UIP: a two-regime approach

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Abstract

This paper adds to evidence that the forward-discount puzzle is at least partly explained as a compensation for taking crash-risk. A number of Central and Eastern European exchange rates are compared. A Hidden Markov Model is used to identify two regimes for most of the exchange rates. These two regimes can be characterised as being either periods of stability or periods of instability. The level of international risk aversion and changes in US interest rates affect the probability of switching from one regime to the other. This model is then used to assess the way that these two factors affect the probability of a currency crisis. While the Czech Republic, Hungary and Bulgaria are very sensitive to international financial conditions, Poland and Romania are relatively immune.

JEL classifications: C24, F31, F32; *Key words:* Exchange rates, uncovered interest parity, foreign exchange risk discount, hidden-Markov model, carry-trade

1 Introduction

The common observation that exchange rate changes deviate systematically from the forward rate so that excess returns are available from borrowing low interest rate or funding currency for an investment in higher rate units or the investment currency is called the forward discount puzzle. This paper uses a two-regime model to understand more about crash-risk by assessing UIP deviations in a range of CEE countries and by using a Hidden Markov

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Model (HMM) to divide the deviations into two categories: those where the high-yield currency does not depreciate (or may even appreciate) against the lower interest rate unit and those where the high-yield currency falls much more than would be anticipated by UIP. In the first case, the conditions are favourable for inflow of capital to the higher interest rate unit. It would be possible to make profitable carry-trades where funds are borrowed in the lower rate unit currency, transferred into the currency with the higher interest rate for the same period as the borrowing and then exchange back and repaid at the end of the contract. These are likely to be times of relative calm in the foreign exchange market. In the second case, these carry-trades would be more likely to lose money and a sharp depreciation of the high yield currency would be associated with exchange rate crisis.

The first regime might be called the period of 'calm' and is a much more common occurrence. The second period is rare and could be called 'crisis'. We use a Hidden Markov Model to identify the probability of switching from a calm regime to one of crisis for each country and also assess the way that these probabilities are affected by exogenous events such as international risk aversion and changes in US interest rates. These exogenous factors increase the risk of experiencing an exchange rate crisis and can be used to compare how vulnerable each country is to these type of external shocks.

This paper adds to the evidence that the forward discount puzzle is explained as a compensation for taking crash-risk, builds on the use of non-linear methods and greatly extends and expands on the use of HMM models for understanding the forward discount puzzle. This paper also contributes to the literature on *sudden stops* as it provides for comparative measures of the probability that a period of calm in the foreign exchange market will switch to one of crisis and it also measures how this probability is affected by changes in international risk appetite and changes in US interest rates. The rest of this paper proceeds as follows, Section 2 discusses the forward discount puzzle, Section 3 presents the Hidden Markov Model, Section 4 Analyses the initial results, Section 5 Considers exogenous influences on the probability of switching from one regime to another and Section 6 Concludes. the response model is the relationship between the observed carry-returns and the unobserved financial regime.

2 Literature

The forward rate today is the rate that currencies can be exchanged in the future. Covered Interest Parity (CIP) asserts that, given the free flow of international capital and competitive markets, the difference between the spot rate and the forward rate must be equal to the interest rate differential for the two currencies for the same period.

$$\frac{F_{t,j}}{S_t} \times (1 + i^*_{t,j}) = (1 + i_{t,j}) \tag{1}$$

where $F_{t,j}$ is the forward exchange rate at time t for domestic currency in terms of overseas for j periods ahead; S_t is spot exchange rate under the same terms at time t; $i_{t,j}$ is the interest rate for the home currency in period t for j periods ahead; $i^*_{t,j}$ is the interest rate for the overseas currency at time t for j periods ahead.

Therefore,

$$\frac{F_{t,j}}{S_t} = \frac{(1 + i_{t,j})}{(1 + i^*_{t,j})} \tag{2}$$

Re-arranging

$$\frac{F_{t,j} - S_t}{S_t} = \frac{(i_{t,j} - i^*_{t,j})}{(1 + i^*_{t,j})} \tag{3}$$

Assuming rational expectations and no financial frictions, the forward rate should be the best unbiased estimate of the future spot rate and therefore, Equation 3 becomes.

$$\frac{E[S_{t+j}] - S_t}{S_t} = \frac{(i_{t,j} - i^*_{t,j})}{(1 + i^*_{t,j})} \tag{4}$$

if i^* is relatively small, this can be approximated by

$$E[s_{t+j}] - s_t = i_{t,j} - i^*_{t,j} \tag{5}$$

where s_t is the log of the exchange rate at time t, s_{t+j} is the log of the exchange rate at t plus j, $i_{t,j}$ is the j-period interest rate at time t and $^*i_{t,j}$ is the the foreign currency j-period interest rate at time t.

Assuming that CIP holds[1] so that the forward rate can account for the interest rate differential, a test of UIP can take the form of

$$\Delta s_{t+j} = \beta_0 + \beta_1 f_{t+j} + \varepsilon \tag{6}$$

where Δs_{t+j} is the change in the log of the exchange rate between period t and j, f_{t+j} is the forward discount expressed as the difference between the logs of the spot rate and the forward rate for j periods ahead of t; ε is an

[1]Since the global financial crisis there is increased evidence that this is not the case as credit and regulatory frictions have become more prominent.

error term, while β_0 and β_1 are the coefficients to be estimated. If UIP holds, β_0 should be equal to zero and β_1 should be equal to one as again the forward rate should be an unbiased estimate of the future exchange rate.

However, this standard test of UIP consistently finds that estimates of β_1 are less than one. A meta-study by Froot and Thaler found that the 75 published estimates had an average value of -0.88 for β_1 Froot (1990). An investment that takes advantage of this deviation by borrowing the low interest rate funding currency for a deposit in the higher interest rate investment currency is called *the carry-trade*. The evidence that the carry-trade is profitable is consistent with the forward discount puzzle.

As UIP is a key component of international financial theory, there has been a tremendous effort to understand this *puzzle*. See Froot (1990), Hodrick (1987), Engel (1996) and Engel (2014) for a summary of the vast literature in this area. Some suggest that it is the lack of stationarity in the series that causes estimation problems Engel (1996) and Roll and Yan (2000); others argue that this is really an issue for developed rather than developing nations Bansal and Dahlquist (1999); tests using 10 year bond yields indicate that the puzzle applies to the short-term but not long-term ratesChinn and Meredith (2004). It may also be associated with factors like heterogeneous market agents (reference), asymmetry of information (reference) or peso problems. There is also a wide ranging discussion over whether, once the assumption of risk-neutrality is abandoned, the apparent breakdown is the result of failing to correctly account for this risk. The last of these is explored here.

2.1 Risk premium

If investors are risk averse and form their expectations rationally, the negative estimate for β could mean that investors required additional return for holding foreign currency assets and Equation 6 can be augmented with a term that would account for this *risk premium.*

$$E[\Delta s_{t+j}] = \beta_0 + \beta_1 f_{t+j} + \beta_2 rp^{re}_{t+j} + \varepsilon \tag{7}$$

where $rp^{re}_{t,j}$ is a rational expectations risk premium at time t for j periods ahead. This seemed reasonable for the original US-centric research but it means that the risk premium is on domestic assets for non-US investors. Canova and Marrinan (1993) find that this risk premium switches from positive to negative as relative interest rates change and that the risk premium has a large variance, serial correlation and has volatility clustering. Fama (1984) finds that the variation in the risk premium is much larger than the variation in the exchange rate or the forward rate and that there is a negative correlation between the premium and the expected spot rate. Froot discovers that short rates consistently predict excess returns on a variety of assets. For foreign exchange, stock, bond and commodity markets, a one percentage annualised increase in the short-term interest rate is associated with about a three percentage point reduction in annualised excess returns Froot (1990). This all seems to undermine the simple risk premium approach to the forward discount puzzle.

In any case,Engel (1996) has identified that this risk premium only holds under rational expectations and market expectations may differ from the information that is used to test Equation 6. These divergences in expectations can be divided into two types: where the econometrician has more information that the agent (the agent gradually learns the model as was the case when floating exchange rates returned in 1973) or where the agent has information that the econometrican does not have (there may be peso problems where a large depreciation is anticipated that has not previously been recorded in the data) Engel (1996). In each of these cases it is usual to expect that the difference should tend to zero over time as market participants or economists acquire more information. However, this does not appear to have happened.

Another strand of the literature will seek to find explanations for the volatility in the risk premium by looking for a relationship with common risk factors. Here the returns are explained by the latent variables such as the forward discounts. Burnside et al. (2011) assess whether carry-returns are correlated with conventional risk factors like the equity risk-premium or real consumption growth and volatility indices. They find limited correlation.McCallum (1994) assumes that rp^{re} is a stochastic process and ask whether there are reasonable grounds for this to be negatively correlated with $E(s_{t+1}-s_t)$ as is found empirically. He concludes that this residual represents all the factors that are not in the model rather than a risk premium.

Engel says that rp^{re} could represent 'expected profit opportunities' (Engel (1996, p.148). As has been seen, these expected profit opportunities are large and highly variable. There are a number of studies that have assessed the skewed nature of the returns that are associated with the carry-trade. These explanations are usually grouped under the terms *peso-phenomenon* or a *crash-risk*. For example, Brunnermeier et al. (2008) develop a general model of *crash risk*, which is due to the sudden unwinding of the carry trade. The crash happens when risk-appetite and funding-liquidity decrease and carry positions are swiftly unwound. A potential carry-trade is calculated

$$z_{t+1} \equiv (i^* - i) - \Delta s_{t+1} \tag{8}$$

where z_{t+1} is the return in excess of the prediction of UIP as i^* is the overseas three month interest rate and i is the domestic three month interest rate and Δs_{s+1} is the change in the log of the exchange rate measured as foreign currency per US dollar Brunnermeier et al. (2008, pp. 8-9). They find that carry trades have large Sharpe Ratios, negative skewness and positive excess kurtosis.

Brunnermeier and Pedersen show how this *crash risk* can be caused by the interaction of illiquidity, margin calls and the evaporation of funding for speculation. When conditions deteriorate, investors seek to exit the carry position, liquidity declines, banks become more cautious about funding speculative positions and an increase in margin requirements together with a reduction in funding lead to spirals of selling and exaggerated price movements Brunnermeier and Pedersen (2009).

Spronk, Vershoor and Zeinkel use a heterogeneous agent model with carry traders in addition to fundamental and chartist traders. Traders adjust their strategy towards those that are most successful. The carry trade is built while conditions are calm. The model is able to replicate the heavy tails, excess volatility and volatility clustering that is evident in foreign exchange returns Spronk et al. (2013).

Jurek (2014) tries to quantify the crash risk by using out-of-the-money put options to hedge the risk of a sharp depreciation of the investment currency. By comparing the performance of hedged and un-hedged positions, he finds that less than one third of carry-trade excess returns can be explained by crash risk. Burnside et al. (2011) use a similar method and find similar results. However, the focus in each of these cases is G20 with only Australia, New Zealand, South Africa and Norway among the traditional carry-trade countries. The variability of funding and the changes in liquidity are likely to be much greater in emerging economies. Hayward and Hölscher (2014) investigated Central and Eastern European exchange rates and found that during periods of high international risk aversion the carry-trade experiences much lower returns and returns that are characterised by large-skew and kurtosis. It appears that the returns that are available under normal or calm conditions are a compensation for taking the risk of large losses when risk aversion rises and liquidity disappears. Returns are available in the calm conditions but they disappear in the crisis period.

3 The Hidden Markov Model

A Markov process depends only on the previous state. A Markov model describes the probability of moving from one set of states to another. Markov models have been used to understand a wide range of phenomena from weather forecasting, speech recognition and internet search. If the states that are to be explained can be observed, the probability of being in a particular state and the probability that there is a switch from one state to another can be estimated from a sample. However, in many cases e states cannot be observed and then the states as well as the transition probabilities must be estimated from the data with a *Hidden Markov Model* (HMM).

Hamilton (1988) used a HMM to incorporate discrete changes in expectations about Fed policy to improve the match between the expectations theory and the term structure of interest rates. This technique assumes that there are unobserved market-expectation regimes and that the adjustment from one state to another is governed by a set of probabilities. Hamilton (1989) analysed the performance of postwar US GNP with adjustments from periods that are termed recessions to those that are called expansions. The parameters of an ARIMA representation of US GNP shift between the two regimes so that in periods of recession the underlying growth rate is three percentage points lower than it is during the expansionary period.

Schaller and Norden (1997) used the same method to assess stock returns in two regimes, uncovering strong evidence of regime-switching between bull and bear markets in the mean and variance of US stock returns. They also find that that the response of stock returns to the price-dividend ratio is asymmetric as adjustment is much swifter during the bull market phase; Dueker (1997) employ a HMM so that switches from periods of low to high volatility explain the evolution of US equities from calm conditions to those that are more like a crisis.

Elliot and Han (2006) use a HMM for the forward discount. They find evidence against UIP and suggest that a three regime model fits the data most effectively. However, the study only assesses the GBP-USD exchange rate and do not consider factors that may affect the transition probabilities. We aim to analyse potential carry-trade profits as arising from one or more regimes. The first step is to determine whether a one, two or three regime model is the most appropriate. A model with more than one regime is often called a *mixture model* as the population of potential carry-trade returns are assumed to come from two or more unobserved sub-populations. In this case, the carry-trade returns are observable and are used to identify the underlying exchange rate.

Figure 1 gives an overview of the system. The HMM has three components: π, A, B:

- the prior model: $P(S_1 = n|\theta_{prior})$ (π)
- the transition model: $P(S_t|S_{t-1}, \theta_{trans})$ (A)

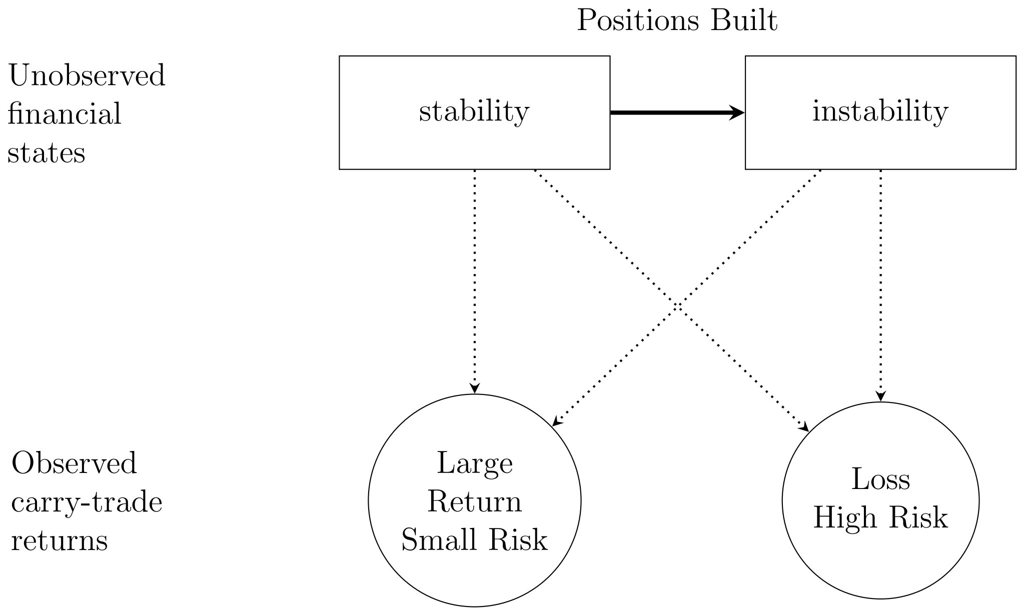

Figure 1: Two-Regime Hidden Markov Model (HMM)

- the response model: $P(Y_t|S_t, \theta_{resp})$ (B)

There are n states or regimes; Y_t are the observed carry-trade returns; and θ_{prior}, θ_{trans} and θ_{resp} are the parameters of the prior, transition and response models respectively. The unobserved financial regimes are modelled as a Markov chain that switches from a period of stability to instability. The returns to the carry-trade are more likely to take particular characteristics according to the underlying regime.

The prior or initial state probabilities give the probability that the system starts in a particular regime; the transition model is the probability of moving from one financial state to another; the response model is the relationship between the observed carry-returns and the unobserved financial regime.

The starting point of the system is given by,

$$P(S_1 = 1), \ldots P(S_1 = N)$$

as the probability of being in one of N regimes, and the state transition matrix for a two-state system is:

$$\begin{bmatrix} P(S_t = 1|S_{t-1} = 1) & P(S_t = 2|S_{t-1} = 1) \\ P(S_t = 1|S_{t-1} = 2) & P(S_t = 2|S_{t-1} = 2) \end{bmatrix}$$

Each element of the matrix shows the probability of being in a particular regime given the previous regime. This is a first-order Markov model. The transition probabilities depend only on the previous state. This seems to be a reasonable assumption in most circumstances. However, to assess whether the risk of a financial crisis is increased by the length of the period of stability, it would be necessary to have a higher order Markov chain, increasing the memory of the system and the computational complexity. In that case an alternative method would be more appropriate.

3.1 Estimation of the parameters

Estimating the HMM involves determining the most likely values for the three sets of parameters in the model.

$$\lambda = (\pi, A_1, B_1) \tag{9}$$

This estimation is done by Maximum Likelihood. As a consequence of the Markov assumption, this is a problem that can be solved with the *Expectation-Maximization (EM) algorithm*. See Dempster et al. (1977) and Hamilton (1989) as well as Visser and Speekenbrink (2010) for full details of the procedure. This is a numerical optimisation. There are two steps. First *the expectation step* will iterate forward from the starting point using prior estimates of the three sets of parameter values to make an initial assessment of the probability of observing each hidden regime given the model parameters. The number of regimes is given but this will be assessed the log-likelihood ratio and other information criteria. The prior estimates of the parameters can either be drawn randomly or they can be determined by some previous information.[2] In this way the most likely unobserved regime for each period can be calculated. A variant of the EM algorithm called *the forward-backward* or *Baum-Welch* algorithm Baum et al. (1970) is used. The Baum-Welch algorithm will find the parameters that maximize the probability of observing the sequence of carry-trade returns.

For the dependent mixture model, the joint likelihood of observation $Y_{1:T}$ and the latent state $S_{1:T}$ given the model parameters is

$$P(Y_{1:T}, S_{1:T}|\theta) = \pi b_{S_t}(Y_1) \prod_{t-1}^{T-1} a_{i,j} b_{s_t}(Y_{t+1}) \tag{10}$$

where π_i is the initial probability of each state; $a_{i,j} = P(S_{t+1} = j|S_t = i$ is the transition probability; and b_{S_t} is the probability of being in a particular state given the observed carry-trade return, $b_j = P(Y_t|S_t = j)$.

[2]In this case the parameters are drawn randomly. However, it would be possible to make initial estimates of the response model from Hayward (2013) using the estimates of the carry-trade returns that are identified.

Secondly, the *maximisation step* then updates the most likely values for the three sets of parameters based on the state sequence that has just been identified. Therefore, the probabilities of starting in a particular regime (π) come from the estimates just used to assess the most likely regime in each period; the transition matrix (A) is estimated from the given sequence; and the factor weights for the response model (B) are also updated. For the expectations part of the iteration, the states are replaced by their expected value given the parameters of the models (θ)

The estimation process then iterates back and forwards between the expectation and maximisation steps until the log-likelihood of the given parameters reaches a peak or the degree of improvement falls below a threshold. It is possible to reach a local maximum so it is usually advisable to run several iterations with different starting values. That has not been done in this case as a seed has been set for reproducibility.

4 Analysis of Results

4.1 Data

The data are a sample of potential carry-trades from investments in Emerging European countries that have been compiled from exchange rate and interest rate data for the period from January 2000 to December 2013. The exchange rates that are assessed are presented in Table 1. The data are taken from the IMF International Financial Statistics (IFS).

The funding currencies are those that have been identified by Baele et al. (2014). They are the euro, the US dollar, the Swiss franc and the Japanese yen. A set of potential carry-trade profits are calculated from the exchange rate and interest rate data using a small modification of the method proposed by Brunnermeier et al. (2008),

$$p_t = \frac{i_t^* - i_t}{\Delta s_t} \tag{11}$$

where i_t^* is the investment currency, i_t is the funding currency, Δs_t is the change in the exchange rate and p_t is the gross profit for potential carry-trades without transaction costs or adjustment for risk. Burnside et al. (2011) constructs a similar series for a panel of 10 exchange rates and finds a very small effect from plausible transaction costs and Sharpe Ratios that compare favourably with equity markets. One month and three month positions are calculated. Only the one month are presented as the results do not differ significantly.

Country	ISO Code	Exchange rate arrangement
Bulgaria	BGN	Currency board
Czech Republic	CZK	Float
Croatia	HRK	Fixed peg
Hungary	HUF	Float
Poland	PLN	Float
Romania	RON	Managed float
Russia	RUB	Fixed peg
Ukraine	UAH	Managed float
Turkey	TRY	Float

The table shows the exchange rates that are considered with the ISO codes that are used from this point onwards and the exchange rate regime. The exchange rate regime is taken from the IMF Classification of Exchange Rate Arrangements and Monetary Policy Frameworks IMF (2009). In many cases there are changes to the arrangements, when that happens the summary here reflects the stance at the end of the period. For more details see Hayward and Hölscher (2014).

Table 1: Countries, ISO codes and exchange rate arrangements

Table 2 shows the descriptive statistics for the potential carry-trade profit series.

4.2 Model Comparison

Initially, there are three models that are applied to each of the carry-trade-profit series.

- Model One (M1) is a standard linear model $p_t = \beta_0 + \varepsilon_t$ where p_t is the carry-trade-profit series, β_0 is the intercept and ε_t is the variation around that level. It provides a mean and standard deviation for the potential carry-trade-profit for the whole series. These are the figures presented in Table 2.

- Model Two (M2) is a standard linear model with two regimes $p_t = \beta_0 + \beta_1 S_n + \varepsilon, \quad n = 1, 2$ therefore a mean and a standard deviation of the potential carry-trade profits for each of the two regimes is produced. The mean and standard deviation of the potential carry-trade profits for each regime are presented in Table 4.

	HUF	PLN	CZK	RON	RUB	BGN	NOK	ISK	UAH	HRK	TRY
Number	167.00	167.00	167.00	167.00	167.00	167.00	167.00	167.00	167.00	159.00	166.00
Mean	12.80	12.41	10.07	11.94	7.21	8.09	8.66	8.47	7.84	10.73	13.56
Sharpe	1.83	1.88	1.56	1.99	1.50	1.36	1.46	1.36	1.89	1.74	2.10
Medium	21.52	13.75	9.32	17.42	9.58	7.64	9.93	7.92	10.06	12.02	16.85
StDev	0.07	0.07	0.07	0.06	0.05	0.06	0.06	0.06	0.04	0.06	0.07
Skew	-0.51	-0.57	-0.21	-0.33	-0.93	-0.02	-0.17	0.16	-1.45	-0.16	-0.67
SES	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19
Kurt	1.64	1.40	0.44	1.36	3.37	1.02	0.56	3.45	7.37	0.90	2.60
SEK	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Max	20.41	18.61	18.09	17.26	16.02	21.58	15.31	30.92	14.76	17.66	18.64
Min	-25.49	-21.84	-19.56	-20.75	-21.11	-18.88	-20.99	-23.80	-21.93	-19.03	-26.68

The table shows the descriptive statistics for the calculated potential carry-trade profits for each country. Number is the number of observations; Mean is the annualised return; Sharpe is the monthly Sharpe Ratio; Median is the median annualised return; StDev is the standard deviation; Skew is the measure of Skewness of the distribution; SES is the standard error of skewness so that a skewness that is twice this size could be considered to be statistically significant at the 5% level; Kurt is the measure of excess kurtosis; SEK is the standard error of kurtosis; Max is the maximum return; Min is the minimum return.

Table 2: Descriptive statistics of potential carry-trade profits

- Model Three (M3) is a standard linear model with three regimes $p_t = \beta_0+\beta_1 S_n+\varepsilon, \quad n = 1, 2, 3$ average and standard deviations are retrieved for three different regimes.

The models are compared using the Akaike Information Criterion (AIC) and the log-likelihood ratios adjusted for the number of parameters estimated. Table 3 summarises the performance of the three base models using these criteria. The models here are tested using the EUR as the funding currency. Results for other funding currencies are very similar and are therefore not reported. A comparison of the mean and variance for the two-regime response models for different funding units is presented in Table 4.

Table 3 compares the three fundamental models with 1, 2 and 3 regimes respectively. The aim is to find the number of regimes that provide the best fit for each of the potential carry-trade profit series for each country. The comparison is based on two criteria: the Akaiki Information Criteria and the Log-likelihood ratio. In each case the test is based on the requirement that the log-likelihood ratio improve by more than would be expected with the increase in the number of explanatory variables.

The first three columns of Table 3 report the AIC for each of the three base models respectively. The next six columns show the Log-likelihood ratio test statistic and the p-value for a comparison of Model One (M1) to Model Two (M2), Model One (M1) to Model Three (M3) and Model (M2) to Model Three (M3) respectively. The null is that the more sophisticated or complex model (two regime rather than one for example) does not improve the explanatory power by more than would be expected with the additional variables. The p-vale is the probability of finding this if the null is true. The final column combines the two tests to present the model that appears to fit the data most effectively.

The two-regime model is superior to the base model with one regime for all countries. However, for the Czech Republic there is ambiguity. For the Czech Republic, the AIC is about equal for the two models and the ratio test is 10.14, giving a χ^2 p-value of 0.0714. The Czech Republic may be better assessed with just a single regime. This case will be investigated further below.

Table 4 presents the estimated response (mean and standard deviation) for each of the two-regimes models. The results are encouraging for the hypothesis that potential carry-trade profits are best modelled as two regimes where one of them is considered the period of calm where carry positions are build and the other is considered the period of crisis where carry-trade positions will be cut back. In the first case, the the ratio p_t should be above one. This is consistent with the estimated value of β_1 in Equation 6 being

	AIC(M1)	ACI(M2)	AIC(M3)	LLR21	LLR21p	LLR31	LLR31p	LLR32	LLR32p	Preferred
HUF	-404.69	-416.94	-407.23	22.25	0.0005	26.50	0.0090	4.30	0.7459	M2
PLN	-423.98	-437.62	-428.82	23.65	0.0003	28.80	0.0042	5.20	0.6357	M2
CZK	-427.23	-427.37	-415.61	10.14	0.0714	12.40	0.4155	2.20	0.9451	M1 or M2
RON	-456.02	-474.81	-464.39	28.79	0.0000	32.40	0.0012	3.60	0.8264	M2
RUB	-523.18	-567.77	-559.95	54.59	0.0000	60.80	0.0000	6.20	0.5188	M2
BGN	-451.98	-454.32	-443.21	12.34	0.0305	15.20	0.2291	2.90	0.8945	M2
UAH	-572.87	-643.36	-636.39	80.49	0.0000	87.50	0.0000	7.00	0.4257	M2
HRK	-431.55	-433.37	-426.58	11.82	0.0373	19.00	0.0877	7.20	0.4068	M2
TRY	-431.05	-441.00	-432.58	19.95	0.0013	25.50	0.0125	5.60	0.5895	M2

Three models are initially tested. Model One (M1) is the base model with carry-trade returns explained by a normal distribution around a linear constant; Model Two (M2) is the linear model with 2 regimes; Model Three (M3) is the linear model with 3 regimes. AIC measures the Akaike Information index for the model identified, LLR is the log-likelihood ratio test statistics that compares the two named and nested models. The p indicates the probability value of the χ^2 test of the log-likelihood ratio with degrees of freedom equal to the number of parameter constraints of the base model. For example LLR32p is the probability value for the ratio between Model Three (M3) and Model Two (M2).

Table 3: Comparison of models

below one. This should also be expected to have smaller standard deviation and less risk because exchange rate conditions are expected to be calm when carry-trade positions can be built. The crisis period should have a mean of less than one and a larger standard deviation to reflect the increased risk that is being taken in this regime. This is when the crash risk happens. The mean and standard deviation for the two regimes are those that are most appropriate for the data. It is certainly possible that the regime with the lower mean could also have the smallest standard deviation. However, this does not happen in any of the cases.

There are two other things to note. First, the funding currency that is selected does not make much difference. The average performance of stability and crash regimes across funding currencies (final column of Table 4) does not change much from the individual currencies. As a consequence, the rest of the results that are presented are based on EUR-funding. Secondly, the results for the Czech Republic support the view that one regime may be a better way to model the data. Though two regimes can be estimated, the differences between the two does not appear to be as clear as it does for the other currencies.

It is possible to look at the evolution of most likely regimes over time. Figure **??** shows the potential carry-trade profits and the two regimes for Poland. It is clear that the Global Financial Crisis is the main regime change that takes place. However, there are a number of smaller changes that take place.

5 Exogenous influences on regime switching probabilities

Periods of foreign exchange crisis are generally referred to as *sudden stops*. See Dornbusch and Werner (1995), Calvo (1998) and Krugman (2000) for more details. Calvo defines the sudden stop as net capital flows moving two standard deviations below their mean level and subsequently returning to within one standard deviation of that point. There are a number of factors that may influence capital flows and exchange rate stability. Exchange rate crisis can be caused by domestic or international factors. Baele et al. (2014) find that exogenous factors that cause sudden stops and flight-to-safety of *hard* currencies are relatively rare events. Their study of of 23 countries finds that most of the events are country specific. They characterise only 25% of the events as *global*. However, it is the exogenous international effects that are the focus here.

Funding	Regime		HUF	PLN	CZK	RON	RUB	TRY	BGN	UAH	HRK	Mean
EUR	Stable	Mean	1.0165	1.0173	1.0129	1.0150	1.0098	1.0151	1.0075	1.0094	1.0091	1.0119
		St-Dev	0.0519	0.0486	0.0542	0.0433	0.0310	0.0460	0.0381	0.0295	0.0251	0.0446
	Crash	Mean	0.9905	0.9862	0.9963	0.9969	0.9962	0.9969	1.0053	0.9673	1.0082	0.9897
		S-Dev	0.1085	0.1026	0.0886	0.0878	0.0779	0.1028	0.0826	0.1116	0.0737	0.0958
USD	Stable	Mean	1.0103	1.0123	1.0072	1.0091	1.0044	1.0087	1.0041	1.0055	1.0054	1.0071
		S-Dev	0.0307	0.0297	0.0305	0.0080	0.0095	0.0314	0.0189	0.0078	0.0187	0.0202
	Crash	Mean	0.9925	0.9845	0.9983	1.0052	1.0004	1.0034	1.0016	0.9932	1.0036	0.9959
		S-Dev	0.0707	0.0641	0.0493	0.0389	0.0407	0.0792	0.0413	0.0635	0.0390	0.0566
CHF	Stable	Mean	1.0052	1.0097	1.0040	1.0085	1.0033	1.0099	1.0012	1.0029	1.0031	1.0051
		S-Dev	0.0181	0.0235	0.0161	0.0179	0.0234	0.0313	0.0083	0.0307	0.0116	0.0205
	Crash	Mean	0.9994	0.9838	0.9934	0.9959	0.9373	0.9952	0.9958	0.9834	0.9916	0.9857
		S-Dev	0.0477	0.0459	0.0380	0.0387	0.0592	0.0792	0.0327	0.0900	0.0384	0.0538
JPY	Stable	Mean	1.0149	1.0157	1.0125	1.0149	1.0074	1.0111	1.0092	1.0094	1.0091	1.0115
		S-Dev	0.0348	0.0359	0.0241	0.0291	0.0226	0.0401	0.0191	0.0307	0.0210	0.0289
	Crash	Mean	0.9843	0.9727	1.0012	0.9972	0.9983	1.0061	1.0002	0.8539	1.0028	0.9801
		S-Dev	0.0767	0.0764	0.0525	0.0580	0.0628	0.0889	0.0487	0.0667	0.0493	0.0668

This table outlines the linear response for Model Two (M2) for each of the regimes for all countries under consideration. Stable is the label given to the regime with the highest mean return. In all cases this is also the regime with the smallest standard deviation. This is not imposed and it is encouraging as, consistent with the idea that the carry-trade is composed of two distinct phases, it suggests that positions are built and profits are available when conditions are stable and that losses are suffered when conditions are more risky. Figures for the Czech Republic should be treated with some caution. The data also suggests that the funding currency does not have a major effect on the performance of the carry-trade.

Table 4: Mean and Standard Deviation of 2 Regime Model

Groen and Peck (2014) consider the effect of changes in global risk aversion on the carry-trade. They find that the initial signal from the US central bank in Fed Chairman Bernanke's May 22 2013 testimony to Congress coincide with an increase in global risk aversion which affected global asset prices. By identifying the performance of exchange rates without a change in risk aversion, they suggest that nearly half of the depreciation of a basket of 45 carry-trade currencies with the largest one-month interest rate relative to a basket of the US dollar and other equally low rate currencies is explained by this increased risk aversion. Nearly all the decline in Emerging market equities is attributable to the increase in risk aversion.

Alexander Klemm and Sosa (2014) use a panel vector autoregression (VAR) method to identify the influence of US monetary policy since 1990 on capital flows to 38 emerging economies, finding evidence that the end to Federal Reserve purchase of government bonds under the quantitative easing programme, while not necessarily leading to capital outflow, could generate *new risk premium shocks* with investors requiring a higher rate of return and therefore lower asset prices.

Consistent with the evidence of an interplay between monetary policy and international risk, and between domestic and international factors, Ahmed and Zlate (2014) show that economic growth, interest rate differentials and the level of global risk appetite are all important determinants of private capital flows to emerging markets. They also suggest that capital flows have been more sensitive to interest rate differentials since the financial crisis of 2007-08. There is also some evidence that US quantitative easing has had some effect on capital flows.

5.1 Additional Models

This section will assess whether exogenous factors influence the potential profitability of the carry-trade or the probability of switching from a regime of foreign exchange calm to one of crisis. Two exogenous forces are to be tested here: international risk aversion and US interest rates. Risk aversion is measured with the VIX index while US interest rate are measured as the 3-month US dollar deposit[3].

These external forces could affect foreign exchange conditions and the carry-trade in two ways: they could have a direct linear influence or they could influence the probability of switching from one regime to another. In

[3]The VIX is an index of implied volatility on options from the S&P 500 index. It is commonly used as a measure of international risk aversion as it signals increased demand by fund managers for option protection of their equity portfolio. See Chicago Board of Trade (2009), Demeterfi et al. (1999) and Diamond (2012) for fuller details.

the first case they could be added as an explanatory variable to the simple linear response model. For example,

$$y_t = \beta_0 + \beta_1 Z_t + \varepsilon \tag{12}$$

where y_t is the carry-trade return and Z_t is the exogenous influence on financial stability (either international risk aversion or US interest rates). This is Model Four (M4) with two versions a and b to signify the effect of risk aversion and US interest rates respectively.

In the second case, the exogenous force is used to explain the transition probabilities using a multinomial logistic regression. For the transition model (A),

$$a_{ij}(t) = P(S_t = j | S_{t-1} = i, z) \tag{13}$$

where $a_{ij}(t)$ is the probability that the system will be on state i at time t when it was in state j in the previous period and covariate z takes a particular value at time t. For a two-regime model, the estimation that is carried out is

$$log(a_{t,n=2})/a_{t-1,n=1} = \beta_0 + \beta_1 z_t \tag{14}$$

State 1 is the baseline category so coefficients are set to zero for that state and the model estimates the relationship between the covariate and probability of switching to the other state.

The logistic function used for this is

$$F(z) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 z)}} \tag{15}$$

giving a probability between zero and one of being in the particular state. Agresti (2014, pp.174-75).

This is Model Five (M5). There are also two version of this model (a and b) that will denote whether the probability of switching from one state to the other is conditional on risk aversion or US interest rates.

Table 5 compares models M4a and M5a against models M1 and M2. Models 4a and 5a use the VIX as a measure of international risk aversion as with an explanatory variable (M4a Equation 12) or as conditioning for the transition probability matrix (M5a Equation 15). Model One (M1) the base model with one state and a linear relationship. There are three criteria used to judge the preferred model: the AIC, log-likelihood ratios and simple linear regression of the additional exogenous explanatory variable. Columns 1 to 4 report the AIC for the M1, M4a, M2 and M5a models respectively.

	AIC(M1)	ACI(M4)	AIC(M2)	AIC(M5)	LLR54	LLR54p	LLR52	LLR52p	Coeff	p-value	Preferred
HUF	-404.69	-402.70	-416.94	-419.50	28.80	0.0001	6.56	0.0377	-0.00	0.9529	M5
PLN	-423.98	-422.03	-437.62	-438.93	28.90	0.0001	5.30	0.0705	-0.00	0.8202	M5/2
CZK	-427.23	-425.25	-427.37	-430.52	17.27	0.0083	7.15	0.0280	0.00	0.8935	M5
RON	-456.02	-454.42	-474.81	-478.07	35.65	0.0000	7.26	0.0265	0.00	0.5307	M5
RUB	-523.18	-449.98	-567.77	-566.68	128.70	0.0000	2.92	0.2328	-0.00	0.9833	M2
BGN	-451.98	-449.98	-454.32	-459.55	21.56	0.0015	9.23	0.0099	-0.00	0.9892	M5
UAH	-572.87	-571.47	-643.36	-647.67	88.20	0.0000	8.31	0.0157	0.00	0.4451	M5
HRK	-431.55	-429.58	-433.37	-439.42	21.84	0.0013	10.05	0.0066	-0.00	0.8590	M5
TRY	-431.05	-429.06	-441.00	-439.26	22.20	0.0011	2.26	0.3232	-0.00	0.9575	M2

The table details the comparison of four models. Here Model One (M1) is the base model; Model Four (M4a) adds the VIX index as a response variable (Equation 12); Model Two (M2) is the simple response model with two regimes; Model Five (M5a) is the two-regime model with the VIX index a a logistic covariate of the transition matrix. LLR is the log-likelihood ratio with the p denoting the p-value for a χ^2 distribution with degrees of freedom equal to the parameter restrictions on the base model that is being compared. Therefore, LLR54p is the p-value for the log-likelihood ratio of Model Five on Model Four. This assesses the performance of the two-regime model with the VIX influencing the transition values against the model where the VIX directly affects the carry-trade return. Coefficient is from Model Four. It is the estimate of β_1 from a regression of Equation 12; p-value is the test of the null hypothesis that β_1 is equal to zero.

Table 5: VIX covariate model table

	-3sd	-2sd	-1sd	Mean	+1sd	+2sd	+3sd
HUF	0.0020	0.0069	0.0242	0.0807	0.2375	0.5249	0.7967
PLN	0.0004	0.0016	0.0063	0.0242	0.0887	0.2766	0.6003
CZK	0.0020	0.0062	0.0234	0.0770	0.2247	0.5019	0.7779
RON	0.0014	0.0043	0.0131	0.0392	0.1119	0.2799	0.5453
RUB	0.0008	0.0022	0.0065	0.0189	0.0537	0.1430	0.3291
BGN	0.0052	0.0145	0.0403	0.1066	0.2533	0.4910	0.7328
HRK	0.0027	0.0073	0.0194	0.0506	0.1257	0.2793	0.5109

Changes in the VIX index affect the transition probabilities. The VIX index has been scaled so that it has a mean of zero and a standard deviation of one. Therefore, the central column shows the probability that the system will switch from a stable regime to one of crisis when the VIX is at its average level. It also shows how this probability changes as the VIX moves one or more standard deviations above and below this average. For example, for Hungary, there is an 8% chance that the system will switch to a period of crash when the VIX is at its average value. This rises to 24% when the VIX is one standard deviation above average and 52% for 2 standard deviations.

Table 6: Assessing the influence of VIX index on financial risk

Columns 5 and 6 report respectively the log likelihood ratio test statistic and the p-value for Model 5 relative to Model 4a; Columns 7 and 8 do the same for the comparison of Model 5a over Model 2. Columns 9 and 10 report the t-statistic and p-value of a line regression of the potential carry-trade profits on a constant and the VIX index and the p-value. Column 11 identifies the model that is most appropriate for each series according to these criteria.

Model 5 is the preferred model in most cases. For Poland, there is a little uncertainty about whether Model Two or Model Five is better. As we shall seen, this is because exogenous factors do not have a major influence here. For Russia and Turkey, while it appears that the level of international risk aversion is best used explaining the probability of switching from one regime to another rather than as an explanation of the level of carry-trade profits, it also appears that the simple two-regime model is superior.

Table 6 provides more detail about Model 5. The changes in the VIX index are normalised to one standard deviation and the base probability of moving from the calm regime when there there is a good chance of achieving carry-trade profits to one of foreign exchange crisis is recorded at various levels of international risk aversion. The average level of international risk aversion (as record by the VIX index) is the central column labelled *Mean*. The probability of switching from a regime of calm to one of crisis is then

	-3sd	-2sd	-1sd	Mean	+1sd	+2sd	+3sd
PLN	0.6601	0.4110	0.2005	0.0827	0.0314	0.0115	0.0042
RON	0.0062	0.0106	0.0182	0.0310	0.0524	0.0871	0.1414
RUB	0.0011	0.0025	0.0056	0.0126	0.0283	0.0620	0.1305

Changes in US 1-month interest rates affect the transition probabilities. The US 1-month interest rate has been scaled so that it has a mean of zero and a standard deviation of one. Therefore, the central column shows the probability that the system will switch from a stable regime to one of crisis when the 1-month interest rate is at its average level. It also shows how this probability changes are the 1-month interest rate moves one or more standard deviations above and below this average. For example, for Romania there is a three percent chance of moving to a foreign exchange crisis when US interest rates are at their mean level. This rises to five, nine and fifteen percent as interest rates move one, two and three standard deviations above average. .

Table 7: Assessing the influence of US 1-month interest rate on financial risk

recorded for one, two and three standard deviations above and below this mean. These are in columns 6, 7 and 8, and 2, 3 and 4 respectively. For example, for HUF, there is an eight percent probability of switching from a period of calm to a period of crisis. This increases to just under twenty-four percent when the VIX is one standard deviation above the mean and reaches nearly eighty percent at three standard deviations above the mean.

Table 5 also gives the best overview of the effect of international factors in the possibility of a financial crisis. The central column shows that when risk aversion is at an average level the probability of a crash will range from about 8% in Hungary to around 2% in Poland and Romania. The 8% reading for Czech, considered one of the pillars of stability in the region is a little surprising. This is probably a function of the analysis ending in 2013. The probability of facing a financial shock increases quite sharply as international risk aversion rises above its average level: at two standard deviations, it is suggested that there is a 50% chance of a financial shock in Hungary and the Czech Republic (again surprising) and a 28% chance in Poland, Romania and Croatia.

Table 7 shows the conditional-transition model that is based on 1-month US interest rates. There are only a few countries where the model works. With Poland, the results are contrary to expectations. As US interest rates move above average levels the risk of moving from a calm to crisis exchange rate regime is reduced. This is probably a result of the limited span of data that is available, the fact that US interest rates were generally falling in the period and the fact that US rate cuts often follow a international financial crisis. Romania and Russian are more conventional and show modest positive effects from increased US interest rates on the risk of suffering a financial shock.

A similar analysis was carried out with the TED spread. This is the spread between tbills and eurodollar rates and is used to capture banking credit risk. There is no case where the TED spread provides any improvement in the standard two-regime model. Again, the failure may be a consequence of the limited data span. There is just one episode where there was a significant increase in the spread (around the time of the Lehman Brothers' bankruptcy in October 2008 where the spread reached 335bp).

6 Conclusions

This study extends the multiple regimes to understand the forward discount puzzle. Potential carry-trade profits are modelled most successfully with two-regimes. There is evidence that compensation for taking the risk of switching to the crisis regime where large losses are likely is part of the explanation for the apparent breakdown in UIP. It is possible to assess how the probability of switching from a regime of exchange rate stability to one of crisis compares across countries and over time. It is also possible to see how the probability of falling into a regime of exchange rate crisis is conditional on international factors - particularly international risk aversion.

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