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Pricing Options with Portfolio-Holding Trading Agents in Direct Double Auction

Sarvar Abdullaev[1] and **Peter McBurney**[2] and **Katarzyna Musial**[3]

Abstract. Options constitute integral part of modern financial trades, and are priced according to the risk associated with buying or selling certain asset in future. Financial literature mostly concentrates on risk-neutral methods of pricing options such as Black-Scholes model. However, it is an emerging field in option pricing theory to use trading agents with utility functions to determine the option's potential payoff for the agent. In this paper, we use one of such methodologies developed by Othman and Sandholm to design portfolio-holding agents that are endowed with popular option portfolios such as bullish spread, butterfly spread, straddle, etc to price options. Agents use their portfolios to evaluate how buying or selling certain option would change their current payoff structure, and form their orders based on this information. We also simulate these agents in a multi-unit direct double auction. The emerging prices are compared to risk-neutral prices under different market conditions. Through an appropriate endowment of option portfolios to agents, we can also mimic market conditions where the population of agents are bearish, bullish, neutral or non-neutral in their beliefs.

1 Introduction

Option is the type of financial derivative that enables its holder (i.e. owner) to buy or sell specified assets at certain future price to writer (i.e. issuer) of the option. Holder of the option buys for an additional cost (i.e option premium) determined by the market or the writer of the option. On the other hand, the writer of the option sells by taking future obligation to trade assets if holder chooses to exercise his right to buy or sell. Option contract must specify the underlying asset to be traded, its *volume*, *strike price* and expiration date. European options can be exercised only on their maturity date, while American options on any date until expiration. We will use only European options in the scope of this paper.

Traders can take different positions with options of different moneyness and create option portfolios which can align with their forecast and at same time limit their loss in case if their forecast is not true. Cohen counts more than 40 option portfolios and classifies them based on their market direction (i.e. bullishness or bearishness), volatility level, riskiness and gain [2]. Let us consider, butterfly spread. This type of spread involves taking positions in options with three different strike prices. In butterfly call spread, trader has an estimate that the price is not going to change sharply, so he wants to stay neutral. He buys 2 call options: one ITM with low K_1 and one OTM with high K_3. At the same time, he sells 2 ATM calls with K_2, where K_2 is halfway between the range of K_1 and K_3. This spread leads to a profit if the asset price will not go far from its current spot price. It will incur in fixed loss if the asset price changes sharply in either directions. Butterfly spread can be created using put options as well.

Pricing options is mostly based on Black-Scholes framework [1] which values options from the perspective of no arbitrage assumption. There is an analytical solution for finding the risk-neutral value of the option. However, in agent-oriented approach, traders can have different assumptions and pricing strategies (not necessarily risk-neutral) based on their private utility functions. Gerber and Pafum described risk-averse traders based on an exponential utility function which could produce a bid-ask spread around risk-neutral option prices [3]. We will use Othman and Sandholm's indifferent option pricing method which takes into account agent's inventory [7].

In this paper, we study how option prices may differ from the risk-neutral prices if the traders come to the market already endowed with some option portfolio. We developed an agent-based system which uses direct double auction mechanism to run option traders. We used inventory-based Logarithmic Market Scoring Rule (LMSR) option trader developed by Othman and Sandholm [7] to enable the option pricing based on the payoff structure of their current portfolio. We endow the LMSR traders with commonly used option portfolios such as *bullish spread*, *bearish spread*, *butterfly spread* etc. and run them in our proposed mechanism. This allows us to set up different sentiment in the market such as more bullish traders (or more bearish traders) and observe the resulted option prices in comparison to risk-neutral price. We explain our key findings from this experiment.

2 Portfolio-holding Trading Agent

In prediction markets, the agents are allowed to change the market maker's payoff structure for a corresponding payment. For example, if market maker is accepting bets for teams A and B on a football match, and his current payoff structure is (300,200) meaning that the aggregator has to pay \$300 in total if team A wins, and \$200 if team B wins. However one would like to bet on team A, and he expects to receive \$50 if his bet is achieved. The aggregator changes his payoff structure to (350,200) by accepting the bet, and he also needs to decide how he can charge the client for accepting his bet. The most common method for evaluating the cost of accepting the bet in prediction markets, LMSR [4] and it is defined as a cost function for the vector of payoffs $\mathbf{x} = \{x_1, x_2, \ldots, x_n\}$ on the probability space of events $\Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$:

$$C(\mathbf{x}) = b \log \left(\sum_i \exp(x_i/b) \right) \quad (1)$$

[1] King's College London, UK, email: sarvar.abdullaev@kcl.ac.uk
[2] King's College London, UK, email: peter.mcburney@kcl.ac.uk
[3] Bournemouth University, UK, email: kmusialgabrys@bournemouth.ac.uk

where $b > 0$ is a liquidity parameter. The larger values of b produce tighter bid/ask spreads, but may also incur larger worst-case losses capped by $b\log(n)$ [8]. The agent who wishes to change the payoff from $\mathbf{x}$ to $\mathbf{y}$ has to pay the difference between the costs $C(\mathbf{y})-C(\mathbf{x})$. In our above example, given that $b = 100$, the aggregator accepting bets must charge the client $C((350, 200)) - C((300, 200)) \approx \$39$ for the bet.

The same principle can be used for the option trader who holds a certain portfolio of options that generate certain payoff for different asset price outcomes in future. The agent can virtually simulate buying or selling particular type of option and compute the changes it makes to its payoff structure. For example, let agent take butterfly call spread with buying ITM call at strike $K_1 = 80$ and OTM call at $K_3 = 120$, and selling 2 ATM calls at $K_2 = 100$. We can compute his discounted payoffs for the range of possible prices where the asset price can end up at time T. Let this payoff structure be $\mathbf{x}$. Trader feels bullish and wants to buy one more call option at strike $K_4 = 130$, so his bid for buying an OTM option at $K_4 = 130$ can be computed from the difference of his payoff structures $C(\mathbf{y}) - C(\mathbf{x})$, and in our particular case, it is \$1.42 given that the liquidity parameter is $b = 2500$. We can also compute the Black-Scholes price of such option given parameters $T = 1$, $r = 0.05$, $S_0 = 100$, $\sigma = 0.02$ and it is \$2.52. This would mean that the trader places a bid for given OTM option less than its risk-neutral value.

3 Experimental Results

We have simulated the asset prices using the Geometric Brownian Motion with a calibrated parameters according to the historic data of NASDAQ-100 index in 2014. The daily mean drift is computed as $\mu = 0.0007$, and and the volatility is $\sigma = 0.0089$. The initial asset price is the same as NASDAQ-100 on 2 January 2014, $S_0 = \$3563.57$, and it is $S_T = \$3597.59$ at the end of the year. We also analyse option with strike \$3563.00 which expires in one year. We use only call options for the simulation, because put prices can be directly computed from the call price using *put-call parity* relationship. Mechanism simulates 365 trading days going up to the point when the option expires. Because mechanism is direct, it clears orders in one round and moves to the next trading day. Every trading day, the traders are re-instantiated with the same distribution of portfolios so they do not remember their previous choices.

LMSR traders create a positive bid-ask spread, which forbids them from trading if the market is uniformly populated with LMSR traders holding the same portfolio. Therefore LMSR trader should be also simulated in mixed groups each holding different set of portfolios, and thus produce different prices. It is also important to note that LMSR trader is deterministic in their pricing, because the only factor which affects their pricing decision is their portfolio and fixed range of events horizon that they use to compute their final payoff. Therefore two LMSR traders holding the same portfolio produce same bids or same asks. To make market more heterogeneous, I use multiple option portfolios to mimic neutral, non-neutral, bullish and bearish traders. Some of the option portfolios used are bearish spread, bullish spread, butterfly spread, ladder, strangle straddle, strip, etc [5]. After running several experiments with LMSR traders, we found out that liquidity $b = 100$ provides reasonable range of bids and asks which are likely to produce trades in the market. LMSR trader picks random quantities between -2000 and 2000 while submitting orders, so agent's decision to buy or sell is uniformly distributed. The negative quantities stand for asks, and the positive ones are bids.

Figure 1 shows the trade between neutral and non-neutral portfolio holders. It can be seen that the prices are volatile around Black-Scholes prices. This is explained using the deterministic nature of LMSR traders. The whole market consists of 2 neutral LMSR traders and 2 non-neutral LMSR traders who output all together 8 different pricing quotes, 4 for bids and 4 for asks. Because mechanism has very few choices to determine the clearing price among mostly homogeneous quotes, the option price for each trading day differs significantly. In the market of bearish traders as shown in Figure 2, the call options are initially underpriced, as they are considered less profitable for the traders expecting the drop in asset prices. However the prices cross the risk-neutral price only after option lives the half of its lifespan. This is because the payoff from the option becomes more certain, as the asset prices continue to grow defying the bearish belief of the trader.

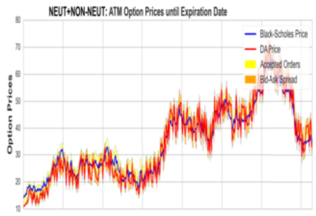

Figure 1. Option prices with neutral and non-neutral LMSR traders

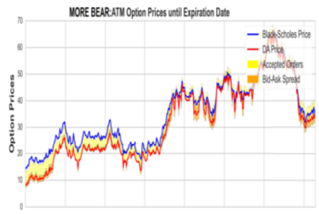

Figure 2. Option prices with more bearish LMSR traders

4 Concluding Remarks

In this paper, we simulated LMSR-based option trading agents in direct double auction mechanism and were able to observe option prices under different population of traders with bearish, bullish, neutral and non-neutral portfolios. Our simulation results have shown that pricing option via double auctions is a valid technique as the obtained prices were close to risk-neutral solution if the market was truly populated with traders having different beliefs. Moreover, we saw that the neutral and the non-neutral traders create volatility as their portfolios consist of opposite payoff structures. Also in more bearish market population, the calls were initially underpriced until the option reached half-way its maturity. This approach can be further improved using other incentive compatible mechanisms such as McAfee's double auction [6] in future researches. Also the traders can be more sophisticated in making buying/selling decisions instead of randomly choosing either action.

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