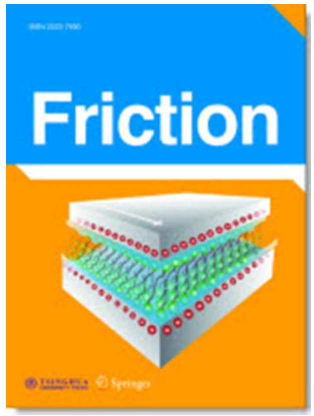

A REVIEW OF FRICTION MODELS IN INTERACTING JOINTS FOR DURABILITY DESIGN

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A REVIEW OF FRICTION MODELS IN INTERACTING JOINTS FOR DURABILITY DESIGN

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Abstract

This paper presents a comprehensive review of friction modelling to provide an understanding of design for durability within interacting systems. Friction is a complex phenomenon and occurs at the interface of two components in relative motion. Over the last several decades, the effects of friction and its modelling techniques have been of significant interests in terms of industrial applications. There is however a need to develop a unified mathematical model for friction to inform design for durability within the context of varying operational conditions. Classical dynamic mechanisms model for the design of control systems has not incorporated friction phenomena due to non-linearity behaviour. Therefore, the tribological performance concurrently with the joint dynamics of a manipulator joint applied in hazardous environments needs to be fully analysed. Previously the dynamics and impact models used in mechanical joints with clearance have also been examined. The inclusion of reliability and durability during the design phase is very important for manipulators which are deployed in harsh environmental and operational conditions. The revolute joint is susceptible to failures such as in heavy manipulators these revolute joints can be represented by lubricated conformal sliding surfaces. The presence of pollutants such as debris and corrosive constituents has the potential to alter the contacting surfaces, would in turn affect the performance of revolute joints, and puts both reliability and durability of the systems at greater risks of failure. Key literature is identified and a review on the latest

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developments of the science of friction modelling is presented here. This review is based on a large volume of knowledge. Gaps in the relevant field have been identified to capitalise on for future developments. Therefore, this review will bring significant benefits to researchers, academics and industrial professionals.

Keywords: Friction, dynamics, joint clearance, numerical models, impact, durability

1. Introduction

Friction is a ubiquitous phenomenon, which occurs at the interface of two surfaces in physical contact and are in relative motion. It may be at times beneficial and or detrimental in other scenarios. The phenomenon of friction is complex, because it has time dependent non-linear characteristics and it is influenced by multiple factors. Friction phenomenon applies to scales ranging from nanometre level interactions to micron level interfaces to large geological interactions [1], [2]. Friction is directly linked to the durability and reliability of interacting systems and if it is not fully optimised then it leads to significant efficiency losses. According to the Jost report of 1966, "a sizeable portion of the GDP of a nation is spent in alleviating friction and its effects namely wear". Although Tribology is a relatively new area, is formed of a confluence of theory and empiricism, continued experimental analyses, mechanics, surface engineering, chemical interactions and more recently computational methodology. Since the phenomenon has both widespread and deep-rooted influence, this review paper seeks to gain an insight into the history of the development of friction and dynamic modelling and to enumerate various friction models and their characteristics.

Friction occurs in both prismatic and revolute mechanisms contacts. In revolute joints, increasing the diameter of the contacts can effectively reduce the contact pressure. However, the sliding distance increases which may result in accelerated wear [3]. The nature of contact in revolute joints in manipulators can vary between conformal and non-conformal contacts depending on whether sliding bearings or anti-friction bearings or have been used. The nature of the clearance existing at the revolute joint contact determines whether the contact is continuous or non-continuous contact during its operation. Continuous contacts can be modelled with a revolute friction model. The non-continuous models require contact models that capture the model dynamics as well as follow energy conservation and is therefore much more tedious to model. Both these models have been examined in sections 4.1and 4.2 of this paper.

The focus of several recent researches has been the modelling of friction in manipulators [4]–[8]. The extended problem also requires the formulation of a suitable control system. Some researchers have tried to use an un-modelled dynamics approach [9], [10]. Friction introduces non-linearity into the dynamics equation, which physically implies phenomena such as stick-slip in relative motion, limit cycles and introduces difficulties in positioning the end effector of the manipulator. However, as of now, friction models are imperative in analysing any mechanism.

Friction modelling has progressed from specific models analysing friction at the interface of geometries [11]–[14], to the analysis of friction at manipulator joints [4], [7], [15] with clearance and their kinematics and dynamic [16]–[25]. Marques et al [26] have recently surveyed friction models in single degree of freedom in planar systems. Lately, researches such as Flores, Mukras et al [27], [28] have analysed the computation of joint wear calculation along with dynamics. The progress of research is seen in the integration of multiple disciplines that include tribology, computational mechanics, control systems, surface interaction and chemical interactions.

This review converges to manipulator joints used in mechanical equipment as excavators and search and rescue smart mechanical systems. Excavator is a commonly deployed platform in disaster sites. However there has been a rise in accident numbers in controlled construction environments with respect to the number of units being used, which has been a major health and safety concern [29]. The manipulator kinematics has been introduced by Koivo et al [30], extended to dynamics by Vaha and Koivo [31], [32]. Subsequent works have followed the modelling approach proposed by these researchers in attempting to develop the dynamics and control methods however 'til date only some studies based on the real arm [33], [34] have shown partial success in implementation. The non-linearity of the dynamics formulation makes the numerical solution both complex and computationally expensive. The computational effort increases with the increase in the degree of freedom e.g. increase in the number of links in the manipulator, transformation from simple open chain manipulator to a closed loop mechanism and with the introduction of the nonlinear friction component into the dynamic equation. According to Haessig and Friedland [78], "friction is the nemesis of precision control". The phenomenon of friction is often ignored in control theory because of its intricacy. For precision control applications, however, the effect of friction cannot be ignored. The main impediment can be attributed to the complexity of dynamics i.e. the non-linearity in the loads and more importantly the question of mimicking a human being. In this respect it is worth noting that Bilandi [4], [35] have studied the friction in an excavator arm.

Moreover, physical failure of a robot is a major obstacle in search and rescue missions [36], [37] and this can only be alleviated through the study of the manipulator mechanism design from the material science viewpoint, the need for which can be substantiated by the growing attention on natural and man-made disasters and the efforts to minimise causalities. The ingress and egress of rescuers is not the only cause for concern in such sites, bringing such sites back to normalcy is part of the post-disaster operation. For these the use of equipment is very much a necessity both to speed up operation and to reduce risk to human beings however the risk reduction also entails focus on the manipulator mechanism to perform in those environments without catastrophic failures. Very few researches have focussed on this aspect since the focus of search and rescue operations has been to detect and replace of live rescuers (human and dogs) with robots. When large quantities of chemicals were found in large radius after the WTC incident [38], the effect of corrosion inducing species on the operating equipment needs to be examined too. Stalwart researchers such as Blau [39] recognise that most appropriate method for determining the effect of friction and its effects and quantifying it, is still in experimentation and analysis also elucidates that the effects of environment on such mechanical joints needs attention. Recently, holistic models combining dynamics, friction and wear have begun to appear in literature.

A multi-disciplinary approach (Figure 1) is needed to fully analyse the problem and to devise a meaningful solution for the dynamics and control of manipulators. Friction and wear effects in the manipulators incorporating environmental effects need to be fully studied. With the increase in available computational power, a transition from simple analytical to complex numerical formulations of friction problem, with an analogous improvement in the range and precision of friction models have been looked. As part of the effort, a detailed literature survey is presented here, which provides an in-depth insight into the modelling of dynamics with a focus on friction Figure 2. A similar review for biodiesels has been presented recently [40]. The future research directions and gaps have been identified and presented for future reference.

Multidisciplinary techniques to advance design methods to improve efficiency, reliability and durability of contacting surfaces is explored in the literature [41]–[44]. The outline of the survey methodology has been given in the next section.

2. Review methodology

The review of literature was conducted with a start from significant literature by citation index and relevance. Amstrong-Helouvry [45] revealed that a growth of 700 articles in tribology is expected yearly. With the available volume of literature, it is an impossible task to encompass all research areas. This review focuses on friction modelling which arises from the specific case of the manipulator arm deployed in harsh environments. The kinematics, dynamics and control of such robotic manipulators have been the subject of interest in the recent few decades. Important keywords are identified (refer to keywords as above) relevant cited publications. A search on (friction models* AND static* AND dynamics*), in June 2016 revealed the following statistics (Figure 3). The focus of this paper is on the development of the techniques of modelling mechanisms, the recognition and the inclusion of friction into dynamic modelling, some necessary aspects of control and the evolution of the modelling methodology of friction along with wear and lubrication which form an integral portion of this science. At the outset, the following aspects are addressed, including (1) identifying the research development and timeline, (2) to identify key review papers, (3) to enumerate the important numerical models and (4) to identify future research directions. The history of the manipulator modelling is outlined in the forthcoming section.

3. History of dynamic modelling

An encapsulated version of the history of mechanics of manipulators and numerical modelling is presented here. The progression in manipulator modelling can be seen with the increase in modelling complexities from the late 1980's to present date. The history of multibody dynamics has been presented by Rahnejat [46] and Schiehlen [47]. Computational dynamics has been presented in the 20th century. The detailed modelling methodology from the robotics and control perspective has been presented by Siciliano and Khatib in their book

[48]. Shigley and Uicker [49] has provided the fundamental theory of mechanisms. Computational aspects have also been described by Groover [50]. Dynamics of parallel manipulator with friction has been presented by Farhat et al [51]. Friction in space manipulator is presented in Hachkowski et al [52]. With the advent of computers and increased availability of computing power, several techniques incorporating engineering design techniques have evolved. The use of CAD and multibody dynamics in the design, simulation and analysis of mechanisms have greatly contributed to the efficiency of the entire process [53].

The free body diagram of a robot with manipulator is given in Figure 4. The first step in the modelling of any mechanism is the development of the kinematic relationship between links, assigning the appropriate relations between links. The planar kinematics of the manipulator arm with three revolute joints has been presented in [30] following the Denavit-Hartenberg [54] convention of coordinate system assignment. The computation of the forward kinematics of such mechanisms is straightforward. However, in the case of the reverse kinematics of multi-link mechanisms multiple solutions exist and therefore the selection of appropriate process is tedious.

The kinematic analysis, which is devoid of force calculations, the development would lead to the dynamic analysis which is presented in literature [55], [56]. The dynamics of mechanisms can be modelled by using Newton Euler method [31], Euler Lagrange Method [57], Gibbs Appel method [58], or Kane's equation [59], [60]. The method used to model the system dynamics depends on the nature of the application and complexity of the design. The dynamic model of the manipulator can be expressed as given in [31]:

$$D_a(\theta)\dot{\theta} + C_a(\theta,\dot{\theta})\dot{\theta} + G_a(\theta) + B_a(\dot{\theta}) = \Gamma\tau_a - F_L \quad (1)$$

where $\theta = [\theta_1\ \theta_2\ \theta_3\ \theta_4]^T$ is the vector representation of joint angles $D_a(\theta)$ represents inertia, $C_a(\theta, \dot{\theta})$ represents Corioli's and centripetal effects, $G_a(\theta)$ represents gravity forces, $B_a(\dot{\theta})$ represents frictional forces, Γ is the input matrix corresponding to joint torques $\tau_a = [\tau_1 \tau_2 \tau_3 \tau_4]^T$, F_L represents soil-tool interaction forces. The first angle, θ_1 represents the rotation of the manipulator about the base of the excavator, which is usually assumed to be null magnitude since the manipulator operation is assumed to be immobile in that degree of freedom. This means that the manipulator remains planar during digging since it does not turn about the base during this task. Therefore, the model complexity and the computational effort are reduced.

From the above equation, it is evident that the friction forces occurring in the revolute joints have not been considered. The soil tool interaction force F_L, is again a highly non-linear component, which acts on the end effector. Several researches are dedicated to the computation of soil-tool interaction forces [61]–[64]. The influence of the soil-tool interactions on the state variables of the manipulator would also affect the friction torque generated at the revolute joints. Therefore, simple friction models would be insufficient to capture the resulting frictional dynamics. In joints mechanism the friction forces may be as high as 20% of the dynamics [7]. Simplification schemes may include (1) simplifying dynamics by ignoring some terms and correcting errors using feedback (e.g. non-linear friction effects, Coriolis's force and centripetal force which can be ignored at low link velocities but constitutes a considerable component of forces at high speeds). The Coriolis's/Centripetal components cannot be corrected by feedback method, or (2) tabulation lookup. Tabulation and interpolation method can be used to create a lookup table for pre-calculated values. Therefore, this technique cannot be employed when non-linear terms occur. Tabulation method cannot be applied to friction due to its high non-linearity. Recursive Newton formulation is more efficient than recursive Lagrangian formulation.

However, they can be brought down to approximately the same computational time, therefore real time solution is possible [65]. A pictorial representation of dynamics and control is given below in Figure 5.

For dynamic systems with clearance, the combination of differential and algebraic equations (DAE) resulting in the equation of motion is given by [66]:

$$\begin{bmatrix} M & \phi_q^T \\ \phi_q & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} g \\ \gamma \end{bmatrix} \quad (2)$$

where M is the mass matrix, and Φ_q is the Jacobian matrix for the constraint equations. $\ddot{q}$ includes the generalised state accelerations. λ denotes the Lagrange multipliers, g is the generalised force vector, γ is the velocity of quadratic velocity terms dependent on velocity, position and time. This equation can be solved by using solution methods which are applicable to algebraic equations in the absence of redundant constraints. The solution method has followed widely Baumgarte stabilisation method. In the case of redundant constrains, the augmented Lagrangian method is employed. Detailed formulation method is presented by Flores in [66] and the model is employed in the majority of subsequent modelling works.

In the next section the progress in friction modelling has been presented.

4. History of friction modelling

While modelling the spatial behaviour, and the dynamics of mechanisms has made significant progress, it is necessary to capture the effects of friction in the joints. In classical modelling, the effect of friction is not considered. However, friction is defined as the tangential reaction force that occurs between two surfaces in contact, dependent on factors that include the contact geometry, the topology, relative velocity of surfaces in contact and displacement of surfaces, load and lubrication [23], [67]–[70]. Friction is a complex phenomenon caused by the interaction of the surface and near surface regions of two interacting components as well as lubricants if present between such surfaces [71]. However, the classical friction model does little more than to give an approximation of friction forces in static analyses.

The selection of friction models is based on the operational condition during application. Several mentioned models include evaluation of physical friction and wear. Olson et al [67] have examined several friction models which are available in the context of automatic control. Within the domain of control theory friction effects are addressed in dynamics by quantifying parameters as noise generated from the ensuing effects [72], however such an approach falls short to address the overall phenomena of friction and wear from the mechanical design approach.

In the forthcoming section, static and dynamic friction models are examined, with their brief history and modelling equations.

4.1. Static friction models

Literatures [2], [73] reveal preliminary inquiries into the nature of friction of interacting bodies. The postulates of friction according to Guillaume Amontons [74] are given as:

- The force of friction is directly proportional to the applied load, i.e. $\mu \propto W$
- The force of friction is independent of the apparent area of contact
- Kinetic friction, μ_k is not proportional to (independent of) the sliding velocity

Therefore, the simplest representation of friction can be given as

$$F_f = \mu W \quad (3)$$

Where the coefficient of friction μ, is dependent on the mating materials in interaction, surface preparation and operating conditions. The force required to initiate movement is known as the static friction force. The force required to maintaining motion is called kinetic friction force. These have different magnitudes with the value of frictional force at limiting conditions have a greater value compared to kinetic condition.

Coulomb proposed the simple roughness model in 1785 which is used for friction force calculation. It is a static model that has neither history nor states [7] and may be explained based on the quasi-static properties of materials. The Coulomb model has no dissipative component to it, which is a drawback [75]. It is given by [76].

$$F_c = \mu F_n sign(v) \tag{4}$$

Also, shear failure is the predominant cause in sliding with friction. For static and Coulomb friction, the friction forces are proportional to the normal load. At low velocities, the shear strength of a solid lubricant film is high compared to the corresponding shear forces of the fluid film build up at higher velocities. The viscous friction can be represented by

$$F_v(v) = \sigma_v v \tag{5}$$

If the lubricating film is sufficient to separate the bodies in contact completely, the hydrodynamic effects become significant i.e. the friction coefficient may increase with the velocity. Therefore, the friction force generated in lubricated systems normally decreases when the velocity increases from zero. This is called the Stribeck effect.

While the Coulomb and viscous friction models account for the fundamental modelling of friction at joints, the addition of Stribeck friction accounts for low velocity, high magnitude friction. The combined effects of the Coulomb, viscous and Stribeck components of friction is shown in Figure 6. The mathematical representation of the combined effects of static friction models is given by

$$F_f(v) = \mu F_n sign(v) + \sigma_v v + F_s(v) \tag{6}$$

These effects, which have been evaluated initially for the linear sliding models, also apply to revolute models as revolute friction torque. Above mentioned model does not accurately capture friction and its effects. The requirement of dynamic friction models is highlighted which have been explained in the next section.

4.2. Dynamic friction models

Phenomenon of friction is being increasingly applied to modelling of dynamic systems and their control. Several models in literature include Dahl model (1968), Karnopp (1985), Bliman Sorin (1995), LuGre (1998) and the Leuven (2000) model. Classical friction model does not accurately predict limit cycle unlike any of the other models. For precision control applications, however the effect of friction cannot be ignored i.e. micron level motion gradient in manipulator junctions may result in positioning various several orders higher at the end effector location. Physics motivated models such as generalised Maxwell slip model, Frenkel Kontrova, Tomlinson, Frenkel Kontrova Tomlinson model, Barridge Knopoff model, and Tomlinson models[76] are not presented here. These models improve accuracy of modelling however cannot be employed for control systems. Several dynamic friction models have been presented in literature incorporating dynamic friction models to enhance capturing the effects of friction. Friction model must account for the transition phenomena between static and kinetic contact, account for hysteresis effect and direction reversals. These models have been introduced to cover the gap in performance found in the static friction models, which have been explained in the previous section. Developments in the dynamic friction models have occurred perhaps from the mid half of the twentieth century. The important models have been summarised Eq (7)-(12).

The key aspects included in these models are the genesis of friction, stick-slip phenomenon, hysteresis, friction lag and friction memory which make it both more complicated and capturing friction effects better compared to static models which are presented in Section 4.1.

Dahl model is among the first of such models [77]. According to Olsson [67] the LuGre model and the Bliman-Sorin both developments on the Dahl model capture most of the friction effects with nearly as much as ease as by the static models. The LuGre model adds the effect of damping to the Dahl model. Piatkowski [78] provides a recent analysis of Dahl and LuGre models. Dynamic friction models are considered by Karnopp [79], Quinn, Kikuuwe [80], [81], Awrejcewicz [82] and Wojewoda [83].

Dahl Model- 1968 [77] [78]

$$F = \sigma_0 z \tag{7}$$

$$\text{where } z = v\, sgn\left(1 - sgn(v)\frac{\sigma_0 z}{F_c}\right)\left|1 - sgn(v)\frac{\sigma_0 z}{F_c}\right|^{\delta_p}$$

Karnopp Friction model-1985, 2008[79] [84]

$$F_{friction} = \begin{cases} -F_C.sgn(\dot{x}) - F_v\,\dot{x} & |\dot{x}| \geq DV \\ -(F_{ext} - k.x) & \begin{cases} |\dot{x}| < DV \\ |F_{ext} - k.x| \leq F_s \end{cases} \\ -F_s.sgn(F_{ext} - k.x) & \begin{cases} |\dot{x}| < DV \\ |F_{ext} - k.x| > F_s \end{cases} \end{cases} \tag{8}$$

Where F_C is the Coulomb friction coefficient, F_v is the viscous friction coefficient, F_s is the static friction coefficient, DV is the limit velocityand the sgn(.) function is given by

$$sgn\,(\alpha) = \begin{cases} 1 & \alpha > 1 \\ 0 & \alpha = 1 \\ -1 & \alpha < 1 \end{cases}$$

Bliman-Sorin Model–1995 [85], [86]

$$\frac{\mathrm{d}x}{\mathrm{d}s} = Ax_s + Bv_s \tag{9}$$

$$\text{Where } F = Cx_s$$

$$A = \begin{pmatrix} \frac{-1}{\eta\varepsilon_f} & 0 \\ 0 & \frac{-1}{\varepsilon_f} \end{pmatrix}, B = \begin{pmatrix} f_1 / (\eta\varepsilon_f) \\ -f_2 / \varepsilon_f \end{pmatrix}, C = (1\ 1)$$

LuGre Model- 1998[67][87]	$F = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{z}$ where $\dot{z} = v - \frac{\sigma_0 \lvert v \rvert}{s(v)} z$ $s(v) = F_c + (F_s - F_c) exp\left((-v/v_s)^{\delta_{vs}}\right)$	(10)
Leuven Model - 2000 [71], [88]	$\frac{\mathrm{dz}}{\mathrm{dt}} = v\left(1 - sgn\left(\frac{F_d(z)}{s(v) - F_b}\right)\left\lvert\frac{F_d(z)}{s(v) - F_b}\right\rvert^n\right)$ $F_f = F_h(z) + \sigma_1 \frac{dz}{dt} + \sigma_2 v$ $F_h(z)$ is the hysteresis force, n is a coefficient used to transitions the shape curves, $s(v)$ is function which models the constant velocity behaviour, given by: $s(v) = sgn(v)(F_c + (F_a - F_e)e^{-\left(\lvert v \rvert / V_a\right)^{\delta}}$	(11)
Seven Parameter Friction model 1994 [45], [76]	$F_f(x) = -k_t x\ (presliding)$ $F_f(v(t), t) = -(F_c + F_v \lvert v(t) \rvert) - sgn(v(t) F_s(v, t_2) \frac{1}{1 + \left[v \frac{(t - \tau)}{v_S}\right]}$ (Coulomb and viscous sliding) $F_{s,a_n} = F_{s,a_{n-1}} + (F_{s,\infty} - F_{s,a_{n-1}}) \frac{t_2}{t_2 + v}$ (Rising static friction -breakaway)	(12)

Piedboeuf et al [7] proposed an algorithm for computing joint friction in robotic simulations which includes Stribeck regime along with the stick-slip process validated against a planar robotic arm. At zero velocity, the friction value may be any value between ± F_s as shown in Figure 6. To alleviate the problem introduced by bi-valued function at zero velocity a gradient is introduced between the transition [89] and is reflected in Figure 6. The solution to overcoming this non unicity is to insert a linear slope across zero crossing where the function becomes bi-valued (applied similarly in [89], [90]). Several researchers have attempted to incorporate the effects of friction.

Dahl Model is similar to the reset integrator model and (1) includes mechanism for zero velocity sticking and (2) application independent design. Disadvantages of the model are that (1) it does not generate a stiction force exceeding the sliding force unlike the other 4 models but can be modified to accommodate it and (2) lower accuracy compared to Karnopp and RI model.

Karnopp used bond-graph method to model the effects of friction for a two body system [79]. While this captures the energy aspect of the system the disadvantage of the bond-graph equation should be formulated for every model. Karnopp model can be represented by Eq. (8). The order of Karnopp model reduces [79], [84], [91] at zero relative velocity between the surfaces. Advantages of this model include (1) stick-slip phenomenon is included in the model and (2) 30% faster execution. The drawbacks of the Karnopp model are that (1) the complexity of the model increases with increasing complexity of the dynamic system and (2) all combination possibilities of motion between the bodies must be considered. More detailed friction models such as the Dahl model and LuGre model (Eq. (10)) which account for the pre-sliding conditions have been presented in literature. Both Dahl and LuGre model are rate dependent because of which they cannot capture the reversal point memory. Swevers et al [71] uses the LuGre model which performs satisfactorily for constant sliding velocity and suggests modification to it. In LuGre model the parameter z can be interpreted as the average bristle deflection. A change in the magnitude of frictional force occurs due to the transition between static and kinetic phase breakaway phenomenon occurring. Therefore, the transitional friction needs to be considered. However, LuGre model does not account for hysteresis behaviour. Swevers et al [71] model includes Stribeck friction in sliding, hysteretic behaviour in pre-sliding, frictional lag, varying breakaway and stick-slip behaviour, supported by experiments but does not account for material characteristics and the effect of loads on material variations. The Bliman Sorin model presented in Eq. (9) is modified form of

LuGre model [76]. Leuven model Eq. (11) is a modification of LuGre model and includes hysteresis with nonlocal memory. The modified Leuven model presented in [88] addresses the issues of memory stack size and frictional force discontinuity at closure of frictional loop with Lueven model. De Wit et al [92] described the loss of performance of high precision manipulators owing to the effects of friction. The effect of friction lag and the existence of a hysteretic relationship between friction and velocity are considered. Breakaway force can also vary according to dynamics of the contact. At microscopic contact dimensions the velocity between the contacts will be non-zero. Stick slip motion is also seen in joints. Friction compensation through observer method is used to develop the control system in such a case. The model captures friction phenomena while maintaining simplicity. The performance of the contact through start of motion to its end and the performance at various velocities have been incorporated into the system.

Haessig and Friedland [93] present two friction models of which one is based on the bristle formulation intended to capture 'sticking' effect (ref. Figure 7) and the other is called 'reset integrator' model which does not encompass sticking but is similar to Karnopp model. The process of initiation of friction is described as the interaction of peaks that initially resembles a spring damper with high stiffness that is reluctant to allow motion. Discontinuity of friction at zero velocity causes very short computational time steps and steep slopes where a linear bypass is implemented.

Reset Integrator (RI) model uses an auxiliary integrator to represent the phenomena of stiction. Advantages of this model are: (1) it is application independent (2) does not require re-derivation to suit each application (3) accurately represents bonding effect of stick-slip friction force (4) this is a logical model (5) loads are calculated to accommodate sticking loads and damping term (6) damping mode is different from bristle in that there are no two separate modes (7) computationally efficient (8) short time steps due to breaking bristles are avoided and (9) selection of parameters is much simpler than that for bristle model. The procedure is listed in Haessig and Friedland [93]. Seven-parameter model Eq. (12) consists of a spring model to capture pre-sliding and Coulomb, Viscous, Stribeck friction and friction lag.

The Bristle model presented in Figure 7 is a simple algorithm which is (1) more efficient and accurate (2) friction represented as many bristles which deflect with stiffness and damping, representative of surface contact at joint (3) frictional force is a function of velocity (4) accurate model (5) number of bristles control the fineness of the model. Disadvantages of the model are that it is (1) not efficient in terms of computational time (2) fine spaced bristles cause successive short computational time steps and can decrease efficiency of the solution method or cause algorithm execution to fail and (3) frictional force can become noisy signals.

Efficiency and accuracy of the models have been compared using fourth order Runge-Kutta method. The order of models in terms of computational efficiency are Dahl followed by Classical friction model, Karnopp, Reset Integral model and the bristle model. The selection of model is a trade-off between accuracy and computational efficiency and the need for further comparison between RI and Dahl models is highlighted. The next section examines the issue of joint clearances in revolute joint contacts.

5. Mechanistic models with clearance and friction

A classification of mechanism models has been presented in Figure 8. While the geometric and kinematic analyses give partial insight into the system performance dynamic models are required to fully describe complex interacting systems. Joints are introduced to provide some constraint on the motion of the mechanism. The joints can be dry or lubricated and contact can be intermittent or continuous and is usually determined by the area of application. Major simulation studies in the area have employed dry friction model. In an ideal mechanism, the joints would have a close fit leading to the classical mechanism model. In actual joints, however the problem of joint clearances exists. Clearances in mechanical systems may occur due to assemblage, manufacturing errors and usually undue wear, performance loss, reduction in stability, noise, dynamic impact loads and affect the transfer of system loads [94]–[96]. This alters the performance of the mechanism and affects the dynamics, control and durability of the mechanism. One example of this is the problem of manipulator end effector positioning [96]. Several factors such as the contact stiffness, surfaces condition, and lubricant need to be considered while developing the contact model for a joint with clearance. In literature the revolute joints are considered either in stand-alone configuration [96] or in assembled form of the slider crank mechanism [97] and four bar mechanism [20]. Wang and Vijaykumar also addressed a similar problem from the perspective of robotic manipulator environmental interaction. Multiple friction contacts in mechanical systems have been analysed in [98].

Modelling and simulation of multibody dynamics with joint clearances is relatively new area of research [94]. Three reviews are identified from literature namely Haines [99] – un lubricated revolute joints (1979), Flores (2010) [21] and Machado et al – compliant contact force models (2012) [100]. The problem of multibody impact with friction was first analysed by Routh in 1891 [98]. The model of dynamic systems with mechanical clearance presented by Dubowsky and Freudenstein [94], [101] in a two part publication and introduced the concept of impact pair model in which the surfaces in contact are modelled as compliant i.e. as spring damper contacts as shown in Figure 9. In 1975, Hunt and Crossley [102] studied the influence of the coefficient of restitution between two impacting bodies based on the force-law approach and recorded the results from the numerical simulations. The issue of impacting multi-bodies with kinematic contacts under the action of impulsive motion or impulsive forces has been addressed by Lankarani and Nikravesh in [103]. Lankarani-

Nikravesh contact model has been subsequently used in literature (Figure 14). Rhee and Akay [104] investigated the revolute joint with clearance for a four bar mechanism and found a non-linear dependence on both the size of the clearance and coefficient of friction with a simple friction model for sliding friction. Friction and impact with joints clearance have been presented by Periera and Nikravesh [105] for intermittent motion with dry friction at the contact.

Modelling contact connection method could use a spring-damper pair, which is activated at the beginning of the contact. Contacting surfaces are initially assumed rigid. Energy dissipation cannot be modelled if the interaction is modelled exclusively by using only a spring connection because the spring has no attached component for dissipation. To analyse the contact-impact the contact should be split into separate contact and departure events where each event is represented by switching function elements as shown in Figure 11. An alternative method of impact modelling for such contacts is either energy or Euler-Lagrangian method. Flores et al, [106] see Figure 10 have used momentum as a state variable of integration for two link pendulum and slider crank systems. Bauchau and Rodriguez [95] present a similar case in which finite element method (FEM) along with dynamic model has been proposed for a slider crank mechanism with flexible links.

Flores et al [106] have analysed the dynamics of a slider-crank mechanism with clearance in the revolute joints. Hertzian contact model and Lankarani-Nikravesh contact impact model is used for calculating the contact parameters [107]. Koshy et al [25] have evaluated the revolute joint with clearance for a rigid link slider crank mechanism focussing on the Hertzian contact model and extending the model to include damping and compared the results with experimental values. Hertz law for contacts is a static model as shown in Figure 14, has only a spring component that prevents energy dissipation. This violates the Law of Conservation of Energy at the contact. Hertz contact law is a nonlinear model. Energy transfer and conservation is a highly complex process. Lankarani-Nikravesh model is widely applicable comparing to pure elastic force law models [25] . Acceleration parameters from the numerical simulations have been compared with values recorded from the slider crank mechanism. Clearance joints have an impact on the performance of mechanism as seen from experimental data so far [108]. It is worth noting that future analyses need to include joints material properties. Flores et al and Zhao et al [97] have concluded that lubrication alone alleviates much of the effects introduced by joint clearance owing to the inherent damping qualities. This implies that the lubricant properties influence the dynamics of the system. Zhao et al [109] presents revolute joint's dynamics with mixed lubrication model by using Lagrange method and incorporating finite element method (FEM) for modelling small end of a connecting rod in an internal combustion engine. Machado et al [110] have compared the performance of various contact force models graphically. Mukras et al [27] has presented a combined model including joint elasticity and viscosity for mechanism dynamics and is shown in Figure 15.

Other models in literature include massless link in 4 bar linkage with clearance [111], three step contact model with three configurations, (1) free flight (2) impact and (3) sliding. [112] presents the three step model of the slider crank mechanism with reaction only on contact. [113] presents a four-link mechanism with three step model, using discontinuous method for pre-collision and post collision momentum balance, three mode approach in which the impact and sliding computed by using a contact force model. Non-ideal joints use force constraints and are modelled by Ravn [114]. According to contact impact pair, flexible mechanisms with multiple clearance [115], joint clearance for massless link and clearance joints have been proposed by Earles and Wu [116]. Slider crank mechanism with multiple clearance joints has been modelled by Yaqubi et al [117]. Flores at el [97] have studied the performance of lubricated journal bearings and slider crank mechanism by using Pinkus-Sternlicht revolute model. Results from the publication shows that the model operates within the bounds of hydrodynamic lubrication theory. Elasto-hydrodynamic lubrication has been considered by Flores et al [66] and Li et al[118]. Several contact force models including Pure elastic Hertzian contact force model, linearised Hertzian contact force model, Force Model, Dissipative Spring Damper Model, Gonthier model, Zhiying and Qishao model, Flores model Visco-elastic Hertzian contact model, Hunt and Crossley model, Lee Wang Hysteresis model, Lankarani-Nikravesh model and Hybrid model are presented in Figure 14. A summary of these models and their successive improvements are presented in a survey of literature [96].

Three challenges that exist in multibody mechanical systems are (1) selection of appropriate constitutive law for the contact-impact event (2) selection of appropriate contact stiffness and damping coefficient and (3) quantification of energy transfer that occurs in such an event which leads to hysteresis [110]. Dynamics of collision may be classified as non-smooth dynamics formulation and the regularised approach see Figure 13.

A solution method for linear complementarity problem which can violate energy conservation principles has been used in [119]. Other solution methods include differential variation inequality (DVI) and Moreau's time stepping algorithm [21], [119]. However the limitation in [119] can be overcome by choosing appropriate friction and degree of non-linearity for complex contact conditions. In addition, the problem of energy dissipation without violating the energy conservation condition is critical since even for low energy impacts, energy is transformed into sound and mechanical vibrations, as a function of the coefficient of restitution whose definition is subjective. The coefficient of restitution, which is representative of energy dissipation, is dependent on factors such as geometry of the contacting surfaces, pre-impact velocity, local material properties, duration of contact, temperature and friction [100]. Machado et al [100] and Flores et al [97], [100] have highlighted the importance of choosing the appropriate model for the mechanism. The

Gonthier model [26] and Flores models are suitable for moderate to low coefficients of restitution. These modified models show enhanced accuracy and the possibility of a unified model requires further investigation.

The best solution technique can be chosen such that the simulation results can be validated. There is sufficient evidence of consistent improvement in available models, which bring simulations closer to reality.

In joints with clearance, the condition of impact with motion exists and the sum of forces at the instance of impact can be represented by

$$F = F_N + F_T \tag{13}$$

Although several impact models are presented in literature to compute the normal force at the point of impact Figure 14, most literature utilises modified Coulomb friction law for computing tangential force at the point of impact. The predominant model for friction employed in dynamic models with clearance in joints is Coulomb dry friction model or modified Coulomb dry friction model [19], [21], [66], [120]. This can be explained by the fact that friction effect is only a minor component of the dynamic contact phenomenon in a joint with clearance and therefore is relegated in the analysis. Modified Coulomb friction law is given by [18]

$$F_T = -c_f c_d F_N \frac{v_T}{\|v_T\|} \tag{14}$$

Where F_T is the tangential friction force, c_f is the friction coefficient, c_d is the dynamic friction coefficient, F_N is the normal force, and v_T is the relative tangential velocity. Dynamic correction coefficient c_d is given by

$$c_d = \begin{cases} 0 & if\ v_T \leq v_0 \\ \frac{V_T - V_0}{v_1 - v_0} & if\ v_0 \leq v_T \leq v_1 \\ 1 & if\ v_T \leq v_1 \end{cases} \tag{15}$$

where v_0 and v_1 are the transition velocities.

The use of dynamic correction factor c_d improves time stepping characteristics of the solution algorithm. The influence of F_N on F_T is therefore determined by impact model. The analysis of journal bearing with clearance joint is presented by Bai and Zhao [96] which incorporates a new contact force model. The essence of applying any such model is to capture the actual physical phenomenon of impact, rebound and movement as shown in Figure 11 and Figure 12. Although several models have been reported in literature, there is scope for improvement in the numerical prediction of joint performances and this requires a case by case evaluation depending on the number of factors affecting contact conditions and performance. Since the contacting surfaces are influenced by several factors, the interacting surfaces and their durability have been examined in the following section.

6. Durability in harsh environments

Durability is the capacity of the mechanism to perform the designated function and fulfil the intended design life without unexpected failure. Durability is critical in all cases especially in mechanisms that are designed for deployment in harsh environmental and operational conditions. Failure of equipment leads to halting progress, loss in revenues and could cause accidents e.g. search and rescue missions in disaster stricken areas. Therefore design life cycle analysis is critical for specialised deployment in high risk environments [121].

The failure of mechanisms deployed in hazardous environments is discussed in literature [36], [122], [123]. Design failure was recognised as a major factor. This also includes the failure of components. Mechanical durability of mechanisms subject to both constant and variable loading is highly desirable. Interacting surfaces of the manipulator revolute joint between various links are affected by multiple factors such as load, sliding speed, lubrication, heat and the influence of external agents such as corrosive fluids and debris. Therefore, failure in interacting surfaces may occur through a variety of modes such as plastic or visco-plastic deformation of material, wear of material through breakdown of lubrication, entry of debris into contact, crack propagation etc.

Durability of interacting surfaces can be enhanced through several methods. According to Ludema [125] there are three methods of modifying surfaces which are: surface treatment [126], surface modification [127] and surface coating [128]. The effects of atmospheric agents on exposed metal alloy surfaces are studied and their durability has been evaluated in [129], [130], [131]. According to Bhushan [132], wear reduction can occur through non-uniformly tall, mean pressure of rounded peaks at the contacts should be lesser than the yield strength of the softer material in the contact. Surface hardening can also improve durability [133]. The use of suitable greases lubricants can also extend the life of the interacting surfaces [134], [135]. The durability of grease can be further enhanced through additives [136]. Erdemir [137] has presented a review of interfaces with attention to solid lubricants which extend wear life.

The enhancement of wear performance through diamond like carbon (DLC) coatings has also been discussed [137]. The use of diamond like carbon coatings has been presented in [138] and significantly reduces friction. Results of nano-composite coating, friction and wear analyses in rolling contact has been presented in [128]. Analysis of interaction at the joint for PEEK and Al 7075 alloys in robotic arms has been presented by Koike et al [139]. Erdemir [140] has recommended the texturing of surfaces to improve the retention of lubrication and provide superior wear resistance in the contact. The addition of surfactants is also expected to enhance the contact durability by modifying the surface characteristics of the contact [141], [142].

Progress has been made from simple surface hardening to the use of surfactants to enhance wear resistance of contacting surfaces. Continued analysis of the influence of external agents in joint contacts and the influence on dynamics at the contact and investigation of the improvement of wear resistance and friction characteristics of the contact surfaces, surface modification and coatings is required for specialised applications. Furthermore, tribological testing is required to ensure the resilience of the interacting joints. Virtual prototyping and simulation [27], [124], along with the tribological experimentation is necessary for accurate prediction and to enhance the durability of interacting surfaces.

A rapid progress is desired in the modelling techniques and simulations, which bring it asymptotically close to actual physical models. In addition, the increasing reliance on simulation packages and virtual prototyping and have been summarised in the forthcoming section.

7. Software packages used in multi-body dynamics

Several commercial software packages are available for the simulation of multibody dynamics [66]. For rigid link multibody mechanics simulation SimMechanics 1st or 2nd generation [53], [143] packages can be used. For joints with clearance packages like ADAMS [25], [144], [145], COMPAMM, NEWEUL, DAP3D [146], MUBODYNA [100]. RAPID and PARASOLIDS have been used [25] for interference detection. The problem can be formulated by using programming languages such as C/C++, Python or MATLAB m-code, depending on the intended application. However, the process of modelling the mechanics from first principles is often too tedious. Increasing reliance on software simulation packages has been observed with increasing model complexity.

8. Conclusions

This paper covers literature in friction and dynamic modelling. It captures the evolution of mechanism and friction dynamics briefly. Key improvements in the area have been identified and presented. The relevance of friction models in manipulator dynamics is seldom discussed because of the difficulty of incorporating all the influencing factors into a single model. However, with the improvements in computational and numerical modelling techniques, the frictional dynamics of mechanisms is more effectively elucidated upon.

Rapid evolution of mechanical modelling methods over the latter half of the 20th century include advances in modelling techniques and computational methods [147]. These enable the modelling and the simulation of complex mechanisms with increased accuracy. The progress from simple kinematics to complex dynamics has been affected through the implementation of several advanced modelling techniques. Improvements in computational capacity has also enhanced the tools available to designers resulting in the reduction of the overall process time.

Friction and wear component requires further research. Friction models have progressed from simple Coulomb, viscous and Stribeck friction models to the more comprehensive dynamic models such as Leuven model. Some models can account for wear in the contact by incorporating finite element computational techniques. High degree of non-linearity of the friction model and factors influencing the contact including the surface conditions, material properties, contact conditions and lubricating conditions are among several other factors and the interlocking nature of these factors ensure that the convergence of a single friction modelling equation does not occur. To add to the complexity, the dynamic modelling of mechanisms involves additional uncertainties such as the end effector trajectories [148] and environmental interaction [98]. Therefore, it is necessary to examine each case individually to determine the important influencing factors in each case.

Progress in dynamic modelling methods with improved friction modelling is envisaged such as a hybrid model that is capable of switching between different regimes. However, the unobservable transition conditions in contact makes this task tedious. Until such a unified model can be derived, smaller unified models addressing specific conditions of contact and frictional force generated in such contacts are useful. This philosophy is consistent with the literature. The influence of environments on contacts determines the life of the mechanism [128]. Improvement of the dynamic modelling techniques and tribo-analysis of the material at the contact is imperative within this context. Further analysis to determine the influence between dynamics of mechanism, material properties, coefficient of friction and the influence of environments is needed (Figure 16).

Tribo-testing subject to load, lubrication conditions and environmental influences would elucidate the dynamics and tribological outcome of such contacts. The interaction between any two or more influencing factors may lead to accelerated failure at the joint or an inordinate rise in frictional resistance which are detrimental to the manipulator operation especially when precision positioning while handling heavy loads is required. The data generated thereof can be used to construct a specific but comprehensive model for the above-mentioned factors.

List of symbols

A	Parameter in Bliman-Sorin model
B	Parameter in Bliman-Sorin model
c_d	Dynamic friction coefficient in modified coulomb friction law

c_f	Friction coefficient in modified coulomb friction law
C	Parameter in Bliman-Sorin model
C_i	Coordinate systems of the multibody with clearance
D	Damping coefficient
DLC	Diamond like coating
DV	Limit velocity in Karnopp model
E^*	Effective modulus of elasticity
e_y	The distance between centres along the ordinate
e_x	The distance between centres along the abscissa
e_{ij}	Vector distance along displacement of centres
F	Friction force generated by friction model
F_1	Force applied on the rigid body 1
F_2	Force applied on the rigid body 2
F_c	Coulomb friction force
F_d	Hysteresis force in the Leuven Model
F_{ext}	Force applied by external actuator
F_f	Friction force (Seven Parameter model)
F_h	Hysteresis force in the Leuven Model
F_n, F_N	Normal force at contact

F_s	Static friction coefficient or Stribeck friction coefficient
$F_{s,a}$	Magnitude of the Stribeck friction at the end of the previous sliding period (Seven Parameter model)
$F_{s,\infty}$	Magnitude of the Stribeck friction after a long time at rest (Seven Parameter model)
F_T	Tangential friction force in modified coulomb model
F_v	Viscous friction force
k	Spring constant
K	Stiffness at contact
M	Mass matrix
n	Normal along the contact
n	Shape curve transitioning coefficient
N	Exponential coefficient
O	Origin of the global coordinate system
O_i	Coordinate system i of the journal
O_j	Coordinate system j of the journal
PEEK	Polyether ether ketone
$\ddot{q}$	Generalised acceleration state vector
r	Clearance between the bodies at contact
R_o	Radius of the outer bearing

R_i	Radius of the journal
RI	Reset Integrator model
s	Space variable in Bliman Sorin model
$s(v)$	Shape transitioning curve
t	Tangent at contact point
t_2	Dwell time (Seven parameter model)
T_L	Time constant of frictional memory
v	Velocity at contact of the moving body
v_0	Threshold values of velocities for dynamic correction factor in the modified Coulomb friction model
v_1	
v_s	State variable in Bliman Sorin model
v_T	Relative tangential velocity at contact
$\dot{x}$	Sliding velocity
$\dot{x}_s$	Characteristic velocity of the Stribeck friction
x	Sliding distance
X	Hysteresis damping factor
X	X axis of the global coordinate system
X_1	Displacement of the rigid body 1
X_2	Displacement of the rigid body 2

Y	Y axis of the global coordinate system
z	State parameter in friction model
α	Baumgarte coefficients
β	Baumgarte coefficients
γ	Time parameter of the rising static friction (Seven parameter model)
$\dot{\delta}$	Time derivative of deflection at contact
δ	Deflection at contact/Penetration depth of journal and bearing
λ	Lagrange multiplier
ϕ_q	Jacobian matrix for constraint equations, the superscript T denotes its transpose
σ_0	Coefficient accompanying the state variable, An equivalent stiffness for position-force relationship at velocity reversal (LuGre model), the tangential stiffness of the static contact
σ_1	Micro-viscous friction coefficient
σ_2, σ_2	Viscous friction coefficient

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List of figures

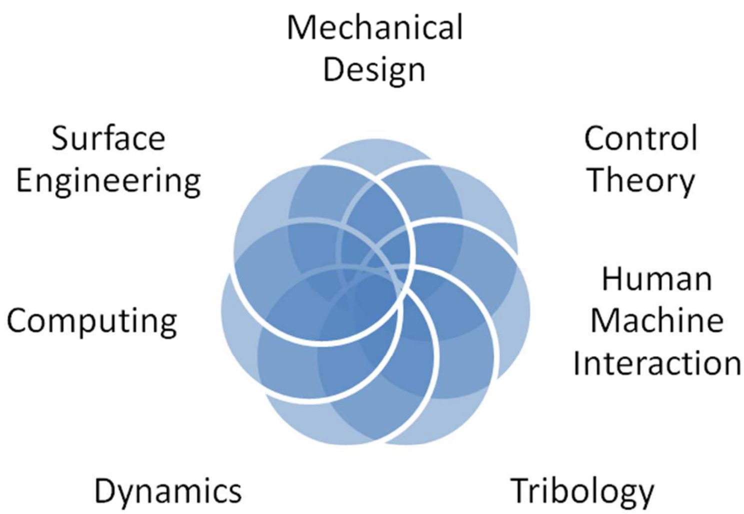


Figure 1 The area of manipulator design modelling and control arises from the confluence of several branches of engineering and science

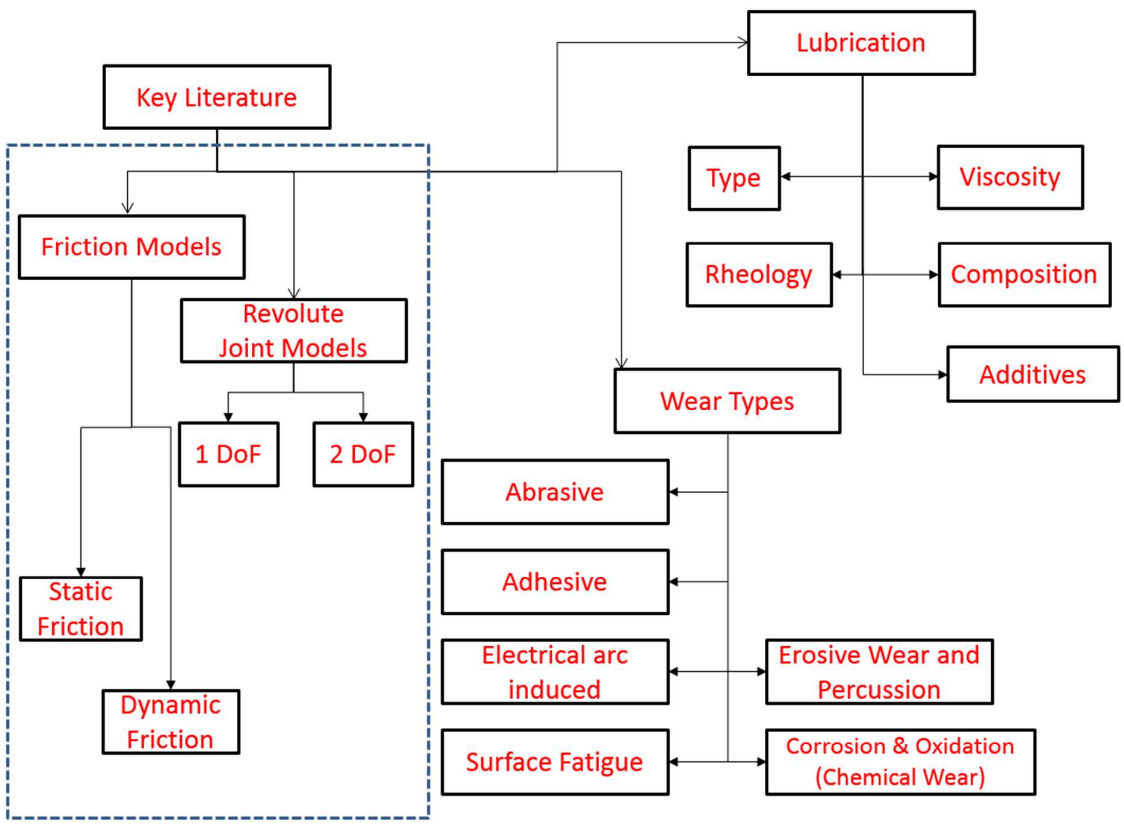
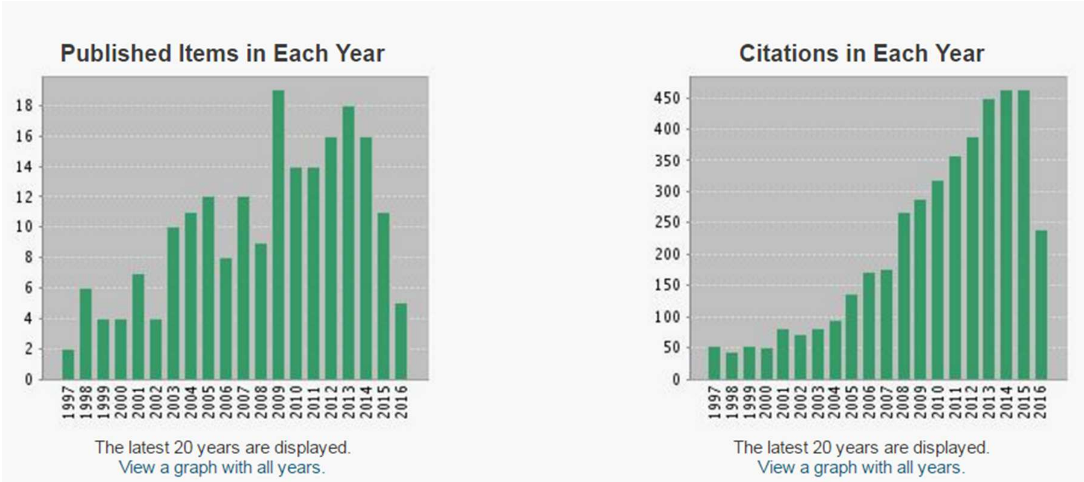


Figure 2 Factors influencing the manipulator mechanism. This blue box highlights the focus areas of this paper i.e. dynamics and friction with reference to manipulator methodology



Results found:	216
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Sum of Times Cited without self-citations [?] :	4312
Citing Articles [?] :	4177
Citing Articles without self-citations [?] :	4148
Average Citations per Item [?] :	20.12
h-index [?] :	34

Figure 3 Articles published in the domain containing keywords of friction models, static and dynamics

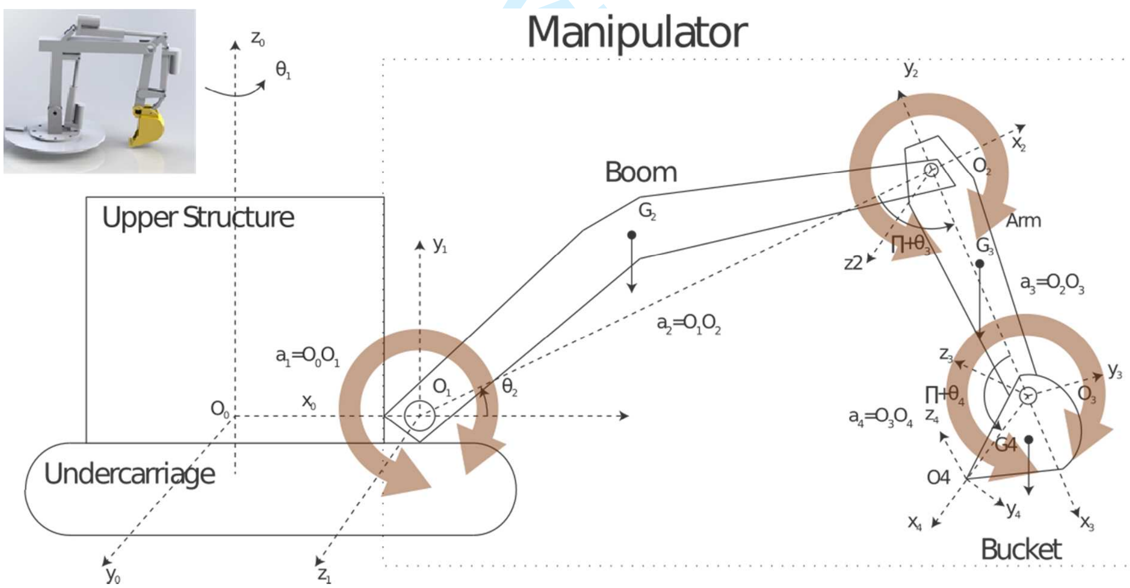

Figure 4 The layout of a robot with a planar manipulator mechanism and the rendered image of a manipulator designed in CAD environment; the arrow overlay depicts the friction torques at rotary joints when the manipulator operates in planar action. The inset of the figure shows the rendering of a manipulator model generated within the computer aided design environment.

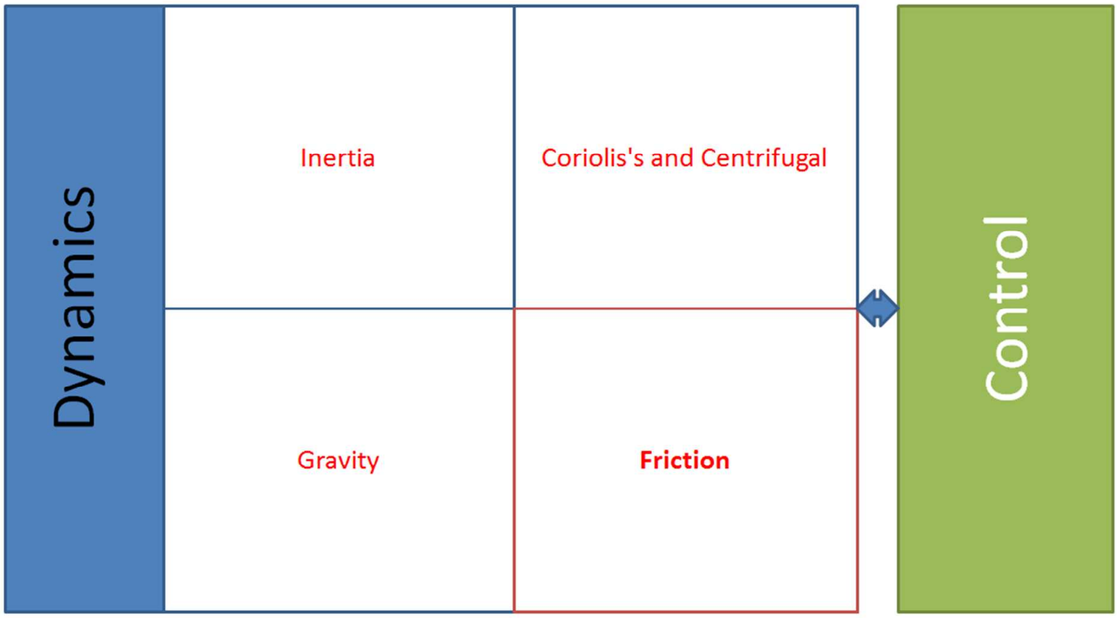

Figure 5 Representation of the dynamics equation and its components

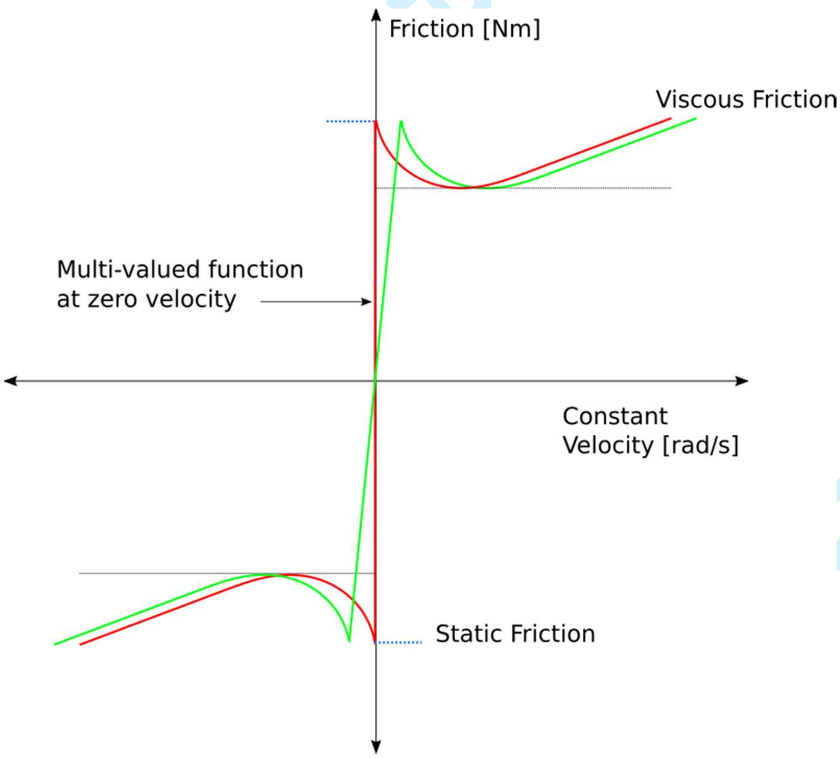

Figure 6 The combined effect of Coulomb friction, Viscous Friction and low velocity Stribeck effect based on [1]

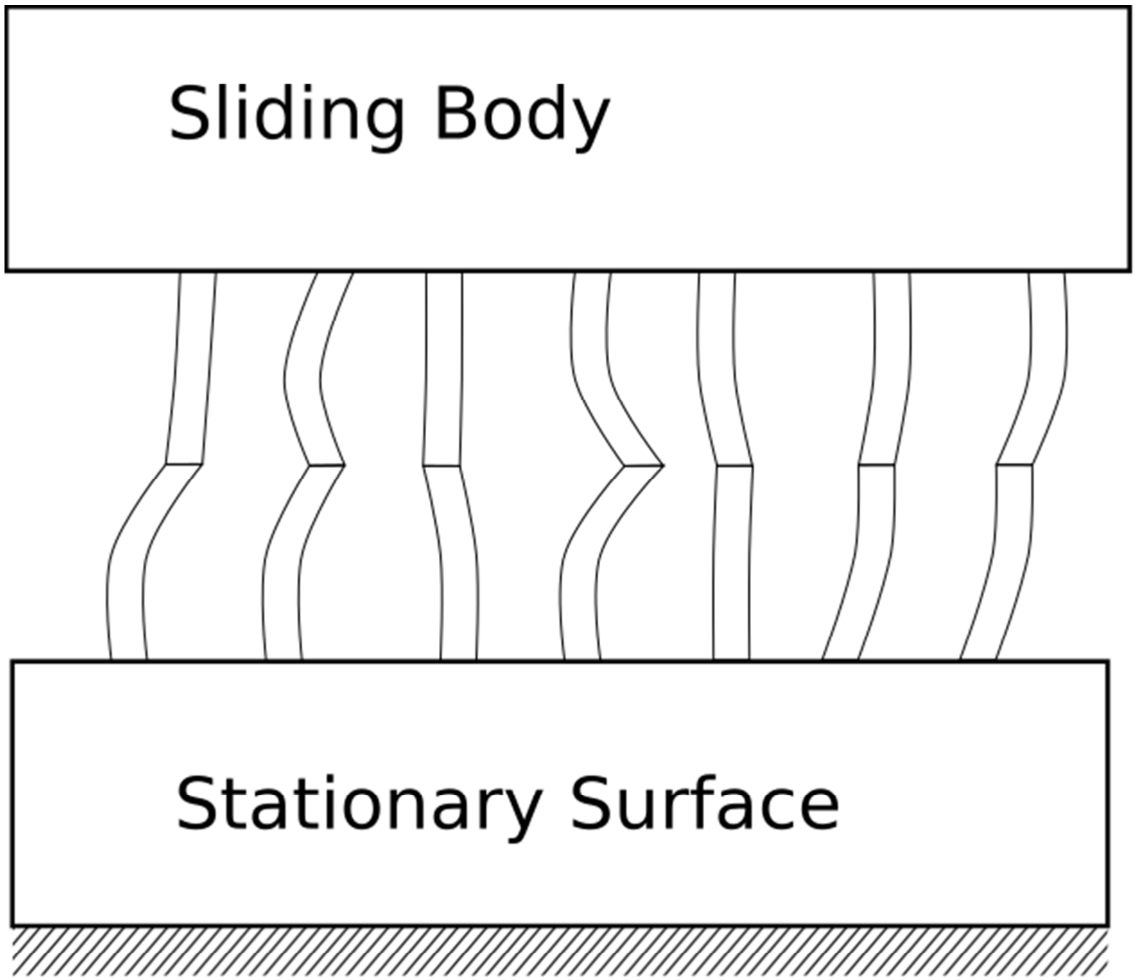

Figure 7 The Bristle model shown above is one in which the contacting surfaces interact through bristles [2]

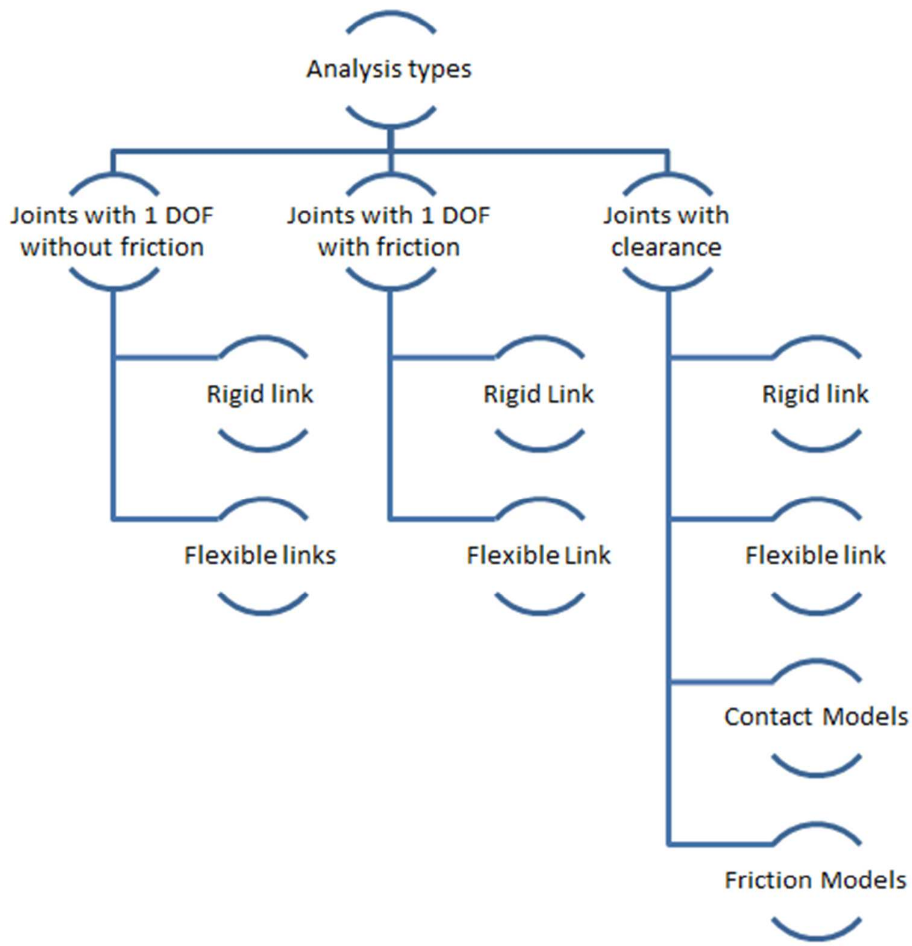

Figure 8 Classification of manipulator dynamics research and problems

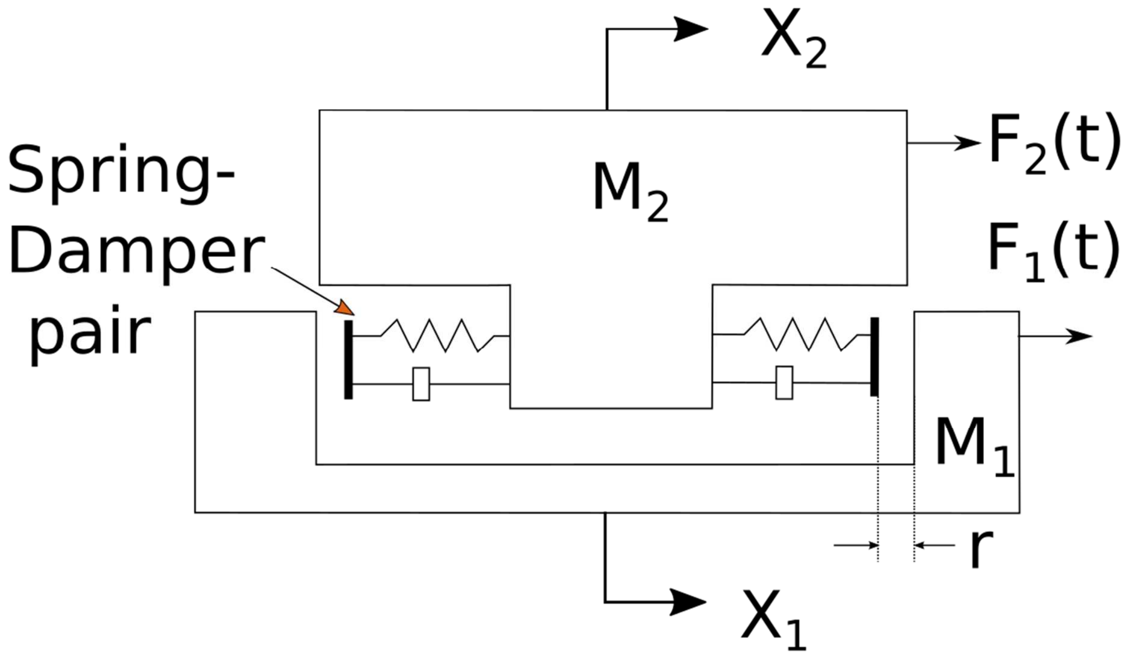

Figure 9 Simple dynamic link coupling with clearance which leads to the 'impact pair' condition based on [3]

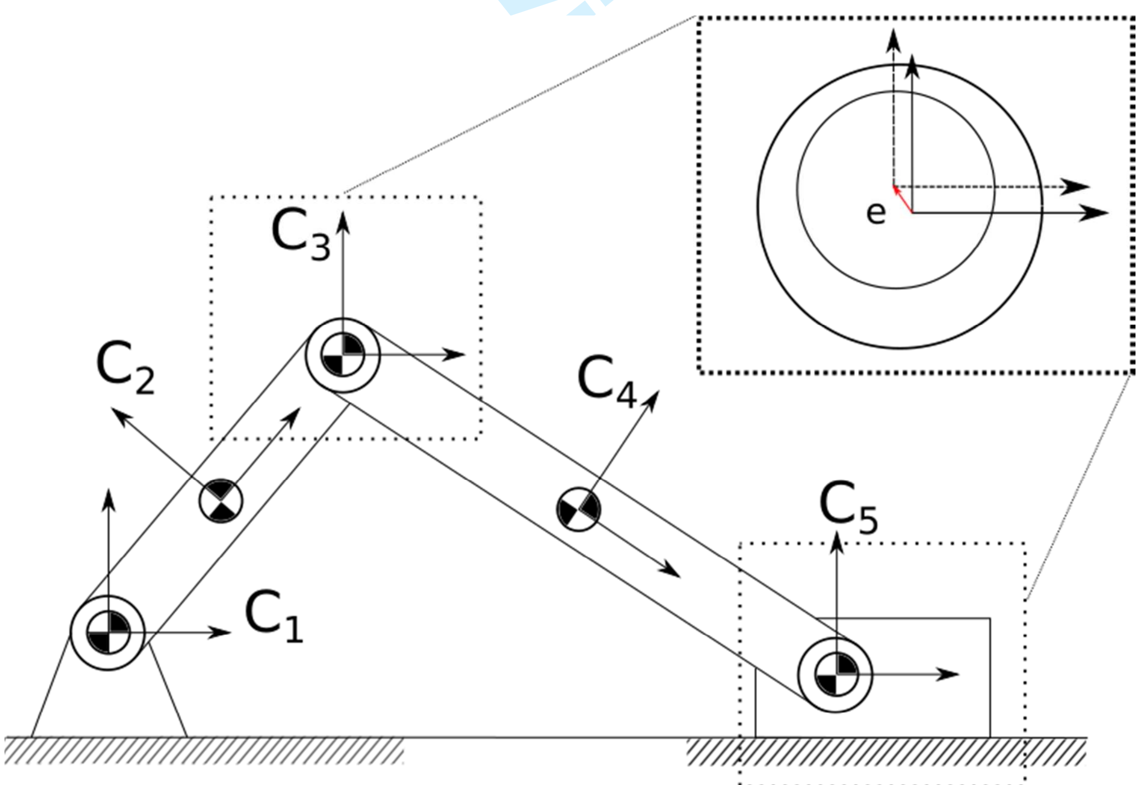

Figure 10 Multibody system using slider crank mechanism with clearance

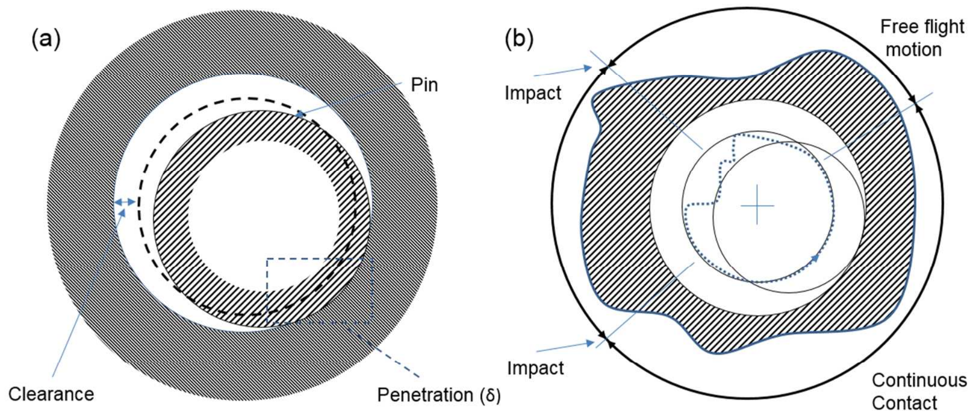

Figure 11 The different modes in a revolute joint with clearance[4]*, and penetration in joints*[5]

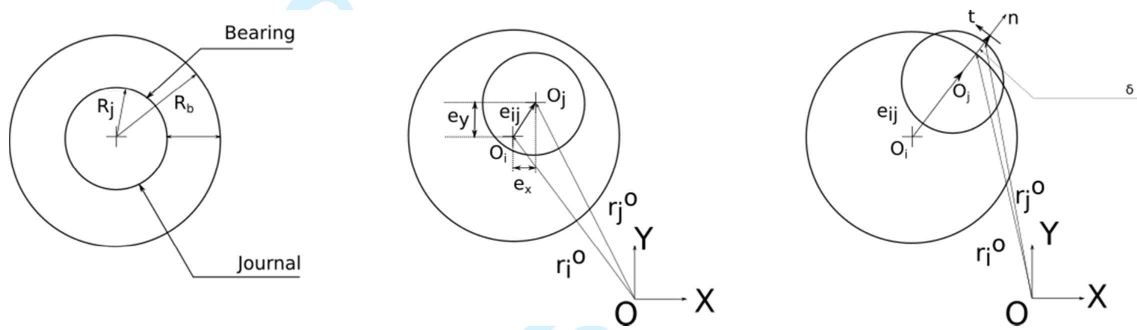

Figure 12 Representation of the journal bearing with clearance for planar case based on [6]

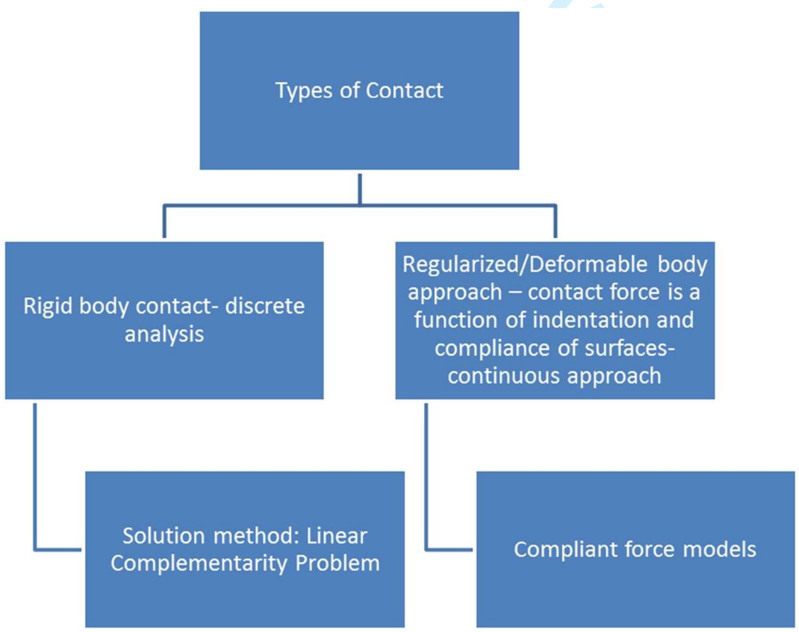

Figure 13 Contact classification according to Machado et al [7]

Pure Elastic Hertz Contact Force Model
$$F_N = K\delta^N$$

Linearized Hertz Contact Force Model
$$F_N = K\delta$$

Force Model
$$F_N = \frac{\pi E^* L\delta}{2}\left(\frac{\delta}{2(c+\delta)}\right)^{0.5}$$

Dissipative Contact Force Model
$$F_N = K\delta + \mathrm{D}\dot{\delta}$$

Visco Elastic Hertz Contact Force Model
$$F_N = K\delta^N + \chi\delta^N\dot{\delta}$$

Hunt and Crossley Elastic Hertz Contact Force Model
$$F_N = K\delta^N\left[1 + \frac{3(1-c_r)}{2}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right]$$

Herbert and McWhannell Model
$$F_N = K\delta^N\left[1 + \frac{6(1-c_r)}{[(2c_r-1)^2+3]}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right]$$

Lee and Wang hysteresis model
$$F_N = K\delta^N\left[1 + \frac{3\,(1-c_r)}{4}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right]$$

Lankarani- Nikravesh Model
$$F_N = K\delta^N\left[1 + \frac{3\,(1-{c_r}^2)}{4}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right]$$

Gonthier Model
$$F_N = K\delta^N\left[1 + \frac{(1-{c_r}^2)}{c_r}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right]$$

Zhiying and Qishao Model
$$F_N = K\delta^N\left[1 + \frac{3(1-{c_r}^2)e^{2(1-c_r)}}{4}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right]$$

Flores Model
$$F_N = K\delta^N\left[1 + \frac{8(1-c_r)}{5c_r}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right]$$

Figure 14 Equations for the different contact models [7]–[9]

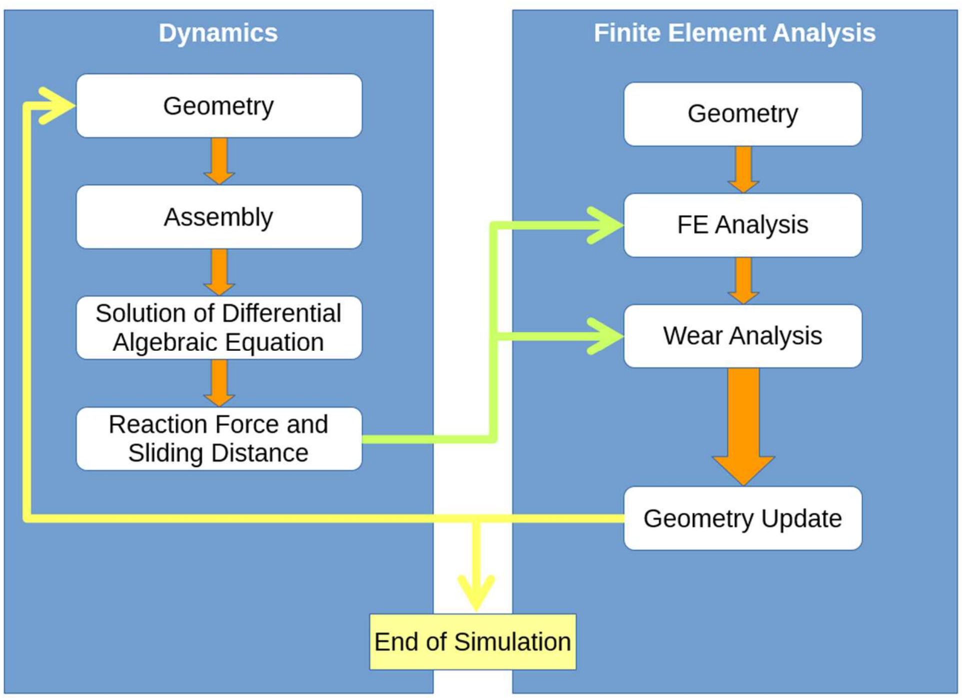

Figure 15 Wear analysis integrated with the dynamics based on [10]

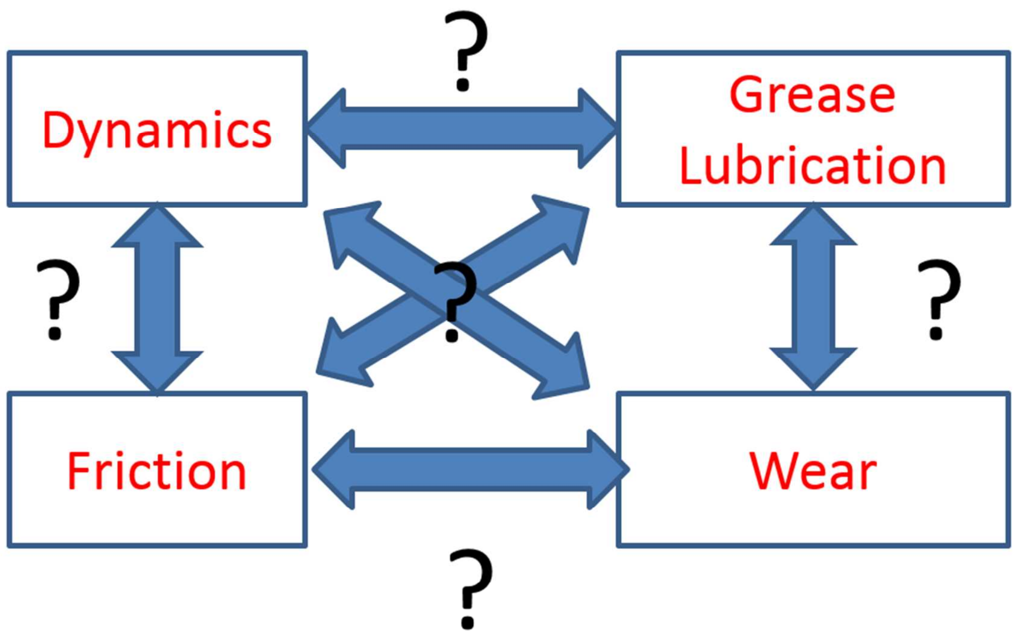

Figure 16 The relationship between various components of a mechanism requires further study. After several decades of research there is no governing principle owing to the sheer complexity of the phenomenon involved.

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