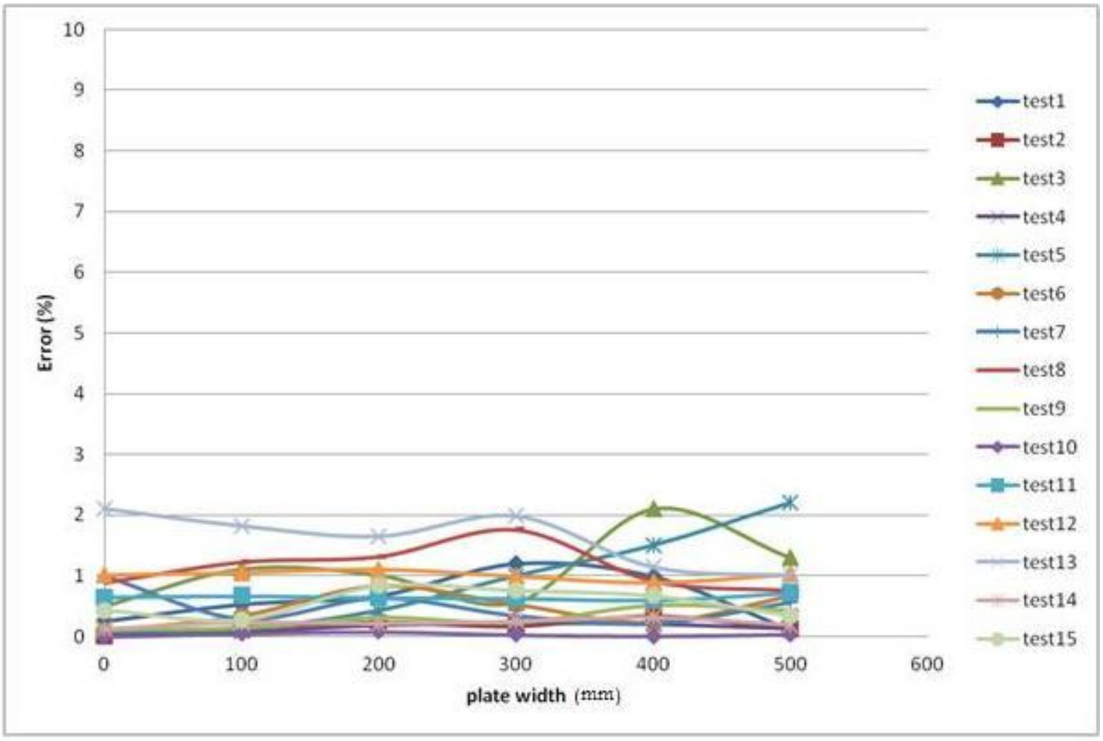

Figure 5.7 Percentage difference of (α) between tests and FE with CFFF boundary condition

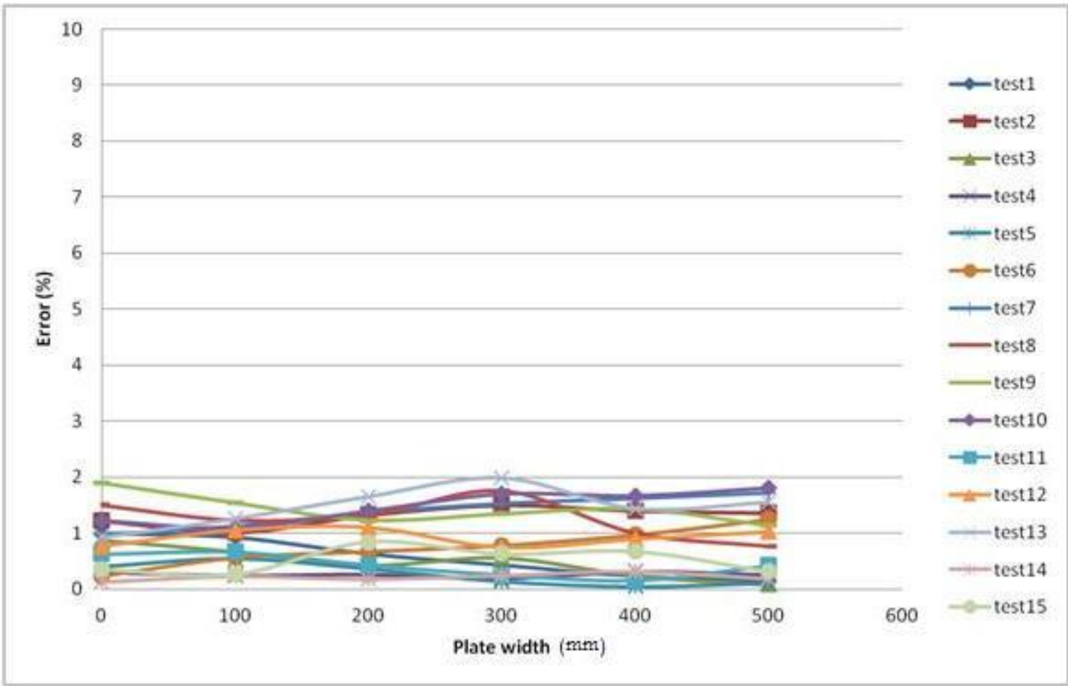

Figure 5.8 Percentage difference of (α) between tests and FE with CCCF boundary condition

Hade et al. (2011) presented an optimisation model based on numerical simulation of laminate composite plate using genetic algorithms with finite difference analyses for simply supported laminated composite plate. The case study is graphite/epoxy materials with the properties of:

$$P_0 = 1000 \ \frac{N}{m^2}, \ \ a = .4 \ m, \ E1 = 100E2 \ Pa, \ E2 = E3, \ G12 = G13 = .5{*}E2, \ \upsilon_{12} = \upsilon_{13} = .25$$

The results of the SA and GA model are shown in Table (5.1) for different ratio of E1/E2. The SA results show good agreement with GA results. In order to be sure that the presented SA is not trapped in local minimum each optimisation method was run 6 times. All the results were the same and the SA model can avoid the local optimum.

Table 5.1 Optimised fibre orientations by GA and SA for different E1/E2 ratio.

$\frac{E1}{E2}$	Optimum lamina orientations	
	GA (Hade et al. 2011)	SA
5	[-45.96,-45.14,-45.73]	[-45.89,-44.99,-45.55]
10	[-49.62,-45.19,-45.13]	[-49.72,-45.29,-44.98]
20	[-44.94,-44.13,-45.48]	[-44.64,-44.43,-45.28]
30	[-48.99,-45.05,-44.76]	[-49.08,-45.50,-44.79]
40	[-41.57,-45.81,-44.92]	[-41.70,-45.61,-44.92]
50	[-44.41,-45.41,-44.72]	[-44.31,-45.76,-45.00]

Here, two other case studies are presented. The first one compares the obtained results with those which were found by Akbulut and Sonmez (2008). The graphite/epoxy materials T300/5308 with the properties of $E_{11} = 40.91$ GPa, $E_{22} = 9.88$ GPa, $G_{12} = 2.84$ GPa, $\vartheta_{12} = 0.292$, $F_{1t} = 779$ MPa, $F_{1c} = -1134$ MPa, $F_{2t} = 19$ MPa, $F_{2c} = -131$ MPa, $F_{12} = 75$ MPa is considered for the first case study.

In this case the maximum acceptable twist angle is $\alpha_{max} = .05$ for square laminated composite plate where each side is 300mm and the boundary condition of CFFF. In Table (5.2) and Table (5.3) the results are compared with previous work. As it is shown in Table (5.2) the number of lay-ups increase for some loads and it shows that in these cases the amount of twist angle is more than α_{max}. (In the notation $[_n]$, Θ is the fibre orientation and n is the number of layers with the fibre orientations of Θ). Clearly by increasing the lay-ups, the safety factor for both Tsai-Wu and Maximum stress will be increased. In Table (5.3) the number of lay-ups and thickness is constant but in some cases the optimum orientations are different from the Akbulut and Sonmez work. Although the safety factor is reduced, when the number of the layers is fixed, it avoids passing the acceptable twist angle and the safety factor is still more than 1, therefore, it is still acceptable. When the maximum twist angle is less than the α_{max} the results are comparable with the work carried out by Akbulut and Sonmez (2008).

Table 5.2 Optimum lamina orientations for Material T300/5308 under different loads

Loading: N_{xx}, N_{yy}, N_{xy} $(MPa\ m)$	Optimum lamina orientations		Safety factor			
	Akbulut , Sonmez (2008)	Present Work	Akbulut, Sonmez (2008)		Present Work	
			Max. Stress	Tsai-Wu	Max. Stress	Tsai-Wu
10/5/0	$[37_{27}/-37_{27}]$	$[39_{29}/-39_{29}]$	1.0277	1.0068	1.1309	1.1001
20/5/0	$[31_{23}/-31_{23}]$	$[36_{27}/-36_{27}]$	1.1985	1.0208	1.3305	1.1560
40/5/0	$[26_{20}/-26_{20}]$	$[26_{22}/-26_{22}]$	1.5381	1.0190	1.6504	1.1903
80/5/0	$[21_{25}/-19_{28}]$	$[21_{25}/-21_{25}]$	1.2213	1.0113	1.2302	1.0120
120/5/0	$[17_{35}/-17_{35}]$	$[17_{35}/-17_{35}]$	1.0950	1.0030	1.0951	1.0030

Table 5.3 Optimum lamina orientations for Material T300/5308 under different loads for constant thickness

Loading: N_{xx}, N_{yy}, N_{xy} $(KPa\ m)$	Optimum lamina orientations		Safety factor	
	Akbulut & Sonmez (2008)	Present Work	Akbulut, Sonmez (2008)	Present Work
200/200/0	$[50.80_4/-49.80_4/26.59_4/-49.73_4]$	$[50.80_4/-49.80_4/26.59_4/-49.73_4]$	2.14	2.14
200/0/200	$[31.72_{16}]$	$[32.40_4/-56.61_4/-7.81_4/33.87_4]$	4.84	1.91
400/200/0	$[30.98_4/-36.57_4/37.67_4/-37.20_4]$	$[-20.12_4/58.01_4/-49.90_4/20.11_4]$	1.64	1.42
200/200/200	$[45_{16}]$	$[30.78_4/-59.20_4/29.09_4/-49.43_4]$	1.11	1.03

In the second case study a highly anisotropic material is considered (material II). The elements of the stiffness matrix are: $D_{1111} = 138000$, $D_{1122} = 44000$, $D_{2222} = 138000$, $D_{1133} = 5000$, $D_{2233} = 6000$, $D_{3333} = 47000$, $D_{1112} = 2000$, $D_{2212} = 8000$, $D_{3312} = 3000$, $D_{1212} = 10000$, $D_{1113} = 0$, $D_{2213} = 3500$, $D_{3313} = 0$, $D_{1213} = 21000$, $D_{1313} = 58000$, $D_{1123} = 0$, $D_{2223} = 1000$, $D_{3323} = 2500$, $D_{1223} = 2000$, $D_{1323} = 4500$, $D_{2323} = 23000$.

In Figure (5.9) the amount of twist angle under a constant load for different ply angles is shown. The amount of $(\theta_{\max})_i$ in equation (5.16) are $(\pm 30, \pm 60)$ which are the local maximums. This test is for the first layer in order to find the $(\theta_{\max})_i$ and the areas which were explained in the penalty function. FEM tests show that these $(\theta_{\max})_i$ are the same by adding the next layers. If the stiffness matrix is symmetric the curves will be symmetric about the y axis. A general stiffness matrix for material II is considered, so there is no mirror about the y axis. Optimum lamina orientations under different loads in this case are shown in the Table (5.4). In this case the pure bending load N_{zz} is also applied to the plate.

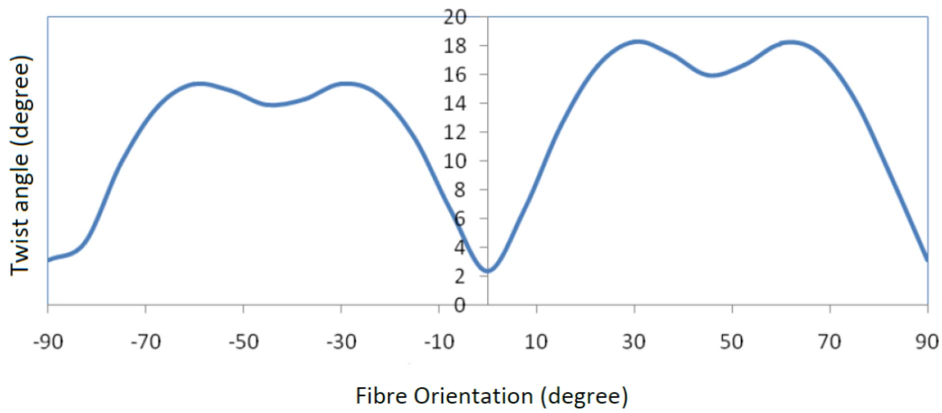

Figure 5.9 Twist angle for a first layer of material (case II)

Table 5.4 Optimum lamina orientations for second case study material under different loads for constant thickness.

Loading: $N_{xx}, N_{yy}, N_{xy}, N_{zz}$ $(KPa\ m)$	Optimum Fibre orientations	Safety factor	
		Max. Stress	Present Work
100/100/0/20	$[-77.68_4/23.54_4/-22.78_4/49.79_4]$	1.2720	1.0931
100/0/100/20	$[53.45_8/-78.23_8]$	1.5402	1.2112
200/100/0/20	$[-22.23_4/49.26_4/-65.33_4/11.23_4]$	1.3200	1.111
100/100/100/20	$[44.8_8/-33.21_8]$	3.3401	2.3765

5.6. Summary

In Chapter 4 a database is generated which included all the results of the laminated composite plates for all possible different orientations for each ply. In order to achieve the best outcome (depending on the designers requirements), using an optimisation method is unavoidable because the fibre orientation in each layer of composite can be in a wide range of [-90 +90] degree and it means the number of combinations of different orientations for laminated plate are considerably large. In this chapter, a SA optimisation method is proposed and used to solve this challenging problem, which is normally non-linear due to anisotropy. The validity of the model is verified by experimental results, a GA and another SA model which were proposed recently. A new penalty function was proposed for the SA in order to overcome the difficulties and shortcomings faced by the previous SA model which was presented by Akbulut & Sonmez (2008). In their work, the effect of transverse shear (induced-twist) was ignored. By the proposed model, the effect of transverse shear and, as a result, out of plane loadings can be solved as well as the in-plane loading. Here, the Tsai-Wu and maximum stress failure criteria are chosen for laminated composite. By applying two failure criteria at the same time the results are more reliable.

CHAPTER 6:

Outcomes and Future Work

6. Outcomes and Future Work

In this Chapter an overview of the steps that are taken in this work are summarised. A discussion of the outcomes of this research at each step, the potential future work that can be undertaken in this area and how this study can help them are proposed.

6.1. Oucomes

As it is mentioned in Chapter 1, the initial idea of this research is formed from previous work which has been undertaken by Maheri et al. (2006, 2007) on controlling the composite structure (wind turbine blade) passively. An advantage of a passive control approach is that it is generally reducing the size of the controlling system, cost of maintenance and the initial cost. In passive control, designers try to employ the structure itself as the controller. On these materials (SMA, piezoelectric, composites) which are normally categorised as smart materials, the external loadings can be generated by heat, electricity, force, etc. For example for wind turbine blades which are made from composite material, wind velocity can be considered as an external load, and potentially this load can be used to control the twist of the blade passively. This increases the efficiency of the wind turbine performance as well as decreasing the cost of controlling the system. On the other hand, the anisotropy of composites produces shear coupling and twist in composite blades. Considering this shear coupling and transverse shear effect on the modelling of composite structures with a complex profile such as a wind turbine blade, is very complicated. In most cases it is possible to simplify and break down these complex structures into a combination of composite plates. Therefore, in this research laminated composite plates were considered and analysed.

Unlike conventional materials, by applying uniform distribution loads on a cantilever anisotropic composite plate, twist deformation is induced as well as bending. The amount of this induced twist can be controlled by changing the fibre orientations in composite plates. Changing fibre orientations changes the stiffness matrix in Hook's law and generally any deformation (including twist) can be calculated from Hook's law.

The aim of this study was to anaswer the following research question:

> *How can a laminated composite plate be designed and controlled by optimising the fibre orientations of the laminae?*

Figure (6.1) shows the flow diagram and a summary of how this research answers this question.

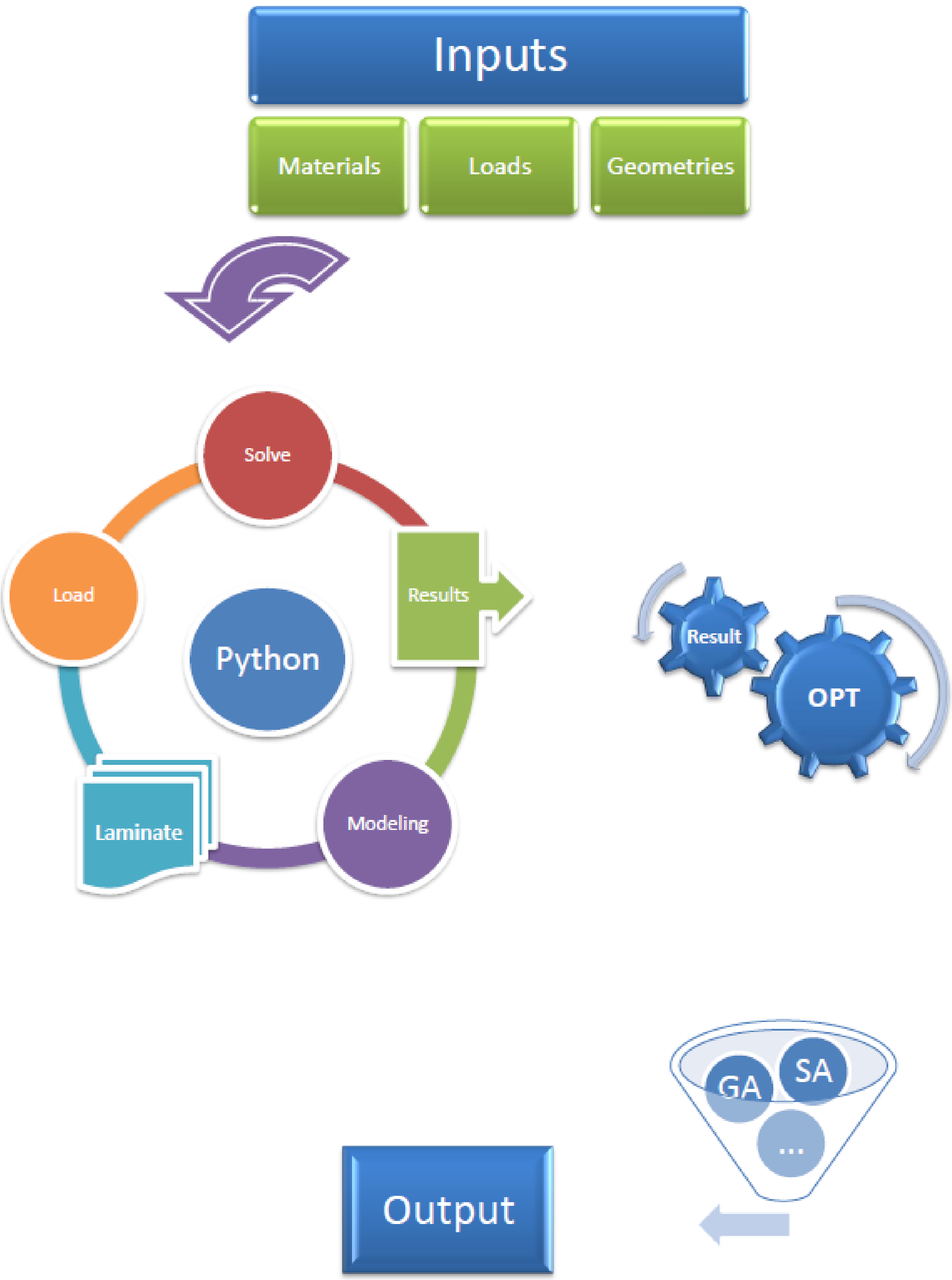

Figure 6.1 Flow diagram of this research

Three major milestones are considered to be accomplished in this project.

- Modelling of Laminated Composite
- FE Modelling, Programming and Experimental Results
- Optimisation

For the first step, a general study about modelling of composite materials was essential. It was also necessary to understand the behaviour of laminated composite plate. But more importantly as one of the critical point which was considered in this research was shear-coupling (twist) in laminated composite plate, the selected method should be able to model and explain transfer shear effect. Due to these reasons, a comprehensive study about analytical models has been performed and the outcome of this study was presented in chapter 2.

Transverse shear not only has an effect on deformation of the laminate composites, it is also important to consider this effect to predict the failure of the composite structures. It is explained that in laminate composite plate stress induced failures occur through different mechanisms. Normally before the in-plane stresses exceed the fibre breakage point, inter-laminar shear stress failure occurs when one layer slips tangentially relative to another. Alternatively, transverse normal stress may increase enough to cause deboning failure where two layers pull away from each other.

In modelling of composites, generally two different approaches have been used to study laminated composite structures analytically: single layer theories and discrete layer theories. In the single layer theory approach, layers in laminated composites are assumed to be one equivalent single layer whereas in the discrete theory approach each layer is considered individually in the analysis. The higher order models are based on an assumption of nonlinear stress variation through the thickness. These theories are capable of representing the section warping in the deformed configuration. The weakness of these models is that some of them do not satisfy the continuity conditions of transverse shear stresses at the layer interfaces. Although the discrete layer theories do not have this concern, they are significantly more labour intensive and computationally slow when solving such problems due to the fact that the order of their governing equations depends on the number of layers. This is the main advantage of single layer theories (and the reason that it is used in this research) compared to discrete layer theories.

It is discussed that because of some simplification assumptions in classical laminate plate theory (normal lines to the mid-plane before deformation remain straight and normal to the plane after deformation), it cannot be used in this research as it is not able to estimate the transverse shear and as a result any induced twist in the laminated composite plate. Other models try to overcome the limitations of CLPT in order to predict the transverse shear more accurately by using more terms in the Taylor series and as a result increasing the order of constitutive equations for the model (i.e first order shear deformation theory, third order shear deformation theory, etc). In some other models such as trigonometric shear deformation and hyperbolic shear deformation theories, different functions are used in order to avoid increasing the order of constitutive equations in composite models. Investigation of different models for laminated composite plates, helps this research and possibly future works in this area to understand how accurately each model can estimate the behaviour of laminated composite products or structures.

In order to cover all modelling of composite materials, an investigation of semi-analytical methods has also been performed as well. Because these models (analytical and semi-analytical models) are the first stage to explain the physical behaviour of composite structures. Furthermore, these methods generally present reliable and quick solutions for simple problems and give an initial idea at an early stage of the design process. They also provide a means of validating the numerical methods and potentially are the tools to enable the further development of new computational models. The main requirement of semi-analytical models is a mathematical technique to solve the complex and non-linear constitutive equations. The Kantorovich method is one of these techniques. It was used to solve the constitutive equations for laminated composite. It was extended later (EKM) and was recently modified by Shufrin et al. (2008) by adding an extra term and named that multi-term extended Kantrovich method. Although this new method is more accurate compared to previous ones, it is modified in this research by using the Newton-Kantorovich-Quadrature (NKQ) method. This method is for solving nonlinear integral equations. It is found out that by presenting the laminated composite constitutive equations in integral format, NKQ method can be used. So, potentially this semi-analytical model can be applied for out of plane loading as there is no simplification made in solving the general constitutive equations for laminated composites. On the other hand, the transverse shear effect is considered in this semi-analytical model. This proposed method is used to reduce the limitation on loading that current semi-analytical models are not able to analyse. The accuracy and convergence of this method has been investigated through a comparison with other semi-analytical solutions and with FEA in order to validate the model. The accuracy of the proposed model is relatively similar to current semi-analytical models. However, unlike most of the other semi-analytical models, the accuracy of this method is independent from the initial chosen functions. This initial function, even if it does not satisfy any of the boundary conditions, does not affect the accuracy of the final solution. Although the proposed model theoretically works, it is not practical to use this model for complex loadings as the computational process increasing exponentially and the model cannot converge. Therefore, due to numerous computations and the number of unknown variables, numerical methods are needed to solve the problem of laminated plates.

Using a numerical method (finite element) was the second step in this research to solve composite problems. In Chapter 4 the finite element method is used to solve the laminated composite plate problem. In order to simulate the composite plate, Abaqus software is used. It is an FEA software with the ability to solve a wide variety of problems and simulate them in order to avoid the expense of manufacturing and prototype making. It is also an open-source software which gives the option to customise the program. Abaqus is chosen as simulation software because the transverse shear effect is considered in Abaqus solver for laminated composites problems. Also, the composite layup editor in Abaqus allows designers to easily add a ply, choose the region to which it is applied, specify its material properties, and define its orientation. It is also able to read the definition of the plies in a layup from the data in a text file. This is convenient when the data are stored in a spreadsheet or were generated by a third-party tool such as an Excel or Notepad file. As the plate does not have a complex shape (simple panel) the automatic meshing which is generated by Abaqus was acceptable in this work. However, the appropriate manual meshing must be considered for more complicated structures. Also, in this work S4R5 and S9R5 elements are used, as these elements are suggested for stress-strain analysis of laminated composite plates in Abaqus. This means 5 degrees of freedom at each node is considered; (u, v, w and two rotations).

The laminated composite plate was under uniform distributed loads (pressure). In order to ensure the experimental results match the FEA, this load is considered to be equal to the weight of the plate which is manufactured. The plate was fixed (clamped) on one side and it is free on the other sides.

In order to customise the simulation for different fibre orientations for each layer a Python program is written. It was essential to write this program as it gave the opportunity to modify the model depending on the requirements. On the other hand, by using this program parameters such as the geometry of the composite plate, material properties, loads and steps of changing fibre angles, can easily be controlled. It also can run the program iteratively for all possible fibre orientations automatically and save all the results in a database. Any numerical or computational method is a means to analyse a practical engineering problem and that analysis is not an end in itself but rather an aid to help the designer. Considering this point the program is modified in order to become a tool for designers. This tool is capable of getting inputs such as material properties and geometries, and then it solves the problem for all possible fibre orientations and also, based on the designer's request, optimises the fibre angles for any type of deformation. The process of how this tool works can be seen in Figure (6.1). This tool is directly linked with Abaqus. The inputs are the material properties, geometry and loads for the plate; where the first two are entered in the tool and the loads and boundary conditions can be defined in the Abaqus program. The output of this tool is a database which contains the results of FEA for all possible fibre orientations, in separate files. Later, this database can be used for any purposes in the design of laminated composite plate. This design can potentially be used by optimising for loads, geometry, thickness or induced twist depending on requirements.

It is important to validate the results which were achieved from FEM. Therefore, experimental tests on laminated composite plates were inevitable. First, the type of resin and fibre were determined. Epoxy was used as a resin because of being environmentally friendly, better adhesive properties and having better performance compared to all other resin types in terms of mechanical properties and resistance. Also, carbon fibre was used because it has better tensile and compressive strength, flexural modulus and fatigue properties in comparison with other type of fibres.

In the manufacturing process Vacuum bagging is very common to produce laminated composite panels. Although this process has its limitation (i.e. the quality of products depend on operator skills), it is used in this work due to equipment availability, higher fibre content laminates, lower void contents, better fibre wet-out because of pressure and easier to employ in terms of health and safety. More than 300 samples were manufactured with this process. A series of experimental tests has been performed.

The average displacement of six identical laminated composite plates (similar in size, fibre orientations, numbers of layers and material properties) were compared with finite element results. The results show less than two percent error for all the samples. Therefore, it can be concluded that there is a good agreement between experimental test and numerical results. This validation shows that the model can be used in more complicated cases where it is hard or practically impossible to test the products.

The final step was to design the laminated composite plate based on fibre orientation. In order to achieve the best outcome an optimisation technique is required to determine the best possible fibre orientations for plies. In Chapter 5, simulated annealing (SA) optimisation methods have been introduced to solve this challenging problem. Stochastic non-linear optimisation methods are utilised for this problem as they can avoid the local optimums. One of the best algorithms in this category is the SA method which is used to solve the problem in this research. A new penalty function was proposed in order to overcome the difficulties and shortcomings faced by the previous research. In the previous models, the effect of transverse shear was neglected. In some applications the induced twist, which is the direct effect of transverse shear, are undesirable. Therefore, the effect of transverse shear is simply ignored and/or the design limited to in-plane stresses. The validity of the proposed model was verified by comparison the results with experimental test data and other GA and SA model which were proposed recently. Different case studies with various boundary conditions and loading were considered. It was shown that the effect of transverse shear is considered in the proposed model and, as a result, out of plane stresses can be solved as well as the in-plane stresses.

The fibre orientation angle in each layer of composite can be in a wide range of [-90 +90] degree and they directly effect the behaviour of the structure. Obviously the wide range of fibre orientation angles in each layer gives many options to design the structure with composite. The modification and improvement of the optimisation method used in this research, not only gives the skills to designers to consider the transfer shear effect, but it also gives them the opportunity to use them as a design parameter. On the other hand, designers can use this induced twist in composite materials as a useful parameter to design special products. This can be a useful displacement to passively control the behaviour of composite structures which is the future outcome of this work for applications such as wind turbine blades, prosthetic feet, etc.

6.2. Conclusion

It is mentioned that the idea of this work is to control the induced twist by optimising the fibre orientations of the laminae. A set of objectives were defined in Chapter 1 in order to complete the goal of this reach. The objectives were to answer the following questions:

- What are the current methods which can model the induced twist deformation in laminate composite plate?
- Is there any possibility to modify the constitutive equation for composite in order to predict the behaviour of laminate composite more accurately?
- How precise the applied model can predict the behaviour of laminated composite plate in comparison with experimental results? (On the other hand the validity of the method must be checked)
- How the fibre orientations in laminated composite plate can be optimised to achieve a certain induced twist displacement?

In the literature review it is shown that current composite models either ignore the induced twist displacement or consider it as an unwanted displacement. It is discussed, because of some simplification assumptions; some composite models such as classical laminate plate theory are not able to estimate the transverse shear and therefore cannot predict the behaviour of laminated composite structure accurately. Not considering a transverse shear effect leads to ignoring the induced twist displacement in composite structures. They are some other models which consider the transfer shear effect. They have been discussed and categorised in Chapter 2. However, the induced twist has not been used in any of the mentioned models to control the composite structures.

A semi analytical model was modified and a new model based on the Newton-Kantorovich-Quadrature method is proposed. The non-linear equations are solved by the NKQ method. The main benefit of this method, compared to current models, is that it can potentially be used for more complicated load and boundary condition cases such as out of plane loads. Although the model considers the transverse shear effect for out of plane loads which leads to estimation of induced twist displacement accurately, the processing time increases exponentially in this model.

The accuracy of the model which is used in this work is studied in Chapter 3. The displacement results of the model are compared to experimental tests. It is shown that there is a good agreement between the applied model and experimental results and there is less than 2% error in laminated composite displacements. Even by considering the out of plane loads the error for induced twist displacement remains under 2%. This validation verifies that this model can be used in optimisation process to achieve a desirable induced twist in laminated composite plate.

In the optimisation of fibre orientations, it is shown that by proposing a new penalty function, the model can be considered for out of plane loads as well as in plane loads. By defining extra terms for the penalty function, unlike current optimisation models, the transverse shear effect and as a result the induced twist displacement were evaluated accurately. On the other hand, this new optimisation model can control the induced twist displacement in laminated composite palte.

6.3. Future works

In this research, the fibre orientations are optimised by considering the effect of transverse shear for laminated composite plates. However, in order to be able to apply this method for industrial applications it needs to be extended to composite beams. Therefore, the next clear steps to continue this work is to expand it for composite thin-walled beams. Although plate and beams are very basic structures, most of complicated structures can be broken down to these two basic structures.

Fibre orientation angle is used as a parameter to design and control the laminated composite plates in this research. The aim of this project is to design the composite structure in order to have a definite deformation (desirable twist) under certain loads. As a result of this work, the plate can be controlled passively. On the other hand, by applying certain loads, certain induced twist is achieved. As it was mentioned, this has the potential to use and categorise anisotropic composite material as smart materials. It gives the opportunity to designers to invent smart structures and products which can be controlled passively. For example carbon fibre composites have been used in the blades of wind turbine generators that potentially can improve power output at a greatly reduced cost. Whenever a wind turbine blade twists, it is directly affecting the changing blade loading, angle of attack, and as a result turbine power. So, twist or pitch angle of the blade in a wind turbine is a key parameter in controlling wind turbine performance.

Passive control of wind turbine blades reduces the size of the controlling system and its initial and maintenance cost. Some mechanical passive controls already exist such as cyclic adjusting load mechanisms or centrifugally loaded mass. However, extra mechanical parts and gadgets are still involved which increase the weight and cost and leads to decreasing the efficiency and performance of wind turbines. Adaptive or smart blades retain the blade itself as the controller to sense the wind velocity (loads) and adjust angle of attack to increase the wind turbine performance.

There are also other aspects of this work which can be further development such as FEM code. Although the Python program is a helpful tool to design laminated composite plates, the running time is very high and it needs to be adjusted especially if it is to be used later for composite beam problems. In order for this design tool to become commercial it need to be linked with the optimisation part directly.

A list of possible future works can be summarised as:

- Analysing, simulating and modelling of thin wall composite beam and expanding this model for composite products and structures.

- Considering the consequences of other parameters such as temperature and humidity on the transverse shear effect.

- Expand the design tool to a commercial composite software design, by improving the interface, processing time, etc.

- Presenting a numerical method based on the proposed NKQ semi-analytical model for composites.

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APPENDIX

The Development of Laminated Composite Plate Theories – A Review

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Abstract:

This study investigates and reviews approaches to modelling laminated composite plates. It explores theories that have been proposed and developed and assesses their suitability and functionality. The particular focus in this study has been on normal stresses and the through-thickness distributions of transverse shear. These are important for composite plates as stress-induced failures can occur in three different ways. Therefore, it is essential, to understand and calculate transverse shear and normal stress through the thickness of the plate accurately. In this study, previous laminated composite plate theories are categorised and reviewed in a general sense, i.e. not problem specific, and the advantages and disadvantages of each model are discussed. This research mainly focuses on how accurate and efficient the models can predict the transverse shear. It starts with displacement-based theories from very basic models such as Classical Laminate Plate Theory (CLPT) to more complicated and Higher Order Shear Deformation Theory (HOSDT). Models are Furthermore, categorised by how the models consider the overall laminate. Here the theories are divided in to two parts; Single Layer Theory (SLT), where the whole plate is considered as one layer, and Layerwise Theory (LT), where each layer is treated separately. The models based on Zig-zag and Discrete Theories are then reviewed and finally the mixed (hybrid) plate theories are studied.

Key words: Laminate theory; Transverse shear; Composites; Layerwise theory

1. Introduction

The demand for high strength, high modulus and low density industrial materials has generated an increased number of applications for fibre laminated composite structures in many different fields such as in submarines, sport equipment, medical instruments, civil engineering, enabling technologies, primary and secondary marine and aerospace structures, astronavigation and many more industries [1]. Composite constructions are usually multilayer produced structures, mostly made of flat and curved panels, built up from several layers or laminae, which are bonded together [2].
In the last half century, the use of composite materials has grown rapidly. These materials are ideal for structural applications that require high strength and low weight. They have good fatigue characteristics and are resistant to corrosion. They provide some flexibility in design through the variation of the fibre orientation or stacking sequence of fibre and matrix materials [3,4]. Another advantage of fibre laminate composites is the capability to design the physical structure and mechanical properties prior to manufacture. The mechanical behaviour of laminates strongly depends on the orientation of fibres and thickness of lamina. Accordingly, the lamina should be designed to satisfy the specific requirements of each particular application in order to obtain the maximum advantages from the directional properties of materials. Accurate and efficient structural analysis, design sensitivity analysis and optimisation procedures for size and shape and the orientation of fibres within the material are also required. This provides a good opportunity to tailor the material properties to the specific application [5,6].

The normal stresses and through-thickness distributions of transverse shear for composite materials are important for many reasons. For example, in laminate composite plate stress-induced failures occur through three mechanisms. If the in-plane stress gets too large, fibre breakage or material yield occurs. However, normally before the in- plane stresses exceed the fibre breakage point, inter-laminar shear stress failure occurs when one layer slips tangentially relative to another. Alternatively, transverse normal stress may increase enough to cause deboning failure where two layers pull away from each other. Therefore, it is essential, to understand and calculate transverse shear and normal stress through the thickness of the plate accurately [7].

Generally, two different approaches have been used to study laminated composite structures: single layer theories and discrete layer theories. In the single layer theory approach, layers in laminated composites are assumed to be one equivalent single layer whereas in the discrete theory approach each layer is considered in the analysis. Also, plate deformation theories can be categorized into two types: stress based and displacement based theories. [8]

A brief review of displacement based theories is given below: Displacement based theories can be divided into two categories: Classical laminated plate theory and shear deformation plate theories [8]. Normally laminated composite plate theories are described in the classical laminated plate theory, the first-order shear deformation theory, the global higher-order theory and the global–local higher shear deformation theory. [9]

In previous studies, many different theories are represented to overcome various issues and explain the behaviour of composite materials more accurately for certain types of problems. In this paper these theories are reviewed, categorised and the advantages, weaknesses and limitations discussed.

2. Classical Laminated Plate Theory (CLPT)

The simplest equivalent single layer (ESL) laminated plate theory, based on the displacement field, is the classical laminated plate theory (CLPT) [10-16]. The two dimensional classical theory of plates was initiated by Kirchhoff [17] in the 19th century, and then was continued by Love [18] and Timoshenko [19] during the early 20th century. The principal assumption in CLPT is that normal lines to the mid-plane before deformation remain straight and normal to the plane after deformation. Although this assumption leads to simple constitutive equations, it is the main deficiency of the theory. The affect of the transverse shear strains on the deformation of the elastic two-dimensional structure are ignored and some of the deformation mode constraints by reducing the model to a single degree of freedom (DoF) results are neglected. This is a consequence of the basic assumptions made. It is also worth mentioning that neglecting shear stresses leads to a reduction or removal of the three natural boundary conditions that should be satisfied along the free edges. These boundary conditions being the normal force, bending moment and twisting couple [20].

Despite its limitations, CLPT is still a common approach used to get quick and simple predictions especially for the behaviour of thin plated structures. The main simplification is that three-dimensional thick structural plates or shells are treated as two-dimensional plate or shells located through mid thickness which results in a significant reduction in the total number of variables and equations, consequently saving a lot of computational time and effort. The governing equations are easier to solve and present in closed-form solutions, which normally provides more physical or practical interpretation. This approach remains popular as it is well-known and has become the foundation for further composite plate analysis methods.

This method works relatively well for structures that are made out of a symmetric and balanced laminate, experiencing pure bending or pure tension. The error induced/introduced by neglecting the effect of transverse shear stresses becomes trivial on or close to the edges and corners of thick-sectioned configurations. The induced error increases for thick plates made of composite layers, for which the ratio of longitudinal to transverse shear elastic modulii is relatively large compared to isotropic materials [21]. It neglects transverse shear strains, underpredicts deflections and overestimates natural frequencies and buckling loads.

In fact, thick composite plates are unsurprisingly focused/sensitive to transverse shear and normal stresses through thickness (3D stress) due to the discontinuous nature of the stack and through-thickness behaviour and its anisotropic nature. [22]. In order to achieve better predictions of the response characteristics, such as bending, buckling stresses, etc. a number of other theories have been developed which are presented in following sections [8, 23-26].

3. First-Order Shear Deformation Theories (FSDT)

Several theories have been proposed to analyse thicker laminated composite plates in order to consider the transfer shear effect. Most of these theories are extensions of the conventional theories developed by

Reissner [20] and Mindlin [27], which are known as the shear deformation plate theories. These theories are based on the assumption that the displacement w is constant through the thickness while the displacements u and v vary linearly through the thickness of each layer (constant cross-sectional rotations w_x and w_y). Generally these theories are known as First-Order Shear Deformation Theories (FSDT) [3]. According to this theory, transverse straight lines before deformation will still be straight after deformation but they are not normal to the mid-plane after deformation. This theory assumes constant transverse shear stress and it needs a shear correction factor in order to satisfy the plate boundary conditions on the lower and upper surface. The shear correction factor adjusts the transverse shear stiffness and thereby, the accuracy of results of the FSDT will be depend significantly on the shear correction factor [8,9]. Further research has since been undertaken to overcome some of the limitations of FSDT without involving higher-order theories in order to avoid increasing the complexity of the equations and computations [21,28,29].

Bhaskar and Varadan [30] used the combination of Navier's approach and a Laplace transform technique to solve the equations of equilibrium. From the results they observed that the dynamic magnification factor for the deflection and the in-plane stress is close to 2.0, and that the interlaminar stresses can reach higher values depending on the geometry of the plate and type of loading [31].

Onsy et al [3] presented a finite strip solution for laminated plates. They used the FSDT and assumed that the displacements u and v vary linearly through the thickness of each layer and are continuous at the interfaces between adjacent layers. They also assumed that the displacement w does not vary through the thickness. These assumptions allow for a more realistic situation where the shear strains across the interfaces between adjacent laminae are not continuous. Two-node and three-node finite strip elements are developed as a result of their work. [3]

In order to calculate transverse shear more accurately, to satisfy all boundary conditions and to analyse the behaviour of more complicated thick composite structures under different loads, the use of higher order shear deformation theories is inevitable.

4. Higher Order Shear Deformation Theories (HOSDT)

The limitations of the classical laminated plate theory and the first order shear deformation theory led researchers to develop a number of global Higher Order Shear Deformation Theories (HOSDT) [9].

The higher order models are based on an assumption of nonlinear stress variation through the thickness. These theories are capable of representing the section warping in the deformed configuration. However, some of these models do not satisfy the continuity conditions of transverse shear stresses at the layer interfaces. Although the discrete layer theories do not have this concern, they are significantly more labour intensive and computationally slow when solving such problems due to the fact that the order of their governing equations depends on the number of layers [32]. This is the main advantage of HOSDT compared to discrete layer theories which will be presented in Section 5.

Whitney attempted to examine the problem with interlaminar normal stress [33]. The calculation of interlaminar normal stress was studied by Pagano [34], a boundary layer theory, by Tang [35] and Tang and Levy [36], the perturbation method by Hsu and Herakovich [37], and approximate elasticity solutions by Pipes and Pagano [38] have been carried out. Later, Pagano, based on assumed in-plane stresses and the use of Reissner's variational principle, developed an approximate theory [39,40]. In most of these models the laminate is assumed to be reasonably long. The stress singularities were considered in a model presented by Wang and Choi [41-44]. They used the Lekhnitskii's [45] stress potential and the theory of anisotropic elasticity and determined the order of stress singularities at the laminate free-edges. The eigen function technique developed by them uses a collocation system at every ply interface in order to satisfy continuity. This limits the application of this method to relatively thin laminate [31].

Ambartsumian [46] proposed a higher order transverse shear stress function in order to explain plate deformation. Soldatos and Timarci [47], suggested a similar approach for dynamic analysis of laminated plates. Various different functions were proposed by Reddy [21], Touratier [48], Karama et al. [49] and Soldatos [50]. The results of some of these methods were compared with work by Aydogdu [23]. Swaminathan and Patil [51] used a higher order method for the free vibration analysis of antisymmetric angle-ply plates [8].

Some models just focus on special cases such as buckling, vibration, etc. For example, a 2D higher-order theory has been developed by Matsunaga [52,53] to investigate buckling in isotropic plates for in-plane loads. Effects of transverse shear and normal deformations have been predicted in his work. General higher order theories, which consider the complete effects of transverse shear, normal deformations and rotary inertia, have been studied for the vibration and stability problems of specific laminates [54,55].

Thermal buckling, modal vibration properties and optimisation have been focused in angle-ply laminated composite plates [6,56-58].

4.1. Third order shear deformation theory (TSDT)

Generally, researchers who have wanted to simulate plates have used the third-order shear-deformation theories first published by Schmidt [59] and then developed by Jemielita [60]. A Third order shear deformation theory (TSDT) is presented by Reddy [16,61]. Some references know this theory as parabolic shear deformation plate theory (PSDPT). Phan and Reddy [62] applied this theory for the free vibration, the bending and the buckling of composite plates [23]. The same unknown displacements as those of the/used in first-order shear deformation theories were used. The theory also satisfies transverse shear free conditions at the outer surfaces. The results for the thick laminates prove that the in-plane stresses are much better than those identified using FSDT, but still have errors when comparing with three-dimensional models. It is noteworthy that this theory is not the layer-wise type, therefore, unlike most of other equivalent single layer theories, it does not satisfy the continuity conditions of transverse shear stresses between layers [9].
Another parabolic distribution of shear strains through the laminated plate thickness was proposed by Dj. Vuksanovic [63]. It has a cubic variation of in-plane displacement. The results confirm that this model can explain/predict the global laminate response better than previous parabolic methods, but it is complex to accurately compute the interlaminar stress distributions [9].
In the third-order theories [21,64], the in-plane displacements are assumed to be a cubic expression of the thickness coordinate while the out-of-plane displacement is a quadratic expression at most. Based on the model that Vlasov presented for equations for bending of plates [65], Carrera presented a third order theory. The reduced third order model with three displacement variables was obtained by imposing homogeneous stress conditions with correspondence to the plate top-surface. This was further modified in the same research for the non-homogeneous stress conditions. Again, a closed form solution result was presented for both stresses and displacements in the case of harmonic loadings and simply supported boundary conditions [66,67]. Idlbi et al. [68] compared TSDT and parabolic shear deformation plate theory with CLPT for the bending of cross-ply plates [23].

4.2. Trigonometric Shear Deformation Plate Theory (TSDPT)

Trigonometric functions are used to describe the shear deformation plate theories called trigonometric shear deformation plate theory (TSDPT). Touratier [48] chose transverse strain distribution as a sine function [23].
Stein [69] developed a 2D theory wherein the displacements are stated by trigonometric series. Trigonometric terms are combined with usual algebraic through-the-thickness terms assumed for the displacements, to provide a better solution. Later the effects of transverse shear on cylindrical bending of laminated composite plates were studied by Stein and Jegley [70]. They show that this theory calculates the stresses more accurately than other theories. A straightforward method to analyse symmetric laminates under tension/compression using the principle of minimum complimentary energy and the force balance method, based on assumed stress distributions was introduced by Kassapogolou and Lagace [71,72]. Afterward, Kassapogolou [73] generalised and modified this approach for general unsymmetric laminate loads (in-plane and out-of-plane common moment and shear loads). This model was more general; however, it does not solve the weaknesses of inequality in Poisson's Ratios and the coefficients of mutual effect that exists between different plies through the thickness [31]. Touratier compared the TSDT proposed by Reddy [21] with the one suggested by Touratier [48] using a sine function to illustrate the warping throughout the thickness of the plate fibre during rotation, by considering the transverse shear [68]. Becker [74] made use of cosine and sine functions for warping deformation of v and w displacement respectively and developed a closed-form higher-order laminated plate theory.
An analytical method was presented by Mortan and Webber [75], using the same approach as Kassapoglou [71] and Becker [74], they considered the thermal effects in their model [31].
An accurate theory for interlaminar stress analysis should consider the transverse shear effect and continuity requirements for both displacements and Interlaminar stresses on the composite interface. In addition to accurate interlaminar stress and transverse shear prediction, it is advantageous if the model is variationally consistent in order to use it for finite element formulation. Considering these points, and to obtain the interlaminar shear stress directly from the constitutive equations, Lu and Liu [76] improved an interlaminar shear stress continuity theory. Although the interlaminar shear stress is obtained directly from the constitutive equations in their model, the deformation in the thickness is neglected and therefore it cannot calculate the interlaminar normal stress directly from the constitutive equations. Despite the conventional analysis for laminated composite materials, where the composite interface is always

assumed to be rigidly bonded due to the low shear modulus and poor bonding, the composite interface can be non-rigid. Later, Lu and Liu [77], in their search to improve their model by investigating the effect of interfacial bonding on the behaviour of composite laminates, developed the interlayer shear slip theory based on a multilayer approach. Finally, a closed-form solution for the general analysis of interlaminar stresses for thin and thick composite laminates under sinusoidal distributed loading was derived by Lee and Liu [78]. Both interlaminar shear stress and interlaminar normal stress at the composite interface were satisfied in this model and also the interlaminar stresses could be calculated directly from the constitutive equations. Touratier [48] proposed a theory based on using certain sinusoidal functions for shear stress. Numerical results for the bending of sandwich plates were presented and compared with the other theories. It was shown that this theory is more accurate than both FSDT and some other higher-order shear deformation theories [31].

4.3. Hyperbolic Shear Deformation Plate Theory (HSDPT)

Hyperbolic shear deformation plate theory (HSDPT) was proposed by Soldatos [50]. Timarci and Soldatos [47] united/combined these shear deformation theories. The basic advantage of this unified theory was the ability to change the transverse strain distribution [23]. Ramalingeswara and Ganesan [79] used parabolic and hyperbolic function to uniform external pressure and a simply supported cylindrical shell for cross ply laminated composite by considering an internal sinusoidal pressure [31]. It is also worth mentioning the Karama model where Karama et al. proposed an exponential function for the transverse strain in his study of the bending of composite [23,49].

4.4. Other HOSDTs

Rohwer [80] presented a comparative study of different higher-order shear deformation theories for multilayer composite plates. The advantages and limitations of the various models were shown with the analysis applied on a rectangular plate with a range of thickness, number of layers, edge ratios and material properties. A double Fourier series was used by Kabir [81,82]. It was tested for symmetric and antisymmetric laminate plates with simply supported boundary conditions at all edges and +45/-45 orientation angles for laminae. Later, by using the same approach and Kirchhofe's theory, a simply supported laminated plate with arbitrary laminations was presented by Kabir [83]. For validation the results were compared with FSDT based on finite element solutions. Based on Kassapogolou and Lagace's [71,72] model, and using boundary layer theory, Ko and Lin [84] proposed a model to analyse the 3D stress distribution around a circular hole in symmetric laminate for in-plane stresses. The laminate was subdivided into interior and boundary layer regions and each stress component was introduced by superposition of the interior and boundary layer stress. All the boundary conditions for each ply and the interface traction continuity were satisfied. Later Ko and Lin [85] extended the work for analysing complete state of stress around a circular hole in symmetric cross-ply laminates under bending and torsion [31].

Wang and Li [86] used 3D anisotropic elasticity and the method of separation of variables to derive the constitutive equations with unknown displacements for each cylindrical lamina of a multilayered shell. They considered mechanical and thermal load with various boundary conditions in their work. The model is able to determine the interlaminar stresses exactly [31]. Matsunaga proposed a global higher-order theory [87]. It was employed to analyse the inter-laminar stress problems of laminated composite and sandwich plates. The advantage of this method is that the total number of unknowns does not increase as the number of layers increases and from the constitutive relations the in-plane stresses can be predicted accurately. By integrating the three -dimensional equilibrium equation, transverse shear and normal stresses can be calculated, which satisfy the free surface boundary conditions and continuity conditions at interfaces [9].

A complex function presented by Soldatos [88] which only dealt with simple bending of a special kind of transversely inhomogeneous monoclinic plates (class of symmetric cross- and angle-ply laminates and functionally graded material). The basis of these methods was the governing equations of the generalised, shear deformable plate theory presented by Soldatos [89]. Later, Soldatos presented a theory which considered the effects of both transverse shear and transverse normal deformation, in a way that allows for multiple, a posteriori, choice of transverse strain distributions to be stated. This function considers the bending-stretching coupling for un-symmetric laminates and inhomogeneous material through-thickness [90,91].

5. Layerwise Theory (LT)

In recent years, the layerwise theories and individual layer theories have been presented to achieve more accurate results. Some of this research has been carried out by Wu and Chen, [92] and Cho et al. [93], Plagianakos and Saravanos [94], and Fares and Elmarghany [95]. These theories require many different unknowns for multilayered plates and are often computationally time-consuming and expensive to obtain accurate results [8,96,97].

A number of layerwise plate models, which can represent the zig-zag behavior of the in-plane displacement through the thickness, have been developed to predict both gross response and the stress distributions [9]. The basic idea is assuming certain displacement and/or stress models in each layer, followed by equilibrium and compatibility equations at the interface to reduce the number of the unknown variables [58]. For example, Di Sciuva and Icardi [97] proposed an eight-noded general quadrilateral plate element with 56 degrees of freedom for anisotropic multi-layered plates based on the third-order zig–zag plate model and demonstrated a good accuracy. Generally in Layerwise theories the number of unknowns increases significantly with the number of layers and consequently the computational weight becomes considerably heavier and higher. To overcome this problem different solutions are suggested. Cho and Parmerter [98] presented a model in which the number of unknowns is independent of the number of layers. They modified their previous model of a composite plate theory for general lamination configurations by superimposing a cubic varying displacement field on a zig–zag linearly varying displacement. This method satisfies transverse shear stress continuity at the layer interfaces and shear-free surface conditions.

There is a lack of accuracy in Equivalent Single Layer (ESL) for thick laminated composite plates, especially in the vicinity of the free edges, corners or other special features such as holes, where the interlaminar shears dominate by the stress field distortion and highly depend on the real stacking sequence. The so-called free edge effect is mainly explained by the mismatch of elastic properties, particularly the Poisson's ratio, at the interface of two consecutive layers. Therefore, some other more accurate layerwise field equations are needed to overcome such limitations. The theories that have been developed to justify through-the-thickness piece-wise behaviour of stresses and displacement are often subject to Zig-Zag theories (ZZ).

The general difference between a single layer plate theory and a multi-layer plate theory is that different tangential elastic compliances of the plies cause the displacement components to show a quick change of their slopes in the thickness direction at each layer interface, the so-called zig–zag effect. To summarise, the in-plane stresses ($\sigma_{11}, \sigma_{22}, \sigma_{12}$) can be discontinuous at each layer interface while the transverse stresses ($\sigma_{13}, \sigma_{23}, \sigma_{33}$), for equilibrium, must be continuous. In ZZ theory the compatibility of the displacements and the inter-laminar equilibrium of the transverse stresses in the thickness direction are assured by defining a new stiffness matrix called C0z [58].

Lekhnitskii in 1968 [99] was one of the pioneers who tried to define a ZZ theory. The main drawback for this approach was the limitation of the approach to multi-layered composite where each layer is isotropic. Ren [100] later improved this model by using an extension of the theory developed by Reissner [101] to multi-layered plates. This approach used a Lagrange function with five parameters, which represent the degrees of freedom of the structure. In fact, each degree of freedom is represented by a function for the whole domain. Normally, the DoFs are:

- two axial in-plane displacements along x and y co-ordinates (namely u_x and u_y)
- transverse displacement w,
- two transverse shear strains γ_{xz} and γ_{yz}.

In uncoupled problems the essential number of degrees of freedom is three. On the other hand when it is possible to separate the in-plane from the transverse response in order to solve the transverse equilibrium equations the shear strains are sometimes replaced by the two rotations of the cross-section about the y and x-axes, therefore the number of DoFs reduced to three [22].

Discrete-layer models which categorise Layerwise theories, have been developed by Noor and Burton [102] and Carrera [103], which can provide very accurate prediction of the displacement and stress. However, increasing the number of layers lead to increasing the number of unknowns. Due to the number of variables depending on the number of layers, they become impractical for engineering application [9].

Di Sciuva [104,105] and then Touratier [48,106] proposed simplified discrete layer models with only five essential variational unknowns to consider the warping in the deformed configuration. Like many early methods these models also could not satisfy the compatibility conditions (both at layer interfaces and at the boundaries). Beakou [107] and Idlbi [108] modified those models to overcome this limitation and then He [109] introduced the Heaviside step function which allows automatic satisfaction of the displacement continuity at interfaces between different layers. Later, based on discrete layer theory and Di Sciuva

[105], He [109] and Ossadzow et al. [110] a new model is presented for laminated composite structure by Karama et al. [32].
Based on Sciuva's theory, new zig-zag models have been proposed by Lee et al. [111], Sciuva [112] and Cho and Parmerter [113]. These models contain the same variables as those given by the FSDT and satisfy transverse shear stresses continuity conditions at interfaces. The number of variables does not depend on the layers of laminated plate. Numerical solutions verify that in-plane stresses compared with 3D solutions are very accurate. However, the transverse shear stresses from the direct constitutive equation method cannot be calculated accurately. In order to accurately predict interlaminar stress, the equilibrium equation approach has to be considered [9].
The discrete-layer theories [114-116] are capable of modeling the warpage of the cross-section during bending and of predicting in-plane responses. A refined shear deformation theory (RSDT) with a simplified discrete-layer model has been presented by He [117] to reduce the total number of dependent variables. The transverse displacement is constant across the thickness and the in-plane displacements assumed to be piecewise linear. The transverse shear strains across any two different layers next to each other are assumed to be linearly dependent on each other as real transverse shear stresses are continuous between layers. Based on those assumptions, governing equations have been derived using the principle of minimum potential energy. The difference between this method and FSDT is the set of governing differential equations which are of the 12^{th} order, which means two orders higher than FSDT. However, the variables are still the same. The limitation on the application of this theory is that the thickness of the shell must be small compared to the principal radii of curvature. Thus, the analytical solutions can be obtained for only a few cases. This model have been tested for various problems such as cylindrical bending of an infinitely long cross-ply laminated strip under sinusoidal loading, and bending of simply supported symmetric and antisymmetric cross-ply rectangular plates under sinusoidal transverse loads [118,119]. It was shown that RSDT is accurate to predict the stresses and deflections by comparing the numerical results of RSDT with FSDT and 3D elasticity theory [120]. Later, based on the same model, He and Zhang presented a simple closed-form solution for antisymmetric angle ply, simply-supported rectangular plates subjected to sinusoidal transverse loads and compared their model with CLPT and FSDT [120,121].
In order to satisfy the continuity of displacements at the interfaces between two adjacent layers Robbins and Reddy [122] introduced a layerwise laminate theory by using a one-dimensional Lagrangian interpolation function associated with a series of n nodes. Using the Lagrangian function for the variation of the displacements through the thickness causes calculations to become greatly heavy when the number of layers increasing [3, 123-125].
Wu and Kuo [126] proposed a local higher-order lamination theory. They used the Lagrange multiplier method and defined the Lagrange multipliers as the interlaminar stresses to evaluate the interlaminar stresses. They also used interlaminar stresses as the primary variables. It guaranteed the equilibrium equation and the displacement continuity constraints at the interface between consecutive layers. Then they used the Fourier series expansion method to analyse the problem [31].
A higher-order layerwise theoretical framework is presented by Plagianakos and Saravanos [94], which was able to predict the response of thick composite plates. The linear approximation by linear layerwise theories is considered. The displacement in each discrete layer through the thickness of the laminate includes quadratic and cubic polynomial distributions of the in-plane displacements. Interlaminar shear stiffness matrices of each discrete layer are presented by considering the interlaminar shear stress compatibility conditions in order to guarantee the continuity of interlaminar shear stresses through the thickness. The main advantage of this model when compared to linear layerwise theories is in the small number of discrete layers used to model the thick composite laminate through-thickness and in the prediction of interlaminar shear stresses at the interface.

6. Mixed Plate Theory

Recently, some researchers have attempted to combine previous models in order to overcome the limitations of each one. Unified equations have been proposed for mixed layerwise and mixed equivalent single layer theories. The main aim is to formulate these unified C0z theories in the most general way for users to be able to choose the approach (equivalent single layer, Layerwise zig–zag, etc); at the same time the order of the expansion of displacements and transverse stresses [58,127].
This class of model has been contemplated over the last few decades. The so-called mixed variational approach-based on the variational principles developed by Hellinger [128] was proposed and then improved by Reissner [101]. The number of variables that must be computed is at least 2N+1 where N is the total number of layers. The number of variables can be significantly reduced by using a weak form of Hooke's Law [129], which shows the variables in terms of the three displacements only. Shimpi et al.

[130] derived two novel formulations with only two variables, which work perfectly for moderately thick isotropic plates. However, it requires ad-hoc calculated shear correction factors for transverse shear stresses in multilayered composite plates [22].
Recently, Tessler et al. [131] have developed a refined Zig-zag theory based on the kinematics of FSDT. The deployment of novel piece-wise linear zig-zag functions provides an accurate and robust approach. The number of computations is relatively low compare to other layerwise theories. By reducing the number of functional degrees of freedom and choosing the appropriate degree of freedom, the function converge quickly and the computational efforts reduced [22].
Cen et al. presented a simple displacement-based, quadrilateral 20 DoF (5 DoF per node) bending element based on the FSDT for analysis of arbitrary laminated composite plates by considering the Timoshenko's beam theory for elements and interpolation for shear strain, rotation and in-plain displacement. By proposing a hybrid procedure the stress solutions especially transverse shear stresses are improved [132].
A Reissner mixed variational theorem (RMVT) based on TSDT was developed for the static analysis of simply-supported, multilayered plates under mechanical loads. Wu et al. based on the observations of these solutions, developed a model for the static analysis of simply supported, multi-layered composite and functionally graded material (FGM) plates. Reddy's third-order displacement model and a layer-wise parabolic function of transverse shear stresses are used as the kinematic and kinetic assumptions, respectively. A set of Euler–Lagrange equations associated with the possible boundary conditions was derived. The results obtained from the present TSDT based on RMVT are compared with those obtained from the published TSDT based on the principle of virtual displacement for single layer orthotropic, multilayered composite and multilayered FGM plates [133].
Another hybrid model, based on FSDT, was presented by Daghia et al. [134] for the analysis of laminated composite plates. This model considered a new quadrilateral four-node finite element from a hybrid stress formulation involving, as primary variables, compatible displacements and element wise equilibrated stress resultants. By the minimum number of parameters, the transverse stresses through the laminate thickness are reconstructed using three-dimensional equilibrium. Their strategy can be adopted with any plate finite element and does not need any correction factor.
Demasi [135-139] presented mixed plate theories based on the Generalized Unified Formulation (GUF) in five parts. In the first part, the governing equations for GUF are explained. GUF is a recent common method which is categorised as a displacement-based theory. GUF is extended for the first time for a mixed variational statement, Reissner's mixed variational theorem [101]. In this technique each of the displacement variables and out-of-plane stresses is independently considered. Also, different orders of expansions for the different unknowns can be chosen. The advantage of this model is that it can easily apply in a single FEM code. This method applied to the case of LT. As it was mentioned before for LT each layer is independently modelled and the compatibility of the displacements and the equilibrium of the transverse stresses between two contiguous layers are enforced a priori. Infinite combinations of the orders used for displacements and out-of-plane stresses can be freely chosen [136]. In addition, GUF is applied to mixed higher order shear deformation theories. The displacements have an equivalent single layer description, whereas the stresses are based on a layerwise category. The compatibility of the displacements and the equilibrium of the transverse stresses between two adjacent layers are considered. The displacement-based ''classical" HSDT results will be obtained by neglecting the out-of-plane stresses and using the static condensation technique in this model [137]. Furthermore, this model is applied for advanced mixed higher order zig-zag theories by adopting of Murakami's zig-zag function [138] and finally the numerical results of this model presented [139].
Bhar et al. [140] presented a method based on least square of error for accurate evaluation of through-the-thickness distribution of transverse stresses in thick composite and sandwich laminates, using a displacement-based C0 higher-order shear deformation theory (HSDT). This model is different from the conventional method of integrating the 3D equilibrium equations for transverse stress in composite laminates. The numerical results compared well with the results from FSDT and HSDT. They used this method to express the transverse shear stress accurately and furthermore expand it for post-processing techniques [140].
Based on the least-squares variational principle, which is an alternative approach to the mixed weak form FEM, Moleiro et al. [141] presented a mixed finite element model for the static analysis of laminated composite plates. This model is a FSDT with displacements and stress resultants as independent variables. By using equal-order C0 Lagrange interpolation functions of high levels along with full integration, the model is proposed and the results for different boundary conditions calculated and compared with previous work by Chou [142]. It is showed that the mixed least-squares model with high-order interpolation functions is insensitive to shear-locking. The model was developed for free vibration and different side-to-thickness ratios (from 10 to 500) [143]. Later, this model is developed in order to be able to choose the finite element approximating spaces independently [144,145].

It is worth noting that Finite Element Methods (FEM) can be used for all different methods which have been mentioned. However, due to numerous computations and number of unknown variables, layerwise theories mostly have been solved by FEM which are presented in previous works [4,16,21,31,97,146-157]. However, FEM is not the only solution for these models. Recently, different meshless methods are presented to solve equation for laminated composite plates [158]. The developments of element-free or meshless methods and their applications in the analysis of composite structures have been reviewed by Liew et al. [159] recently.

7. Conclusion and Future Work

In this review, composite laminate plate theories have been generally categorised. The advantages and limitations of each model have been discussed. This research mainly focused on how accurately and efficiently the models can predict the transverse shear effects. It is explained that CLPT and FSDT are unable to accurately compute transverse shear stresses of both moderately thick and thick laminated plates. To obtain reasonable transverse shear stresses, the global displacement theories with very higher order shear deformation should be adopted. Also, the zig-zag theories satisfying interlaminar continuity of transverse shear stresses at interfaces is unable to accurately compute transverse shear stresses directly from constitutive equations. To accurately obtain transverse shear stresses, 3D equilibrium equations have to be adopted which require heavy computational processing because of the increasing number of variables. Due to the number of variables depending on the number of layers, they become impractical for engineering applications.

Some recent researchers have tried to use the transverse shear and warping effect in highly anisotropic composite to passively control the composite structure. Smart passive adaptive structures are a new approach for developing smart and predictable composite materials with wide ranging applications. They can be used to exploit the effects of shear and elastic coupling and link stretching to bending to twisting of the structure. This requires a much better and more deterministic knowledge of interlaminar shear. Such passively smart structures can have a wide range of applications such as in the new generation of bend-twist adaptive wind turbine blades, smart helicopter blades of passive adaptive fan and compressor blades. Therefore, predicting transverse shear effect accurately and in practical way for engineering applications is essential. Novel approaches are being proposed and tested by the authors and the results will be the subject of the future publications.

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A Semi-Analytical Model for Buckling of Laminated plates with the NKQ Method

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Abstract. A semi-analytical approach for analysis of laminated plates with general boundary conditions under a general distribution of loads is developed. The non-linear equations are solved by the Newton-Kantorovich-Quadrature (NKQ) method which is a combination of well-known Newton-Kantorovich method and the Quadrature method. This method attempts to solve a sequence of linear integral equations. In this paper this method is used to propose a semi-analytical model for buckling of laminated plates. The convergence of the proposed method is investigated and the validation of the method is explored through numerical examples and the results compared with finite element method (FEM). There is a good agreement between the NKQ model and FEM results.

1. Introduction

In the last half century, the use of composite materials has grown rapidly. These materials are ideal for structural applications that require high strength and low weight. They have good fatigue characteristics and are resistant to corrosion [1]. Understanding the mechanical behaviour of composite plates is essential for efficient and reliable design and for the safe use of structural elements. The complex behaviour of laminated plate structures normally need a non-linear model to describe them. Moreover, anisotropic and coupled material behaviour add more non-linearity to analysis. The non-linear analysis of buckling of laminated plates has been the subject of many research projects. Also, various semi-analytical and numerical methods for the description and response of laminated plates have been developed. The common analytical non-linear theories for laminated composites such as classical laminated plate theory [2,3] and first shear deformation plate theory [3,4] generally use the Rayleigh-Ritz method [5] or the Galerkin method [6]. The accuracy of these analytical models depends on the trial functions. The semi-analytical non-linear methods are an essential tool that provides perception to the physical non-linear behaviour of the composite plate structure, present fast and reliable solutions during the preliminary design phase and also provide a means of validations the numerical methods and enable the development of new computational models.

The buckling behaviour of elastic plates is a geometric non-linear problem. The non-linear large deflection behaviour of a plate can be analysed by solving the Von-Kármán non-linear equations [1,7] together with the appropriate boundary conditions. Unfortunately, the Von-Kármán equations are coupled and fourth-order and thus no rigorous solutions are available. This has prepared the ground for the development of the approximate methods and semi-analytical models. Ovesy and Assaee presented a buckling model by considering The effects of bend–twist coupling of composite laminated plates and using semi-energy finite strip approach [7]. Shufrin et al. proposed a semi analytical approach for Buckling of symmetrically laminated rectangular plates.

The solution of the partial differential buckling equations has been reduced to a solution of a set of ordinary differential equations using the multi-term extended Kantorovich method [8].
Bisagni and Vescovini focused on analytical formulation for local buckling and post-buckling analysis of stiffened laminated panels [9]. Lopatin and Morozov worked on Buckling of the rectangular orthotropic plate subjected to linearly varying in-plane loading with certain boundary condition (SSCF) [10]. Later The effect of anisotropy on post-buckling behaviour of laminated plates is studied by Assaee et al. They used semi-energy finite strip method on their work. The NKQ method is used by authors to present a semi-analytical model for solving the constitutive equation of laminate plate with general boundary conditions, in this paper the NKQ method is used to propose a semi-analytical model of buckling of laminated plates.

2. Governing Equations

2.1. General composite equations

The state of stress at a point in a general continuum can be represented by nine stress components σ_{ij} $(i,j = 1,2,3)$ acting on the sides of an elemental cube with sides parallel to the axes of a reference coordinate system. In the most general case the stress and strain components are related by the generalised Hook's law as follows [1]:

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} \quad (i,j,k,l = 1,2,3) \tag{1}$$

where C_{ijkl} is the stiffness components [11].
In the classical laminate theory, it is assumed that straight lines normal to the middle surface remain straight and normal to that surface after deformation. These assumptions are not valid in the case of thicker laminates and laminates with low stiffness central plies undergoing significant transverse shear deformations. In the following, referred to as first-order shear deformation laminate plate theory, the assumption of normality of straight lines is removed [1,3].

$$\begin{bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} dz \quad \begin{bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} z dz \quad \begin{bmatrix} V_q \\ V_r \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \tau_{xz} \\ \tau_{yz} \end{bmatrix} dz \Rightarrow \begin{bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \\ V_q \\ V_r \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} & 0 & 0 \\ A_{12} & A_{22} & A_{23} & B_{12} & B_{22} & B_{23} & 0 & 0 \\ A_{13} & A_{23} & A_{33} & B_{13} & B_{23} & B_{33} & 0 & 0 \\ B_{11} & B_{12} & B_{13} & D_{11} & D_{12} & D_{13} & 0 & 0 \\ B_{12} & B_{22} & B_{23} & D_{12} & D_{22} & D_{23} & 0 & 0 \\ B_{13} & B_{23} & B_{33} & D_{13} & D_{23} & D_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & E_{11} & E_{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & E_{12} & E_{22} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \\ k_1 \\ k_2 \\ k_{12} \\ \gamma_{13} \\ \gamma_{23} \end{bmatrix} \tag{2}$$

where the components of this section stiffness matrix are given by:

$$\begin{aligned} (A_{ij}, B_{ij}, D_{ij}) &= \int_{-h/2}^{h/2} \overline{Q}_{ij}^{m} (1, z, z^2) dz \quad (i,j = 1,2,3) \\ E_{ij} &= \int_{-h/2}^{h/2} \overline{Q}_{\alpha\beta}^{m} k_i k_j dz \quad (i,j = 1,2 \ and \ \alpha,\beta = i+4, j+4). \end{aligned} \tag{3}$$

$\overline{Q}_{ij}$ are the off-axis stiffness components, which can be explained in terms of principal stiffness components, Q_{ij}, which are defined in reference [1,11]

2.2. Basic NKQ Equations

The nonlinear integral equation in the Urysohn form is defined as [12]:

$$y(x) = f(x) + \int_{\Omega} K(x,t,y(t))dt \qquad a \le x \le b \tag{4}$$

If $\Omega = (a, x)$, it is named a nonlinear Volterra integral equation and if $\Omega = (a, b)$, it is named the nonlinear Fredholm integral equation. To approximate the right-hand integral in equation (4), the usual quadrature methods similar to the ones used to approximate the linear integral equations that lead to the following nonlinear systems. For further information on quadrature methods in this respect, see [12].

$$y(x_i) = f(x_i) + \sum_{j=0}^{n} w_j K(x_i, x_j, y(x_j)), \qquad i = 0,1,2,,...,n \tag{5}$$

$$\begin{cases} y(x_0) = f(x_0) \\ y(x_i) = f(x_i) + \sum_{j=0}^{i} w_{ij} K(x_i, x_j, y(x_j)), \qquad i = 0,1,2,,...,n \end{cases} \tag{6}$$

where $w_{ij}s$ and $w_j s$ are weights of the integration formula. For further information on the Newton-Kantorovich method, see [12].

3. Application of NKQ

As it is mentioned the general form of the nonlinear Volterra integral equations of the Urysohn form is:

$$y(x) = f(x) + \int_a^x K(x,t,y(t))dt \qquad a \le x \le b \tag{7}$$

By considering the equation (5,6) and by integrating $\phi_{k-1}(x)$ with $y_k(x) - y_{k-1}(x)$:

$$\begin{cases} y_k(x_0) = f(x_0) \\ y(x_i) = f(x_i) + \sum_{j=0}^{i} w_{ij} K(x_i, x_j, y_{k-1}(x_j)) + \\ \qquad \sum_{j=0}^{i} w_{ij} K'_y(x_i, x_j, y_{k-1}(x_j)) \left[y_k(x_j) - y_{k-1}(x_j) \right] \qquad i = 1,2,\ldots,n \end{cases} \tag{8}$$

Consider:

$$\left(F^{(k-1)}\right)_{i+1} = \begin{cases} f(x_0) & i = 0 \\ f(x_i) + \sum_{j=0}^{i} w_{ij} K(x_i, x_j, y_{k-1}(x_j)) - \sum_{j=0}^{i} w_{ij} K'_y(x_i, x_j, y_{k-1}(x_j)) y_{k-1}(x_j) & i = 1,2,...,n \end{cases}$$

(9)

$$\left(A^{(k-1)}\right)_{i+1 \quad j+1} = \begin{cases} w_{ij} K'_y(x_i, x_j, y_{k-1}(x_j)) & i = 1,2,...,n \quad j = 0,1,...,i \\ 0 & otherwise \end{cases} \tag{10}$$

$$\left(Y^{(k)}\right)_{i+1} = y_k(x_i) \qquad i = 0,1,2,...,n \tag{11}$$

This equation can be solved by considering an initial solution $y_0(x)$ and constructing the $Y^{(0)}, A^{(0)}, F^{(0)}$ and also using the following repetition sequence (for further details see [12]):

$$\left(I - A^{(k-1)}\right) Y^{(k)} = F^{(k-1)} \qquad k = 1,2,...,n \tag{12}$$

On the other hand, by considering an initial solution $y_0(x)$, $(Y^{(0)})_i$ would be $y_0(x_i)$ and by using equation (9) and (10) $F^{(0)}, A^{(0)}$ are obtained respectively. Then by solving the system $\left(I - A^{(0)}\right)Y^{(1)} = F^{(0)}$, $Y^{(1)}$ is obtained. By repeating this procedure and next using equation (12), the values of $Y^{(1)}, Y^{(2)}, Y^{(3)}, ...Y^{(m)}$ are calculated for $m \in N$. m is a constant value which can be increased for higher n. Depending on n, an approximate solution for equation (10) is presented.

To evaluate buckling load capacity of a square laminated plate the minimization of total potential energy approach is considered. For more details of parameters see [7]. The total potential energy of plate is presented as following equation:

$$V_s = \frac{1}{2}\int\int \left\{[\varepsilon_L]^T [A][\varepsilon_L] + 2[\varepsilon_{nL}]^T [A][\varepsilon_L] + [\phi]^T [D][\phi] + [\varepsilon_{nL}]^T [A][\varepsilon_{nL}]\right\} dxdy \qquad (13)$$

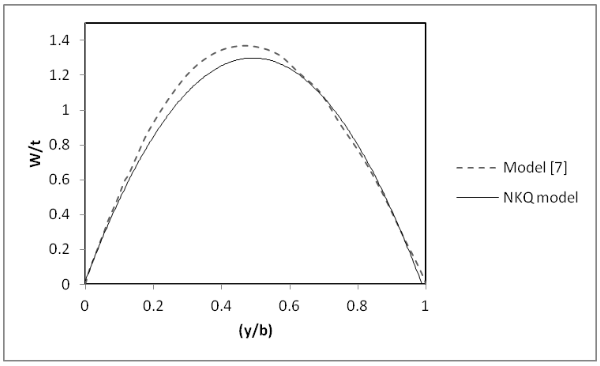
NKQ method run a series of buckling displacement functions to converge till it converge and error become less than a certain desirable amount.

4. Verification Study

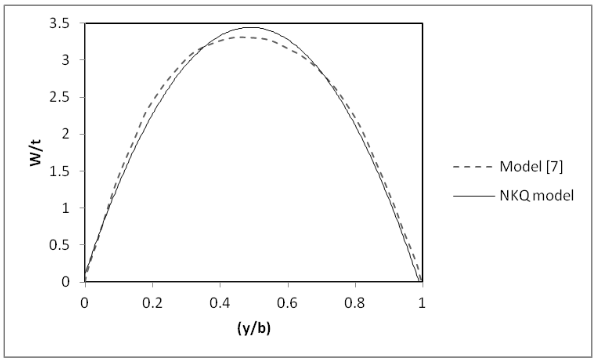
In order to verify the NKQ method numerical examples are solved and compared with previous research. In the first example, a square plate with all edges simply supported out-of-plane, and with the unloaded edges free to move in plane is considered. The square plate is assumed to be of 200mm length and 2mm thickness. Moreover, the plate is from high strength carbon-epoxy with $[(+45)_2/(-45)_2/(+45)_2/(-45)_2]_s$ lay-ups and the material properties of a single ply as follows:

$E_1 = 140Gpa, \quad E_2 = 10Gpa, \quad v_{12} = 0.3 \quad t = 0.125mm \quad G_{12} = 5Gpa$

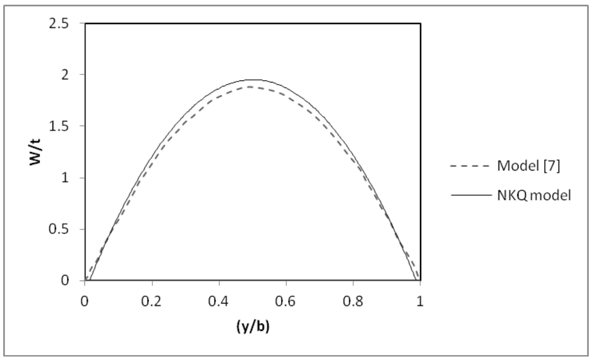
The results is shown in Figure 1a to 1d after five iterations and compared with the results presented by Ovesy and Assaee [7].



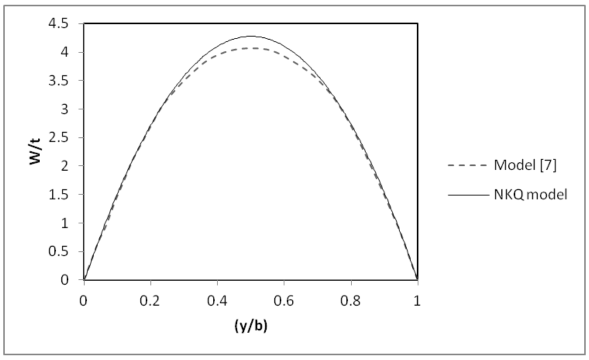
(a)



(b)



(c)



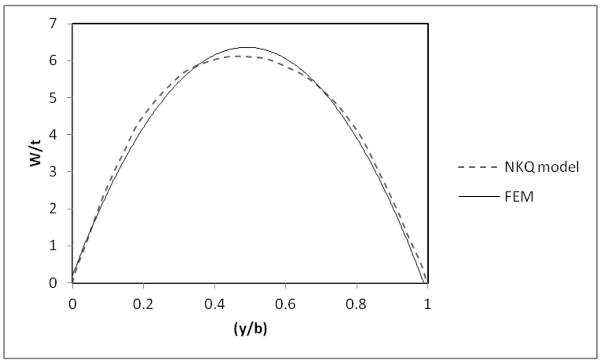
(d)

Fig. 1. Variation of out-of plane deflection across the plate at a) x=L/4, P=1.212P_{co}, b) x=L/4, P=2.027P_{co}, c) x=L/2, P=1.212P_{co}, d) x=L/2, P=2.027P_{co},

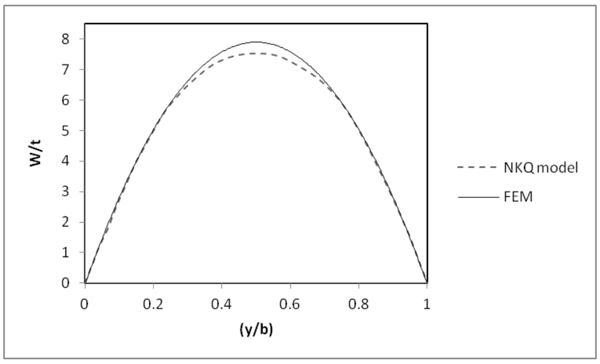
In the next example a square laminated plate with a length of 50cm under uniform loading is considered. The material properties are:

$$E_1 = 43.5Gpa, \quad E_2 = E_3 = 11.5Gpa, \quad \nu_{12} = \nu_{13} = .27, \quad \nu_{23} = .4, \quad G_{12} = G_{13} = 3.45Gpa \quad G_{23} = 4.12Gpa$$

As it is shown in Figures 2a and 2b there is a good agreement between the model and FEM results.



(a)



(b)

Fig. 2. Variation of out-of plane deflection across the plate at a) x=L/4, P=3.9112P_{co}, b) x=L/2, P=3.9112P_{co}

5. Conclusion

It is very important to include the effects of bend–twist coupling terms in the appropriate analysis of buckling of composite plates. Conventional analytical models are not able to consider this effect. In this paper a semi analytical model presented to consider the transverse shear for buckling of composite plates. Based on the concept of semi-energy method, for the postbuckling analysis of geometrically perfect thin-walled composite laminated plates under uniform end shortening is presented in this paper. The non-linear equations are solved by Newton-Kantorovich-Quadrature (NKQ) method. This method breaks down the laminated composite plats equations to series of sequential equations and attempts to solve iterative linear integral equations. The convergence of the proposed method is compared with other method. Furthermore, there is a good agreement between the model and FEM results.

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Optimum Design of Fibre Orientation in Composite Laminate Plates for Out-plane Stresses

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ABSTRACT

Previous studies have shown that composite fibre orientation angles can be optimised for specific load cases such as longitudinal or in-plane loading. However, the methodologies utilised in these studies cannot be used for general analysis of such problems. In this research an extra term is added to the optimisation penalty function in order to consider the transverse shear effect. This modified penalty function leads to a new methodology whereby the thickness of laminated composite plate is minimised by optimising the fibre orientation angles for different load cases. Therefore, the effect of transverse shear forces is considered in this study.

Simulated annealing (SA), which is a stochastic optimisation method, is used to search for the optimal design. This optimisation algorithm has been shown to be reliable as it is not based on the starting point and it can escape from the local optimum points. In accordance with the annealing process where temperature decreases gradually, this algorithm converges to the global minimum.

In this research, the Tsai-Wu failure and maximum stress criteria for composite laminate are chosen. Tsai-Wu failure is operationally simple and readily amenable to computational procedures. In addition, this criterion shows the difference between tensile and compressive strengths clearly, through its linear terms. By applying two failure criteria at the same time the results are more reliable.

Experimentally generated results show a very good agreement with the numerical results, validating the simulated model used. Finally, to validate the methodology the numerical results are compared to the results of previous research with specific loading.

Key Word: Composite Laminate Plate, Optimisation, Transverse Shear, Out-plane Stresses

1. INTRODUCTION

The demand for high strength, high modulus and low density industrial materials has generated an increased number of applications for fibre laminated composite structures in many different fields such as in submarines, sport equipment, medical instruments, civil engineering, enabling technologies, primary and secondary marine and aerospace structures, astronavigation and many more industries [1]. Composite constructions are usually multilayer produced structures, mostly made of flat and curved panels, built up from several layers or laminae, which are bonded together [2].

In the last half century, the use of composite materials has grown rapidly. These materials are ideal for structural applications that require high strength and low weight. They have good fatigue characteristics and are resistant to corrosion. They provide some flexibility in design through the variation of the fibre orientation or stacking sequence of fibre and matrix materials [3,4]. Another advantage of fibre laminate composites is the capability to design the physical structure and mechanical properties prior to manufacture. The mechanical behaviour of laminates strongly depends on the orientation of fibres and thickness of lamina. Accordingly, the lamina should be designed to satisfy the specific requirements of each particular application in order to obtain the maximum advantages from the directional properties of materials. Accurate and efficient structural analysis, design sensitivity analysis and optimisation procedures for size and shape and the orientation of fibres within the material are also required. This provides a good opportunity to tailor the material properties to the specific application [5,6]. However, it increases the complexity of the design problem. This complexity exists, not only because of numerous design variables, but also because of having a multimodal and variable-dimensional optimisation problem with unattainable or costly derivatives [7].

Optimum strength designs of continuous fibre-reinforced composite laminates have been used since the early days of these materials. The first research to investigate the fibre orientation of a unidirectional lamina yielding maximum strength under in-plane stress conditions has been carried out by Sandhu and Brandmaier [8]. Brandmaier found that the strength of a unidirectional lamina under in-plane stresses could be maximised analytically with respect to the fibre orientation [9]. The results based upon Tsai-Hill failure criterion indicated that the optimum fibre orientation depended upon the stress state and the relative value of the transverse and in-plane shear strengths of the lamina. When the strength of a multidirectional composite laminate is to be maximised, more complicated and explicit optimisation techniques are needed [10]. Chao et al. were the pioneers that sought the optimum strength design of multidirectional laminates using a search technique [11]. Latterly, many studies have been devoted to the optimum strength design of multidirectional laminate. Among these are the works by Park [12], Fukunaga and Chou [13], Miravete [14], Fukunaga and Vanderplaats [15], Haftka, Gurdal et al. [16,17], Spallino et al. [18], Weaver [19], Chattopadhyay [20], Luersen [21] and Ghiasi et al. [22].

Previous studies have shown that composite fibre orientation angles can be optimised by different optmisation methods for specific load cases such as longitudinal or in-plane loading [23-35].

In this Paper the thickness of laminated composite plates is minimised by optimising the fibre orientation angles for different load cases. The novelty of the research presented in this paper is that the effect of transverse shear forces and therefore the induced twist angle are considered.

2. Governing Equation

2.1. ANALYSIS OF A LAMINATE COMPOSITE PLATE

The state of stress at a point in a general continuum can be represented by nine stress components σ_{ij} $(i, j = 1,2,3)$ acting on the sides of an elemental cube with sides parallel to the axes of a reference coordinate system (Fig. 1).

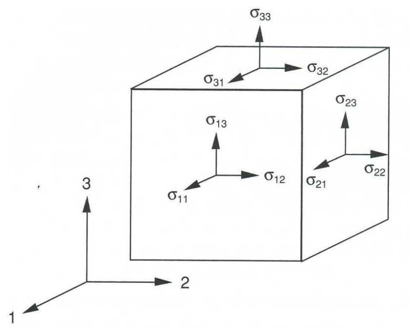

Figure 1. State of stress at a point of a continuum [36]

In the most general case the stress and strain components are related by the generalised Hook's law as follows:

$$\begin{bmatrix}\sigma_{11}\\\sigma_{22}\\\sigma_{33}\\\sigma_{23}\\\sigma_{31}\\\sigma_{12}\\\sigma_{32}\\\sigma_{13}\\\sigma_{21}\end{bmatrix} = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1123} & C_{1131} & C_{1112} & C_{1132} & C_{1113} & C_{1121} \\ C_{2211} & C_{2222} & C_{2233} & C_{2223} & C_{2231} & C_{2212} & C_{2232} & C_{2213} & C_{2221} \\ C_{3311} & C_{3322} & C_{3333} & C_{3323} & C_{3331} & C_{3312} & C_{3332} & C_{3313} & C_{3321} \\ C_{2311} & C_{2322} & C_{2333} & C_{2323} & C_{2331} & C_{2312} & C_{2332} & C_{2313} & C_{2321} \\ C_{3111} & C_{3122} & C_{3133} & C_{3123} & C_{3131} & C_{3112} & C_{3132} & C_{3113} & C_{3121} \\ C_{1211} & C_{1222} & C_{1233} & C_{1223} & C_{1231} & C_{1212} & C_{1232} & C_{1213} & C_{1221} \\ C_{3211} & C_{3222} & C_{3233} & C_{3223} & C_{3231} & C_{3212} & C_{3232} & C_{3213} & C_{3221} \\ C_{1311} & C_{1322} & C_{1333} & C_{1323} & C_{1331} & C_{1312} & C_{1332} & C_{1313} & C_{1321} \\ C_{2111} & C_{2122} & C_{2133} & C_{2123} & C_{2131} & C_{2112} & C_{2132} & C_{2113} & C_{2121} \end{bmatrix} \begin{bmatrix}\epsilon_{11}\\\epsilon_{22}\\\epsilon_{33}\\\epsilon_{23}\\\epsilon_{31}\\\epsilon_{12}\\\epsilon_{32}\\\epsilon_{13}\\\epsilon_{21}\end{bmatrix} \quad (1)$$

or,

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} \quad (i, j, k, l = 1,2,3) \qquad (2)$$

where C_{ijkl} is the stiffness components [30].

By considering the symmetry of the stress and strain tensors and the energy relations, it is proven that the stiffness matrices are symmetric. Thus the state of stress (strain) at a point can be described by six components of stress (strain), and the stress-strain equations are expressed in terms of 21 independent stiffness constants [36].

2.2. IN-PLANE STRESS

The simplest equivalent single layer (ESL) laminated plate theory, based on the displacement field, is the classical laminated plate theory (CLPT) [37-43]. The two dimensional classical theory of plates was initiated by Kirchhoff [44] in the 19th century, and then was continued by Love [45] and Timoshenko [46] during the early 20th century. The principal assumption in CLPT is that normal lines to the mid-plane before deformation remain straight and normal to the plane after deformation. Although this assumption leads to simple constitutive equations, it is the main deficiency of the theory. The affect of the transverse shear strains on the deformation of the elastic two-dimensional structure are ignored and some of the deformation mode constraints by reducing the model to a single degree of freedom (DoF) results are neglected. This is a consequence of the basic assumptions made. It is also worth mentioning that neglecting shear stresses leads to a reduction or removal of the three natural boundary conditions that should be satisfied along the free edges. These boundary conditions being the normal force, bending moment and twisting couple [47].

For solving in-plane stress normally the classical laminate theory is used. It is assumed that plane stress components are taken as zero. With respect to the coordinate system shown in (Fig. 2) the in-plane stress components are related to the strain components as:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{bmatrix} \qquad (3)$$

where k is the lamina number, $\overline{Q}_{ij}$ are the off-axis stiffness components, which can be explained in terms of principal stiffness components, Q_{ij}, using the tensor transformation rules [48] as

$$\overline{Q}_{11} = Q_{11}cos^4\theta + 2(Q_{12} + 2Q_{66})sin^2\theta cos^2\theta + Q_{22}sin^4\theta$$
$$\overline{Q}_{22} = Q_{11}sin^4\theta + 2(Q_{12} + 2Q_{66})sin^2\theta cos^2\theta + Q_{22}cos^4\theta$$
$$\overline{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})sin^2\theta cos^2\theta + Q_{12}(sin^4\theta + cos^4\theta)$$
$$\overline{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})sin\theta cos^3\theta + (Q_{12} - Q_{22} + 2Q_{66})sin^3\theta cos\theta) \quad (4)$$
$$\overline{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})sin^3\theta cos\theta + (Q_{12} - Q_{22} + 2Q_{66})sin\theta cos^3\theta)$$
$$\overline{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})sin^2\theta cos^2\theta + Q_{66}(sin^4\theta + cos^4\theta)$$

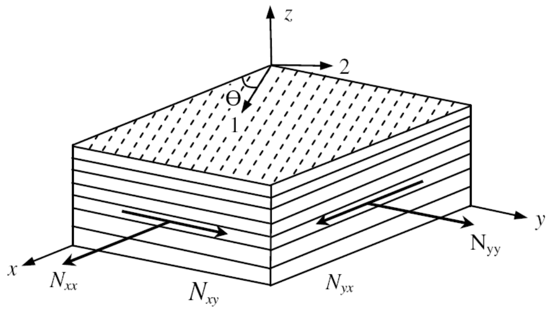

Figure 2. A scheme of composite plate under in-plane stress [35]

The principal stiffness terms, Q_{ij}, are related to elastic properties of the material along the principal directions, E_1, E_2, G_{12}, ϑ_{12}, and ϑ_{21} [48]. The effect of transverse shear stress is not considered by previous work [35] because in their work the laminate is only subject to in-plane loads. Therefore, strain components defined with respect to x–y–z coordinate system are the same for each ply regardless of the fibre orientation. For the same reason, the mechanical response of the laminate is independent of the stacking sequence. Stress resultants, or forces per unit length of the cross section, are obtained as:

$$\begin{bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} \mathrm{dz} = 2\sum_{k=1}^{m} n_k t_0 \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix}_k \quad (5)$$

Here m is the number of distinct laminae, n_k is the number of plies in the kth lamina. Here, lamina is meant to be a group of plies with the same orientation angle. Substituting the stress–strain relation given by Eq. (3) into Eq. (5):

$$\begin{bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{bmatrix} \quad (6)$$

where A_{ij}, components of extensional stiffness matrix, are given by

$$A_{ij} = 2\sum_{k=1}^{m} n_k t_0 (\overline{Q}_{ij})_k \quad (7)$$

Principal stress components can be obtained using the following transformation [35]:

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \tau_{12}\end{bmatrix}=\begin{bmatrix}cos^2\theta_k & sin^2\theta_k & 2cos\theta_k sin\theta_k\\ sin^2\theta_k & cos^2\theta_k & -2cos\theta_k sin\theta_k\\ -cos\theta_k sin\theta_k & cos\theta_k sin\theta_k & cos^2\theta_k - sin^2\theta_k\end{bmatrix}_k\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix} \quad (8)$$

Despite its limitations, CLPT is still a common approach utilised to determine quick and simple predictions especially for the behaviour of thin plated structures. The main simplification is that three-dimensional thick structural plates or shells are treated as two-dimensional plate or shells located through mid thickness which results in a significant reduction in the total number of variables and equations, consequently saving a lot of computational time and effort. The governing equations are easier to solve and present in closed-form solutions, which normally provides more physical or practical interpretation. This approach remains popular as it is well-known and has become the foundation for further composite plate analysis methods.

This method works relatively well for structures that are made out of a symmetric and balanced laminate, experiencing pure bending or pure tension. The error induced/introduced by neglecting the effect of transverse shear stresses becomes trivial on or close to the edges and corners of thick-sectioned configurations. The induced error increases for thick plates made of composite layers, for which the ratio of longitudinal to transverse shear elastic modulii is relatively large compared to isotropic materials [49]. It neglects transverse shear strains, underpredicts deflections and overestimates natural frequencies and buckling loads.

2.3. OUT-PLANE STRESS

As discussed in Section 2.2, in the classical lamination theory, it was assumed that the laminate is thin compared to its lateral dimensions and that straight lines normal to the middle surface remain straight and normal to that surface after deformation. Therefore, the transverse shear stress and strain are neglected. These assumptions are not valid in the case of thicker laminates and laminates with low stiffness central plies undergoing significant transverse shear deformations. In order to overcome these limitations several theories have been proposed to analyse thicker laminated composite plates in order to consider the transfer shear effect. Most of these theories are extensions of the conventional theories developed by Reissner [47] and Mindlin [50], which are known as the shear deformation plate theories. These theories are based on the assumption that the displacement w is constant through the thickness while the displacements u and v vary linearly through the thickness of each layer (constant cross-sectional rotations w_x and w_y). Generally these theories are known as First-Order Shear Deformation Theories (FSDT) [3]. According to this theory, transverse straight lines before deformation will still be straight after deformation but they are not normal to the mid-plane after deformation. This theory assumes constant transverse shear stress.

In the following, referred to as first-order shear deformation laminate plate theory, the assumption of normality of straight lines is removed compare to CLPT. On the other hand straight lines normal to the middle surface remain straight but not normal to that surface after deformation [43].

For out-plane stress, the equation (4), (5) and (6) are described as (9), (10) and (11) respectively.

$$
\begin{aligned}
\overline{Q}_{11} &= Q_{11}cos^4\theta + 2(Q_{12} + 2Q_{66})sin^2\theta cos^2\theta + Q_{22}sin^4\theta \\
\overline{Q}_{22} &= Q_{11}sin^4\theta + 2(Q_{12} + 2Q_{66})sin^2\theta cos^2\theta + Q_{22}cos^4\theta \\
\overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})sin^2\theta cos^2\theta + Q_{12}(sin^4\theta + cos^4\theta) \\
\overline{Q}_{13} &= (Q_{11} - Q_{12} - 2Q_{33})sin\theta cos^3\theta + (Q_{12} - Q_{22} + 2Q_{33})sin^3\theta cos\theta) \\
\overline{Q}_{23} &= (Q_{11} - Q_{12} - 2Q_{33})sin^3\theta cos\theta + (Q_{12} - Q_{22} + 2Q_{33})sin\theta cos^3\theta) \quad (9) \\
\overline{Q}_{33} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{33})sin^2\theta cos^2\theta + Q_{33}(sin^4\theta + cos^4\theta) \\
\overline{Q}_{55} &= Q_{55}cos^2\theta - Q_{66}sin^2\theta \\
\overline{Q}_{56} &= Q_{55}sin\theta cos\theta - Q_{66}sin\theta cos\theta \\
\overline{Q}_{66} &= Q_{55}sin^2\theta - Q_{66}cos^2\theta
\end{aligned}
$$

$$
\begin{aligned}
\begin{bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{bmatrix} &= \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} dz \\
\begin{bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{bmatrix} &= \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} zdz \qquad (10) \\
\begin{bmatrix} V_q \\ V_r \end{bmatrix} &= \int_{-h/2}^{h/2} \begin{bmatrix} \tau_{xz} \\ \tau_{yz} \end{bmatrix} dz
\end{aligned}
$$

$$
\begin{bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \\ V_q \\ V_r \end{bmatrix} = \begin{bmatrix} N_1 \\ N_2 \\ N_{12} \\ M_1 \\ M_2 \\ M_{12} \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} & 0 & 0 \\ A_{12} & A_{22} & A_{23} & B_{12} & B_{22} & B_{23} & 0 & 0 \\ A_{13} & A_{23} & A_{33} & B_{13} & B_{23} & B_{33} & 0 & 0 \\ B_{11} & B_{12} & B_{13} & D_{11} & D_{12} & D_{13} & 0 & 0 \\ B_{12} & B_{22} & B_{23} & D_{12} & D_{22} & D_{23} & 0 & 0 \\ B_{13} & B_{23} & B_{33} & D_{13} & D_{23} & D_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & E_{11} & E_{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & E_{12} & E_{22} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \\ k_1 \\ k_2 \\ k_{12} \\ \gamma_{13} \\ \gamma_{23} \end{bmatrix} \quad (11)
$$

where the components of this section stiffness matrix are given by:

$$
\begin{aligned}
(A_{ij}, B_{ij}, D_{ij}) &= \int_{-h/2}^{h/2} \overline{Q}_{ij}^{m} (1, z, z^2)dz \quad (i, j = 1,2,3) \qquad (12) \\
E_{ij} &= \int_{-h/2}^{h/2} \overline{Q}_{\alpha\beta}^{m} k_i k_j dz \quad (i, j = 1,2 \text{ and } \alpha, \beta = i + 4, j + 4).
\end{aligned}
$$

2.4. FAILURE CRITERIA

As it is shown in previous research [35] using one failure method is not reliable enough for evaluating the results. Also in each fibre orientation the type of the failure is switched between fibre, shear and transverse shear failure [43]. Therefore, it is logical that the results would be checked by at least two failure criteria. In this research Maximum stress and Tsai-Wu criteria are chosen.

2.4.1. Maximum stress criterion

Maximum stress criterion is one of the simplest failure methods to apply. According to this criterion, failure is predicted whenever one of the principal stress components exceeds its corresponding strength. It is expressed in the form of the following subcriteria:

$$\sigma_1 = \begin{cases} F_{1t} & when\ \sigma_1 > 0 \\ -F_{1c} & when\ \sigma_1 < 0 \end{cases} \quad (13.1)$$

$$\sigma_2 = \begin{cases} F_{2t} & when\ \sigma_2 > 0 \\ -F_{2c} & when\ \sigma_2 < 0 \end{cases} \quad (13.2)$$

$$\sigma_3 = \begin{cases} F_{3t} & when\ \sigma_3 > 0 \\ -F_{3c} & when\ \sigma_3 < 0 \end{cases} \quad (13.3)$$

$$|\tau_4| = F_4 \quad (13.4)$$

$$|\tau_5| = F_5 \quad (13.5)$$

$$|\tau_6| = F_6 \quad (13.6)$$

F_{1t} and F_{1c} are the longitudinal tensile and compressive strengths, F_{2t} and F_{2c} are the transverse longitudinal tensile and compressive strengths and F_6 is the in-plane shear strength. Four additional lamina strength parameters, which are relevant in three-dimensional analysis, are the out-plane or interlaminar tensile, compressive, and shear strengths, F_{3t}, F_{3c}, F_4 and F_5.

2.4.2. Tsai-Wu criterion

The Tsai-Wu failure criterion is one of the most reliable static failure criteria as it provides a simple analytical expression taking components. Tsai-Wu [43] proposed a modified tensor polynomial theory by assuming the existence of a failure in the stress space. In contracted notation it takes the form:

$$f_i\sigma_i + f_{ij}\sigma_i\sigma_j = 1 \quad (14)$$

where f_i and f_{ij} are second- and fourth-order strength tensors, and $i, j = 1,2, \ldots ,6$ [36]. By applying assumptions some of f_i and f_{ij} are identified. Finally it is reduced to a failure envelope for constant values of shear stress $\tau_6 = kF_6$

$$f_1\sigma_1 + f_2\sigma_2 + f_{11}\sigma_1^2 + f_{22}\sigma_2^2 + 2f_{12}\sigma_1\sigma_2 = 1 - k^2 \quad (15)$$

or equivalently

$$\frac{\sigma_{11}^2}{F_{1t}|F_{1c}|} + \frac{\sigma_{22}^2}{F_{2t}|F_{2c}|} + \frac{\tau_{12}^2}{F_6} - \frac{\sigma_{11}\sigma_{22}}{\sqrt{F_{1t}F_{1c}F_{2t}F_{2c}}} + \left(\frac{1}{F_{1t}} - \frac{1}{|F_{1c}|}\right)\sigma_{11} + \left(\frac{1}{F_{2t}} - \frac{1}{|F_{2c}|}\right)\sigma_{22} < 1$$
(18)

3. OPTIMISATION

An optimised composite laminate requires finding the minimum number of layers, and the best fibre orientation and thickness for each layer. Several optimisation methods have been introduced to solve this challenging problem, which is often non-linear, non-convex, multimodal, and multidimensional. Nowadays usually stochastic non-linear optmisation methods are utilised for this problem as they can avoid the local minimums.

One of the best algorithms in this category is Simulated Annealing (SA) method which is used in similar problems [7].

3.1 SIMULATED ANNEALING ALGORITHM

Kirkpatrick et al. [51] proposed simulated annealing as a powerful stochastic search technique in 1983. The method gets its name from the physical process whereby the temperature of a solid is raised to a melting point, where the atoms can move freely and then slowly cooled. The method attempts to model the behaviour of the atoms in forming arrangements in solid material during annealing. Although the atoms move randomly, as their natural behaviour they tend to form lower-energy configurations [52]. However, this is a time driven process. When a material is crystallised from its liquid phase, it must be cooled slowly if it is to assume its highly ordered, lowest-energy, perfect crystalline state. At each temperature level during this annealing process, the material should reach equilibrium. As the temperature decreases, the arrangement of the atoms gets closer and closer to the lower energy state. This process continues until the temperature finally reaches freezing point [52]. The temperature is initially assigned a higher value, which corresponds to more probability of accepting a bad move and is gradually reduced by a user-defined cooling schedule. Retaining the best solution is recommended in order to preserve the good solution [52].

At each iteration of the simulated annealing algorithm, a new point is randomly generated. The distance of the new point from the current point, or the extent of the search, is based on a probability distribution with a scale proportional to the temperature. The algorithm accepts all new points that lower the objective, but also, with a certain probability, points that raise the objective. The algorithm avoids being trapped in local minima, by accepting points that raise the objective, and is able to explore globally for more possible solutions. An annealing schedule is selected to systematically decrease the temperature as the algorithm proceeds. As the temperature decreases, the algorithm reduces the extent of its search to converge to a minimum.

If a set of configurations is considered, in each iteration the speed convergency would be increased. In this research the SA proposed by Erdal et al. [52] is applied. The number of these configurations depends on the dimension of the problem.

$$N = 7(n + 1) \quad (17)$$

where n is the dimension of the problem, i.e. the number of design variables [52].

3.2 PENALTY FUNCTION and OPTIMISATION PROCEDURE

In this step a penalty function is expressed and then this function has to be optimised.

$$F = \sum_{k=1}^{m} n_k + w_1 P_{MS} + w_2 P_{TW} - w_1 SF_{MS} - w_2 SF_{TW} + \frac{w_{3i}}{k} \cos(\Delta\theta) \quad (18)$$

n_k is the number of plies in the k^{th} lamina, in which the orientation angle is θ_k; m is the total number of distinct lamina; the second and third terms represent the penalty values introduced to increase the value of the objective function for designs for which failure is predicted and thus to restrict the search to the feasible design space; P_{MS} and P_{TW} are penalty values calculated based on the maximum stress criterion and the Tsai–Wu criterion, respectively. SF_{MS} and $\mathrm{SF}_{\mathrm{TW}}$ are equal to the safety factors according to the maximum stress and Tsai–Wu criteria, respectively, if they are greater than 1.0,

otherwise these terms are equal to zero; w_i are suitable coefficients [35]. This penalty function is the same as the one defined by Akbulut and Sonmez [35] in 2008 except the last term of equation (18). In their work, the ply angles are optimised for in-plane plate and the effect of shear stress and as a result induced twist angle of the plate was neglected. However in this research the induced twist angles is considered. Therefore the maximum acceptable twist angle is defined and by assuming this maximum twist angle, the appropriate coefficient for w_{3i} is obtained.
Implying several tests by Finite Element Method (FEM) software shows that maximum twist for each material happens at the specific angles $(\theta_{max})_i$ $i = 1,2,3, ...$ i is the number of possible θ_{max}. The range of fibre orientations $[-90, 90]$is divided into several areas, each θ_{max} is the centre of the area and in each iteration the program find the $\theta_k$to the range that it belongs to and then the program works with the appropriate θ_{max} and corresponding w_{3i}. Except for the related w_{3i}, the other w_{3i} are equal to zero. $\Delta\theta$ is defined as:

$$\Delta\theta = \theta_{max} - \theta_k \qquad (19)$$

So when the ply angle in each layer (θ_k) would be close to θ_{max}, the amount of $cos(\Delta\theta)$ is bigger and thus the last term of penalty function and as a result penalty function would be larger. For a proper w_3, the amount of induced twist (α) always will be less than maximum acceptable twist (α_{max}) .
The reason that the objective is reduced for safe designs is that there may be many feasible designs with the same minimum thickness. Of these designs, the optimum is defined as the one with the largest failure load. Accordingly, the objective function is linearly reduced in proportion to the failure margin [53]. Similarly in another study [54], the margins to initial failure were maximized with the minimum feasible number of laminae. The safety factor of the laminate according to the maximum stress criterion, SF_{MS}, is calculated as follows [35]:

$$SF_{MS}^k = \min \text{ of } \begin{cases} SF_X^k = \begin{cases} F_{1t}/\sigma_{11} & if\ \sigma_{11} > 0 \\ F_{1c}/\sigma_{11} & if\ \sigma_{11} < 0 \end{cases} \\ SF_Y^k = \begin{cases} F_{2t}/\sigma_{22} & if\ \sigma_{22} > 0 \\ F_{2c}/\sigma_{22} & if\ \sigma_{22} < 0 \end{cases} \\ SF_S^k = S/|\tau_{12}| \end{cases} \qquad (20)$$

$$SF_{MS} = \min \text{ of } SF_{MS}^k \quad for\ k = 1,2, \dots, m-1, m \qquad (21)$$

The safety factor for the k^{th} lamina, SF_{TW}^k, according to the Tsai– Wu criterion is defined as the multiplier of the stress components at lamina k, σ_{ij}^k, that makes the right hand side of Eq. (16) equal to 1 then it turns into [35]:

$$a\left(SF_{TW}^k\right)^2 + b\left(SF_{TW}^k\right) = 1 \qquad (22)$$

where

$$a = \frac{(\sigma_{11}^k)^2}{F_{1t}|F_{1c}|} + \frac{(\sigma_{22}^k)^2}{F_{2t}|F_{2c}|} + \frac{(\tau_{12}^k)^2}{F_6^2} - \frac{(\sigma_{11}^k)(\sigma_{22}^k)}{\sqrt{(F_{1t}F_{1c}F_{2t}F_{2c})}}$$
$$b = \left(\frac{1}{F_{1t}} - \frac{1}{|F_{1c}|}\right)\sigma_{11}^k + \left(\frac{1}{F_{2t}} - \frac{1}{|F_{2c}|}\right)\sigma_{22}^k \quad (23)$$

The root of the above equation gives the safety factor. Because a negative safety factor is not physically meaningful, the absolute value of the first root is considered as the actual safety factor.

$$SF_{Tw}^k = \left|\frac{-b \pm \sqrt{b^2+4a}}{2a}\right| \quad (24)$$

Then, the minimum of SF_{TW}^k is chosen as the safety factor of the laminate.

$$SF_{TW} = \text{min of } SF_{TW}^k \quad for\ k = 1,2,\dots,m-1,m \quad (25)$$

In equation (24), the $\left|\frac{-b+\sqrt{b^2+4a}}{2a}\right|$ can be considered, as b is always positive and the aim is to find the minimum of SF_{TW}^k.
The penalty value due to the violation of the maximum stress and Tsai-Wu criteria are calculated in equations (26) and (27) respectively:

$$P_x^k = \begin{cases} 0 & if\ SF_x^k \geq 1 \\ (1/SF_x^k) - 1 & if\ SF_x^k < 1 \end{cases}$$
$$P_y^k = \begin{cases} 0 & if\ SF_y^k \geq 1 \\ (1/SF_y^k) - 1 & if\ SF_y^k < 1 \end{cases} \quad (26)$$
$$P_s^k = \begin{cases} 0 & if\ SF_s^k \geq 1 \\ (1/SF_s^k) - 1 & if\ SF_s^k < 1 \end{cases}$$

$$P_{TW}^k = \begin{cases} 0 & if\ SF_{TW}^k \geq 1 \\ (1/SF_{TW}^k) - 1 & if\ SF_{TW}^k < 1 \end{cases} \quad (27)$$

The total penalty value for the laminate due to the violation of the maximum stress and Tsai-Wu criteria are then calculated by summing up the penalty values calculated for each lamina.

$$P_{MS} = \sum_{k=1}^m P_x^k + P_y^k + P_s^k \quad (28)$$
$$P_{TW} = \sum_{k=1}^m P_{TW}^k \quad (29)$$

4. Results

4.1. Experimental Results

In order to validate the FEM model some experimental tests have been performed. For each case six similar laminated plates are manufactured. One of the samples is shown in figure (3). Carbon fibre is used for all laminated experimental tests and the size of plate is 500*500 (mm).

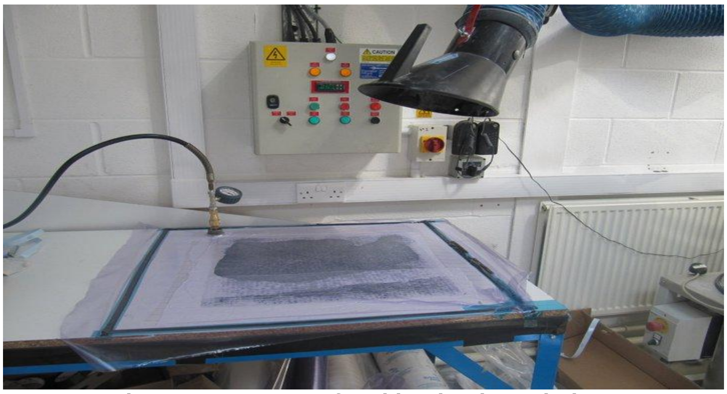

Figure 3. Process of making laminated plate

In figure (4) t_1 to t_6 are the experimental test results for six similar plates under the constant load. Six plates in each case have the same layup and geometry; t_a is the average of t_1 to t_6 and t_F is the FEM results.

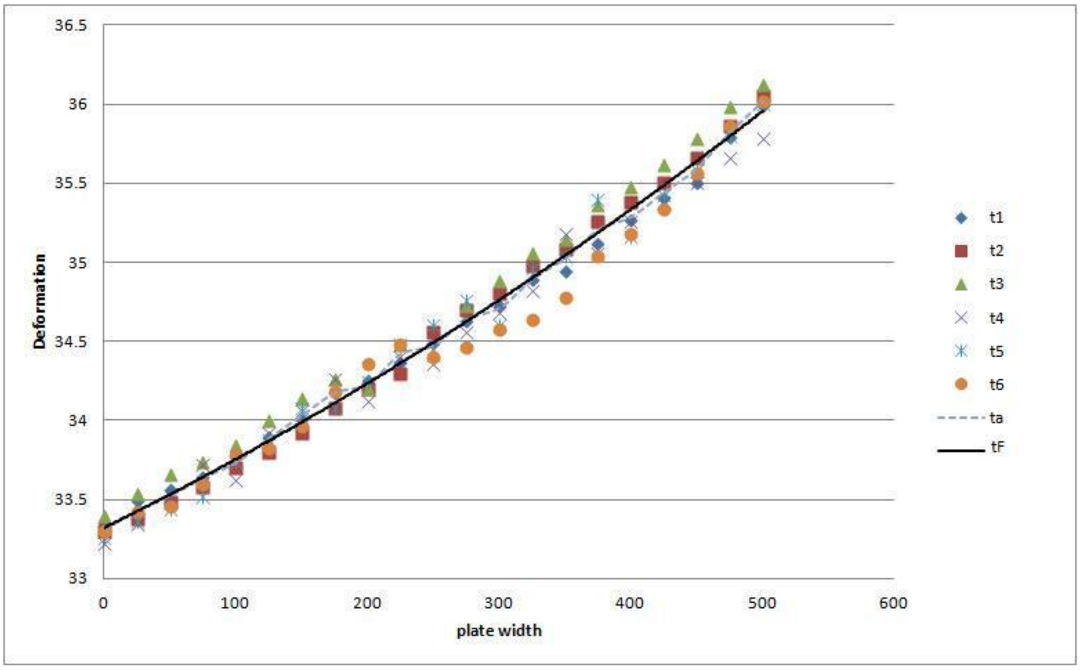

Figure 4. An example for one case; t_1 to t_6 are the experimental test results for six similar plates under constant load. t_a is the average of t_1 to t_6 and t_F is the FEM outputs.

The experimental tests have been carried out for 15 different cases with different loads and layups. In figure (5) the percentage difference of deformation between tests and FEM results and in figure (6) the percentage difference of α are shown. As is shown in figures (5,6) there is a very good agreement between FEM and experimental results. In figures (5,6) the axis which shows error is zoomed in to 10% to distinguish the differences between each case.

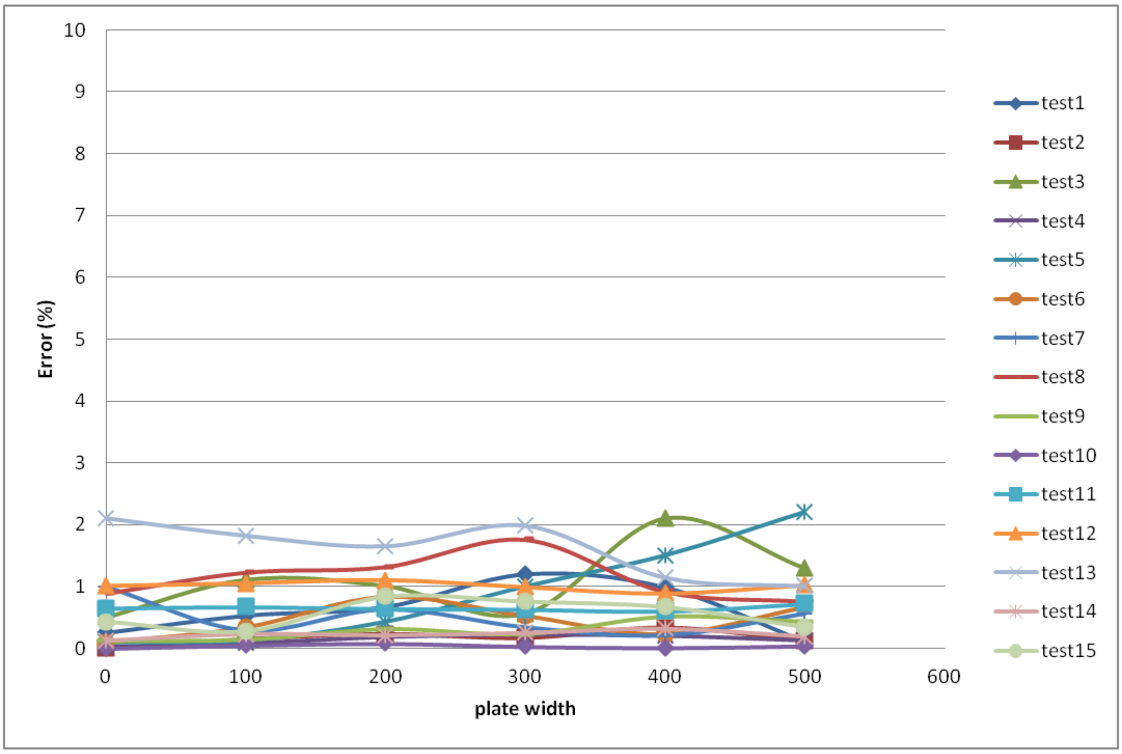

Figure 5. Percentage difference of deformation between tests and FEM results

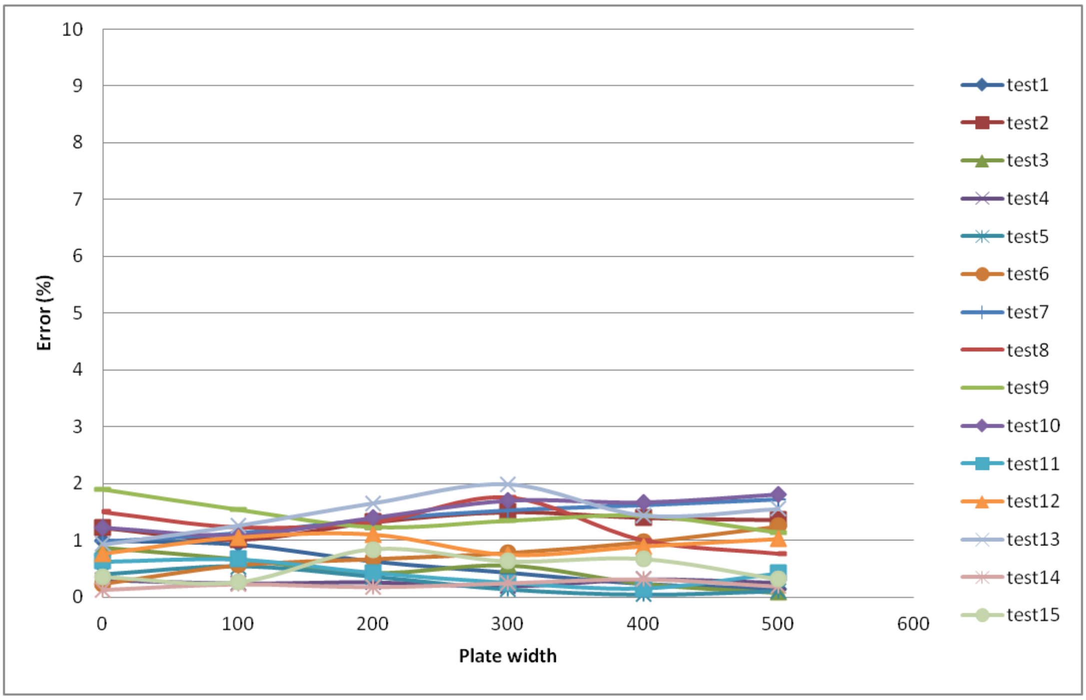

Figure 6. Percentage difference of (α) between tests and FEM results

4.2. Case Study Results

In this Section, two case studies are considered. The first case study compares the obtained results with those which were found by Akbulut and Sonmez [35] to validate the proposed optimisation method. The graphite/epoxy materials T300/5308 with the properties of E_{11} = 40.91 GPa, E_{22} = 9.88 GPa, G_12 = 2.84 GPa, ϑ_{12} = 0.292, F_{1t} = 779 MPa, F_{1c} = −1134 MPa, F_{2t} = 19 MPa, F_{2c} = −131 MPa, S = 75 MPa is considered for the first case study.

In this case the maximum acceptable twist angle is $\alpha_{max} = .01$. In table (1) and table (2) the results are compared with previous work [35]. As it is shown in table (1) the number of lay-up increase for some loads and it shows that in these cases the amount of twist angle is more than α_{max}. Clearly by increasing the lay-ups, safety factor for both Tsai-Wu and Maximum stress will be increased. In table (2) the number of lay-ups and thickness is constant but in some cases the optimum orientation angles is different from the previous work. Although the safety factor is reduced, it avoids passing the acceptable twist angle and the safety factor is still more than 1, therefore, it is still acceptable. When the maximum twist angle is less than the α_{max} the results are comparable with the work has been done by Akbulut and Sonmez [35].

Table 1. Optimum lamina orientations for Material T300/5308 under different loads

Loading: N_{xx}, N_{yy}, N_{xy} $(MPa\ m)$ (Fig. 2)	Optimum lamina orientations		Safety factor			
	Akbulut & Sonmez [35]	Present Work	Akbulut et al. [35]		Present Work	
			Max. Stress	Tsai-Wu	Max. Stress	Tsai-Wu
10/5/0	$[37_{27}/-37_{27}]$	$[39_{29}/-39_{29}]$	1.0277	1.0068	1.1309	1.1001
20/5/0	$[31_{23}/-31_{23}]$	$[36_{27}/-36_{27}]$	1.1985	1.0208	1.3305	1.1560
40/5/0	$[26_{20}/-26_{20}]$	$[26_{22}/-26_{22}]$	1.5381	1.0190	1.6504	1.1903
80/5/0	$[21_{25}/-19_{28}]$	$[21_{25}/-21_{25}]$	1.2213	1.0113	1.2302	1.0120
120/5/0	$[17_{35}/-17_{35}]$	$[17_{35}/-17_{35}]$	1.0950	1.0030	1.0951	1.0030

Table 2. Optimum lamina orientations for Material T300/5308 under different loads for constant thickness

Loading: N_{xx}, N_{yy}, N_{xy} $(KPa\ m)$ (Fig. 2)	Optimum lamina orientations		Safety factor	
	Akbulut & Sonmez [35]	Present Work	Akbulut & Sonmez [35]	Present Work
200/200/0	$[50.80_4/-49.80_4/26.59_4/-49.73_4]$	$[50.80_4/-49.80_4/26.59_4/-49.73_4]$	2.14	2.14
200/0/200	$[31.72_{16}]$	$[32.40_4/-56.61_4/-7.81_4/33.87_4]$	4.84	1.91
400/200/0	$[30.98_4/-36.57_4/37.67_4/-37.20_4]$	$[-20.12_4/58.01_4/-49.90_4/20.11_4]$	1.64	1.42
200/200/200	$[45_{16}]$	$[45_{16}]$	1.11×10^{16}	1.11×10^{16}

In the second case study a highly anisotropic material is considered (material II). The elements of stiffness matrix are: $D_{1111} = 138000$, $D_{1122} = 44000$, $D_{2222} = 138000$, $D_{1133} = 5000$, $D_{2233} = 6000$, $D_{3333} = 47000$, $D_{1112} = 2000$, $D_{2212} = 8000$, $D_{3312} = 3000$, $D_{1212} = 10000$, $D_{1113} = 0$, $D_{2213} = 3500$, $D_{3313} = 0$, $D_{1213} = 21000$, $D_{1313} = 58000$, $D_{1123} = 0$, $D_{2223} = 1000$, $D_{3323} = 2500$, $D_{1223} = 2000$, $D_{1323} = 4500$, $D_{2323} = 23000$.

In figure (7) the amount of twist angle under a constant load for different ply angles is shown. So the amount of $(\theta_{\max})_i$ in equation (20) are $(\pm 30, \pm 60)$ which are the local maximums.

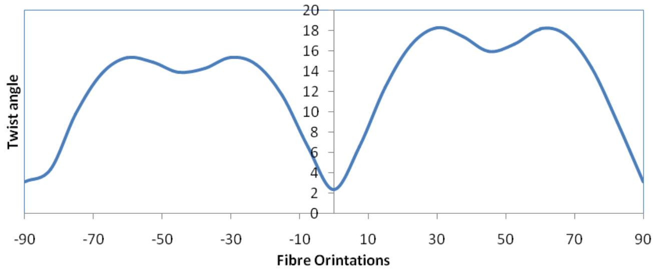

Figure 7. Twist angle for a first layer of material (II)

Table 3. Optimum lamina orientations for second case study material under different loads for constant thickness.

Loading: $N_{xx}, N_{yy}, N_{xy}, N_{zz}$ $(KPa\ m)$	Optimum lamina orientations	Safety factor	
		Max. Stress	Present Work
100/100/0/20	$[-77.68_4/23.54_4/-22.78_4/49.79_4]$	1.2720	1.0931
100/0/100/20	$[53.45_8/-78.23_8]$	1.5402	1.2112
200/100/0/20	$[-22.23_4/49.26_4/-65.33_4/11.23_4]$	1.3200	1.111
100/100/100/20	$[44.8_8/-33.21_8]$	3.3401	2.3765

This test is for the first layer in order to find the $(\theta_{\max})_i$ and the areas which were explained in Section 3.2. FEM tests show that these $(\theta_{\max})_i$ are the same by adding the next layers. If the stiffness matrix is symmetric the curves will be symmetric about y axis. A general stiffness matrix for material II is considered, so there is no mirror about y axis. Optimum lamina orientations under different loads in this case are shown in the table (3). In this case the pure bending load N_{zz} is also applied to the plate.

4. CONCLUSION

In this study, an optimisation methodology of composite plates was presented. A method was proposed in order to overcome the difficulties and shortcomings faced by the previous research. In previous work the effect of transverse shear was neglected and therefore the induced twist angle is ignored. In some applications the twist angle, which is the direct effect of transverse shear, is undesirable. Therefore, in this research, after optimising the fibre orientations, by considering the induced twist angle as well as safety factor, the induced twist angle always stays less than the acceptable twist angle. One of the other weakness in previous work was that the plate was optimised under specific loads, such as longitudinal or in-plane loading. By the proposed method in this research the out-plane stress optimisation can be solved as well as the in-plane stresses. In order to have a reliable optimisation, simulated annealing, which is one of the stochastic optimisation methods and can escape the local minima is applied and the penalty function for this optimisation method is modified. This modified penalty function forces the induced twist to stay under a predefined induced twist. In addition, two Tsai–Wu and maximum stress failure criteria are used in the algorithm individually to avoid false optimal design.

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A Semi-Analytical Model for Deflection Analysis of Laminated plates with the Newton-Kantorovich-Quadrature Method

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Abstract:

A semi-analytical approach for analysis of laminated plates with general boundary conditions under a general distribution of loads is developed. The non-linear equations are solved by the Newton-Kantorovich-Quadrature (NKQ) method which is a combination of well-known Newton-Kantorovich method and the Quadrature method. This method attempts to solve a sequence of linear integral equations. The convergence of the proposed method is compared with other semi-analytical methods. The validation of the method is explored through various numerical examples and the results compared with finite element method (FEM) and experimental tests. Good agreement between the NKQ model, FEM and experimental results are shown to validate the model.

Key words: composites; laminates; semi-analytical model

1. Introduction

In the last half century, the use of composite materials has grown rapidly. These materials are ideal for structural applications that require high strength and low weight. They have good fatigue characteristics and are resistant to corrosion [1].

Understanding the mechanical behaviour of composite plates is essential for efficient and reliable design and for the safe use of structural elements. The complex behaviour of laminated plate structures normally need a non-linear model to describe them. Moreover, anisotropic and coupled material behaviour add more non-linearity to analysis. In general, there is no closed-form exact solution for the non-linear problem of composite plates for large deformations with arbitrary boundary conditions. The non-linear analysis of laminated plates has been the subject of many research projects. Also, various semi-analytical and numerical methods for the description and response of laminated plates have been developed. A comprehensive summary of the solutions for the geometrically non-linear analysis of isotropic and composite laminated plates was given by Chia [2]. The common analytical non-linear theories for laminated composites such as classical laminated plate theory [3-9] and first shear deformation plate theory [9-11] generally use the Rayleigh-Ritz method [12] or the Galerkin method [13-15]. The accuracy of these analytical models depends on the trial functions which they choose and they have to satisfy at least the kinematic boundary conditions. For certain boundary conditions and out-of-plane loadings these methods are so complicated and time consuming [16]. There are also some numerical methods to analyse laminated plates for the large deflection including the finite strip method [17,18]; the differential quadrature technique [19]; the method of lines [20]; Finite Element Method (FEM) [21-32]. Different meshless methods are also presented to solve the equations for laminated composite plates [33]. The development of element-free or meshless methods and their applications in the analysis of composite structures have been reviewed by Liew et al. [34] recently.

Due to numerous computations and the number of unknown variables, numerical methods are needed to solve problem of laminated plates. However, the analytical and semi-analytical non-linear methods are an essential tool that provides perception to the physical non-linear behaviour of the composite plate structure. Furthermore, these methods normally present fast and reliable solutions during the preliminary design phase. They also provide a means of validations the numerical methods and enable the development of new computational models. Therefore, the development of the semi-analytical methods has been growing rapidly [16].

The aim of this work is to achieve a semi-analytical approach for the non-linear model of laminated plates with arbitrary boundary conditions for general out-of-plane loadings. A Newton-Kantorovich-Quadrature (NKQ) method was proposed recently, by Saberi-Najafi and Heidari [35], for solving nonlinear integral equations in the Urysohn form. This method is expanded and used in this paper to present a semi-analytical model for laminated composite plates. Different extended Kantorovich methods (EKM) have been used by researchers to analyse the free-edge strength of composite laminates [36], the bending of thick laminated plates [37], buckling of symmetrically laminated composite plates [38] and laminated rectangular plates under general out-of-plane loading [16]. The multi-term extended Kantorovich method assumes a solution of the two-dimensional problem in the form of a sum of products of functions in one direction and functions in the other direction. As a result, the problem is reduced to a set of non-linear ordinary differential equations in the second direction. The solution of the resulting one-dimensional problem is then used as the assumed functions and the problem is solved again for the first direction. These iterations are repeated until convergence is completed. Unlike most of the other semi-analytical methods the accuracy of the solution is independent of the initial chosen functions. This initial function, even if it does not satisfy any of the boundary conditions [39,40], does not affect the accuracy of the solution. The EKM was applied by Soong [41] to the large deflection analysis of thin rectangular isotropic plates subjected to uniform loading. Some solutions [40,41] used only one-term for expansion yielding of isotropic plates. However, it is showed that one term formulation is not enough to predict the behaviour of anisotropic plates [40]. In this study, the NKQ is used to overcome these shortcomings and the model for out-of-plane loading as well. The accuracy and convergence of the method has been investigated through a comparison with other semi-analytical solutions and with finite element analysis (FEA) using a number of numerical examples in order to validate the model [16].

2. Governing Equations

2.1. General composite equations

The state of stress at a point in a general continuum can be represented by nine stress components σ_{ij} $(i, j = 1,2,3)$ acting on the sides of an elemental cube with sides parallel to the axes of a reference coordinate system (Fig. 1).

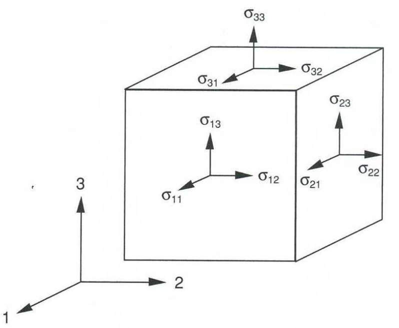

Figure 1. State of stress at a point in a general continuum [42]

In the most general case the stress and strain components are related by the generalised Hook's law as follows [1]:

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} \quad (i, j, k, l = 1,2,3) \tag{1}$$

where C_{ijkl} is the stiffness components [42]. Thus in general, it would require 81 elastic constants to characterize a material fully. However, by considering the symmetry of the stress and strain tensors and the energy relations, it is proven that the stiffness matrices are symmetric. Thus the state of stress (strain) at a point can be described by six components of stress (strain), and the stress-strain equations are expressed in terms of 21 independent stiffness constants [42].

2.1.1. IN-PLANE STRESS

The classical laminate theory is used to analyze the mechanical behaviour of the composite laminate. It is assumed that plane stress components are taken as zero. The in-plane stress components are related to the strain components as:

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix}_k = \begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\end{bmatrix}\begin{bmatrix}\epsilon_{xx}\\ \epsilon_{yy}\\ \gamma_{xy}\end{bmatrix} \tag{2}$$

where k is the lamina number, $\overline{Q}_{ij}$ are the off-axis stiffness components, which can be explained in terms of principal stiffness components, Q_{ij}, which are defined in reference [1,42]

Stress resultants, or forces per unit length of the cross section, are obtained as:

$$\begin{bmatrix}N_{xx}\\ N_{yy}\\ N_{xy}\end{bmatrix} = \int_{-h/2}^{h/2}\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix}dz = 2\sum_{k=1}^{m} n_k t_0 \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix}_k \tag{3}$$

Here m is the number of distinct laminae, n_k is the number of plies in the kth lamina. Here, lamina is meant to be a group of plies with the same orientation angle. Substituting the stress–strain relation given by Eq. (2) into Eq. (3) [1]:

$$\begin{bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{bmatrix} \tag{4}$$

where A_{ij}, components of extensional stiffness matrix, are given by:

$$A_{ij} = 2 \sum_{k=1}^{m} n_k t_0 (\overline{Q}_{ij})_k \tag{5}$$

Principal stress components can be obtained using the following transformation [29]:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} cos^2\theta_k & sin^2\theta_k & 2cos\theta_k sin\theta_k \\ sin^2\theta_k & cos^2\theta_k & -2cos\theta_k sin\theta_k \\ -cos\theta_k sin\theta_k & cos\theta_k sin\theta_k & cos^2\theta_k - sin^2\theta_k \end{bmatrix}_k \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} \tag{6}$$

2.1.2 OUT-of-PLANE STRESS

In the classical laminate theory, it is assumed that straight lines normal to the middle surface remain straight and normal to that surface after deformation. These assumptions are not valid in the case of thicker laminates and laminates with low stiffness central plies undergoing significant transverse shear deformations. In the following, referred to as first-order shear deformation laminate plate theory, the assumption of normality of straight lines is removed. On the other hand straight lines normal to the middle surface remain straight but not normal to that surface after deformation [1,9]. Q_{ij} can be found in reference [1,42]:

$$\begin{gathered} \begin{bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} dz \\ \begin{bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} zdz \\ \begin{bmatrix} V_q \\ V_r \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \tau_{xz} \\ \tau_{yz} \end{bmatrix} dz \end{gathered} \tag{7}$$

$$\begin{bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \\ V_q \\ V_r \end{bmatrix} = \begin{bmatrix} N_1 \\ N_2 \\ N_{12} \\ M_1 \\ M_2 \\ M_{12} \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} & 0 & 0 \\ A_{12} & A_{22} & A_{23} & B_{12} & B_{22} & B_{23} & 0 & 0 \\ A_{13} & A_{23} & A_{33} & B_{13} & B_{23} & B_{33} & 0 & 0 \\ B_{11} & B_{12} & B_{13} & D_{11} & D_{12} & D_{13} & 0 & 0 \\ B_{12} & B_{22} & B_{23} & D_{12} & D_{22} & D_{23} & 0 & 0 \\ B_{13} & B_{23} & B_{33} & D_{13} & D_{23} & D_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & E_{11} & E_{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & E_{12} & E_{22} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \\ k_1 \\ k_2 \\ k_{12} \\ \gamma_{13} \\ \gamma_{23} \end{bmatrix} \tag{8}$$

where the components of this section stiffness matrix are given by:

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-h/2}^{h/2} \overline{Q}_{ij}^{m} (1, z, z^2) dz \quad (i, j = 1,2,3)$$
$$E_{ij} = \int_{-h/2}^{h/2} \overline{Q}_{\alpha\beta}^{m} k_i k_j dz \quad (i, j = 1,2 \ and \ \alpha, \beta = i + 4, j + 4). \tag{9}$$

The out-of-plane boundary conditions include three cases: simply supported (S), clamped (C), and free (F) edges. The four possible in-plane restraints along the plate edges are shown in Fig. 2, and they are denoted by a subscript index [16].

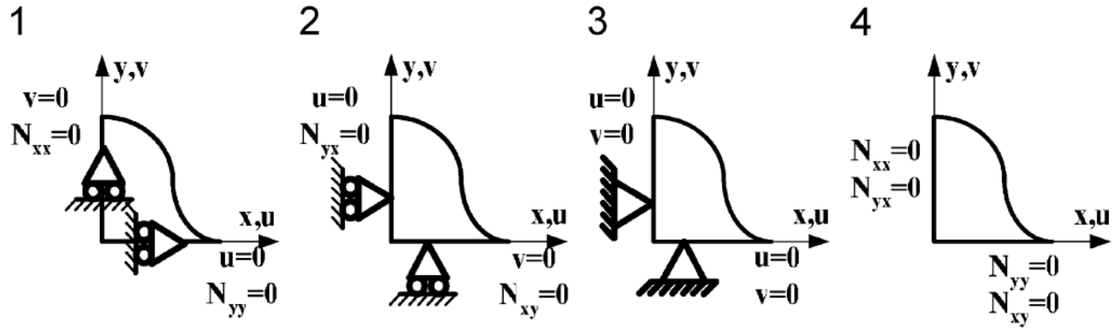

Figure 2. Boundary conditions [16]

2.2. Basic NKQ Equations

The nonlinear integral equation in the Urysohn form is defined as [35]:

$$y(x) = f(x) + \int_{\Omega} K(x, t, y(t)) dt \qquad a \le x \le b \tag{10}$$

If $\Omega = (a, x)$, it is named a nonlinear Volterra integral equation and if $\Omega = (a, b)$, it is named the nonlinear Fredholm integral equation. To approximate the right-hand integral in (10), the usual quadrature methods similar to the ones used to approximate the linear integral equations that lead to the following nonlinear systems for Fredholm and Volterra equations are used, respectively. For further information on quadrature methods in this respect, see [35,43-49].

$$y(x_i) = f(x_i) + \sum_{j=0}^{n} w_j K\left(x_i, x_j, y(x_j)\right), \qquad i = 0,1,2,,...,n \tag{11}$$

$$\begin{cases} y(x_0) = f(x_0) \\ y(x_i) = f(x_i) + \sum_{j=0}^{i} w_{ij} K\left(x_i, x_j, y(x_j)\right), \qquad i = 0,1,2,,...,n \end{cases} \tag{12}$$

where $w_{ij}s$ and $w_j s$ are weights of the integration formula.

In the Newton-Kantorovich method, an initial solution for $y(x)$ is considered. The following iteration method is used to solve the following sequence of linear integral equations instead of a nonlinear integral equation. For further information on the Newton-Kantorovich method, see [35,50,51].

$$\begin{cases} y_k(x) = y_{k-1}(x) + \phi_{k-1}(x) \\ \phi_{k-1}(x) = \varepsilon_{k-1}(x) + \int_\Omega K'_y(x,t,y_{k-1}(t))\phi_{k-1}(t)dt \\ \varepsilon_{k-1}(x) = f(x) - y_{k-1}(x) + \int_\Omega K(x,t,y_{k-1}(t))dt \end{cases} \quad (13)$$

where $K'_y(x,t,y) = \frac{\partial}{\partial y} K'(x,t,y)$.

In NKQ method which is used in this paper, equations (11-13) are combined by Saberi-Najafi and Heidari (2010) to solve the nonlinear integral equations.

3. Application of NKQ

As it is mentioned the general form of the nonlinear Volterra integral equations of the Urysohn form is:

$$y(x) = f(x) + \int_a^x K(x,t,y(t))dt \qquad a \le x \le b \quad (14)$$

By considering the equation (11-13) and by integrating $\phi_{k-1}(x)$ with $y_k(x) - y_{k-1}(x)$:

$$\begin{cases} y_k(x_0) = f(x_0) \\ y(x_i) = f(x_i) + \sum_{j=0}^{i} w_{ij} K(x_i, x_j, y_{k-1}(x_j)) + \sum_{j=0}^{i} w_{ij} K'_y(x_i, x_j, y_{k-1}(x_j))[y_k(x_j) - y_{k-1}(x_j)] \quad i = 1,2,\ldots,n \end{cases}$$

(15)

Consider:

$$\left(F^{(k-1)}\right)_{i+1} = \begin{cases} f(x_0) \quad i = 0 \\ f(x_i) + \sum_{j=0}^{i} w_{ij} K(x_i, x_j, y_{k-1}(x_j)) - \sum_{j=0}^{i} w_{ij} K'_y(x_i, x_j, y_{k-1}(x_j)) y_{k-1}(x_j) \quad i = 1,2,\ldots,n \end{cases}$$

(16)

$$\left(A^{(k-1)}\right)_{i+1 \quad j+1} = \begin{cases} w_{ij} K'_y(x_i, x_j, y_{k-1}(x_j)) \quad i = 1,2,\ldots,n \quad j = 0,1,\ldots,i \\ 0 \qquad otherwise \end{cases} \quad (17)$$

$$\left(Y^{(k)}\right)_{i+1} = y_k(x_i) \qquad i = 0,1,2,\ldots,n \quad (18)$$

This equation can be solved by considering an initial solution $y_0(x)$ and constructing the $Y^{(0)}, A^{(0)}, F^{(0)}$ and also using the following repetition sequence (for further details see [35]):

$$\left(I - A^{(k-1)}\right)Y^{(k)} = F^{(k-1)} \qquad k = 1,2,...,n \tag{19}$$

On the other hand, by considering an initial solution $y_0(x)$, $(Y^{(0)})_i$ would be $y_0(x_i)$ and by using equation (16) and (17) $F^{(0)}, A^{(0)}$ are obtained respectively. Then by solving the system $\left(I - A^{(0)}\right)Y^{(1)} = F^{(0)}$, $Y^{(1)}$ is obtained. By repeating this procedure and next using equation (19), the values of $Y^{(1)}, Y^{(2)}, Y^{(3)}, ...Y^{(m)}$ are calculated for $m \in N$. m is a constant value which can be increased for higher n. Depending on n an approximate solution for equation (10) is presented. Noticeably, by increasing m, the solution tends to be more accurate with respect to n. However it is shown that to achieve good results it is not necessary to increase m significantly.

The general basic equations for laminated composite plate are [9]:

$$-\left(\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y}\right) + I_0 \frac{\partial^2 u_0}{\partial t^2} + I_1 \frac{\partial^2 \phi_x}{\partial t^2} = 0 \tag{20}$$

$$-\left(\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y}\right) + I_0 \frac{\partial^2 v_0}{\partial t^2} + I_1 \frac{\partial^2 \phi_y}{\partial t^2} = 0 \tag{21}$$

$$-\left(\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y}\right) - \mathrm{N}(u_0, v_0, w_0) - q + I_0 \frac{\partial^2 w_0}{\partial t^2} = 0 \tag{22}$$

$$-\left(\frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y}\right) + Q_x + I_2 \frac{\partial^2 \phi_x}{\partial t^2} + I_1 \frac{\partial^2 u_0}{\partial t^2} = 0 \tag{23}$$

$$-\left(\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y}\right) + Q_y + I_2 \frac{\partial^2 \phi_y}{\partial t^2} + I_1 \frac{\partial^2 v_0}{\partial t^2} = 0 \tag{24}$$

Where:

$$\mathrm{N}(u_0, v_0, w_0) = \frac{\partial}{\partial x}\left(N_{xx} \frac{\partial w_0}{\partial x} + N_{xy} \frac{\partial w_0}{\partial y}\right) + \frac{\partial}{\partial y}\left(N_{xy} \frac{\partial w_0}{\partial x} + N_{yy} \frac{\partial w_0}{\partial y}\right) \tag{25}$$

$$\begin{Bmatrix} I_0 \\ I_1 \\ I_2 \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} 1 \\ z \\ z^2 \end{Bmatrix} \rho_0 dz \tag{26}$$

And equations (7-9) are simplified to [9]:

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \end{Bmatrix} \quad (27)$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \end{Bmatrix} \quad (28)$$

$$\begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} = K_s \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \frac{\partial w_0}{\partial y} + \phi_y \\ \frac{\partial w_0}{\partial x} + \phi_x \end{Bmatrix} \quad (29)$$

Where

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-h/2}^{h/2} \overline{Q}_{ij}\left(1, z, z^2\right) dz \qquad i, j = 1,2,..,6 \quad (30)$$

where $\overline{Q}_{ij}$ $(i, j = 1,2,...,6)$ are the transformed plane-stress stiffness coefficients.
By adopting the variation principle of virtual work and applying the NKQ the equations (31) to (35) are derived:

$$\begin{aligned} &\int_{\Omega^e} \Big(\frac{\partial \delta u_0}{\partial x}\{A_{11}\left[\frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right] + A_{12}\left[\frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2\right] \\ &+ A_{16}\left[\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right] + B_{11}\frac{\partial \phi_x}{\partial x} + B_{12}\frac{\partial \phi_y}{\partial y} + B_{16}\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right)\} \\ &+ \frac{\partial \delta u_0}{\partial y}\{A_{16}\left[\frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right] + A_{26}\left[\frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2\right] \\ &+ A_{66}\left[\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right] + B_{16}\frac{\partial \phi_x}{\partial x} + B_{26}\frac{\partial \phi_y}{\partial y} + B_{66}\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right)\}\Big) dxdy \\ &- \oint_{\Gamma^e} N_n \delta u_{0n} ds + \int_{\Omega^e}\left(I_0 \frac{\partial^2 u_0}{\partial t^2} + I_1 \frac{\partial^2 \phi_x}{\partial t^2}\right)\delta u_0 dxdy = 0 \end{aligned} \quad (31)$$

$$
\begin{aligned}
&\int_{\Omega^e} \Big(\frac{\partial \delta v_0}{\partial y}\{A_{12}\left[\frac{\partial u_0}{\partial x}+\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right]+A_{22}\left[\frac{\partial v_0}{\partial y}+\frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2\right] \\
&+A_{26}\left[\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}+\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right]+B_{12}\frac{\partial \phi_x}{\partial x}+B_{22}\frac{\partial \phi_y}{\partial y}+B_{26}\left(\frac{\partial \phi_x}{\partial y}+\frac{\partial \phi_y}{\partial x}\right)\} \\
&+\frac{\partial \delta v_0}{\partial x}\{A_{16}\left[\frac{\partial u_0}{\partial x}+\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right]+A_{26}\left[\frac{\partial v_0}{\partial y}+\frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2\right] \qquad (32) \\
&+A_{66}\left[\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}+\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right]+B_{16}\frac{\partial \phi_x}{\partial x}+B_{26}\frac{\partial \phi_y}{\partial y}+B_{66}\left(\frac{\partial \phi_x}{\partial y}+\frac{\partial \phi_y}{\partial x}\right)\}\Big)dxdy \\
&-\oint_{\Gamma^e} N_s \delta u_{0s}\, ds+\int_{\Omega^e}\left(I_0\frac{\partial^2 v_0}{\partial t^2}+I_1\frac{\partial^2 \phi_y}{\partial t^2}\right)\delta v_0\, dxdy=0
\end{aligned}
$$

$$K_s \int_{\Omega^e} \{\frac{\partial \delta w_0}{\partial x}(A_{55}\left[\frac{\partial w_0}{\partial x}+\phi_x\right]+A_{45}\left[\frac{\partial w_0}{\partial y}+\phi_y\right])$$

$$+\frac{\partial \delta w_0}{\partial y}(A_{45}\left[\frac{\partial w_0}{\partial x}+\phi_x\right]+A_{44}\left[\frac{\partial w_0}{\partial y}+\phi_y\right])\}dxdy$$

$$\int_{\Omega^e} \{\frac{\partial \delta w_0}{\partial x}[\frac{\partial \delta w_0}{\partial x}(A_{11}\left[\frac{\partial u_0}{\partial x}+\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right]+A_{12}\left[\frac{\partial v_0}{\partial y}+\frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2\right]$$

$$+A_{16}\left[\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}+\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right]+B_{11}\frac{\partial \phi_x}{\partial x}+B_{12}\frac{\partial \phi_y}{\partial y}+B_{16}\left(\frac{\partial \phi_x}{\partial y}+\frac{\partial \phi_y}{\partial x}\right))$$

$$+\frac{\partial \delta w_0}{\partial y}\{A_{16}\left[\frac{\partial u_0}{\partial x}+\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right]+A_{26}\left[\frac{\partial v_0}{\partial y}+\frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2\right]$$

$$+A_{66}\left[\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}+\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right]+B_{16}\frac{\partial \phi_x}{\partial x}+B_{26}\frac{\partial \phi_y}{\partial y}+B_{66}\left(\frac{\partial \phi_x}{\partial y}+\frac{\partial \phi_y}{\partial x}\right)\}]$$

$$\frac{\partial \delta w_0}{\partial y}[\frac{\partial \delta w_0}{\partial y}(A_{12}\left[\frac{\partial u_0}{\partial x}+\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right]+A_{22}\left[\frac{\partial v_0}{\partial y}+\frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2\right]$$

$$+A_{26}\left[\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}+\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right]+B_{12}\frac{\partial \phi_x}{\partial x}+B_{22}\frac{\partial \phi_y}{\partial y}+B_{26}\left(\frac{\partial \phi_x}{\partial y}+\frac{\partial \phi_y}{\partial x}\right))$$

$$+\frac{\partial \delta w_0}{\partial x}\{A_{16}\left[\frac{\partial u_0}{\partial x}+\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right]+A_{26}\left[\frac{\partial v_0}{\partial y}+\frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2\right] \qquad (33)$$

$$+A_{66}\left[\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}+\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right]+B_{16}\frac{\partial \phi_x}{\partial x}+B_{26}\frac{\partial \phi_y}{\partial y}+B_{66}\left(\frac{\partial \phi_x}{\partial y}+\frac{\partial \phi_y}{\partial x}\right)\}]$$

$$-\oint_{\Gamma^e} V_n \delta w_0 ds+\int_{\Omega^e} I_0 \frac{\partial^2 w_0}{\partial t^2}\delta w_0 dxdy=0$$

$$\int_{\Omega^e} (\frac{\partial\delta\phi_x}{\partial x}\{B_{11}\left[\frac{\partial u_0}{\partial x}+\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right]+B_{12}\left[\frac{\partial v_0}{\partial y}+\frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2\right]$$
$$+B_{16}\left[\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}+\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right]+D_{11}\frac{\partial\phi_x}{\partial x}+D_{12}\frac{\partial\phi_y}{\partial y}+D_{16}\left(\frac{\partial\phi_x}{\partial y}+\frac{\partial\phi_y}{\partial x}\right)\}$$
$$+\frac{\partial\delta\phi_x}{\partial y}\{B_{16}\left[\frac{\partial u_0}{\partial x}+\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right]+B_{26}\left[\frac{\partial v_0}{\partial y}+\frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2\right]$$
$$+B_{66}\left[\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}+\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right]+D_{16}\frac{\partial\phi_x}{\partial x}+D_{26}\frac{\partial\phi_y}{\partial y}+D_{66}\left(\frac{\partial\phi_x}{\partial y}+\frac{\partial\phi_y}{\partial x}\right)\}$$
$$+K_s\delta\phi_x(A_{55}\left[\frac{\partial w_0}{\partial x}+\phi_x\right]+A_{45}\left[\frac{\partial w_0}{\partial y}+\phi_y\right])dxdy$$
$$-\oint_{\Gamma^e} M_n\delta\phi_n ds+\int_{\Omega^e}\left(I_2\frac{\partial^2\phi_x}{\partial t^2}+I_1\frac{\partial^2 u_0}{\partial t^2}\right)\delta\phi_x dxdy=0 \qquad (34)$$

$$\int_{\Omega^e} (\frac{\partial\delta\phi_y}{\partial y}\{B_{12}\left[\frac{\partial u_0}{\partial x}+\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right]+B_{22}\left[\frac{\partial v_0}{\partial y}+\frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2\right]$$
$$+B_{26}\left[\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}+\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right]+D_{12}\frac{\partial\phi_x}{\partial x}+D_{22}\frac{\partial\phi_y}{\partial y}+D_{26}\left(\frac{\partial\phi_x}{\partial y}+\frac{\partial\phi_y}{\partial x}\right)\}$$
$$+\frac{\partial\delta\phi_y}{\partial x}\{B_{16}\left[\frac{\partial u_0}{\partial x}+\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right]+B_{26}\left[\frac{\partial v_0}{\partial y}+\frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2\right]$$
$$+B_{66}\left[\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}+\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right]+D_{16}\frac{\partial\phi_x}{\partial x}+D_{26}\frac{\partial\phi_y}{\partial y}+D_{66}\left(\frac{\partial\phi_x}{\partial y}+\frac{\partial\phi_y}{\partial x}\right)\}$$
$$+K_s\delta\phi_y(A_{45}\left[\frac{\partial w_0}{\partial x}+\phi_x\right]+A_{44}\left[\frac{\partial w_0}{\partial y}+\phi_y\right])dxdy$$
$$-\oint_{\Gamma^e} M_s\delta\phi_s ds+\int_{\Omega^e}\left(I_2\frac{\partial^2\phi_y}{\partial t^2}+I_1\frac{\partial^2 v_0}{\partial t^2}\right)\delta\phi_y dxdy=0 \qquad (35)$$

Where the secondary variables of the formulation are:

$$N_n \equiv N_{xx}n_x+N_{xy}n_y$$
$$N_s \equiv N_{xy}n_x+N_{yy}n_y \qquad (36)$$

$$M_n \equiv M_{xx}n_x+M_{xy}n_y$$
$$M_s \equiv M_{xy}n_x+M_{yy}n_y \qquad (37)$$

$$V_n \equiv \left(Q_x + N_{xx} \frac{\partial w_0}{\partial x} + N_{xy} \frac{\partial w_0}{\partial y} \right) n_x + \left(Q_y + N_{xy} \frac{\partial w_0}{\partial x} + N_{yy} \frac{\partial w_0}{\partial y} \right) n_y \qquad (38)$$

Equations (31) to (35) can be estimated by considering them as Urysohn form and an initial solution $y_0(x)$ and consequently the $Y^{(0)}, A^{(0)}, F^{(0)}$ and also repeating the sequences for equation (19).

4. Verification Study

In order to verify the NKQ method a number of numerical examples are solved and compared with previous research. In the first example, a four layer glass/epoxy laminate $[0°, 90°]_s$ with ply properties [52] is studied:

$$\begin{aligned} &E_1 = 43.5Gpa, \quad E_2 = E_3 = 11.5Gpa, \quad \nu_{12} = \nu_{13} = .27, \\ &\nu_{23} = .4 \quad G_{12} = G_{13} = 3.45Gpa \quad G_{23} = 4.12Gpa \end{aligned} \qquad (39)$$

The plate is a square with 0.5m length and 0.01m thickness. A trigonometric function is chosen for initial guess ($y_0(x) = \sin(\pi x / l)$). It is shown in Figure (3) that it is converged after five iterations. Aghdam and Falahatgar [37] used an Extended Kantorovich method EKM for analysing the thick composite plate. By choosing a trigonometric function as an initial guess, the model converges after 4 iterations.

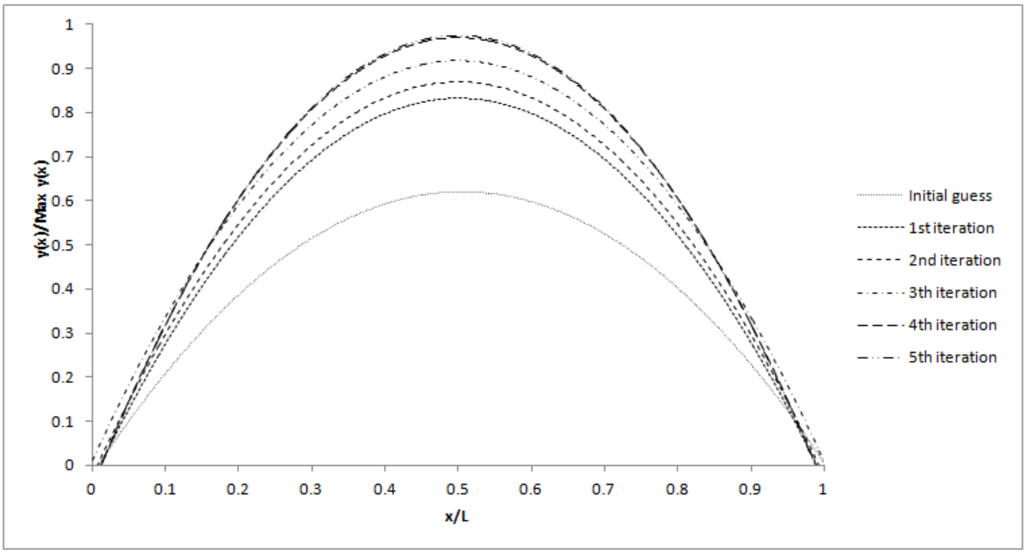

Figure 3. Convergence of NKQ method

In Table (1) the number of iterations, which are needed for convergence, for three different initial functions are shown. As it is mentioned earlier the initial guess does not have to satisfy the boundary conditions, so any initial function can be selected. Furthermore, as it is shown in Table (1) that the NKQ method is relatively quick and does not significantly depend on the initial value. The main advantage of this method, compared to EKM, is that it can be used for more complicated cases such as out-of-plane loading and different boundary conditions.

Table 1. Number of iteration for convergence of NKQ and EKM

Initial guess	Number of iteration in order to converge for NKQ	Number of iteration in order to converge for EKM [37]
Trigonometric function $y_0(x) = \sin(x)$	5	4
Polynomial function $y_0(x) = 1 + x + x^2$	4	NA
Exponential function $y_0(x) = e(x)$	4	NA

In the next study the material properties are:

$$E_1 = 215Gpa \quad E_2 = E_3 = 23.6Gpa, \quad \nu_{12} = \nu_{13} = .17,$$
$$\nu_{23} = .28 \quad G_{12} = G_{13} = 5.4Gpa \quad G_{23} = 2.1Gpa$$

The plate is a square and each length is 0.25m, the thickness is 0.006m and the lay ups are $[0°, 90°, 0°]_s$. In Table (2) the relative error between the NKQ method and FEM for different numbers of iterations are shown for a plate clamped on one side (C) and free (F) on the other three sides (CFFF). This example was then repeated for SSSS and CCCC boundary conditions and the results are shown in Tables (3) and (4) respectively.

Table 2. Relative error for CFFF boundary condition

Number of iterations	1	2	3	4	5	6	7	8	9	10
Relative error (%) for u	29.4	8.3	3.4	1.9	1.1	0.6	0.3	0.1	0.0	0.0
Relative error (%) for v	34.5	9.2	5.1	2.1	1.2	0.8	0.3	0.1	0.0	0.0
Relative error (%) for w	53.4	11.3	5.4	2.1	1.2	0.8	0.4	0.2	0.0	0.0

Table 3. Relative error for SSSS boundary condition

Number of iterations	1	2	3	4	5	6	7	8	9	10
Relative error (%) for u	25.4	9.1	3.9	1.9	1.2	0.7	0.4	0.1	0.0	0.0
Relative error (%) for v	47.2	11.0	5.6	2.5	1.5	1.0	0.5	0.2	0.1	0.0
Relative error (%) for w	76.6	21.2	7.0	2.9	1.5	0.9	0.5	0.2	0.1	0.1

Table 4. Relative error for CCCC boundary condition

Number of iterations	1	2	3	4	5	6	7	8	9	10
Relative error (%) for u	92.2	30.4	14.1	8.3	5.3	3.2	1.8	1.0	0.4	0.1
Relative error (%) for v	63.8	19.2	9.2	5.1	3.7	1.9	0.9	0.4	0.1	0.0
Relative error (%) for w	77.1	24.5	11.1	6.6	4.9	2.2	1.3	0.7	0.3	0.1

In Table (5), the dimensionless deflection at the centre of plate is compared between FEM, multi-term extended Kantorovich method (MTEKM) and NKQ method under different levels of load (patch out-of-plane load) [16]. As shown, the NKQ results generally show a reasonable agreement with FEM. Semi-analytical models (MTEKM and NKQ) illustrate less than 3% error. The structure is a square plate with CFCC boundary conditions. The angle-ply laminated plate has four symmetric layers [45, -45].

Table 5. Dimensionless W/h for CFCC square laminated plate under different loads

Q	MTEKM (W)[16]	NKQ (W)	ABAQUS	%MTEKM Error[16]	%NKQ Error
2488	.746	.741	.763	2.23	2.88
4975	1.092	1.090	1.115	2.05	2.24
7463	1.328	1.325	1.335	2.02	1.50
9950	1.512	1.559	1.541	1.81	1.16
12438	1.667	1.667	1.695	1.66	1.66
14925	1.800	1.802	1.827	1.54	1.36
17413	1.919	1.913	1.947	1.43	1.74
19900	2.027	2.025	2.053	1.33	1.36

In the next example a square laminated plate with a length of 0.5m under uniform loading is considered. The plate is clamped at one side and the deformation at the other edge is measured. Because of the different lay-ups (anisotropic) and out-of-plane loading there is an induced twist at the free edge of the plate. The aim of this example is to find out if the NKQ model can estimate this induced twist. The results are shown for an experiment, FEM and NKQ method in Figure (5). Carbon fibre is used for all laminated experimental tests and the size of plate is 500*500 (mm). The experimental test results which are shown in Figure 4 are the average results of six identical plates under the constant load.

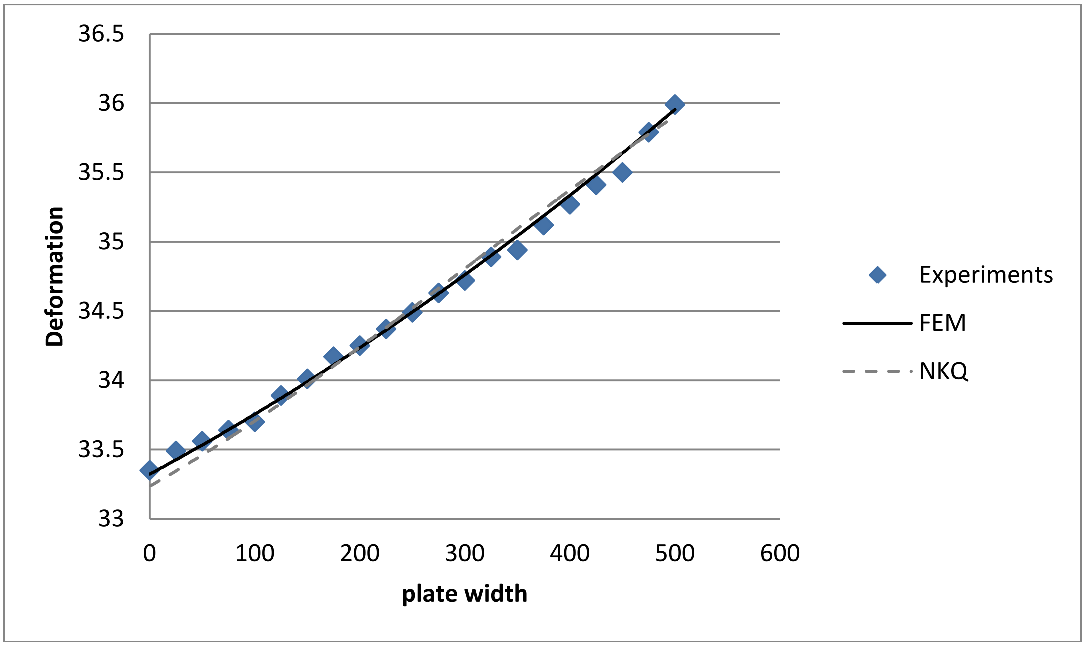

Fig4. Deformation at free edge of anisotropic laminated plate

5. Conclusion

The constitutive equations of the laminated composite plates are non-linear. The semi-analytical non-linear methods are an essential tool that provides perception to the physical non-linear behaviour of the composite plate structure, present fast and reliable solutions during the preliminary design phase and also provide a means of validations the numerical methods and enable the development of new computational models. In this paper a semi-analytical approach for the analysis of laminated plates with general boundary conditions and distribution of loads is proposed. The non-linear equations are solved by Newton-Kantorovich-Quadrature (NKQ) method. This method breaks down the laminated composite plats equations to series of sequential equations and attempts to solve iterative linear integral equations. The convergence of the proposed method is compared with other semi-analytical methods (EKM and MTEKM). Various numerical examples with different boundary conditions and loadings are studied. Good agreement between the NKQ model, FEM and experimental results are shown to validate the model.

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FE Design Tool for Laminated Composite Plates

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Abstract. Considering the non-linearity, complexity and anisotropy of constitutive equations in composite materials, numerical methods are essential to evaluate the behaviour of this material. The finite element method (FEM) is a powerful computational technique for the solution of differential and integral equations that arise in various fields of engineering and applied science such as composite materials. Here, an FEM tool is designed to analyse non-linearity in the behaviour of composites caused by the effect of transverse shear and twist in laminated composite plates. The tool is established by using FEM for composites in ABAQUS combined with programming in Python to run the tests for all possible fibre orientations in laminated composite plates. It is shown that the tool has the ability to design laminated composite plates by considering the effect of transverse shear and the tool's output provides results for all different fibre orientations. It is demonstrated that there is good agreement between numerical results obtained from this tool and experimental results. The advantages of the tool give designers the opportunity to use this tool for wide range of products.

Introduction

Composite materials have been used in a wide range of products and industries due to their excellent mechanical properties. Various studies have been carried out to analyse composite materials. Reddy [1] presented an overview of the literature on finite element (FE) modelling of laminated composite plates up to 1985. Kapania [2] gave a review of studies up to 1989 on the analysis of laminated shells. Engblom et al. [3] presented a model in which shear deformation is considered in a parametric plate and shell element with respect to the shear effects by allowing mid surface displacements to be independent of the rotations. The model was based on eight-noded quadrilateral geometries with four corner nodes and four midside nodes located at the mid surface of the element with six degrees of freedom per node. Wu and Kuo [4] presented a method based on the local higher-order lamination theory to analyse thick laminated composite plates. In their theory, the displacement continuity at the interface between layers was described by considering the potential energy functional of the laminates and applying Lagrange multipliers. Noor et al. [5] presented a computational model based on first order shear deformation theory (FSDT) for accurate determination of transverse shear stresses and their sensitivity coefficients in laminated composite panels for mechanical loads. Bose and Reddy [6] presented finite element methods (FEM) of various shear deformation theories for the evaluation of composite plates and also compared transverse displacements and through the thickness distributions of transverse stresses and in-plane stress. Another model which decreases the order of differentiation by one as compared to the standard equilibrium approach was shown by Rolfes et al. [7]. The model needed only quadratic shape functions for analysing the required derivatives at the element level and also the computational effort was low since it must only have stiffness continuity in shape functions in the FE code. Recently, development of laminated composite plate theories was reviewed by Khandan et al. [8].

Shell elements generally have been used to model structures in which one dimension (the thickness) is significantly smaller than the other dimensions. In ABAQUS 6.10-1 (SIMULIA), general three-dimensional shell elements are available with three different formulations: general-purpose, thin-only, and thick-only. Here, S9R5 elements in ABAQUS are used which means the element has nine nodes and five degrees of freedom. It is important to consider that any numerical or computational method is a way to analyse a practical engineering problem and that analysis is not an end in itself but rather an aid to design. This is the reason, the FE methods in ABAQUS is used in this work to present a laminated composite design tool.

Governing Equations

Classical laminate plate theory (CLPT) was the first model to analyse composites. However, it is not able to explain transverse shear effect in laminated composites. In the classical laminate theory, it is assumed that straight lines normal to the middle surface remain straight and normal to that surface after deformation. These assumptions are not valid in the case of thicker laminates and laminates with low stiffness central plies undergoing significant transverse shear deformations. By considering the transverse shear effect, the complexity and non-linearity of constitutive equations for composites increase significantly and therefore using a numerical method such as FEM to solve these complicated equations is necessary. In order to consider the effect of transverse shear, the following equations (Eq. 1-3) are considered (Q_{ij} can be found in references [9,10]):

$$\begin{bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} dz$$

$$\begin{bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} zdz \qquad (1)$$

$$\begin{bmatrix} V_q \\ V_r \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \tau_{xz} \\ \tau_{yz} \end{bmatrix} dz$$

$$\begin{bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \\ V_q \\ V_r \end{bmatrix} = \begin{bmatrix} N_1 \\ N_2 \\ N_{12} \\ M_1 \\ M_2 \\ M_{12} \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} & 0 & 0 \\ A_{12} & A_{22} & A_{23} & B_{12} & B_{22} & B_{23} & 0 & 0 \\ A_{13} & A_{23} & A_{33} & B_{13} & B_{23} & B_{33} & 0 & 0 \\ B_{11} & B_{12} & B_{13} & D_{11} & D_{12} & D_{13} & 0 & 0 \\ B_{12} & B_{22} & B_{23} & D_{12} & D_{22} & D_{23} & 0 & 0 \\ B_{13} & B_{23} & B_{33} & D_{13} & D_{23} & D_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & E_{11} & E_{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & E_{12} & E_{22} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \\ k_1 \\ k_2 \\ k_{12} \\ \gamma_{13} \\ \gamma_{23} \end{bmatrix} \qquad (2)$$

where the components of this stiffness matrix are given by:

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-h/2}^{h/2} \overline{Q}_{ij}^{m} (1, z, z^2) dz \quad (i, j = 1,2,3)$$
$$E_{ij} = \int_{-h/2}^{h/2} \overline{Q}_{\alpha\beta}^{m} k_i k_j dz \quad (i, j = 1,2 \; and \; \alpha, \beta = i + 4, j + 4). \tag{3}$$

Laminated Composite Tool

The results for different fibre orientations in each layer of the laminated composite plate are needed and can be helpful for designers in order to evaluate potential acceptance cases and choose the best option, depending on the final product requirements. Therefore, it is essential in ABAQUS to change the fibre orientations by small steps in each layer. It is very time-consuming manual process and as a number of layers increases, it becomes almost impossible to perform manually. A Python programming script was written for this purpose. After writing the Python program to run iteratively for a laminated composite plate for all different possible fibre orientations, the program is expanded and modified in order to be able to use as a design tool for laminated composite plate. The graphical user interface of this tool is shown in Fig. 1.

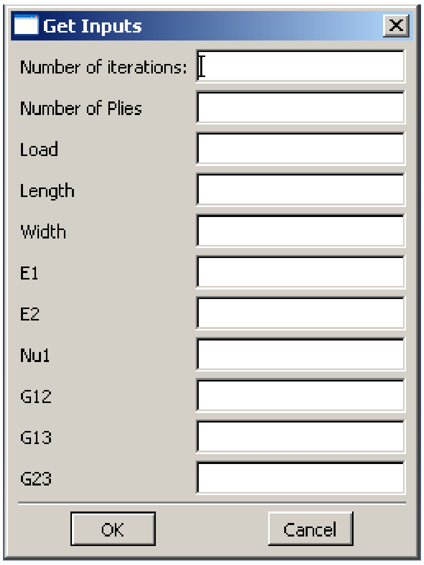

Fig 1. Laminated composite plate tool.

As it is shown in Fig. 1, the inputs are the number of iterations required, number of plies, load, geometry of the plate (width and length) and material properties (E1, E2, Nu1, G12, G13, G23). The material properties and geometry are entered directly in the design tool and the loads and boundary conditions must be defined in ABAQUS. The output of this tool is a database of ABAQUS results (stress, strain, etc) for certain all possible fibre orientations.

Experimental Tests

In order to validate the results of this design tool, some experimental tests were performed. For each layup six laminated plates with identical geometry were manufactured. Carbon fibre with material properties of E1=40.91 GPa, E2=9.88 GPa, Nu1=0.292 G12=2.84 GPa, G13=1.13 GPa and G23=0.91 GPa was used for all laminated experimental tests in this work. Square laminated composite panels (500 x

500 mm) were produced by vacuum bagging. In order to minimise the error, the average results of these six plates were calculated to be compared with FEM results obtained from the design tool. As it is shown in Fig. 2, the plates are fixed on one side (a) and displacement in z direction is measured on the opposite free side of the plate (b) due to the effect of gravity only.

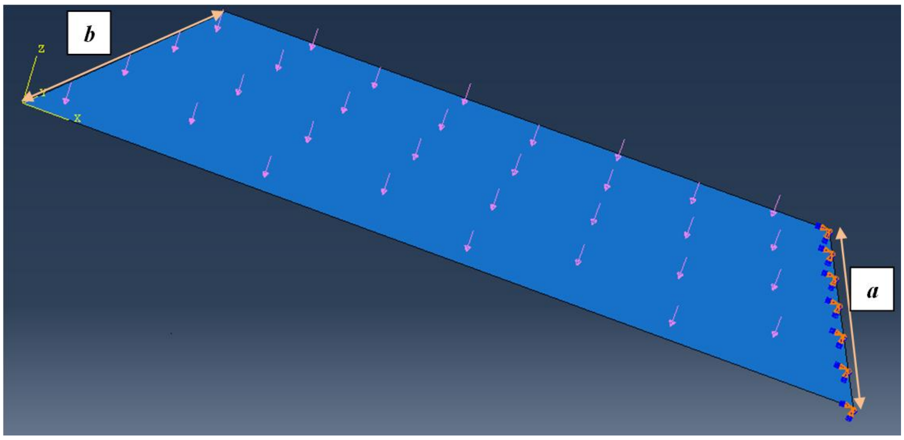

Fig 2. Boundary condition and loading (the plate is clamped on side *a*. Displacements are measured on side *b*).

An example of the results obtained from a plate with four layers and fibre orientation of [0/45/0/90] is shown in Fig. 3. The results of five other random samples with varying fibre orientation layups are shown in Fig. 4. It is observed that for all the samples, the errors between the experimental and FEM displacements were less than 2%.

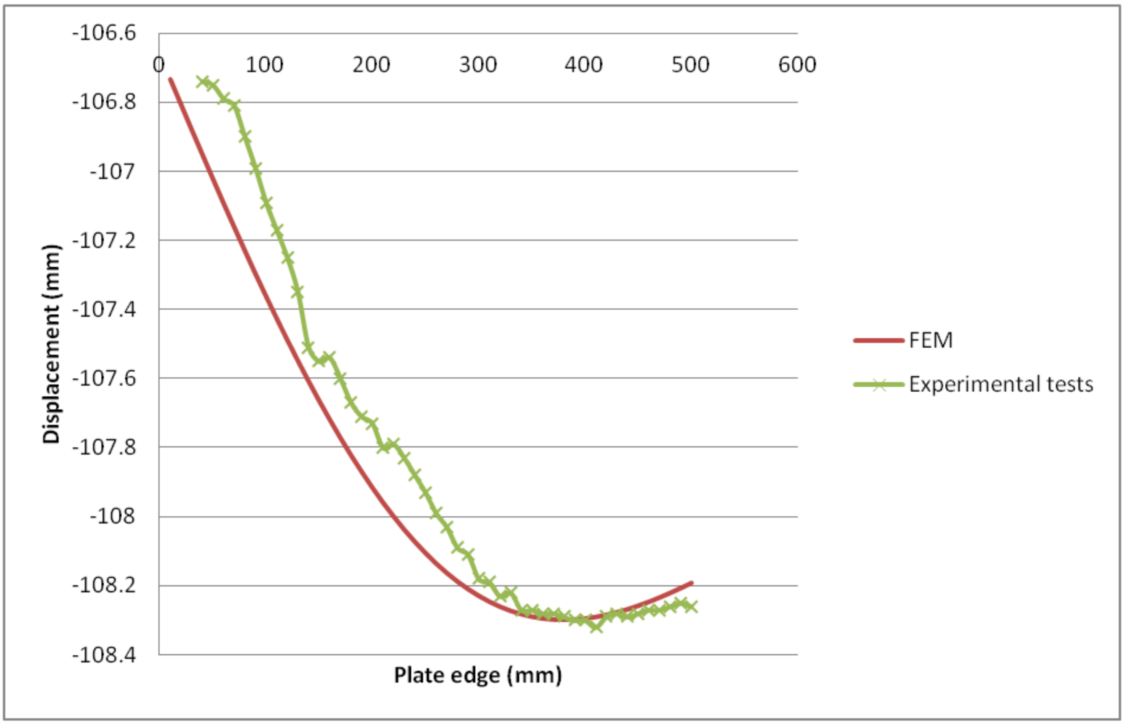

Fig 3. Displacement for the free edge of [0/45/0/90] plate with FEM and experiment.

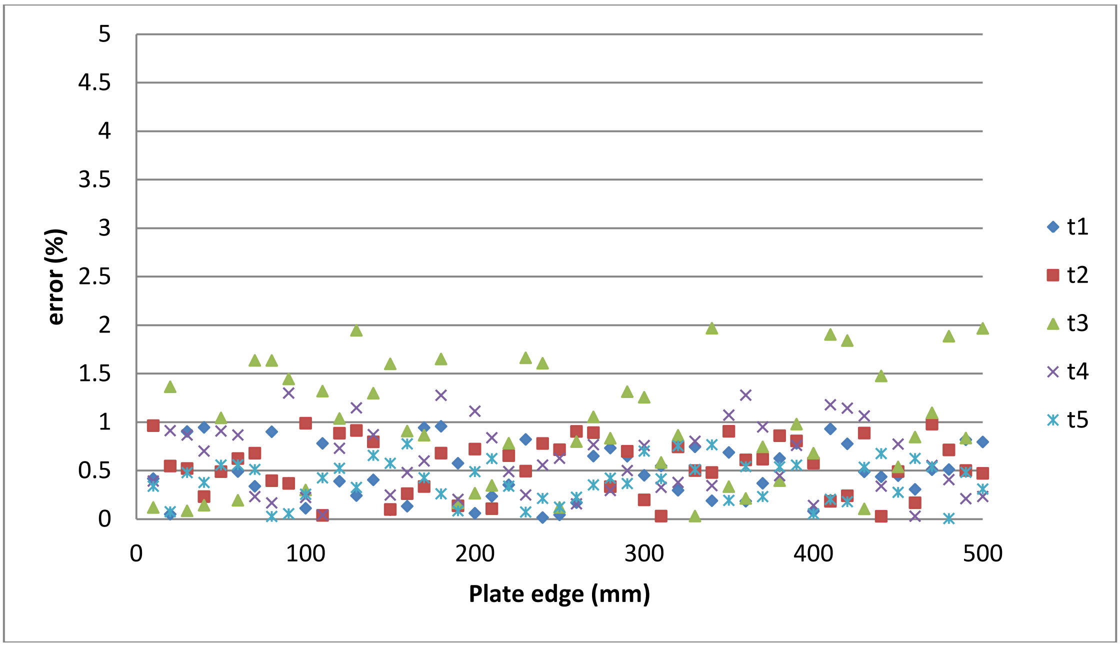

Fig 4. Percentage error between FEM and experimental tests (t1-t5).

Conclusion

The aim of this work is to present a design tool to evaluate laminated composite plates for different fibre orientations. Here, the finite element method is used to solve non-linear equations of composite materials by considering the effect of transverse shear in a laminated composite plate. In order to validate the FE model and the output of the design tool, a series of experimental tests has been performed and there is a good agreement between numerical and experimental results. It is shown that there is less than two percent error between the result of the designed composite tool and experimental results. The tool has the ability to accept material properties, geometries of the plate and loads as an input. The output is a database which contains the results of FEA for all possible fibre orientations and can potentially link with any optimisation technique to optimise the composite for load, geometry, thickness, induced twist or cost, depending on the final product's function.

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Overview of Abaqus

The interaction with Abaqus is through the main window and the look of the window change as you work through the modelling process. Figure (1) shows the components that appear in the main window (Abaqus 6.10).

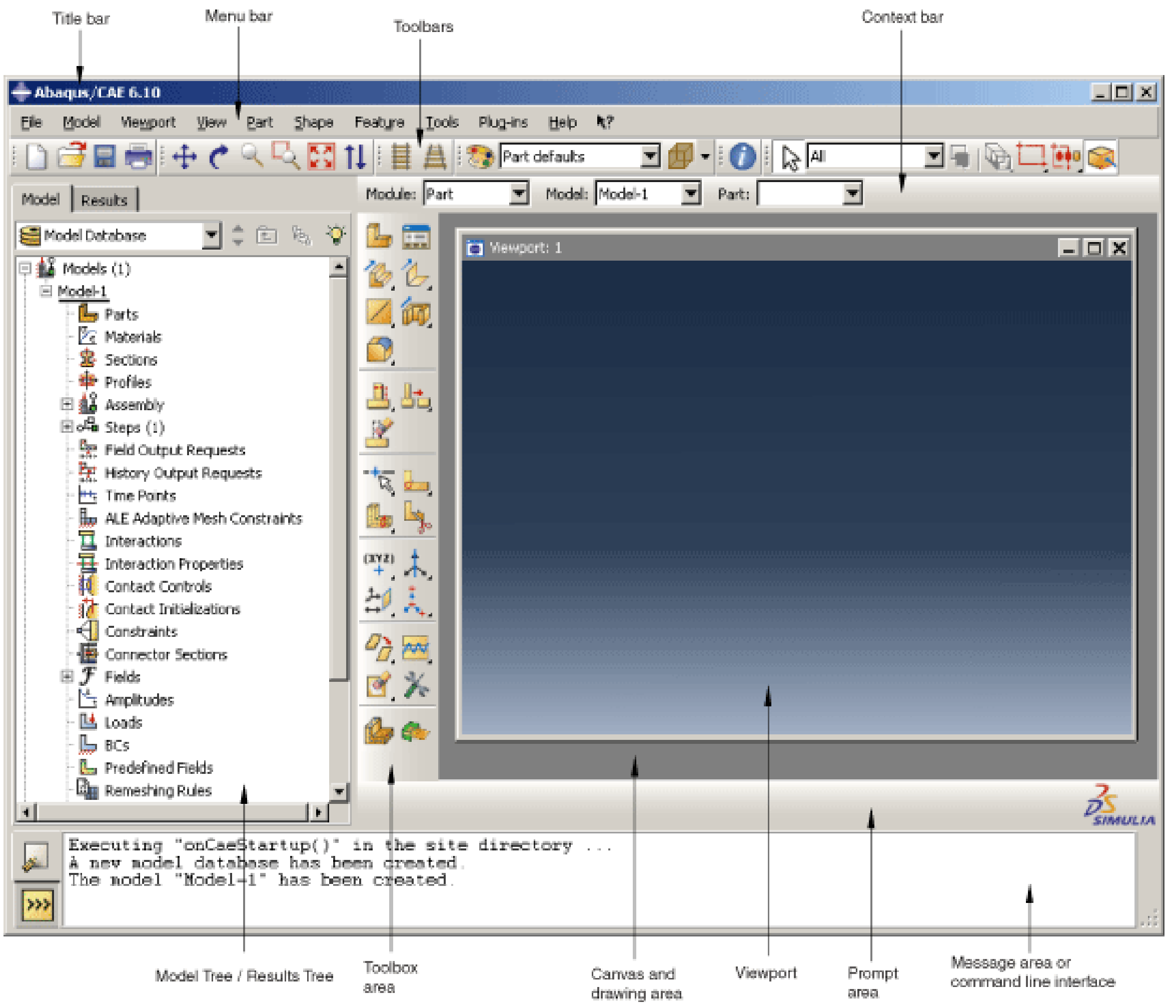

Figure 1 Components of the main window (Abaqus 6.10)

Menu bar

The menu bar includes all the possible menus; the menus give access to all the functionality in the product. Different menus appear in the menu bar depending on which module is selected by user from the context bar.

Toolbars

The toolbars is for quick access to items that are also available in the menus.

Model Tree

The Model Tree makes using Abaqus more user friendly, as it provides a graphical overview of the model and the objects that it contains, such as materials, parts, loads, steps and output requests. The Model Tree also offers a convenient, tool for moving between modules and for managing objects.

Results Tree

The Results Tree gives users a graphical overview of the output databases and other session-specific data such as X–Y plots. Especially if there is more than one output database open in the session, user caneasily use the Results Tree to move between output databases.

Toolbox area

When user enters a module, the toolbox area displays tools in the toolbox which are appropriate and relevant for that specific module. The toolbox allows quick access to many of the module functions. These functions are also available from the menu bar.

Viewport

Viewports are windows which Abaqus displays your model.

Command line interface

Users can write the command in command line interface with type Python, which will be explained later, to customise the existence model.

Modelling

The Model Tree is convenient tool for searching and controlling the models. The Model Tree provides a visual description of items in a model. For example, Figure (2) shows the Model Tree in one case.

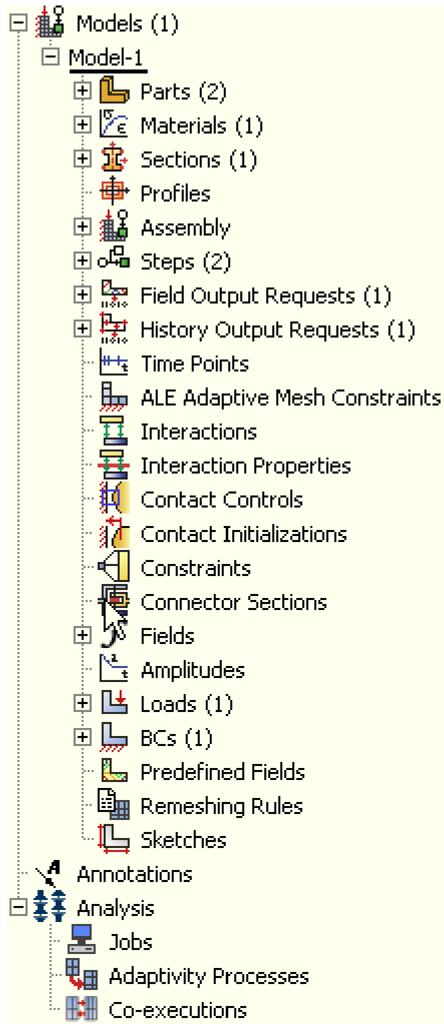

Figure 2 The Model Tree

In the Parts container we draw the model and all parts of it. In our case it is a simple panel. In Loads and BCs containers, the Force/Loads and boundary conditions and their types are defined respectively. In this research tests, loads are uniform and the square laminated composite panel fixed (clamped) in one side. In Material container the materials, laminate parameters such as number of layers, fibre orientations and thickness of each layer are defined which will be explained in the following sections.

Worth to mention, that Abaqus has no built-in system of units. The consistent units that are used in Abaqus are defined in Table (1).

Table 1 Consistent units

Quantity	SI	SI (mm)	US Unit (ft)	US Unit (inch)
Length	m	mm	ft	in
Force	N	N	lbf	lbf
Mass	kg	tonne (103 kg)	slug	lbf s2/in
Time	s	s	s	s
Stress	Pa (N/m2)	MPa (N/mm2)	lbf/ft2	psi (lbf/in2)
Energy	J	mJ (10–3 J)	ft lbf	in lbf
Density	kg/m3	tonne/mm3	slug/ft3	lbf s2/in4

Mesh/Ellement

A wide range of elements is available in Abaqus. This extensive element library provides a powerful set of tools for solving many different problems. Each element in Abaqus has a unique name, such as T2D2, S4R, or C3D8I. The element name identifies each aspects of an element. Figure (3) shows the element families most commonly used in a stress analysis. One of the major distinctions between different element families is the geometry type that each family assumes. For composite material analysis, we used shell element in this research which is the most common element to analyse the laminated composite plates.

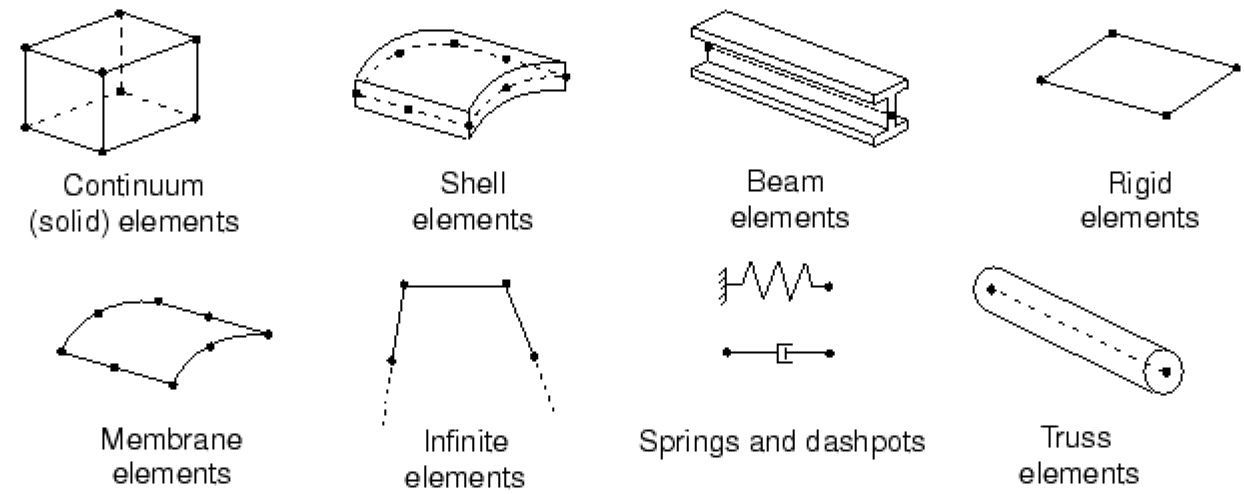

Figure 3 Commonly used element families (Abaqus 6.10)

The degrees of freedom (dof) are the fundamental variables calculated during the analysis. For a stress/displacement simulation the degrees of freedom are the translations at each node. Some element families, such as the beam and shell families, have rotational degrees of freedom as well.

Temperatures, rotations, displacements and the other degrees of freedom are calculated only at the nodes of the element. At any other point in the element, the displacements are acheived by interpolating from the nodal displacements. Usually the interpolation order is recognised by the number of nodes used in the element. Three different types of element are shown in Figure (4).

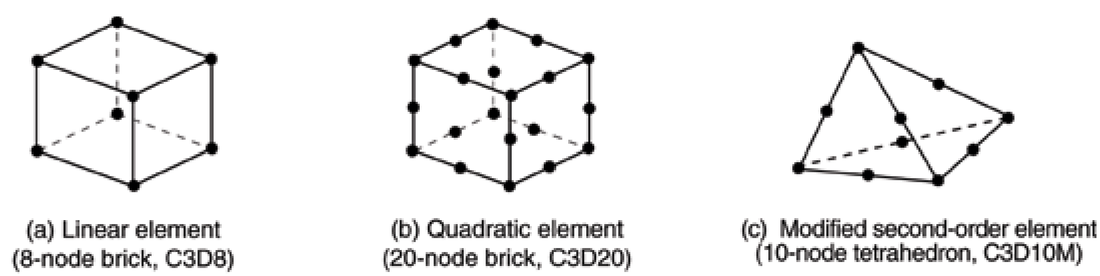

Figure 4 Linear brick, quadratic brick, and modified tetrahedral elements.

Shell elements are used to model structures in which one dimension (the thickness) is significantly smaller than the other dimensions. In Abaqus general three-dimensional shell elements are available with three different formulations: general-purpose, thin-only, and thick-only. The general-purpose shells and the axisymmetric shells with asymmetric deformation account for finite membrane strains and arbitrarily large rotations. The three-dimensional "thick" and "thin" element types provide for arbitrarily large rotations but only small strains. Table (2) summarises the shell elements available in Abaqus.

Table 2 Three classes of shell elements in Abaqus

General-Purpose Shells	**S4, S4R, S3/S3R, SAX1, SAX2, SAX2T, SC6R, SC8R**
Thin-Only Shells	STRI3, STRI65, S4R5, S8R5, S9R5, SAXA
Thick-Only Shells	S8R, S8RT

The three-dimensional elements in Abaqus whose names end in the number "5" (e.g., S4R5, STRI65) have 5 degrees of freedom at each node: three translations and two in-plane rotations (i.e., no rotations about the shell normal). However, all six degrees of freedom are activated at a node if required; for example, if rotational boundary conditions are applied or if the node is on a fold line of the shell. Here, we used S4R5 and S9R5 elements.

Composite layups

This section provides information on modelling composite layups with Abaqus. Composite layups in Abaqus are designed to help you manage a large number of plies in a typical composite model. Composite sections are a product of finite element analysis and may be difficult to apply to a real-world application. Unless your model is relatively simple and all plies cover the same region, you will find it increasingly difficult to define your model using composite sections as you increase the number of plies. It can also be cumbersome to add new plies or remove or reposition existing plies. We start with a basic shape and then we add plies of different materials and thicknesses to selected regions and orient the plies to provide the greatest strength in a particular direction. The Abaqus composite layup editor allows you to easily add a ply, choose the region to which it is applied, specify its material properties, and define its orientation. We can also read the definition of the plies in a layup from the data in a text file. This is convenient when the data are stored in a spreadsheet or were generated by a third-party tool.

It is important to specify the correct orientation of the fibres in a ply. Abaqus allows us to define a reference orientation for the layup as well as a reference orientation for each ply in the layup. In addition, we can specify the direction of the fibres in a ply relative to the reference orientation of the ply. By default, the coordinate system of a layup is the same as the coordinate system of the part; similarly, the coordinate system of a ply is the same as the coordinate system of the layup.

Figure (5) shows the Edit Composite Layup window in Abaqus. We can control and edit the ply name, region, material, thickness for each layer, coordinate system, fibre rotation angle (fibre orientation) and integration points.

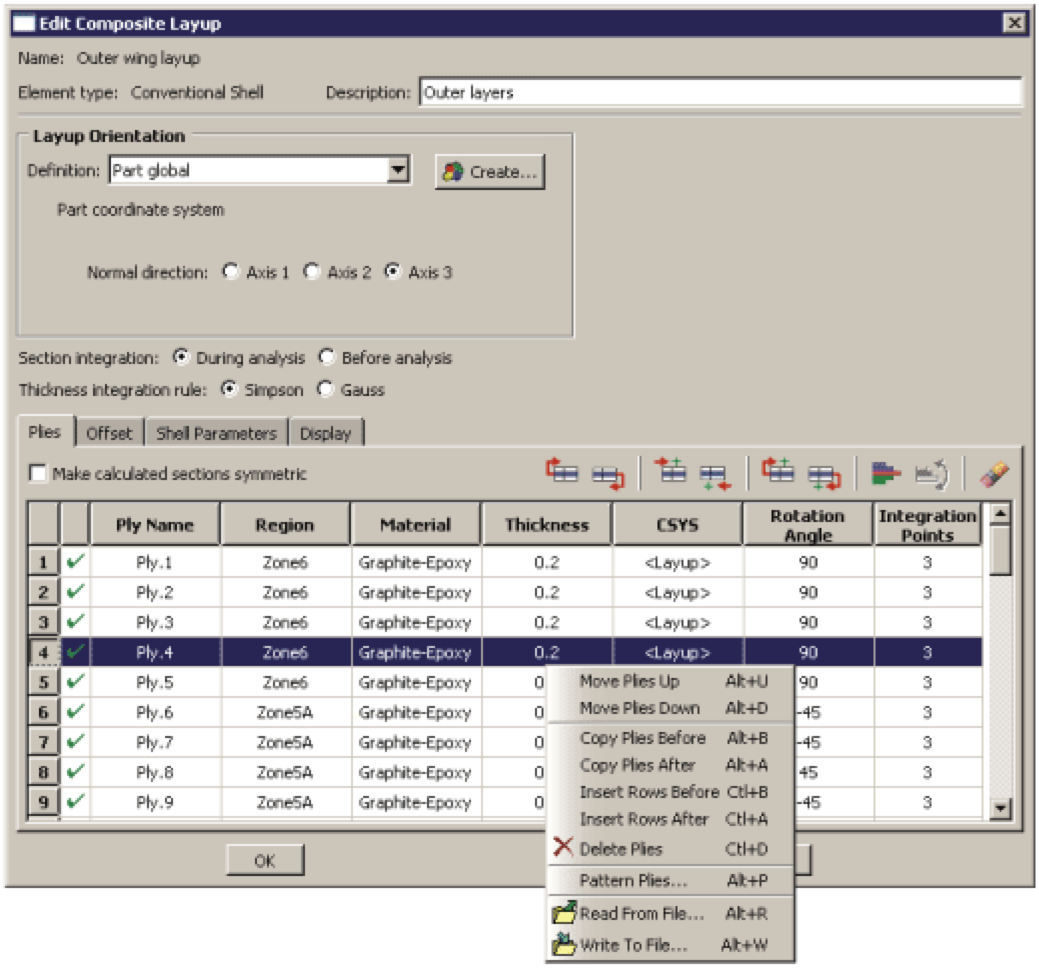

Figure 5 The ply table in the composite layup editor.

Region

Select the region to which the ply is assigned. We can choose faces from the viewport, or can choose a set that refers to a face.

Material

Materials can be chosen for each ply. It can be selected from Edit Material from the menu that appears, and select the desired material from the list of available materials or create a new material.

Depend on the type of composite we consider, Laminae, orthotropic or anisotropic, we should enter 6, 9 or 21 material properties (Figure 6).

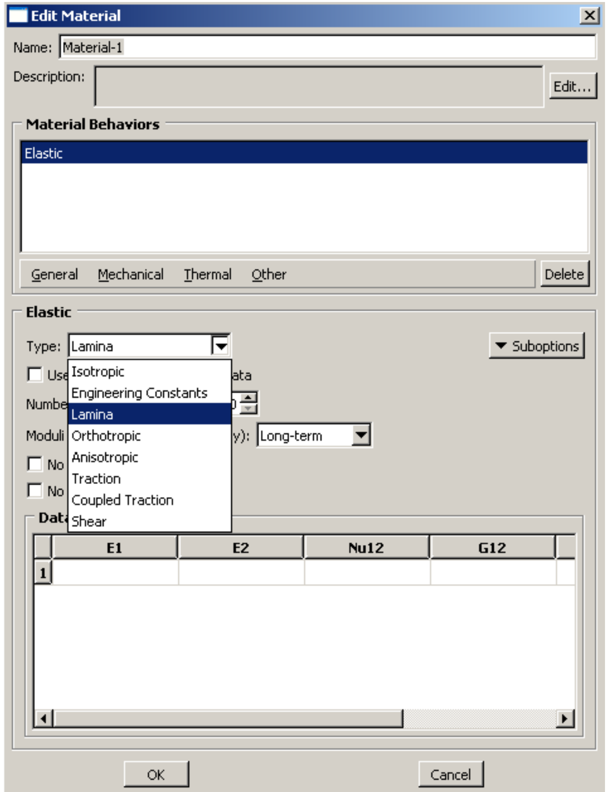

Figure 6 Input material properties

Rotation Angle

It describes the rotation (positive clockwise about the normal direction) for the ply's reference orientation.

Python

Python is a powerful programming language. It has efficient high-level data structures and a simple but effective approach to object-oriented programming. Rather than needing all desired functionality to be completely built into the language's core, Python is designed to be highly extensible. Also, built-in modules can be written in C or C++. Python can also be used as an extension language for existing modules and applications that require a programmable interface. The following chart shows the Abaqus scripting interface commands (Figure 1).

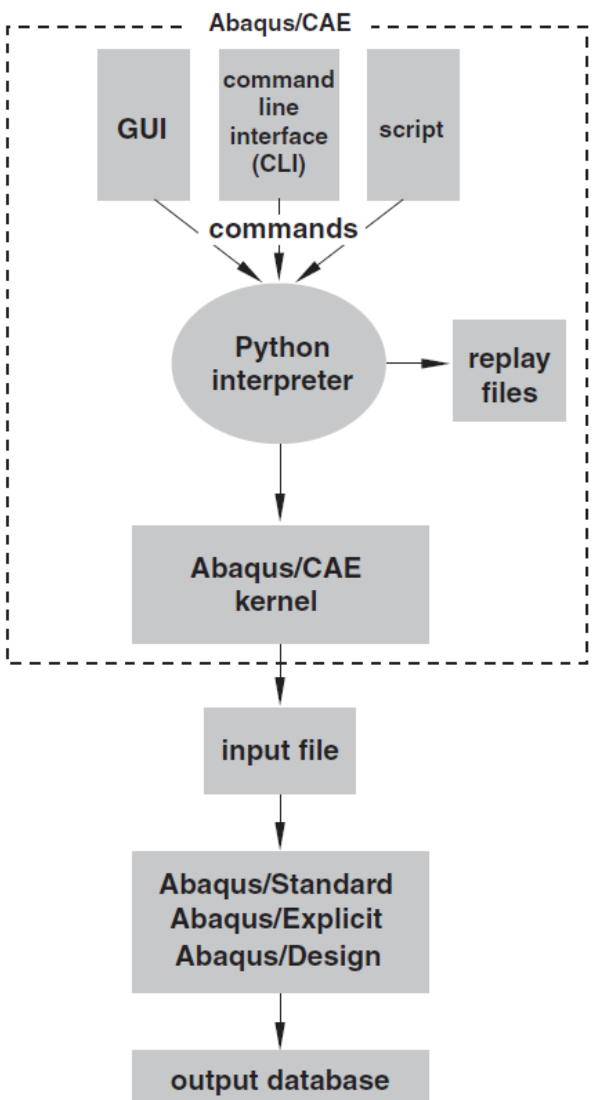

Figure 1 Abaqus Scripting Interface commands

The program which is written to link with Abaqus and run the tests iteratively is presented in appendix (B). It is worth to mention some key points about how to work with Python and some concepts of it.

Traditional procedural languages, such as C and FORTRAN, are based around sub-routines or functions that perform actions. Unlike them, object-oriented programming languages, such as Python and C++, are based around objects. An object gathers and summarises some data and functions that are used to manipulate those data. These data are called the members of the object. The functions which manipulate the data are called methods. Modelling the objects can be varying from a tree (real- world problem) to an array of nodes (abstract data from a tree object such as its height, diameter and age).

Running a script

A script can be run when you start an Abaqus session by typing the following command:

abaqus cae script=myscript.py

"myscript.py" is the name of the file which the script is written by that. The script can also be run selecting *File→Run Script* from the main menu bar in Abaqus. Abaqus shows the *Run Script* dialog box, and you choose the file containing the script.

Python data types

Integer:

An integer is based on a C program and also can be compared to a FORTRAN integer. For extremely large integer values, user should state a long integer. The size of a long integer is unlimited and depends on the integer which is used. The "L" at the end of the number shows that it is a long integer.

Float:

Floats represent floating-point numbers or real numbers. It can be used exponential notation for floats.

Complex:

A complex number has two members, the real member and the imaginary member respectively. Complex numbers use the “j” notation to indicate the imaginary part of the number.

Sequences:

Sequences include strings, lists, tuples, and arrays.

Loop

A Python script can include a loop. The start and end of a loop is controlled by indentation in the script. It is a general and fundamental rule for Python that should be considered by users. Start and end of each loop and function is controlled by indentation in script. Unlike other program, Python is not using a special character, such as “}”, to signify the end of a control block such as an *If* statement. Instead, Python uses indentation to indicate the end of a control block. Users define the indentation that governs a block and should be really careful with that. When your script returns to the original indentation, the block ends. Like other language programs, Python has commands such as *while, break, if, for* and *elif, else* to write and control loops.

Abaqus object model

The object model is a concept in object-oriented programming such as Python. The object model provides:

- A definition of each Abaqus Scripting Interface object which are containing the methods and data members.
- Definitions of the relationships and functions between different objects. These relationships and functions form the structure of the object model.

Container/ Singular object

A Container is an object that takes in objects of a similar type. Objects that are not containers are described as a Singular object. A singular object includes no other objects of a similar type.

Session/ Mdb/ Odb

Session objects are objects that are not saved between Abaqus sessions; for example, the objects that define user-defined views, remote queues and viewports, as shown in Figure (2). The statement from Abaqus imports an instance of the Mdb object. Mdb objects are objects which are saved in a model database and can be recovered between Abaqus sessions. Mdb objects contain the Job object and the Model object (Figure 3). .odb objects are saved in an output database and include both results and model data, as shown in Figure (4).

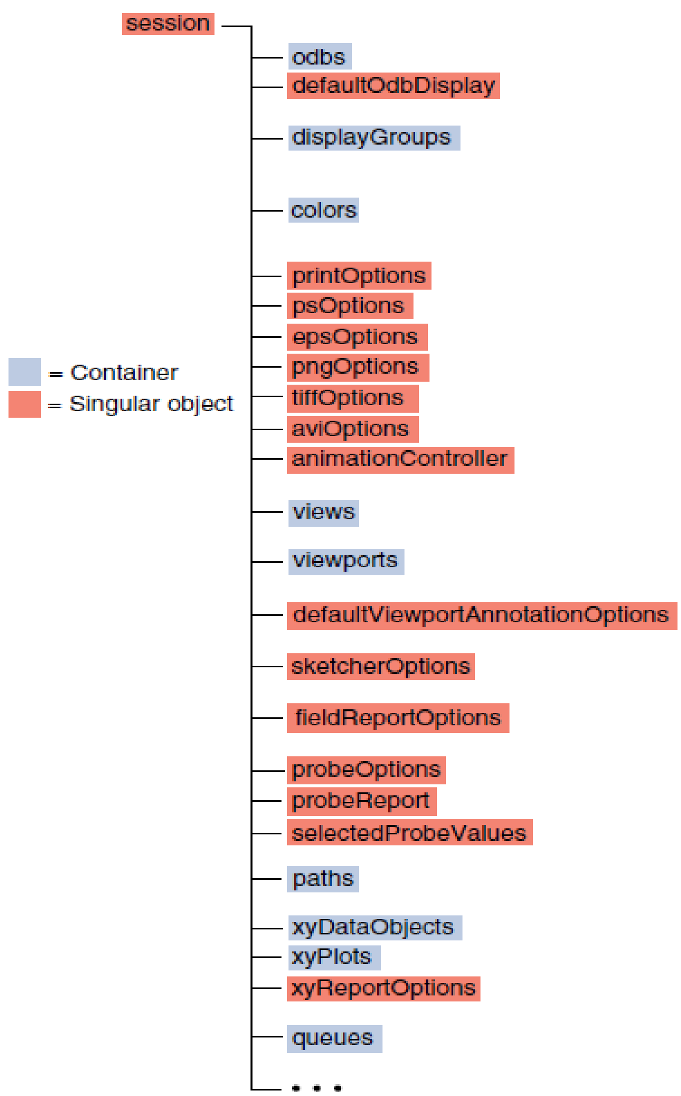

Figure 2 The Session object model

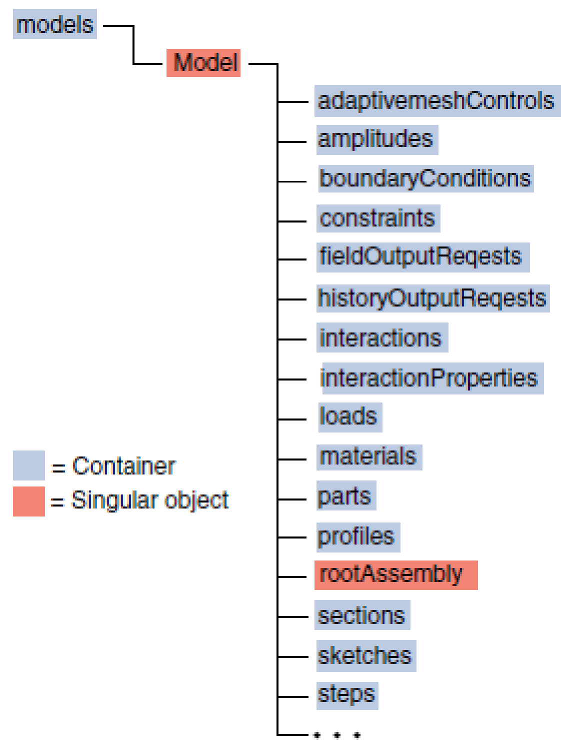

Figure 3 The Mdb object model

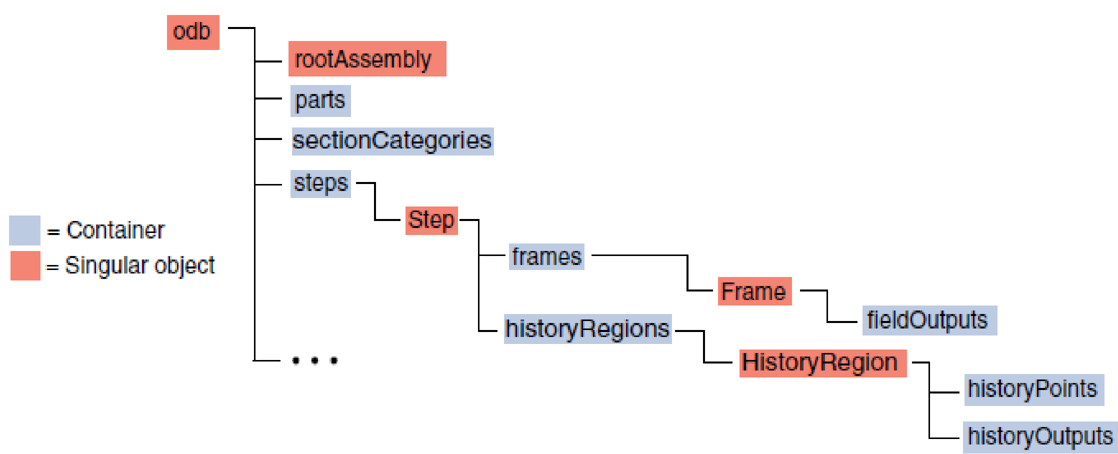

Figure 4 The Odb object model

Composite Processing

There are many different material options to choose as composite materials. From fibres, cores and resins, which each one has its own unique set of properties such as stiffness, toughness, strength, heat resistance, production, rate cost, etc. However, the final properties of a composite part produced from these different materials are not only a function of the individual properties of the resin matrix and fibre it is also dependent on the way that the materials themselves are designed into the products and the way that they are processed. A number of the commonly used composite production methods are compared. The advantages and disadvantages of each process are classified and sorted. By considering these points the production method is selected and the tests are designed.

Spray Lay-up

Fibre is chopped in a hand-held gun and fed into a spray of catalysed resin directed at the mould. Then the materials are left to cure under standard atmospheric conditions. The resin is normally polyester in this process and the fibres are glass roving. The main advantages of this method are, widely used for many years, low cost in tooling depositing fibre and resin. The disadvantage of spray lay-up are, only short fibres are incorporated and laminates tend to be very resin-rich and therefore heavy, which limits the mechanical properties of the laminate, the high styrene contents of spray lay-up resins means that they are potentially harmful and their lower viscosity means that they can penetrate in clothing etc and finally resins need to be low viscosity to be sprayable. This really compromises their mechanical thermal properties. This method normally is used to fabricate products such as bathtubs, simple enclosures, shower trays and some small dinghies.

Wet Lay-up/Hand Lay-up

In this process resins are soaked into fibres which can be in the form of knitted, unidirectional, woven or bonded fabrics, by hand. This is normally have been done by rollers or brushes, with a special type of roller (nip-roller) to impregnate resin into the fabrics better. Here, like spray Lay-up process, laminates are left to cure under standard atmospheric conditions.

In this process any kind of resins and fibres can be used. However, Aramid fabrics are not been suggested as they can be hard to wet-out by hand. This method is widely being used in the production of boats, standard wind-turbine blades and architectural mouldings. Advantage and disadvantage of this method is listed in Table (1).

Table 1 advantage and disadvantage of Hand lay-up method

Advantage	• **Simple principles to teach** • **Widely used for many years** • **Low cost tooling** • **Wide choice of suppliers and material types** • **Higher fibre contents, and longer fibres than with spray lay-up**
Disadvantage	• Resin mixing, laminate resin contents, and laminate quality are very dependent on the skills of laminators. Low resin content laminates cannot usually be achieved without the incorporation of excessive quantities of voids. • Health and safety considerations of resins. The lower molecular weight of hand lay-up resins generally means that they have the potential to be more harmful than higher molecular weight products. The lower viscosity of the resins also means that they have an increased tendency to penetrate clothing etc. • Limiting airborne styrene concentrations to legislated levels from polyesters and vinylesters is becoming increasingly hard without expensive extraction systems. • Resins need to be low in viscosity to be workable by hand. This generally compromises their mechanical/thermal properties due to the need for high styrene levels.

Vacuum Bagging

Vacuum bagging is an extension of the wet lay-up process where pressure is applied to the laminate once laid-Up in order to improve its consolidation. This method is used in this research and samples are made by vacuum bagging process. Vacuum bagging is performed by sealing a plastic film over the wet laid-up laminate and onto the tool. Air under the bag is extracted by a vacuum pump and thus up to one atmosphere of pressure is applied to the laminate to consolidate it. By applying this process, the laminate cures relatively faster and also the resin spread in the laminate almost equally. In this process, extra resin will be absorbed from the laminate, therefore we have more accurate ratio of fibre to resin. The main applications are, race car components, core-bonding in production boats and large one-off cruising boats. Advantage and disadvantage of this method is listed in Table (2).

Table 2 advantage and disadvantage of vacuum bagging method

Advantage	• **Higher fibre content laminates can usually be achieved than with standard wet lay-up techniques.** • **Lower void contents are achieved than with wet lay-up.** • **Belter fibre wet-out due to pressure and resin flow into bagging materials.** • **Health and safety: The vacuum bag reduces the amount of volatiles emitted during cure.**
Disadvantage	• The extra process adds cost both in labour and in disposable bagging materials A higher level of skill is required by the operators • Mixing and control of resin content still largely determined by operator skill.

Filament Winding

This process is initially used for hollow and generally circular or oval sectioned components, such as pipes and tanks. The main advantage of this process is that the orientations of the fibre can be controlled very accurately. Fibre tows are passed through a resin bath before being wound onto a mandrel in a variety of orientations, controlled by the fibre feeding mechanism, and rate of rotation of the mandrel. Pipelines, gas cylinders, Storage tanks and fire-fighters breathing tanks are produced with this method.

Resin Transfer Moulding (RTM)

Fabrics are normally laid up as a dry stack of materials. These fabrics are usually prepared and pre-pressed to the mould shape, and held together by a binder. Then, these prepared materials are laid into the mould tool. A second mould tool is then clamped over the first, and resin is injected into the cavity. Sometimes a vacuum is also applied to the mould cavity to help resin in being drawn into the fabrics. This is known as Vacuum Assisted RTM (VARTM) or Vacuum Assisted Resin Injection (VARI). After all the fabric is wet out, the resin inlets have been closed, the laminates are allowed to cure. Cure and injection often happen at ambient or elevated temperature. Train seats and small complex aircraft components are normally made with RTM.

Prepregs

Fabrics and fibres are pre-impregnated under heat and pressure or with solvent, with a pre-catalysed resin by the materials manufacturer. The materials are normally stored frozen. The reason for that is the resin is usually a near-solid at ambient temperatures, so the pre-impregnated materials (Prepregs) have a light sticky feel to them, such as that of adhesive tape. It is in spite of the fact that the catalyst itself is largely latent at ambient temperatures and giving the materials several weeks (even months) of useful life when defrosted. The prepregs are laid up by machine or hand onto a mould surface, vacuum bagged and then heated to typically 120-180 Celsius. This allows the resin to reflow and eventually to cure. Addttional pressure for the moulding is usually provided by an autoclave (effectively a pressurised oven) which can apply up to 5 atmospheres to the laminate. F1 racing cars, aircraft structural components (e.g. wings and tail sections), sporting goods such as tennis racquets and skis are produced by this method.

In Tables (3) to (5) the advantage and disadvantage of filament winding, resin transfer moulding and prepregs methods are listed. There are some other methods such as pultrusion, low temperature curing prepregs, resin film infusion (RFI) which are not as common as the above methods.

Table 3 advantage and disadvantage of filament winding method

Advantage	• **This can be a very fast and therefore economic method of laying material down. Resin content can be controlled by metering the resin onto each fibre through dies.** • **Fibre cost is minimised since there is no secondary process to convert fibre into fabric prior to use.** • **Structural properties of laminates can be very good since straight fibres can be laid in a complex pattern to match the applied loads**
Disadvantage	• The process is limited to convex shaped components • Fibre cannot easily be laid exactly along the length of a component. • Mandrel costs for large components can be high. • The external surface of the component is unmoulded, and therefore cosmetically unattractive. • Low viscosity resins usually need to be used with their attendant lower mechanical and health and safety properties generally compromises their mechanical/thermal properties due to the need for high styrene levels.

Table 4 advantage and disadvantage of resin transfer moulding method

Advantage	• **High fibre volume laminates can be obtained with very low void contents.** • **Good health and safety, and environmental control due to enclosure of resin.** • **Possible labour reductions.** • **Both sides of the component have a moulded surface.**
Disadvantage	• Matched tooling is expensive and heavy in order to withstand pressures. • Generally limited to smaller components. • Unimpregnated areas can occur resulting in very expensive scrap parts.

Table 5 advantage and disadvantage of prepregs method

Advantage	• **Resin catalisation and the resin content in the fibre is accurately set by the materials manufacturer.** • **High fibre contents can be safely achieved.** • **The materials have excellent health and safety characteristics and are clean to work with.** • **Fibre cost is minimised in unidirectional tapes since there is no secondary process to convert fibre into fabric prior to use.** • **Resin chemistry can be optimised for mechanical and thermal pertormance, with the high viscosity being impregnatable due to the materials manufacturer's process.** • **The extended working times of up to several months at room temperatures means that structurally optimised, complex lay-ups can be readily achieved.** • **Potential for automation and labour saving**
Disadvantage	• Materials cost is higher for preimpregnated fabrics • Autoclaves are usually required to cure the component. These are expensive, slow to operate and limited in size. • Tooling needs to be able to withstand the process temperatures involved • Core materials need to be able to withstand the process temperatures and pressures.

Python program:

Part I.

```
# -*- coding: mbcs -*-
#x=getInputs(((('Number of iterations:\t valid numbers,81,625',''),('Number of
        Plies',''),('Load',''),('Lenght',''),('Width',''),('E1', ''),('E2',''),('Nu1',''),('G12',''),('G13',''),('G23',''),)))
#N=x[0]
```

Part II.

```
import sys,math
import visualization
from abaqus import *
from abaqus import getInput
from abaqusConstants import *
from part import *
from material import *
from section import *
from assembly import *
from step import *
from interaction import *
from load import *
from mesh import *
from job import *
from sketch import *
from visualization import *
from connectorBehavior import *
import regionToolset
from odbAccess import *
```

Part III.

```
mdb.models['Model-1'].ConstrainedSketch(name='__profile__', sheetSize=200.0)
mdb.models['Model-1'].sketches['__profile__'].rectangle(point1=(0.0, 0.0),
   point2=(500.0, 500.0))
mdb.models['Model-1'].Part(dimensionality=THREE_D, name='Part-1', type=
   DEFORMABLE_BODY)
mdb.models['Model-1'].parts['Part-1'].BaseShell(sketch=
```

```
    mdb.models['Model-1'].sketches['__profile__'])
del mdb.models['Model-1'].sketches['__profile__']
mdb.models['Model-1'].Material(name='Material-1')
mdb.models['Model-1'].materials['Material-1'].Elastic(table=((
    40910000000.0, 9880000000.0, 0.292, 2840000000.0, 1580000000.0,
    980000000.0), ), type=LAMINA)
mdb.models['Model-1'].parts['Part-1'].CompositeLayup(description='',
    elementType=SHELL, name='CompositeLayup-1', offsetType=MIDDLE_SURFACE,
    symmetric=False, thicknessAssignment=FROM_SECTION)
mdb.models['Model-1'].parts['Part-1'].compositeLayups['CompositeLayup-1'].Section(
    integrationRule=SIMPSON, poissonDefinition=DEFAULT, preIntegrate=OFF,
    temperature=GRADIENT, thicknessType=UNIFORM, useDensity=OFF)
mdb.models['Model-1'].parts['Part-1'].compositeLayups['CompositeLayup-1'].ReferenceOrientation(
    additionalRotationType=ROTATION_NONE, angle=0.0, axis=AXIS_3, fieldName='',
    localCsys=None, orientationType=GLOBAL)
```

Part IV.

```
N=625
n=int (N)
if n==81:
   myInputFile = open('c:/temp/ABAQUS/Number_input_81_-90_0_90.txt','r')
if n==625:
   myInputFile = open('C:\Users\Rasoul\Desktop\Abaqus_Rasoul\Number_input_625_-90_-
        45_0_45_90.txt','r')
```

Part V.

```
v=myInputFile.readlines()
mystring=[]

q=n*4
for a in range (q):
    string=[int(v[a])]
    mystring=mystring+string
#print mystring
myInputFile.close()
```

```
myInputFile2 = open('C:\Users\Rasoul\Desktop\Abaqus_Rasoul\Number_nodes_end.txt','r')
v=myInputFile2.readlines()
mystring2=[]
for a in range (51):
  string2=[int(v[a])]
  mystring2=mystring2+string2
myInputFile.close()
```

Part VI.

```
for i in range(0,625):
    a=mystring[(4*i)]
    b=mystring[(4*i)+1]
    c=mystring[(4*i)+2]
    d=mystring[(4*i)+3]

    mdb.models['Model-1'].parts['Part-1'].compositeLayups['CompositeLayup-1'].CompositePly(
    additionalRotationField='', additionalRotationType=ROTATION_NONE, angle=0.0
    , axis=AXIS_3, material='Material-1', numIntPoints=3, orientationType=
    SPECIFY_ORIENT, orientationValue=a, plyName='Ply-1', region=Region(
    faces=mdb.models['Model-1'].parts['Part-1'].faces.getSequenceFromMask(
    mask=('[#1 ]', ), )), suppressed=False, thickness=0.2, thicknessType=
    SPECIFY_THICKNESS)

    mdb.models['Model-1'].parts['Part-1'].compositeLayups['CompositeLayup-1'].CompositePly(
    additionalRotationField='', additionalRotationType=ROTATION_NONE, angle=0.0
    , axis=AXIS_3, material='Material-1', numIntPoints=3, orientationType=
    SPECIFY_ORIENT, orientationValue=b, plyName='Ply-2', region=Region(
    faces=mdb.models['Model-1'].parts['Part-1'].faces.getSequenceFromMask(
    mask=('[#1 ]', ), )), suppressed=False, thickness=0.2, thicknessType=
    SPECIFY_THICKNESS)
    mdb.models['Model-1'].parts['Part-1'].compositeLayups['CompositeLayup-1'].CompositePly(
    additionalRotationField='', additionalRotationType=ROTATION_NONE, angle=0.0
    , axis=AXIS_3, material='Material-1', numIntPoints=3, orientationType=
    SPECIFY_ORIENT, orientationValue=c, plyName='Ply-3', region=Region(
    faces=mdb.models['Model-1'].parts['Part-1'].faces.getSequenceFromMask(
```

```
mask=('[#1 ]', ), )), suppressed=False, thickness=0.2, thicknessType=
SPECIFY_THICKNESS)

mdb.models['Model-1'].parts['Part-1'].compositeLayups['CompositeLayup-1'].CompositePly(
additionalRotationField='', additionalRotationType=ROTATION_NONE, angle=0.0
, axis=AXIS_3, material='Material-1', numIntPoints=3, orientationType=
SPECIFY_ORIENT, orientationValue=d, plyName='Ply-4', region=Region(
faces=mdb.models['Model-1'].parts['Part-1'].faces.getSequenceFromMask(
mask=('[#1 ]', ), )), suppressed=False, thickness=0.2, thicknessType=
SPECIFY_THICKNESS)


mdb.models['Model-1'].HomogeneousSolidSection(material='Material-1', name='Section-1',
    thickness=None)
mdb.models['Model-1'].StaticStep(description='load', name='Step1', previous='Initial')
mdb.models['Model-1'].rootAssembly.DatumCsysByDefault(CARTESIAN)
mdb.models['Model-1'].rootAssembly.Instance(dependent=ON, name='Part-1-1',
part=mdb.models['Model-1'].parts['Part-1'])
mdb.models['Model-1'].EncastreBC(createStepName='Initial', name='fix', region=
Region(
edges=mdb.models['Model-1'].rootAssembly.instances['Part-1-1'].edges.getSequenceFromMask(
mask=('[#2 ]', ), )))

mdb.models['Model-1'].Pressure(amplitude=UNSET, createStepName='Step1',
distributionType=UNIFORM, field='', magnitude=14.20, name='p', region=Region(
side1Faces=mdb.models['Model-1'].rootAssembly.instances['Part-1-1'].faces.getSequenceFromMask(
mask=('[#1 ]', ), )))
mdb.models['Model-1'].parts['Part-1'].setElementType(elemTypes=(ElemType(
elemCode=S8R5, elemLibrary=STANDARD), ElemType(elemCode=STRI65,
elemLibrary=STANDARD)), regions=(
mdb.models['Model-1'].parts['Part-1'].faces.getSequenceFromMask(('[#1 ]', ), ), ))

mdb.models['Model-1'].parts['Part-1'].seedPart(deviationFactor=0.1, size=10)
mdb.models['Model-1'].parts['Part-1'].generateMesh()
mdb.models['Model-1'].rootAssembly.regenerate()
print (a,b,c,d)
jobName='JOB_%s_%s_%s_%s' % (a,b,c,d)
```

```
print jobName

mdb.Job(atTime=None, contactPrint=OFF, description='', echoPrint=OFF,
explicitPrecision=SINGLE, getMemoryFromAnalysis=True, historyPrint=OFF,
memory=50, memoryUnits=PERCENTAGE, model='Model-1', modelPrint=OFF,
multiprocessingMode=DEFAULT, name=jobName,nodalOutputPrecision=SINGLE,
numCpus=1, queue=None, scratch='', type=ANALYSIS, userSubroutine='',
waitHours=0, waitMinutes=0)

mdb.jobs[jobName].submit()
mdb.jobs[jobName].waitForCompletion()
myOdb=openOdb(path=jobName + '.odb')
myOdb = visualization.openOdb(path=jobName + '.odb')
outputFile = open('disp_U_all_'+jobName + '.txt','w')
lastFrame = myOdb.steps['Step1'].frames[1]
displacement=lastFrame.fieldOutputs['U']
field_displacement_Values=displacement.values
outputFile.write('Node\t\tU[1]\t\tU[2]\t\tU[3]\n')
```

Part VII.

```
for v in field_displacement_Values:
    outputFile.write('%d\t%.9e\t%.9e\t%.9e\n' % (v.nodeLabel, v.data[0], v.data[1], v.data[2] ))

outputFile.close()
outputFile = open('disp_U_end_nodes_'+jobName + '.txt','w')
outputFile.write('Node\t\tU[1]\t\tU[2]\t\tU[3]\n')

for i in mystring2:
    v=field_displacement_Values[i-1]
    outputFile.write('%d\t%.9e\t%.9e\t%.9e\n' % (v.nodeLabel, v.data[0], v.data[1], v.data[2] ))

outputFile.close()
del mdb.models['Model-1'].parts['Part-1'].compositeLayups['CompositeLayup-1']
```

Part VIII.

```
mdb.models['Model-1'].parts['Part-1'].CompositeLayup(description='',
elementType=SHELL, name='CompositeLayup-1', offsetType=MIDDLE_SURFACE,
symmetric=False, thicknessAssignment=FROM_SECTION)
mdb.models['Model-1'].parts['Part-1'].compositeLayups['CompositeLayup-1'].Section(
integrationRule=SIMPSON, poissonDefinition=DEFAULT, preIntegrate=OFF,
temperature=GRADIENT, thicknessType=UNIFORM, useDensity=OFF)
mdb.models['Model-1'].parts['Part-1'].compositeLayups['CompositeLayup-1'].ReferenceOrientation(
additionalRotationType=ROTATION_NONE, angle=0.0, axis=AXIS_3, fieldName='',
localCsys=None, orientationType=GLOBAL)
```

Here, different parts of the written Python program are explained. In part (I) the program ask the user to enter the inputs such as number of the iterations which user wants the program to perform. The number of iterations depends on the step change for fibre orientations. For example if the plate is 4 layers and the -90, 0 and +90 degrees are considered for fibre orientations, it means 3 fibre orientation for each layer, which means the number of iterations would be 3*3*3*3 or 81. If the fibre orientations of -45 and +45 degrees are also consider for this case, the number of iterations are increasing to 5*5*5*5 or 625. The other inputs are the number of plies, geometry of the laminated composite plate and material properties.

In part (II), it is essential to recall different parts of Abaqus in Python. In Part (III), Modelling of the laminated composite plate has been defined. If there is no input in each parameter, a default input number is considered. In Part (IV) an example of how the program works for different inputs is shown. The fibre orientations are in separate notepad files. The program, depends on the input, is calling the related file and uses the fibre orientations from the file. This way is much faster to generate the fibre orientations in the program.

The program needs to have files to save the output results on them. The name of the file must be changed in each iteration to avoid rewriting the results on the same file. Also, in order to be able to use this file later in a database, it is necessary to know, what fibre orientations is presenting in each file. Therefore, to recognise the results which are saved in the file belongs to which fibre orientations; the name of the files must somehow represent the fibre orientations of the laminated composite plate. Also as the names of the files are in string format, they must be changed to integer format in order to be counted as a number. This number in the name of the files shows the fibre orientation for plies. For example the file name “45_0_-60_90” shows that the results in the file belong to a laminated composite plate with 4 layers and fibre orientations of [45 0 -60 90]. Part (V) in the program is written for this purpose.

Part (IV) is a loop command which is changing the orientations for each layer. The number of layers is considered 4 in this example. It can be simply extended for more layers. In Part (IIV) the results are saved in output files which were created in part (V). In the shown program the output is displacements. It is important to notice that output jobs (.odb) are Abaqus files and the results are not limited to the ones which are mentioned in the Python program. The .odb file can be opened by Abaqus at any time and the results (stress, strain, displacements, etc.) can be checked. On the other hand, when the job is completed by Python, all different output results can be viewed in Abaqus without submitting the job again each time.

At each iteration the model is redefined and at the end of the job it is deleted. It is because the fibre orientation is part of modelling and to change the fibre orientations, the whole modelling part must be changed. As the model deleted at the end of each iteration, the model must be defined again at the end of the program which is presented in part (VIII).