

**FAILURE ANALYSIS OF FLOW-INDUCED VIBRATION PROBLEM OF IN-SERVICED  
DUPLEX STAINLESS STEEL PIPING SYSTEM IN OIL AND GAS INDUSTRY**

**K.K. Kong\*<sup>1</sup>, S.Y. Khoo<sup>1</sup>, Z.C. Ong<sup>1,6</sup>, H.C. Eng<sup>2</sup>, Z. Ismail<sup>3,6</sup>, W.T. Chong<sup>1</sup>, S. Noroozi<sup>4</sup>,  
A.G.A. Rahman<sup>5</sup>;**

<sup>1</sup>Department of Mechanical Engineering, Engineering Faculty, University of Malaya, 50603  
Kuala Lumpur, Malaysia

<sup>2</sup>Quadrant 2 Technologies Sdn. Bhd., Kuala Lumpur, Malaysia

<sup>3</sup>Department of Civil Engineering, Engineering Faculty, University of Malaya, 50603 Kuala  
Lumpur, Malaysia

<sup>4</sup>School of Design, Engineering & Computing, Bournemouth University, Poole, Dorset, BH12  
5BB, UK

<sup>5</sup>Faculty of Mechanical Engineering, University of Malaysia Pahang, 26600 Pekan, Pahang,  
Malaysia

<sup>6</sup>Advanced Shock and Vibration Research Group, Applied Vibration Laboratory, Block R,  
Faculty of Engineering, University of Malaya, Malaysia

*\*Corresponding author: keenkuan@yahoo.com*

In oil and gas industry, piping system provides transport for a wide range of substances such as petrochemicals and water. They are required to operate nonstop for a schedule of 24/7. Flow-induced vibration (FIV) of the piping system is the most common causes of high cycle fatigue. Besides, excessive load caused by unfavourable operating condition may increase the probability of failure occurrence. Duplex Stainless Steel (DSS) is

commonly used for piping system in oil and gas industry due to its reasonable high endurance limit for dynamic stress intensity, high corrosion resistance and low cost. Failure of the DSS piping system can have disastrous effects, leading to injuries and fatalities as well as to substantial cost to industry and the environment. Therefore, there is a need to perform failure analysis of this kind of flow-induced vibration problem. In this study, novel method of failure analysis of DSS piping system due to flow-induced vibration, while in-service, is proposed. The proposed non-destructive technique is able to determine a suitable operating condition for continuous operation without failure. The technique relies on the combined operation of Operational Modal Analysis (OMA), Operating Deflection Shape (ODS) analysis and linear elastic Finite Element Analysis (FEA). Modal parameters obtained from OMA are used to correlate with FE model, while the ODS analysis result is used as initial displacement boundary condition to measure dynamic stresses through the FEA. The effect of different operating conditions (i.e. flow rates at 275mmscfd, 300mmscfd, 325mmscfd and 350mmscfd) for two distinct valve opening cases (i.e. fully-open and partially-opened) on the dynamic stress is examined and they are utilised for forecasting purpose in failure analysis. The result shows that maximum operating conditions are 360mmscfd and 400mmscfd for fully-opened and partially-opened Flow Control Valves (FCVs) respectively. Beyond this limit, the piping system most likely will fail.

Keywords: Duplex Stainless Steel, failure analysis, finite element analysis, flow-induced vibration, in-serviced pipe, non-destructive, operating deflection shape, operational modal analysis, piping system, stress analysis

## **Introduction**

In oil and gas industry, pipelines and piping system provides transport for a wide range of substances such as petrochemicals and water and they fulfil safety functions – e.g. cooling systems in nuclear power plants. They are required to operate nonstop for a schedule of 24/7. Duplex Stainless Steel (DSS) is commonly used for piping system in oil and gas industry due to its reasonable high endurance limit for dynamic stress intensity, high corrosion resistance and low cost. Failure of DSS piping systems can have disastrous effects, leading to injuries and fatalities as well as to substantial cost to industry and the environment. Besides, piping vibration problems in operating plants have resulted in costly unscheduled outages and backfits.<sup>1</sup> Piping vibration failures have been one of the major causes of downtime, fires and explosions in industrial plant over the past 30 years. For example, one piping failure at a petrochemical plant in 1974 caused over \$114,000,000 in property damage due to an explosion.<sup>2</sup>

Vibration loading, typically mechanical or flow-induced, are the most common causes of high cycle fatigue.<sup>3</sup> Besides, excessive load caused by unfavourable operating condition may increase the probability of failure occurrence. Besides, on a survey conducted by Kutsu and Scholl<sup>4</sup>, pipe cracking was identified as the most frequently recurring problem, the most significant cause of which was determined to be piping vibration. Mechanical vibration was the cause of 22.3% of all reportable occurrences involving pipes and fittings. One of the main causes of the unpredictable behaviour of pipes is the induced vibrations due to the interaction between the structure (walls of the pipe as well as the pipe supports) and the fluid flowing through the pipe. Generally the fluid behaves as a turbulent flow and exerts random pressures on the wall of the pipe.<sup>5</sup>

Fluid-structure interaction, turbulent flow fluctuations and unsteady pressure induces random excitation of the pipe and support structure which may result in resonant vibrations. It has been shown that the fluid-structure interaction phenomenon induces a significant dynamic response in the structure which alternates the fluid forces acting on the inside walls of the pipes.<sup>6</sup>

Failure analysis focuses on collecting and analysing data to determine the cause of a failure, includes recommending the failure prevention methods. A lot of studies had been done on finding the root cause of failure.<sup>7-10</sup> Previous studies<sup>11-14</sup> showed that Flow-Induced vibration (FIV) is the major root cause of many piping failure. Therefore, the current study focuses on the prevention method for FIV-caused problem. Prevention of FIV problem can be done by monitoring the structural health condition. Non-destructive structural health monitoring is of primary important for in-serviced piping system. It can be done in several ways, such as damage detection method with system identification method<sup>15-17</sup> and force identification method<sup>18, 19</sup>.

The use of vibration analysis and modal parameters is already common place in system identification and force identification. Extracting modal parameters while the system is in operation is not possible but it is highly desirable. This is well known as Operational Modal Analysis (OMA) as demonstrated by references<sup>20-22</sup> which utilises ambient forces generated due to their own operation. In addition, other types of OMA named Impact-Synchronous Modal Analysis (ISMA) has been implemented widely, which utilises artificial impact excitation.<sup>23, 24</sup> Another form of checking structures for signs of excess movement is Operating Deflection Shape (ODS) analysis which is a non-invasive and non-destructive approach used to monitor the overall dynamics and the

condition of a system while in operation.<sup>25-27</sup> This method is useful when classical condition monitoring is not possible or when a full 3D visualization of the dynamics of the motion is desirable.<sup>27</sup> With the growth in the power of digital computers, Finite Element Analysis has been utilised in many analysis, such as stress analysis<sup>28-30</sup>.

In this paper, a non-destructive method of failure analysis for an in-serviced DSS piping system due to flow-induced vibration is proposed. The technique relies on the combined application of OMA, ODS analysis and Finite Element Analysis (FEA). The highest operating flow rate for the piping system in two distinct valve opening cases (i.e. fully-opened and partially-opened) could be determined based on the results of dynamic stress analysis. Hence, this information is utilized to ensure that the pipe operates under the allowable dynamic stress for a “theoretically infinite” life cycle.

## **Theory**

### **Experimental Modal Analysis (EMA) vs Operational Modal Analysis (OMA)**

Modal analysis is used to measure the dynamic characteristic of a system such as natural frequency, damping and mode shape. It can be divided into two types: Experimental Modal Analysis (EMA) and OMA. The former analysis requires a complete ‘shutdown’ situation of the system with no unaccounted excitation force.<sup>31</sup> EMA is conducted by using an artificial excitation with impact hammer or shaker.<sup>32</sup> Instead of field testing with large and complete system, EMA is used for lab testing of individual components or parts of the system. In this case, actual boundary conditions in lab environment may vary with the one in field environment. Therefore, boundary conditions need to be reasonably

simulated when using EMA.<sup>33</sup> If EMA is performed under a ‘operation’ situation, the induced response will be a linear superimposition of the responses due to artificial excitation, unaccounted operating forces, ambient forces, and so forth.<sup>24</sup> This will cause error to the measured Frequency Response Function (FRF) which is the transfer function between output response and input force. In fact, FRF contains the information of dynamic characteristics of a system and modal parameters can be extracted from FRF by using curve fitting algorithm. As a result, conducting EMA under an ‘operation’ situation always cause poor coherence result primary due to the unaccountable operating forces.

Compared to EMA, OMA is performed under an ‘operation’ situation. In this case, output-only measurement (i.e. response measurement) due to the operating forces and ambient forces is needed for OMA. In this way, OMA is cheap and fast to conduct because it needs no elaborate excitation equipment and boundary condition simulation.<sup>33</sup> Besides, dynamic characteristics of a complete system can be obtained under real ‘operation’ situation via OMA. The operating forces acting on the in-serviced system usually cannot be measured (i.e. sinusoidal force due to motor type excitation and broadband random excitation due to flow-induced vibration). Presently, operational modal analysis procedures are limited to the case when excitation to the system is white stationary noise.<sup>34</sup> According to Zhang<sup>33</sup>, system characteristics under real loading can be linearized due to broadband random excitations.

### **Operating Deflection Shapes (ODS) analysis**

ODS can be defined as any forced motion of two or more DOFs (points & directions) on an in-serviced machine or structure.<sup>35</sup> ODS can be divided into three categories: time-

based ODS, frequency-based ODS and run-up/down ODS. Time-based ODS is extremely useful in giving an overall ODS, which can be planar, orbital or 3D for a non-stationary signal such as a transient signal compared to frequency-based ODS. It is recommended to perform frequency-based ODS for stationary signal such as a steady state signal under a constant operating conditions.

In general, two methods of measurement are used to acquire the ODS: simultaneous method and measurement set method.<sup>35</sup> The former method is preferred for a small scale test object as all channels of data can be acquired at once by using multi-channel acquisition system and hence very time efficient. However, for a large scale test object, it is not able to acquire all data at a time. In this case, measurement set method is used. The most common measurement set method is the ODS FRF.<sup>36</sup> It is formed by the magnitude of auto spectrum of a roving response and the phase of the cross spectrum between the roving response and the fixed reference response. From application point of view, the frequency-based ODS with ODS FRF is the most common method to be used for vibration monitoring of piping system in oil and gas field. Furthermore, ODS is useful in defining areas of structural weakness and also mechanical "looseness".<sup>37</sup>

## **Material and methods**

### **Example piping system**

The piping system which was used as an example in this study was highly pressurized gas transporting pipeline in an offshore platform at Malaysia. The piping system was supported by 5 pipe supports and its total length was approximately 30m (98.425ft.). The

pipe began from long distance transporting pipeline from other platform and it ended in a storage tank. The operating pressure was approximately 4.5MPa (0.653ksi) and the corresponding temperature of the pipeline was approximately 23°C (73.4°F).

The material of the pipe was DSS and there was no insulation. There were two types of piping used, which are one with nominal outer diameter of the piping cross section was 609.6mm (24in.) with the nominal value of wall thickness was 17.48mm (0.688in.) and nominal outer diameter of the piping cross section was 406.4mm (16in.) with the nominal value of wall thickness of 12.70mm (0.5in.). Again, there were two nominal radius of curvature of pipe bends, which were 914.4mm (36in.) and 609.6mm (24in.), measured from the central line of the cross section of the pipe.

### **Measurement procedure**

The need for a non-invasive testing and evaluation approach was justified by the “need to know” whether or not any high vibrations based primarily on the vibratory stresses introduced into the piping by its running conditions, i.e. the fluid pipe interaction, may affect the overall structural integrity of the piping system and result in a catastrophic failure. The fluid flow generated random excitation to the pipe and hence there was no issue of resonance. Many times the apparently high vibration in pipes may not cause excessive stresses in the piping, but could cause excessive stresses to piping system that were attached to the vibrating pipe.



To assess the problem, measurements had been performed using 4 channels real-time data acquisition system which consisted of sensor (i.e. a tri-axial accelerometer and an uni-axis accelerometer), data acquisition hardware (i.e. analog-to-digital converter), data acquisition and post-processing software (i.e. DASyLab and ME'scope software) plus other accessories (i.e. cables and magnetic base). All the measurement locations were taken using tri-axial accelerometer in 3 principal directions namely X, Y and Z. Most of the measurement locations were linked to obtain a wire mesh model in software to represent the overview of DSS pipe. After inserting the measured data into database, this model can be used to animate and visualize the vibration movement of the piping system.

Several sets of vibration measurements were performed on piping systems and their support structures under different operation conditions: partially-opened Flow Control Valves (FCVs) A & B @ 17% and fully-opened FCVs A & B with a flow rate of 275mmscfd, 300mmscfd, 325mmscfd and 350mmscfd.

## **Result and Discussion**

### **Operational Modal Analysis (OMS) & Finite Element Analysis (FEA) Correlation**

By using the behaviour of broadband random excitation due to fluid flow, OMA is used to measure the dynamic characteristic of the piping system. OMA result shows that the 1st mode appears at natural frequency of 3.60Hz while the 2nd modes at 4.56Hz. The mode shapes are vibrating in Y-direction and X-direction for 1st and 2nd modes respectively. Correlation between OMA and FEA by using 1st and 2nd vibration modes are shown in Fig. 1. A good correlation between the results of OMA and FEA correlation

validate the reliability of the FE model in further analysis (i.e. stress analysis). As a rule of thumb, percentages of errors of natural frequency less than 10% indicates a reasonable agreement between OMA and FEA results.<sup>38</sup> Fig. 1 shows the FEA model exhibits 3.91Hz and 4.57Hz which are vibrating at Y-direction and X-direction for 1st and 2nd modes respectively. Errors of 8.61% and 0.22% are obtained for the identified natural frequency at 1st and 2nd modes respectively. Thus, the FE model is valid and hence suitable to be used for stress analysis.

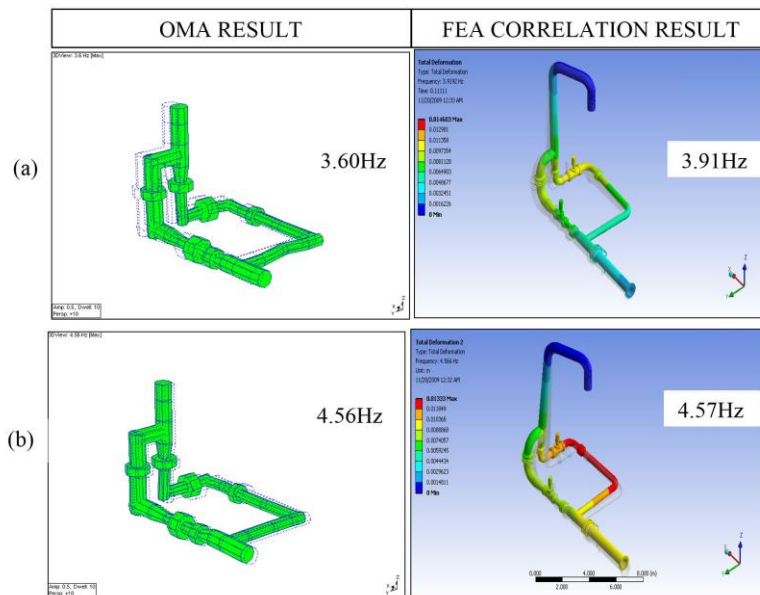


Figure 1: Correlation between OMA and FEA by using (a) 1st and (b) 2nd Vibration modes

### Operating Deflection Shape (ODS) Analysis

Several sets of ODS analysis are performed on the DSS pipe under different operation conditions. For simplify purpose, this study only shows a set of ODS analysis result for partially-opened FCVs A & B @ 17% with a flow rate of 275mmscfd and another set of ODS analysis result for fully-opened FCVs A & B with a flow rate of 300mmscfd. Fig. 2

shows the overlaid ODS spectrums for the DSS pipe. The Allowable Piping Vibration Level versus Frequency for comparison is provided for preliminary screening purpose as follows reference<sup>39, 40</sup>. It is observed that the movement are dominated by two frequencies which are 3.60Hz and 4.56Hz. The entire ODS spectrums are imported to FE environment for further dynamic stress analysis.

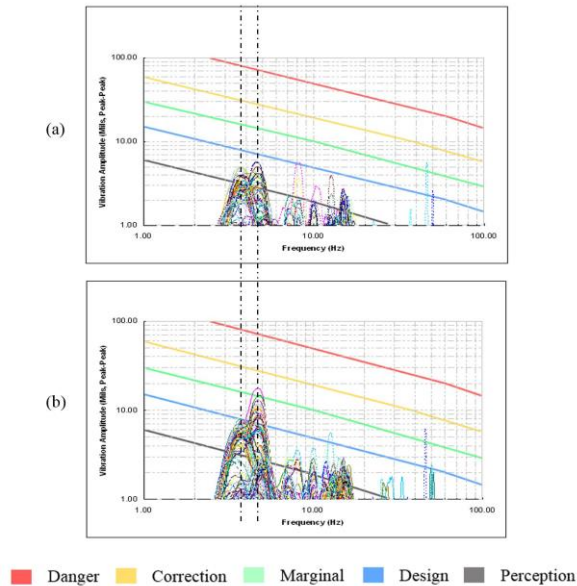


Figure 2 Overlaid ODS Result for DSS Pipe While Operating at (a) 275mmscfd with partially-opened valve (b) 350mmscfd with fully-opened valve

### Dynamics Stress Analysis

Displacements are obtained by ODS analysis which gives the displacement pattern of the total structure. Dynamics Stress are calculated using FEA based on the displacement values in X, Y and Z directions for 1<sup>st</sup> and 2<sup>nd</sup> vibration modes obtained from ODS analysis.

Fig. 3(a) shows the dynamics stress results obtain from the first 2 vibration modes of 275mmscfd (partially-opened valve). For dynamic stress at 3.60Hz, it is observed the

maximum stress register at 28.24MPa and located at location labelled I as shown in Fig. 3(a). For dynamic stress at 4.56Hz, it is observed the maximum stress register at 47.94MPa and located at location labelled II as shown in Fig. 3(a).

Fig. 3(b) shows the dynamics stresses obtain from the first 2 vibration modes of 350mmscfd (fully-opened valve). For dynamic stress at 3.60Hz, it is observed the maximum stress register at 35.58MPa and located at location labelled III as shown in Fig. 3(b). For dynamic stress at 4.56Hz, it is observed the maximum stress register at 78.58MPa and located at location labelled IV as shown in Fig. 3(b).

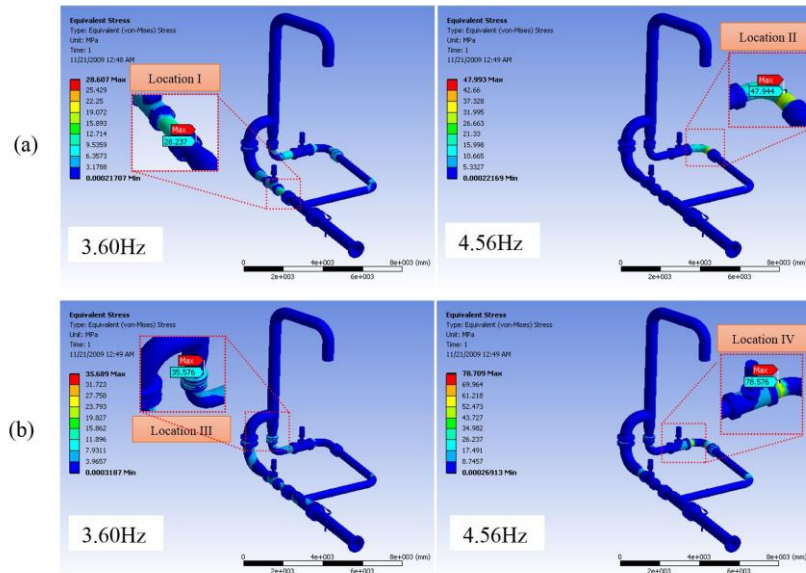


Figure 3: Dynamics Stress of piping operating at 3.60Hz and 4.56Hz respectively for various flow rates: (a) 275mmscfd (partially-opened valve) (b) 350mmscfd (fully-opened valve)

Table 1 shows the high stress location at the for each vibration mode. The summation of the stresses for mode 1 and mode 2 at the same location will give the total

stress values. Overall, the total stress for all the locations are under the pipe stress allowable limit (93.8MPa) for indefinite lifecycle.

Table 1: Summary of Dynamic Stress Level for various flow rates: (a) 275mmscfd (partially-opened valve) (b) 350mmscfd (fully-opened valve)

Condition	Location	Stress Level		Total Stress (Mode 1 + Mode 2)
		Mode 1 (3.60Hz)	Mode 2 (4.56Hz)	
275mmscfd (partially-opened valve)	I	28.24MPa (Max. for Mode 1)	13.18MPa	41.42MPa
	II	3.88MPa	47.94MPa (Max. for Mode 2)	51.82MPa
350mmscfd (fully-opened valve)	III	35.58MPa (Max. for Mode 1)	32.22MPa	67.80MPa
	IV	8.82MPa	78.58MPa (Max. for Mode 2)	87.40MPa

### **Dynamics Stress Analysis Forecast For Maximum Operating Condition Before Failure**

Maximum Dynamic Stress for the partially-opened FCVs A & B @ 17% and the fully-opened FCVs A & B with a flow rate of 275mmscfd, 300mmscfd, 325mmscfd and 350mmscfd is plotted to forecast the maximum operating condition before failure. Peak performance (in terms of flow rate) for the piping system in two distinct valve opening cases (i.e. fully-opened and partially-opened) is forecasted, where maximum operating conditions are 360mmscfd and 400mmscfd for fully-opened and partially-opened FCVs respectively. Beyond this limit, the piping system most likely will fail.

## Conclusion

A non-destructive method of failure analysis has been presented for an in-serviced DSS piping system due to flow-induced vibration using the combined application of OMA, ODS analysis and FEA. The method can be used to forecast the maximum operating condition for a piping system. The highest operating flow rate for the piping system are 360mmscfd and 400mmscfd for fully-opened and partially-opened FCVs respectively. Beyond this limit, the piping system most likely will fail.

## Acknowledgement

The authors wish to acknowledge the financial support and advice given by Postgraduate Research Fund (PV086-2011A), University of Malaya Research Grant (RP022D-2013AET) and Advanced Shock and Vibration Research (ASVR) Group of University of Malaya

## References

1. D. E. Olson: 'Pipe Vibration Testing and Analysis', Companion Guide to the ASME Boiler & Pressure Vessel Code, New York, 2002, ASME Press.
2. W. G. Garrison. Major fires and explosions analyzed for 30-year period, *Hydrocarbon processing*, 1988, **67**(9), 115-118.
3. O. N. E. Agency: 'Nuclear Power Plant Operating Experiences from the IAEA/NEA Incident Reporting System 2002-2005', 1-56; 2006,
4. O. Kustu and R. Scholl: 'Research Needs for Resolving the Significant Problems of Light-Water Reactor Piping Systems', Proceedings of ANS/EMS Topical Meeting—Thermal Reactor Safety, Knoxville, TN, USA, 1980.
5. R. D. Blevins: 'Flow-induced vibration'; 1990, New York, Van Nostrand Reinhold.
6. H. L. Dai, L. Wang, Q. Qian, and J. Gan, *Vibration analysis of three-dimensional pipes conveying fluid with consideration of steady combined force by transfer matrix method*, in *Applied Mathematics and Computation*. 2012. p. 2453-2464.
7. M.-B. Lin, K. Gao, C.-J. Wang, and A. A. Volinsky, *Failure analysis of the oil transport spiral welded pipe*, in *Engineering Failure Analysis*. 2012. p. 169-174.
8. G. Atxaga and A. M. Irisarri. Study of the failure of a duplex stainless steel valve, *Engineering Failure Analysis*, 2009, **16**(5), 1412-1419.

9. S. S. M. Tavares, C. Scandian, J. M. Pardal, T. S. Luz, and F. J. da Silva, *Failure analysis of duplex stainless steel weld used in flexible pipes in off shore oil production*, in *Engineering Failure Analysis*. 2010. p. 1500-1506.
10. Z. A. Majid, R. Mohsin, Z. Yaacob, and Z. Hassan, *Failure analysis of natural gas pipes*, in *Engineering Failure Analysis*. 2010. p. 818-837.
11. A. D. Dimarogonas. Vibration of cracked structures: A state of the art review, *Engineering Fracture Mechanics*, 1996, **55**(5), 831-857.
12. B. Guo, S. Song, A. Ghalambor, and T. R. Lin: 'Chapter 18 - Pipeline Vibration and Condition Based Maintenance', in 'Offshore Pipelines (Second Edition)', 299-337; 2014, Boston, Gulf Professional Publishing.
13. Y. H. Choi and S. Y. Choi. Socket weld integrity in nuclear piping under fatigue loading condition, *Nuclear Engineering and Design*, 2007, **237**(2), 213-218.
14. M. P. Paidoussis. Real-life experiences with flow-induced vibration, *Journal of Fluids and Structures*, 2006, **22**(6-7), 741-755.
15. M. M. Fayyadh, H. A. Razak, and Z. Ismail, *Combined modal parameters-based index for damage identification in a beamlike structure: theoretical development and verification*, in *Archives of Civil and Mechanical Engineering*. 2011. p. 587-609.
16. Z. C. Ong, A. G. A. Rahman, and Z. Ismail. Determination of Damage Severity on Rotor Shaft Due to Crack Using Damage Index Derived from Experimental Modal Data, *Experimental Techniques*, 2012, **38**(5), 18-30.
17. A. G. A. Rahman, Z. Ismail, S. Noroozi, and O. Z. Chao, *Study of open crack in rotor shaft using changes in frequency response function phase*, in *International Journal of Damage Mechanics*. 2013. p. 791-807.
18. S. Y. Khoo, Z. Ismail, K. K. Kong, Z. C. Ong, S. Noroozi, W. T. Chong, and A. G. A. Rahman, *Impact force identification with pseudo-inverse method on a lightweight structure for under-determined, even-determined and over-determined cases*, in *International Journal of Impact Engineering*. 2014. p. 52-62.
19. N. Hu and H. Fukunaga. A new approach for health monitoring of composite structures through identification of impact force, *Journal of Advanced Science*, 2005, **17**(1/2), 82-89.
20. S. V. Modak, C. Rawal, and T. K. Kundra, *Harmonics elimination algorithm for operational modal analysis using random decrement technique*, in *Mechanical Systems and Signal Processing*. 2010. p. 922-944.
21. A. De Vivo, C. Brutti, and J. L. Leofanti, *Modal shape identification of large structure exposed to wind excitation by operational modal analysis technique*, in *Mechanical Systems and Signal Processing*. 2013. p. 195-206.
22. E. Reynders and G. D. Roeck. Reference-based combined deterministic–stochastic subspace identification for experimental and operational modal analysis, *Mechanical Systems and Signal Processing*, 2008, **22**(3), 617-637.
23. A. G. A. Rahman, O. Z. Chao, and Z. Ismail, *Effectiveness of Impact-Synchronous Time Averaging in determination of dynamic characteristics of a rotor dynamic system*, in *Measurement*. 2010.
24. A. G. A. Rahman, Z. C. Ong, and Z. Ismail, *Enhancement of coherence functions using time signals in Modal Analysis*, in *Measurement*. 2011. p. 2112-2123.
25. C. Devriendt, G. Steenackers, G. De Sitter, and P. Guillaume. From operating deflection shapes towards mode shapes using transmissibility measurements, *Mechanical Systems and Signal Processing*, 2010, **24**(3), 665-677.

26. W. D. Marscher and C.-W. Jen: 'Use of operating deflection and mode shapes for machinery diagnostics', Proceedings of 17th International Modal Analysis conference, Kissimmee, Florida, 1999, 65-71.
27. B. H. Tongue: 'Principles of vibration'; 2002, New York, Oxford University Press
28. H. Kim, Y. Hwang, and H. Yoon: 'Dynamic Stress Analysis of a Bus Systems', Proceedings of the 2nd Worldwide Automotive Conference, MSC Software Corporation, Dearborn, MI, October, 2000, 9-11.
29. A. S. Blicblau, M. Singh, E. McConnell, and M. Pleaner. Stress analysis of a novice windsurfer sail by finite element analysis, *Mathematical and Computer Modelling*, 2008, **47**(11–12), 1108-1116.
30. H. Lee: 'Finite element analysis of a buried pipeline', PhD thesis, The University of Manchester, Manchester, UK, 2010.
31. D. J. Ewins: 'Modal testing: theory, practice and application'; 2000, Research studies press Baldock.
32. B. J. Schwarz and M. H. Richardson. Experimental modal analysis, *CSI Reliability week*, 1999.
33. L. Zhang, T. Wang, and Y. Tamura, *A frequency–spatial domain decomposition (FSDD) method for operational modal analysis*, in *Mechanical systems and signal processing*. 2010. p. 1227-1239.
34. P. Mohanty and D. Rixen. A modified Ibrahim time domain algorithm for operational modal analysis including harmonic excitation, *Journal of Sound and Vibration*, 2004, **275**(1), 375-390.
35. H. Vold, B. Schwarz, and M. Richardson: 'Measuring operating deflection shapes under non-stationary conditions', Proceedings of the 18th International Modal Analysis Conference, Kissimmee, Florida, 2000.
36. B. J. Schwarz and M. H. Richardson. Introduction to operating deflection shapes, *CSI Reliability Week, Orlando, FL*, 1999.
37. R. J. Sayer: 'Operating Deflection Shapes, Part 2: Case History Applications', Proceedings of the Vibration Institute Training Conference and 37th Annual Meeting, Jacksonville, Florida, 2013.
38. J. P. De Clerck and J. A. Cafeo: 'Vibration Model Validation Based on Statistical Confidence of Modal Property Predictions', IMAC-XXI: Conference & Exposition on Structural Dynamics, Orlando, Floridas, 2003.
39. J. Wachel and D. Smith: 'Vibration Troubleshooting of Existing Piping Systems', Engineering Dynamics Inc. Seminar Presentation, 1991.
40. J. Wachel: 'Piping Vibration and Stress', Machinery Vibration Monitoring and Analysis Seminar, New Orleans, 1981, 1-20.