Hybrid Function Representation with Distance Properties

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Abstract

This paper describes a novel framework allowing for a hybrid representation of heterogeneous objects. We consider advantages and drawbacks of the conventional representations based on scalar fields of different kinds. The main result is introducing a hybrid representation called Hybrid Function Representation (HFRep) that preserves the advantages of the Function Representation (FRep) and Signed Distance Fields (SDFs) without their drawbacks. This new representation allows for obtaining a continuous smooth distance field in the Euclidean space for the FRep. We present the mathematical basics for our approach that uses the Discrete Distance Transform (DDT) and a step-function. The procedure for generation HFRep using continuous interpolation and smoothing techniques are also described. A few examples show how the approach works in practice.

CCS Concepts

• ***Shape modelling*** → *Volumetric models ;*

1. Introduction

Heterogeneous volumetric object modelling [PAC08] is a rapidly developing field and has a variety of different applications. One of the main advantages of heterogeneous objects is the presence of the internal structure within which different physical properties of the real object can be set up. In the interior of the object those properties are represented as multi-material distribution, micro-structures, material density, optical properties and other physically based properties.

This paper describes a novel representational framework for heterogeneous objects on the basis of scalar fields. The object is defined by a discrete or continuous function. Its value at the given point can define the distance to the object boundary as well as the material properties, density and other interior properties.

The most widely used scalar fields are Sign Distance Fields (SDFs), Function Representation (FRep), Adaptively Sampled Distance Fields (ADFs), Interior Distance Fields (IDFs) and some others. These representations have their own advantages, restrictions and areas of applications.

SDF is a class of scalar fields where the scalar value represents an Euclidean distance to the source object, and the sign of the function value is changing on the boundary of the object which leads to the precise boundary definition. In solid and heterogeneous modelling, SDFs are usually continuous, i.e. defined at every point in the domain. The distance property of the field allows for the extensive set of operation and efficient rendering of the object, and as a result SDFs have a wide range of possible applications [JBS06]. However, this representation can produce errors of different types in case of complex geometry and topology. The SDFs can have C^1 discontinuities even when the source surface is smooth, especially when using Boolean operations based on min/max functions. C^1 continuity of the field is crucial for heterogeneous multi-material modelling to avoid stress concentrations [BST04].

Function Representation [PASS95] is a generalised class of procedurally defined objects in an implicit form. In the FRep framework a geometric object is described using a continuous function $F(x_1, x_2, ..., x_n)$ defined on a closed subset of n-dimensional Euclidean space E^n. It takes positive values inside the object, zero on the boundary of the object and negative values outside of the object. It covers traditional solids, scalar fields, heterogeneous objects, static and time-dependent volumes. Unfortunately, in general case, FRep is more suitable for solid modelling than for heterogeneous modelling. The scalar field based models created with the FRep do not always have a distance property and therefore a wide range of methods for the material definition that rely on distance to the surface cannot be used without computationally expensive operations on the FRep aimed to approximate distance.

Adaptively Sampled Distance Fields (ADFs) [FPRJ00] provide distances to the boundary of the object in the exterior and interior points as the SDFs do. The main advantage of the ADFs is adaptive sampling of the object according to the local details which significantly speeds up its generation and reduces memory consumption. In addition, they are stored in a spatial hierarchy for efficient processing of the geometry. This representation, however, is known to produce C^1 discontinuities at cell boundaries and even C^0 discontinuities where cells of different size appear.

Interior Distance Fields (IDFs) is a generalisation of SDFs with

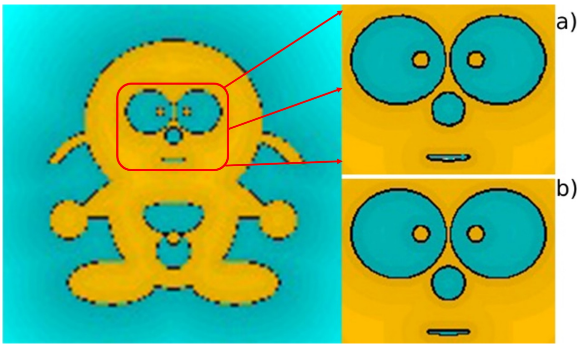

Figure 1: *The zoomed regions of the DDT field obtained for the FRep defined character. Left image: the result of calculating the sparse DDT on the grid* 128×128 *points. Right picture: a) the zoomed region of the sparse DDT after applying the bicubic interpolation to it, the final grid is* 512×512 *points, b) the zoomed region of the DDT obtained on the grid* 512×512 *points.*

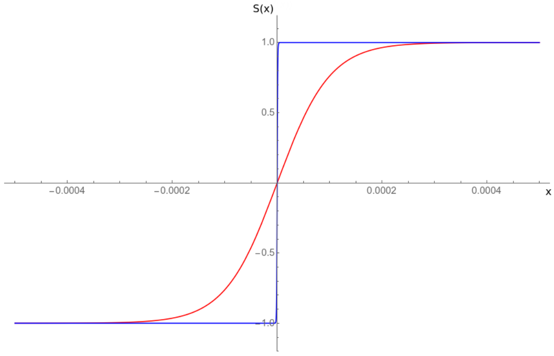

Figure 2: *Plots of two sigmoid functions which coincide: hyperbolic tangent sigmoid and hyperbolic tangent with according slope values* s_l*. Red line:* $S_{sig}(x)$, $s_l = 10^{-4}$ *or* $S_{tgh}(x)$, $s_l = 10^4$*. Blue line:* $S_{sig}(x), s_l = 10^{-5}$ *or* $S_{tgh}(x), s_l = 10^6$.

different metrics such as geodesic on the surface of the source object or an intrinsic distance inside the object. This type of distance fields can be used for setting up material distribution inside the object [FSP15] and also provides a precise boundary definition. At the same time, they are more computationally expensive than other representations.

All the mentioned representations have their advantages and drawbacks. In this work we introduce a theoretical framework based on a novel object representation called Hybrid FRep (HFRep). It is based on combining of different scalar field representations and aims to unify their advantages and compensate for their drawbacks. In this paper we focus on two representations discussed above, namely the FRep and the SDF, because SDFs allow for efficient material modelling inside the object, whilst FRep provides smoothness of the unified representation. We will show that the HFRep allows for obtaining both discrete and continuous distance properties of the resulting field and provides the distance properties. In the next sections we will describe how the HFRep representation is defined.

2. Hybrid Function Representation

Let us give the exact problem statement: for the given shape it is essential to obtain a real function with (1) sufficiently accurate distance property approximation everywhere in Euclidean space without C^0 and C^1 discontinuities anywhere, (2) with exact zero values only at the object boundary and (3) changing its sign on the boundary. Let us outline the solution that combines two representations, namely the SDF and the FRep, of the same object specifically taking advantage of both SDF's distance property and FRep's continuity for the defining function.

Let the object O be defined using the FRep defining function with the equation $F(x_1, x_2, ..., x_n) \geq 0$. First, we use the Discrete Distance Transform (DDT) to obtain a distance field $dist_O(P)$ in a finite set of points by calculating the unsigned distance to the surface of FRep object in these points. Then, we convert this discrete distance field into a continuous field by using the appropriate smoothing operations. Finally, to obtain the sign for the DDT we use the step-function $S(x)$, the values x of which are calculated using the FRep field. Therefore HFRep can be defined as

$$F_h(x_1, x_2, ..., x_n) = S(F(x_1, x_2, ..., x_n)) \cdot dist_O(P) \qquad (1)$$

2.1. Distance Transform

The operation which allows for obtaining the distance property for an arbitrary scalar field is called the distance transform. Formally, it is a mapping from one scalar field F into another scalar field D such that $D(\mathbf{x}) = inf_{p:F(\mathbf{p}=0)} |\mathbf{x} - \mathbf{p}|$. The distance obtained by distance transform is usually unsigned.

In a general case, to get the distance transform for an arbitrary object represented by the FRep can be problematic because the boundary of the object is not explicit. There are various methods to tackle this problem, some of them are more precise in terms of the obtained distance values, and some being more computationally efficient. For example, the distance can be calculated by numerically solving re-initialisation equation [SSO94]. However, that method is quite inefficient when the distance needs to be computed in a large number of points. Therefore in practice, many methods use the Discrete Distance Transform (DDT) as a more efficient alternative. In this case, the value of the field is calculated on a regular grid by multiple traversing distance values in it to obtain the final approximation of an Euclidean distance to the approximated surface. Note that the Discrete Distance Transform results in a distance to the object computed with a certain tolerance and cannot be considered as an exact operation.

The field obtained after applying the DDT is a discrete field, as it is calculated in a finite set of points, normally on a regular grid. To obtain the continuous function, interpolation techniques should be used. In our method, we use the cubic spline interpolation applied to the appropriate domain. Thus, in 2D we interpolate values of the sparse grid with a bicubic interpolation [Kno00] which allows for producing accurate results for a smooth continuous function while being easy to compute (see Fig. 1).

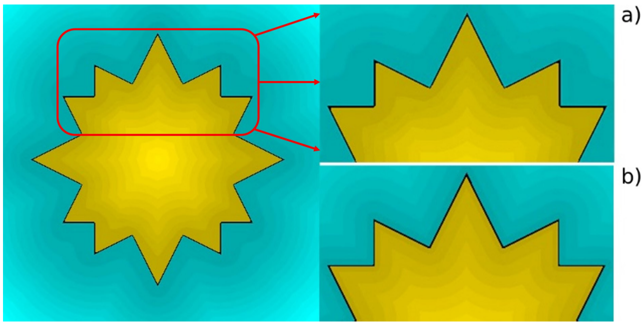

Figure 3: *Demonstration of the HFRep continuity in comparison to the DDT, where the black line is the zero level-set (the left image is a star obtained by set-theoretical operations applied to the FRep-defined half-planes; a) the zoomed region of HFRep, $s_l = 0.2$; b) the same zoomed region of the DDT).*

On the left image of the Fig. 1 we show the sparse DDT field calculated at each fourth point of the full resolution grid. In the right part of this figure we show the difference between the DDT calculated in full resolution (b) and the result we obtain after applying the bicubic interpolation to the sparse grid (a). The quality of the interpolated field is nearly the same as in the Fig. 1, (b).

2.2. Step-function

As it follows from equation (1), we need to define the sign to distinguish between interior and exterior of the object O. We suggest to obtain the sign using a step-function. The step-function should satisfy the following requirements: it is equal -1 when it coincides with the exterior of FRep object, on the boundary of the FRep object it should be equal to 0, and inside the object it should be equal to 1. This step-function should be continuous everywhere, it just modifies the values of the DDT and does not produce additional zeroes in the resulting field by multiplying it by values of the DDT.

The most suitable class of functions which satisfies the stated requirements is *sigmoid functions*. We have chosen two functions from this class: the *hyperbolic tangent sigmoid function* [VMR*88] and the *hyperbolic tangent function*, which can be defined as follows:

$$S_{sig}(x) = \frac{r}{1+exp(-2x/s_l)} - \frac{r}{2} \tag{2}$$

$$S_{tgh}(x) = tanh(s_l \cdot x) \tag{3}$$

where r controls the range of the $S(x)$ along y-axes and s_l controls the slope of the function. After setting $r = 2$ the function (3) will be propagated from -1 to 1 along the y-axes. The smoothness of the DDT can be controlled by varying the s_l parameter. We present their plots in Fig. 2. Depending on the value of the slope parameter s_l they can quite accurately approximate the step-function behavior near the zero level-set of the FRep.

In Fig. 3 we show a star-shape with sharp features defined by set-theoretical operations using R-functions [PASS95]. From the Fig. 3, (b) follows that the DDT suffers C^1 discontinuities in the interior of the object as the isolines are not smooth inside. In case of the HFRep, Fig. 3, (a) shows that by setting up parameter $s_l = 0.2$ in equation (3) the function in the interior of the object remain to be continuous and smooth.

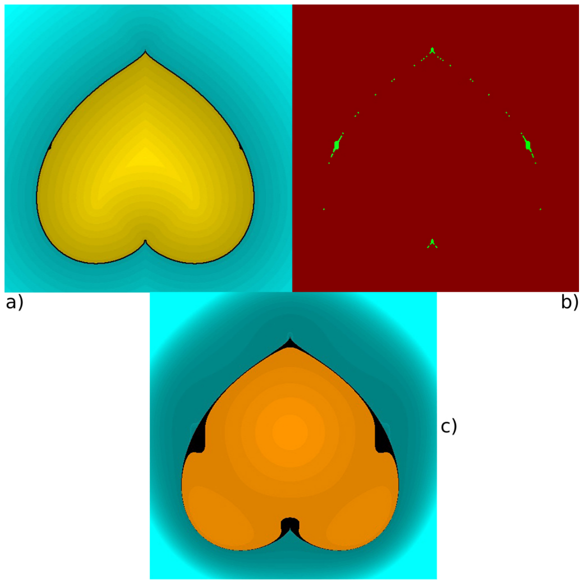

Figure 4: *HFRep and FRep fields obtained for the implicit surface (4) defining the heart modelled on the grid with resolution* 512×512*, where the black line is the zero level-set: a) the HFRep field; b) the calculated difference between the HFRep and the DDT fields; c) the FRep field.*

3. Implementation and Results

In this section we will discuss implementation of the proposed method in 2D. We have implemented our method using *OpenCV* library and $c++$.

At the initial step we calculate the FRep field, which defines the heart shape (Fig. 4, (c)) according to the following formulae:

$$f(x,y) = ((x-x_0)^2 + (y-y0)^2 - 1.0)^3 - (x-x_0)^2(y-y0)^3 \tag{4}$$

After the function field was generated, we use it as an input to the procedure of calculating the DDT. The negative values of the FRep field serve as an indicator of the exterior of the object, zero and positive values are indicating the boundary and the interior of the object. The DDT has been implemented as an 8-Points Sequential Euclidean Distance Transformation algorithm from [LL92]. This DDT algorithm requires two grids that are traversed multiple times in order to compare the distance written in the current point with the distances stored in eight neighbour points around it. Finally, two grids are merged together and the result is stored in the final DDT grid.

To distinguish between the exterior, the boundary of the object and its interior we need to define the sign for the obtained DDT. As it was mentioned in subsection 2.2, we suggest to use step-function $S(x)$ defined by equation (3), where x are the values of the FRep

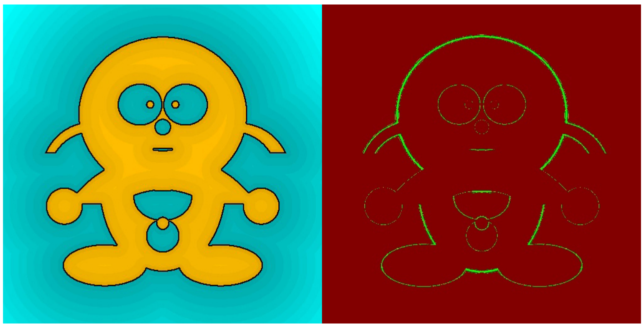

Figure 5: *HFRep field obtained for the functionally defined character on the grid* 512×512 *(left) and the calculated difference between the HFRep and the DDT field (right). The black line on the left image is the zero level-set.(generation time: 0.85 sec., Intel Corei7-6700HQ 2.60 GHz, 16 GB RAM)*

field obtained at the initial step. To obtain the final Hybrid Function Representation (HFRep) we need to use the equation (1). The final field can be seen in Fig. 4, (a). To graphically demonstrate in what extent the HFRep and the DDT are different from each other, we calculate the difference between them and show it in Fig. 4, (b). The step-function (3) with parameter $s_l = 0.001$ slightly modifies the DDT values on the boundary of the object, where the FRep values are equal to 0. It can be explained by the requirements we mentioned for the step-function in subsection 2.2. One of them is that this function should be continuous, mimics the FRep field behaviour as close as possible and barely modifies the DDT field.

Comparing the FRep field (Fig. 4, (c)) and the HFRep field (Fig. 4, (a)) we can clearly see that the FRep field does not completely follow the shape of the object it is defined for, while the HFRep field follows it and provides distances to the object. As the operations defined for the FRep field are at least C^1 continuous, the FRep field can be modified first and then HFRep can be recalculated and the final field will be continuous everywhere in Euclidean space as it is demonstrated in Fig. 3, (a).

We calculate the DDT at each fourth point of the grid of the size 512×512 and apply the bicubic interpolation after it to obtain the in-between values. The sufficiently good result is shown in Fig. 1. Using sparse DDT grids allows for speeding up the generation of the final HFRep preserving continuous properties of the field.

In Fig. 5 we show a more sophisticated example of using the HFRep. The character shown in the figure is defined using the FRep set-theoretic operations defined by R-functions which are applied to the simple primitives, namely ellipsoid, torus, elliptic cylinder and sphere. Totally the model was created using 30 primitives and 42 operations. Then we have applied the proposed method to the final FRep field and obtain the smooth HFRep shown in Fig. 5, left. To minimise the difference between the HFRep and the DDT fields, which can be seen in Fig. 5, right image, it can be essential (especially in case of more complicated HFRep fields) to modify s_l parameter in equation of the step-function (3). For the current example it was set up as $s_l = 0.00001$ to obtain more accurate result along the border of the final object.

4. Conclusions and Future Work

In this work we have introduced a theoretical framework for heterogeneous objects on the basis of a new unifying hybrid representation called HFRep. It unifies the FRep and SDFs and allows for exploiting their advantages whilst neglecting their drawbacks. Most importantly, it allows for obtaining distances to the object for the Function Representation, which does not provide them by default. The FRep object can be conveniently modified using established operations for FRep which are at least C_1 continuous. Using the modified FRep for generating step-function values $S(x)$ and controlling s_l parameter of this function, the resulting HFRep can be obtained as a continuous smooth distance field. We have demonstrated that applying the bicubic interpolation to the sparse DDT field for obtaining in-between values is still producing well approximated field which is close to the original one. As the introduced new hybrid representation provides distances in interior and exterior of the object, it can be used for setting up materials and other interior properties of the object, for example, as a parametrisation of the space by the distance from the material features [BST04].

In the future work we are going to extend the approach to the 3D case setting up materials in the interior of heterogeneous object and provide mathematically defined operations for this representation. Some other smoothing functions will also be explored. Investigation of the most efficient numerical procedures to obtain distances is also on our agenda.

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