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AND YET ANOTHER METHOD FOR THE IDENTIFICATION OF MODAL CONSTANTS IN EXPERIMENTAL MODAL ANALYSIS

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ABSTRACT

Modal Identification from Frequency Response Functions (FRFs) is a chapter of Experimental Modal Analysis (EMA) that many would consider as something from the past. Yet, in a previous work [2], a new approach to determine the modal damping factors from FRFs was proposed. Contrary to other modal identification methods which are based on the dynamic motion governing equations, the method used the dissipated energy per cycle of vibration as a starting point. The method used a plot of the sine of the phase of the receptance against its amplitude. In this paper, it will be shown that near resonant frequencies, its shape is elliptical, whereby the modal constants can be determined.

Keywords: experimental modal analysis (EMA); modal identification; modal constants.

1. THEORETICAL DEVELOPMENT

If the modes of a dynamic system are sufficiently spaced, the influence from other modes is small when compared to the resonant mode [2]. In such a case, the receptance of an MDOF in the vicinity of a resonance ω_r resembles the equation of a SDOF[†] and can be expressed as [1]:

$$\alpha_{\omega\to\omega_r} \cong \frac{A_R + iA_I}{\omega_r^2 - \omega^2 + i\eta_r\omega_r^2} \quad (1)$$

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[†] In fact, a SDOF has no complex modal constant and it equals unity. It should be emphasized that this arbitrary SDOF corresponds to the approximation where the influence of other modes (in a MDOF) can be neglected and the mode under analysis can be accepted as a SDOF system.

where A_R and A_I are the real and imaginary parts of the complex modal constant, ω_r is the angular natural frequency and η_r is the modal damping factor for mode r. The whole development is based on this assumption: that mode shapes are conveniently well spaced in the frequency spectrum.

Condition 1: $|\omega_r^2 - \omega^2| \gg 0$

Away from the natural frequency, and considering, for better convenience, a lightly damped system where $\eta_r \cong 0$, equation (1) is simplified to:

$$\alpha_{\omega \ll \gg \omega_r} \cong \frac{A_R + iA_I}{\omega_r^2 - \omega^2} = \frac{A_R}{\omega_r^2 - \omega^2} + i\frac{A_I}{\omega_r^2 - \omega^2} \tag{2}$$

If the receptance is represented in the Argand plane, then the phase $\theta_{\omega \ll \gg \omega_r}$ is related to the imaginary $\alpha_{I_{\omega \ll \gg \omega_r}}$ and real $\alpha_{R_{\omega \ll \gg \omega_r}}$ parts of the receptance by:

$$\tan\left[\theta_{\omega \ll \gg \omega_r}\right] = \frac{\alpha_{I_{\omega \ll \gg \omega_r}}}{\alpha_{R_{\omega \ll \gg \omega_r}}} = \frac{A_I}{A_R} \Rightarrow \theta_{\omega \ll \gg \omega_r} = \tan^{-1}\left(\frac{A_I}{A_R}\right) \tag{3}$$

Therefore, in the $x \equiv H$ *vs* $y \equiv \sin(\theta)$ plane this becomes:

$$\sin\left[\theta_{\omega \ll \gg \omega_r}\right] = \sin\left[\tan^{-1}\left(\frac{\mathrm{A_I}}{\mathrm{A_R}}\right)\right] \tag{4}$$

Condition 2: $|\omega_r^2 - \omega^2| = 0$

When at the natural frequency, i.e., when $\omega = \omega_r$, equation (1) will achieve its maximum value:

$$\alpha_{\omega = \omega_r} = \frac{A_R + iA_I}{i\eta_r\omega_r^2} = \frac{A_I}{\eta_r\omega_r^2} - i\frac{A_R}{\eta_r\omega_r^2} \tag{5}$$

which, when solved for A_R, becomes:

$$A_R = \sqrt{H_{\omega=\omega_r}^2 \eta_r^2 \omega_r^4 - A_I^2} \tag{6}$$

Equation (6) allows determining the real part of the modal constant from its complex counterpart, which must be determined somehow. Therefore, if one solves equation (4) for A_I when having equation (6) in consideration, and after some mathematical manipulation, this results in:

$$A_I = \sqrt{\frac{H_{\omega=\omega_r}^2 \eta_r^2 \omega_r^4}{\left[\tan[\sin^{-1}(\theta_{\omega \ll \gg \omega_r})]\right]^{-2} + 1}} \tag{7}$$

Equations (6) and (7) allow determining the real and imaginary parts of the modal constant from the plot of the receptance in the $x \equiv H$ *vs* $y \equiv \sin(\theta)$ plane.

2. PROPERTIES OF THE RECEPTANCE IN THE $x \equiv H$ *VS* $y \equiv \sin(\theta)$ PLANE

Let us consider the example of an arbitrary SDOF system with complex modal constant $\bar{A}_r = 1000 + 500i$, 20.4 Hz natural frequency and 0.01 modal damping factor. Let us also assume that the receptance was experimentally measured in the 0 to 40 Hz frequency range with a period of acquisition of 2 s (i.e., a 0.5 Hz frequency resolution). Plots of the amplitude H and phase θ of the receptance so obtained in the frequency domain and the same function in the $x \equiv H$ *vs* $y \equiv \sin(\theta)$ plane are shown in figure 1.

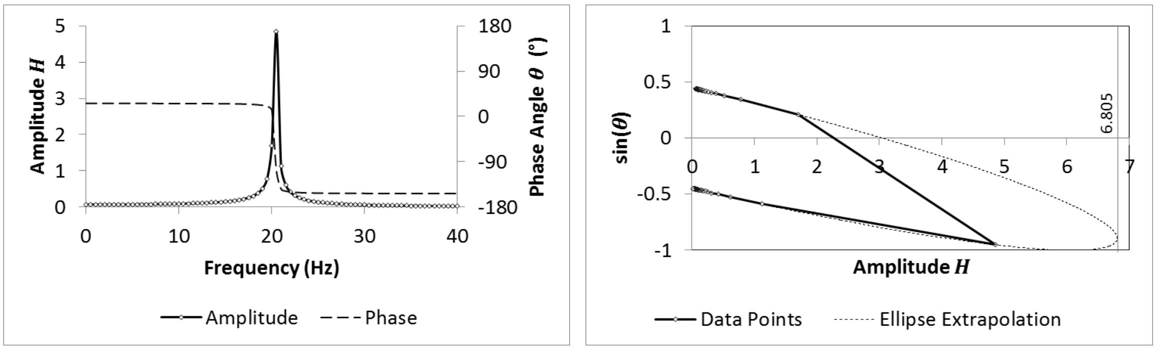

Figure 1. Example of the amplitude and phase of a SDOF receptance in the frequency domain (left) and the same SDOF receptance represented in the $x \equiv H$ *vs* $y \equiv sin(\theta)$ plane (right).

The first observation to note is that the receptance data points when plotted in the $x \equiv H$ *vs* $y \equiv \sin(\theta)$ plane describe a loop that can be fitted with the half of an ellipse. In this case, since the data has no noise and it is a SDOF, a perfect correlation between the data and the fit was obtained (figure 1 to the right). This ellipse has some important properties, namely:

1. The ellipse is centred at (0, 0) and its slope depends on the damping coefficient h (0, 0) [9. 10].
2. The function is limited between 1 and -1 since it depends on a sinusoidal function. In the example shown, where half an ellipse is represented, the ellipse is tangent at $y = -1$;
3. Since the natural frequency (20.4 Hz) is not a multiple of the frequency resolution (0.5 Hz) in the example shown, the frequency spectrum does not show the exact amplitude at the natural frequency. The amplitude at the resonance for this given example is determined to be 6.805 from both equation (1) and the ellipse in figure 1 to the right.
4. The value $\theta_{\omega \ll\gg \omega_r}$ required to determine the imaginary part A_I of the modal constant (equation 7) can also be determined from the ellipse, since this is when the ellipse crosses the y axis, i.e. $\theta_{\omega \ll\gg \omega_r} = \sin^{-1}[\sin(\theta_{x=0})]$.

3. NUMERICAL EXAMPLES AND RESULTS

A total of eight different cases were used to illustrate the proposed method, which results are all listed in the full paper. Some of these are listed in table 1. The results from the modal identification following the process described herewith are shown in table 2. It is shown that there is good agreement between the theoretical models and the results obtained from the modal identification, at least for SDOF systems. The MDOF produced slightly worse results, namely for the second mode shape. This might be due to the fact that this mode shape has a considerably high imaginary part and also because of the influence of the other mode shape in the vicinity. Figure 3 shows the MDOF case 7 in the x ≡ H vs y ≡ sin(θ) plane, where the fitting ellipses can be clearly seen.

Table 1. Numerical models' theoretical properties.

	Mode 1					Mode 2				
	Modal Constant 1		$f(Hz)$	η (%)	Amp	Modal Constant 2		$f(Hz)$	η (%)	Amp
Case	Real	Imag				Real	Imag			
1	1000	0	20.4	1	6.087	-	-	-	-	-
5	1000	-500	20.4	1	6.805	-	-	-	-	-
7	1000	-500	20.4	1	6.805	2000	-1200	50.25	5	0.4679

Table 2. Results from the modal identification.

Case	Mode 1					Mode 2				
	Modal Constant 1		$f\,(Hz)$	η (%)	Amp	Modal Constant 2		$f\,(Hz)$	η (%)	Amp
	Real	Imag				Real	Imag			
1	1019	1033	20.40	1.000	6.204	-	-	-	-	-
5	1004	-502.1	20.35	1.009	6.805	-	-	-	-	-
7	996.3	-485.0	20.22	0.939	7.312	2636	-1803	49.64	0.07189	0.4670

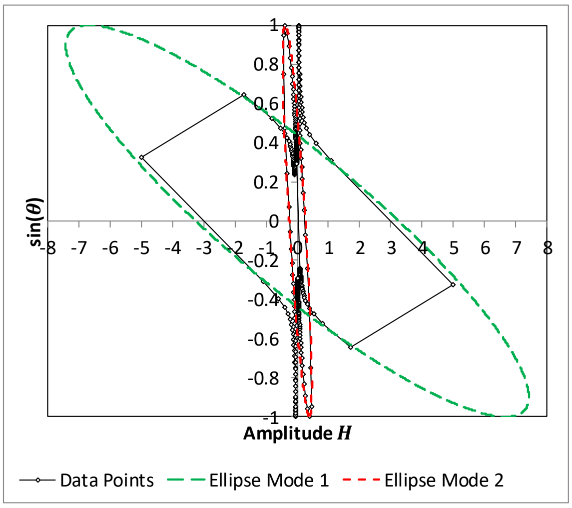

Figure 3. MDOF receptance for case 7 represented in the $x \equiv H$ vs $y \equiv sin(\theta)$ plane with two ellipses fitting the date at the vicinity of the mode shapes.

4. CONCLUSIONS

A novel method for the identification of the modal constants from FRFs for lightly damped systems with conveniently spaced mode shapes was presented. In previous works [2], it was shown that this method can provide a better estimate of the modal damping factors than the method of the inverse. However, with respect to the modal constants, more research is still required. Firstly, the method should be capable to "isolate" the already identified mode shapes to reduce the degree of influence from the other mode shapes. This is something that can easily be achieved programmatically and is expected to increase its accuracy. Secondly, the method should be tested on actual experimental data to assess how robust it is in real situations where noise and uncertainty are present. Thirdly, the fitting of the ellipse is not easy to achieve and must be improved, especially when the frequency resolution is coarse, as most of the data points in the ellipse will be shifted towards the origin. Finally, the method is yet to be tested on the modal identification of multiple FRF functions.

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REFERENCES

[1] N.M.M. Maia, J.M.M. Silva, Theoretical and Experimental Modal Analysis, Research Studies Press and John Wiley and Sons, Somerset, 1997.

[2] D. Montalvão, J.M.M. Silva, An alternative method to the identification of the modal damping factor based on the dissipated energy. Mech. Syst. Signal Process., 54-55 (2015) 108-123.